Modeling Stability and Turbulence in Tokamak Fusion Reactors

E. M. Davis [1], M. Porkolab [1], J. W. Hughes [1],
B. LaBombard [1], P. B. Snyder [2], and X. Q. Xu [3]

[1]MIT PSFC, [2]GA, and [3]LLNL

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Outline

- Fusion background
  - Reactions of interest
  - Requirements for a reactor
- Tokamaks
  - What are they? How do they initiate fusion? What currently limits their operation?
  - Details of MIT's Alcator C-Mod
- Modeling
  - Equations and physical intuition
  - Linear stability
  - Nonlinear turbulence
- Conclusions and future work

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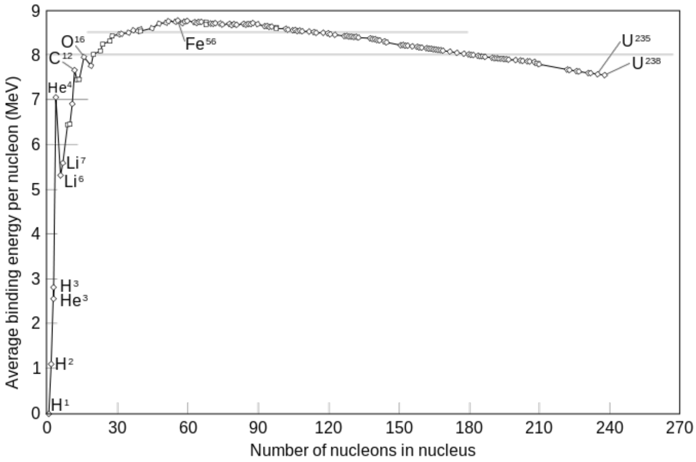
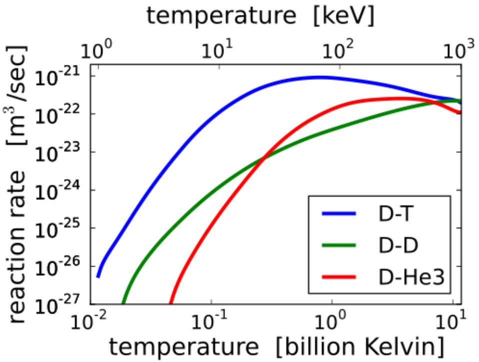
Fusion of light nuclei produces energy via Einstein's $E = mc^2$



[1]

[1] Wikimedia Commons

Many experiments focus on fusing hydrogen isotopes, mostly because of their high reaction rates



[2]

Deuterium-Tritium:

$$D + T \rightarrow {}^4_2\text{He} + n + 17.6\,\text{MeV}$$

Deuterium-Deuterium:

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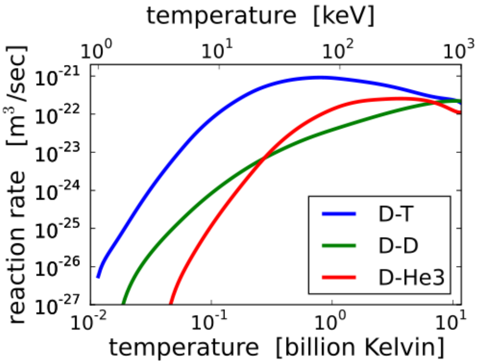

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The fusion triple product $nT\tau_E$ sets a *necessary* condition for ignition

Ignition: Fusion byproducts sufficiently heat plasma to sustain fusion reactions

Balancing the energy released by D-T fusion with Bremsstrahlung emission yields

$$nT\tau_E \geq 4.7\,\text{atm-s}$$

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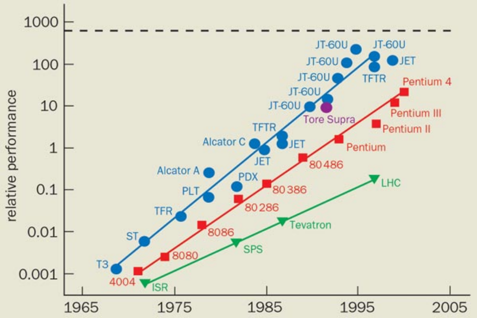

$nT\tau_E$ has outpaced Moore's Law! [3]

[3] Physics World (1/04)

The tokamak is the leading concept for a magnetic fusion reactor

Tokamak: Toroidal chamber with
axial magnetic field

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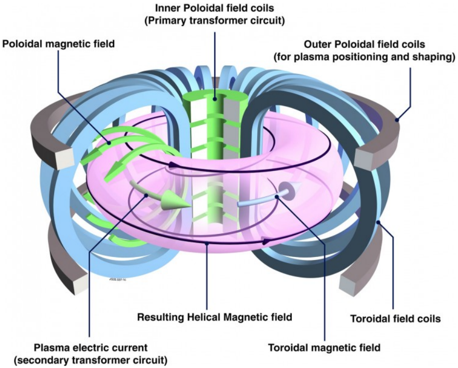

