An Asymptotic, Homogenized, Simplified P_2 Approximation to the Neutron Transport Equation

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STEWARDSHIP SCIENCE GRADUATE FELLOWSHIP

Outline

1. Introduction to Neutron Transport
2. Asymptotic Analysis
3. Results
4. Conclusions and Future Work

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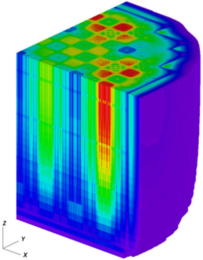
Neutron Transport

The study of the motions and interactions of neutrons with materials.

Applications:

- Reactor physics
- Shielding
- Radiation monitoring



Neutron Transport - Quantities of Interest

- Angular neutron flux:

$$\psi(\boldsymbol{x}, \boldsymbol{\Omega}, E) = v(E)\, n(\boldsymbol{x}, \boldsymbol{\Omega}, E)$$

- Scalar neutron flux:

$$\phi(\boldsymbol{x}, E) = \int_{4\pi} \psi(\boldsymbol{x}, \boldsymbol{\Omega}, E)\, d\boldsymbol{\Omega}$$

- Macroscopic cross section for reaction j:

$$\Sigma_j(\boldsymbol{x}, E)$$

- Reaction rates:

$$\Sigma_j(\boldsymbol{x}, E)\, \phi(\boldsymbol{x}, E)$$

- Core eigenvalue (multiplication factor):

$$\lambda = \frac{1}{k}$$

Boltzmann Transport Equation

$$\begin{aligned}\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\psi(\boldsymbol{x},\boldsymbol{\Omega},E)+\Sigma_t\left(\boldsymbol{x},E\right)\psi\left(\boldsymbol{x},\boldsymbol{\Omega},E\right)\\ =\int_0^\infty\int_{4\pi}\Sigma_s\left(\boldsymbol{x},\boldsymbol{\Omega}'\to\boldsymbol{\Omega},E'\to E\right)\psi\left(\boldsymbol{x},\boldsymbol{\Omega}',E'\right)d\Omega' dE'\\ +\lambda\,\chi\left(E\right)\int_0^\infty\int_{4\pi}\nu\Sigma_f\left(\boldsymbol{x},\boldsymbol{\Omega}',E'\right)\psi\left(\boldsymbol{x},\boldsymbol{\Omega}',E'\right)d\Omega' dE'\end{aligned}$$

- Difficult problem to solve:
  - 6-dimensional solution space (7-D with time)
  - The solution is not smooth
  - Cross sections are measured quantities with uncertainties

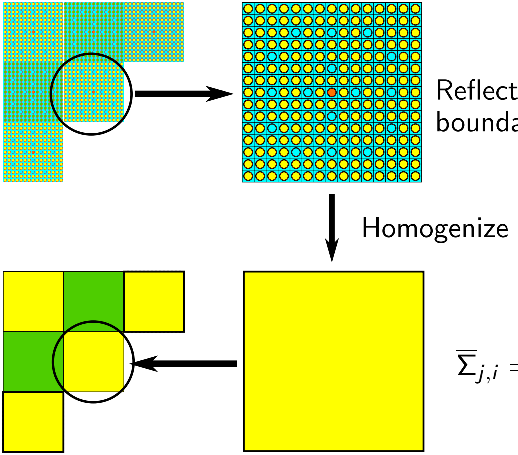
Numerical Methods

- Monte Carlo
  - Random numbers are used to simulate individual neutron behavior
  - Works because the transport equation describes the *average* behavior of a distribution of neutrons
- Transport
  - Discretize (in space, angle, and energy) the transport equation and solve the resulting system of equations
  - Examples: Discrete Ordinates (S_N), or the Method-of-Characteristics (MOC)
- Spherical Harmonics-based (P_N)
  - Eliminate the angular variable with a spherical harmonic expansion
  - More equations, smaller phase-space