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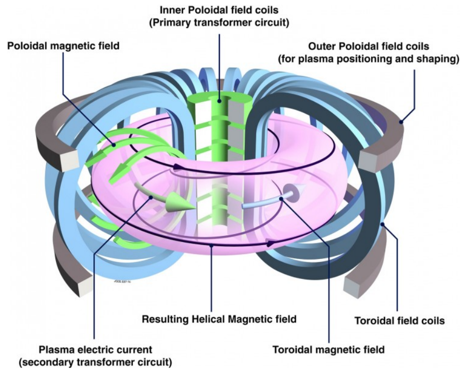

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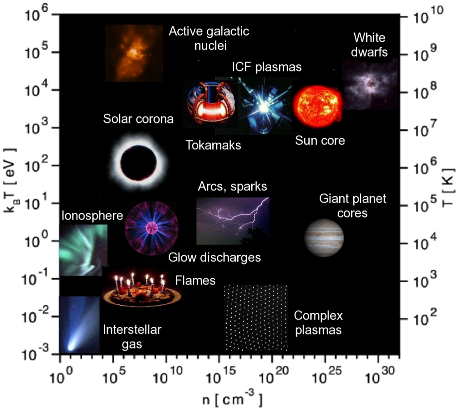

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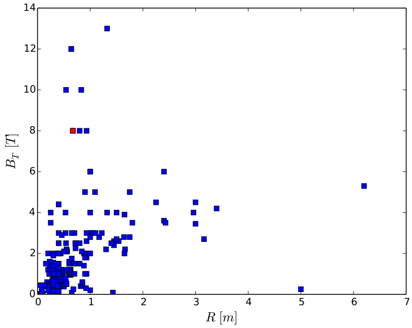
MIT's Alcator C-Mod tokamak is uniquely poised to address many reactor-relevant issues



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- C-Mod is a compact, high magnetic field ($B \leq 8.1$ T) tokamak
  - $n\tau_E \sim B^3$
  - Cost $\sim R^3$
- All-metal plasma facing components
  - Substantially changes RF coupling and edge recycling
- Reactor-relevant particle densities and heat fluxes
  - $n \sim 10^{20}$ m^{-3}
  - $q_{||} \sim 500$ MW/m^{-2}
- Workforce development

[6] All-the-World's Tokamaks

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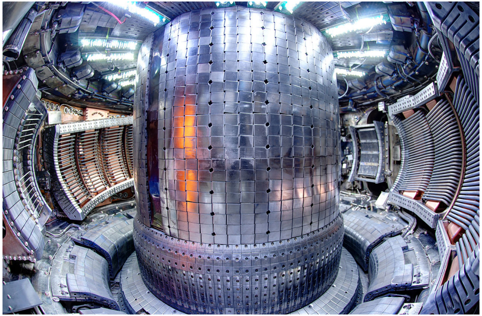

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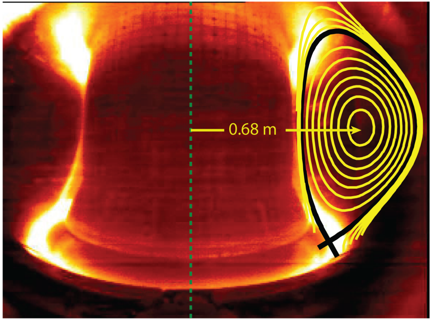

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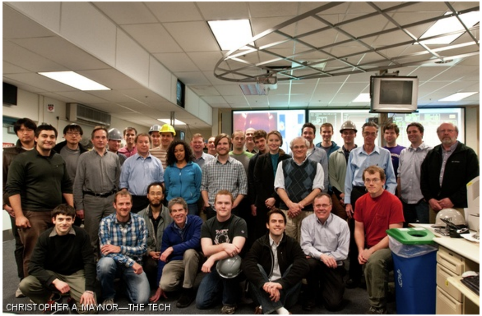

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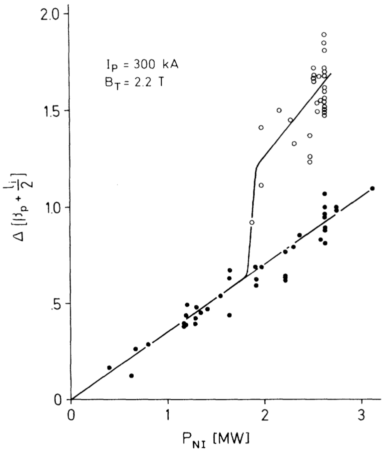
[6] The Tech

H-mode operational regime dramatically improves core performance



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[7] Wagner et al. PRL '82

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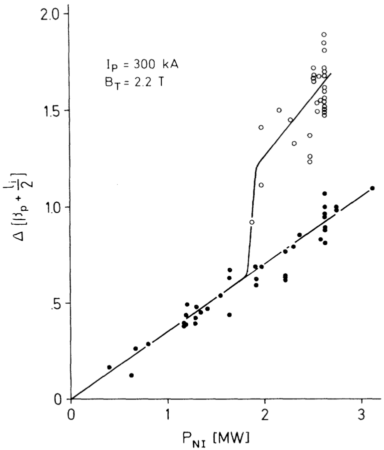