Homogenized Neutron Transport

Full core detailed transport calculation

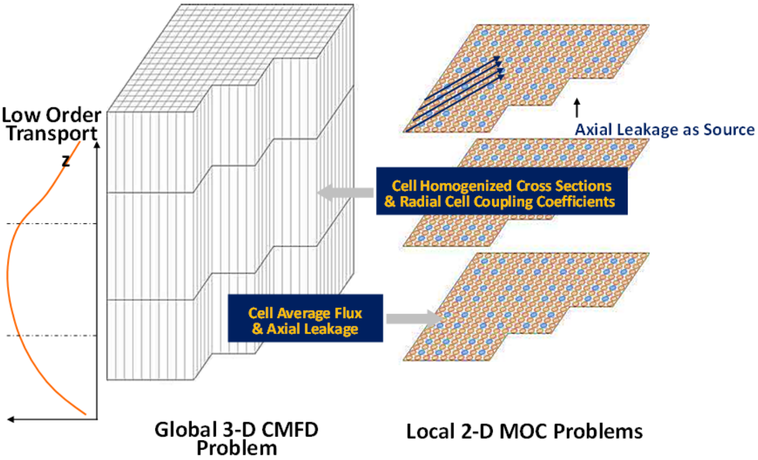
Reflecting (Zero-Current) boundary conditions



Full core homogenized diffusion calculation

$$\overline{\Sigma}_{j,i} = \frac{\int_{x_{i-1/2}}^{x_{i+1/2}} \Sigma_j(x)\phi(x)dx}{\int_{x_{i-1/2}}^{x_{i+1/2}} \phi(x)dx}$$

2-D/1-D Method


Low Order Transport
z
Cell Homogenized Cross Sections & Radial Cell Coupling Coefficients
Axial Leakage as Source
Cell Average Flux & Axial Leakage
Global 3-D CMFD Problem
Local 2-D MOC Problems

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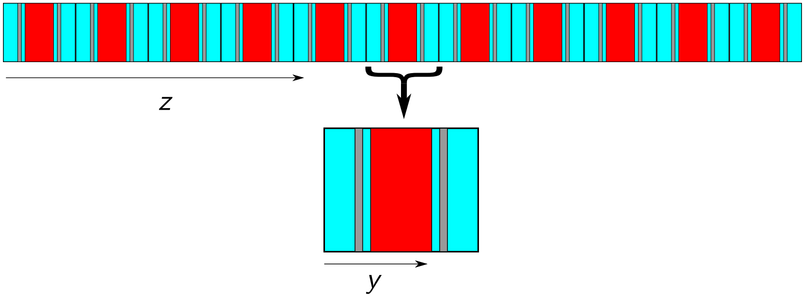
Asymptotic Derivation

- Assume the system consists of a symmetric cell (e.g. a pin or an assembly) repeated N times, with N large enough such that $\epsilon = 1/N \ll 1$
- The system can be represented by two spatial variables
  - "Fast" variable y - variations in the cell
  - "Slow" variable z - variations in the core



Expanded Transport Equation

We can therefore rewrite ψ:

$$\psi(x,\mu) = \Psi(y,z,\mu)$$

and expand Ψ in ϵ:

$$\Psi(y,z,\mu) = \Psi_0(y,z,\mu) + \epsilon\Psi_1(y,z,\mu) + \epsilon^2\Psi_2(y,z,\mu) + \dots$$

Substituting the expanded angular flux into the transport equation:

$$\begin{aligned} &\mu\frac{\partial}{\partial y}\left[\Psi_0(y,z,\mu) + \epsilon\Psi_1(y,z,\mu) + \epsilon^2\Psi_2(y,z,\mu) + \dots\right] + \epsilon\mu\frac{\partial}{\partial z}\left[\Psi_0(y,z,\mu) + \epsilon\Psi_1(y,z,\mu) + \epsilon^2\Psi_2(y,z,\mu) + \dots\right] \\ &\quad + \Sigma_t(y)\frac{\partial}{\partial y}\left[\Psi_0(y,z,\mu) + \epsilon\Psi_1(y,z,\mu) + \epsilon^2\Psi_2(y,z,\mu) + \dots\right] \\ &\qquad = \frac{1}{2}\left(\Sigma_s(y) + \left(\lambda_0 + \epsilon^2\lambda_2\right)\nu\Sigma_f(y)\right)\int_{-1}^{1}\left[\Psi_0(y,z,\mu') + \epsilon\Psi_1(y,z,\mu') + \epsilon^2\Psi_2(y,z,\mu') + \dots\right]d\mu' \end{aligned}$$

Equate by Order of ϵ

ϵ^0:

$$L\Psi_0(y,z,\mu) = 0$$

ϵ^1:

$$L\Psi_1(y,z,\mu) = -\mu\frac{\partial}{\partial z}\Psi_0(y,z,\mu)$$

ϵ^n for $n \geq 2$:

$$L\Psi_n(y,z,\mu) = -\mu\frac{\partial}{\partial z}\Psi_{n-1}(y,z,\mu) + \lambda_2\frac{\nu\Sigma_f(y)}{2}\int_{-1}^{1}\Psi_{n-2}(y,z,\mu')\,d\mu'$$

where L is the infinite-lattice transport operator,

$$\begin{aligned} L\Psi(y,z,\mu) =& \mu\frac{\partial}{\partial y}\Psi(y,z,\mu) + \Sigma_t(y)\Psi(y,z,\mu) \\ &- \frac{1}{2}\left(\Sigma_s(y) + \lambda_0\nu\Sigma_f(y)\right)\int_{-1}^{1}\Psi(y,z,\mu')\,d\mu' \end{aligned}$$

Asymptotic Homogenized Diffusion

From the solvability conditions for Ψ_2 and Ψ_3, we obtain an asymptotic homogenized diffusion equation:

$$-\overline{D}_0 \frac{\partial^2}{\partial x^2}\phi_0(x) + \overline{\Sigma_a}\phi_0(x) = \lambda\overline{\nu\Sigma_f}\phi_0(x)$$

The diffusion angular flux includes the first two terms of the asymptotic expansion:

$$\psi(x,\mu) = f_0(x,\mu)\,\phi_0(x) - f_1(x,\mu)\frac{\partial}{\partial x}\phi_0(x)$$

where $f_n(x,\mu)$ is the solution to the lattice equation

$$Lf_n(x,\mu) = g(x,\mu)$$

The standard diffusion angular flux only includes the first term:

$$\psi(x,\mu) = f_0(x,\mu)\,\phi_0(x)$$

Asymptotic Homogenized SP_2

From the solvability conditions for Ψ_2 through Ψ_5, we obtain an asymptotic homogenized SP_2 equation:

$$\boxed{-\left[\overline{D}_0 + \left(\lambda\overline{\nu\Sigma_f} - \overline{\Sigma}_a\right)\overline{D}_2\right]\frac{\partial^2}{\partial x^2}\phi\left(x\right) + \overline{\Sigma}_a\phi\left(x\right) = \lambda\overline{\nu\Sigma_f}\phi\left(x\right)}$$

The SP_2 angular flux includes the first three terms of the asymptotic expansion:

$$\begin{aligned}\psi\left(x,\mu\right) =& f_0\left(x,\mu\right)\phi\left(x\right) - f_1\left(x,\mu\right)\frac{\partial}{\partial x}\phi\left(x\right) + f_3\left(x,\mu\right)\frac{\partial^2}{\partial x^2}\phi\left(x\right)\\ &+ \left(\lambda - \lambda_0\right)f_2\left(x,\mu\right)\phi\left(x\right)\end{aligned}$$

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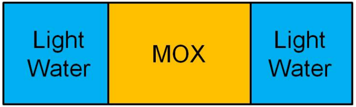
C5G7 Unreflected MOX

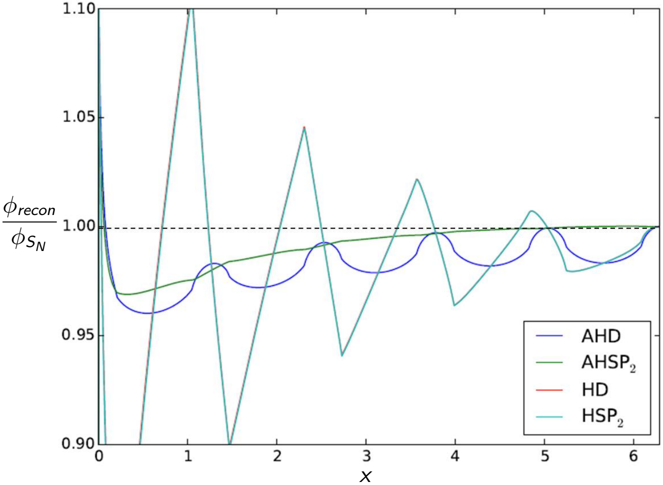
- Single pin cell:



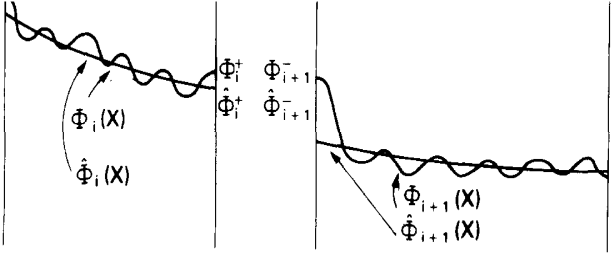
- We are comparing the four methods: standard homogenized diffusion (HD), standard SP_2 (HSP_2), asymptotic homogenized diffusion (AHD), and asymptotic SP_2 ($AHSP_2$):

	Ref k_{eff}	**Δk_{eff} (pcm)**			
# of Pins	**S_N**	**HD**	**HSP_2**	**AHD**	**$AHSP_2$**
5	1.442140	-1949.3	-1499.8	-1599.5	-802.6
10	1.588159	-363.8	-309.1	-217.9	-121.7
20	1.640565	-70.5	-65.8	-24.7	-16.5
40	1.655860	-15.4	-15.1	-2.8	-2.2

C5G7 Unreflected MOX - Fluxes



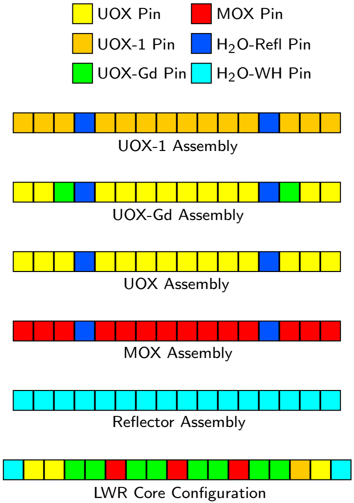
Discontinuity Factors

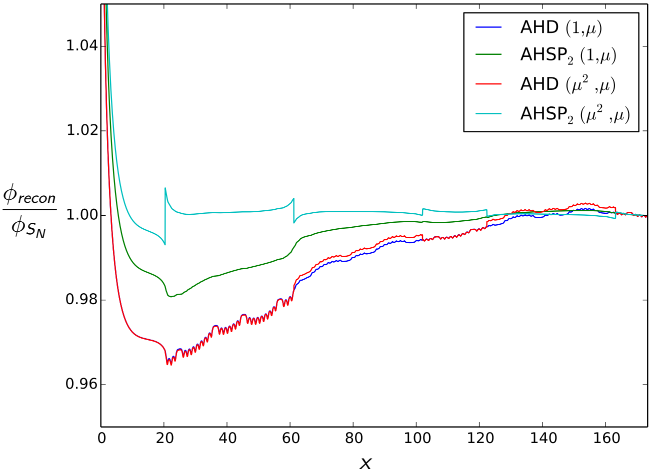


- Allow an additional degree of freedom
- Ensure an angular moment of the angular flux is continuous:

$$\int_{-1}^{1} g(\mu)\,\psi\left(x_0^-,\mu\right)d\mu = \int_{-1}^{1} g(\mu)\,\psi\left(x_0^+,\mu\right)d\mu$$

LWR Test Case





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Conclusions

1. Derived an asymptotic homogenized SP_2 equation with a corresponding expression for reconstructed angular flux

2. In a series of 1-D test problems, asymptotic SP_2 consistently proved more accurate than asymptotic diffusion, standard SP_2, and standard diffusion at calculating the eigenvalue and reconstructed flux

3. The choice of discontinuity factor can have a significant impact and must be carefully chosen

Future Work

1. Extension to multigroup. This has been done for diffusion and requires using an ansatz. A similar process should be successful with SP_2
2. Application to the 2-D/1-D method
3. Graduate?

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Questions?

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