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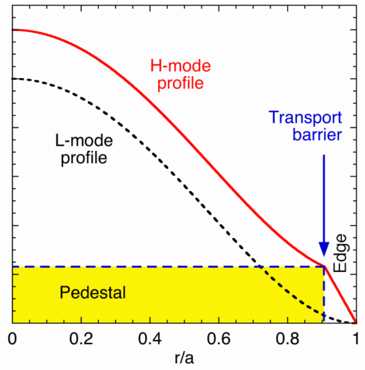

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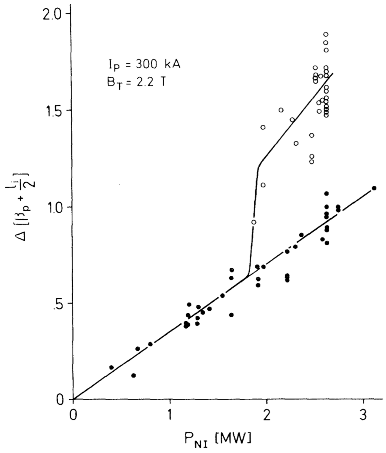

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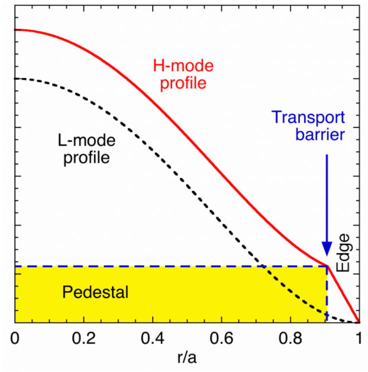

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$$\langle \sigma v \rangle \propto T^{-2/3} \exp\left(-bT^{-1/3}\right)$$

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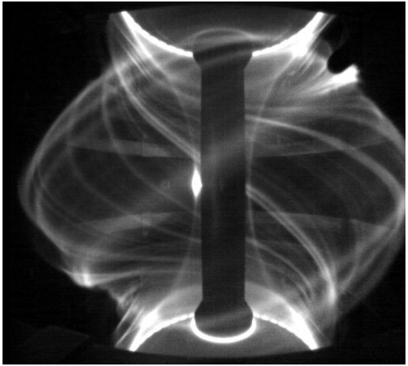
Edge Localized Modes (ELMs) flush impurities from the H-mode core; ELM onset is readily predicted by peeling-ballooning (PB) theory

An ELM on MAST



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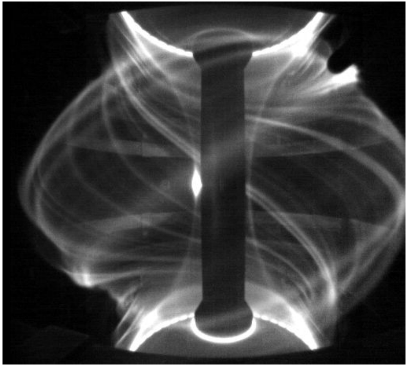

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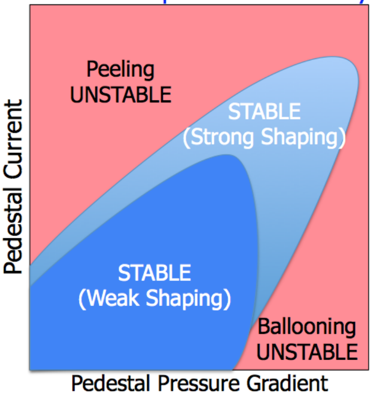

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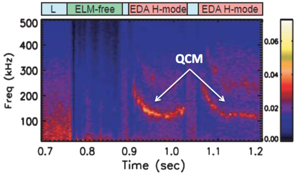
C-Mod's Enhanced D$_\alpha$ (EDA) H-mode expels impurities with a continuous edge Quasicoherent Mode (QCM) rather than peridodic ELMs

- $\sim$ 100 kHz QCM expels impurities



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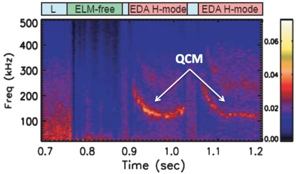

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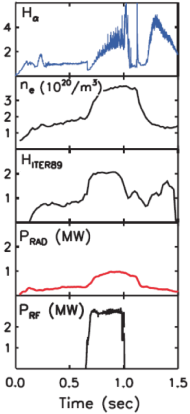

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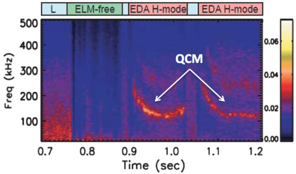

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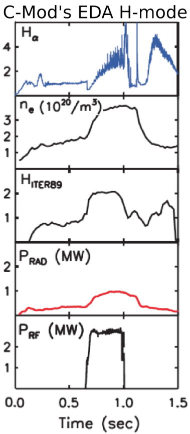

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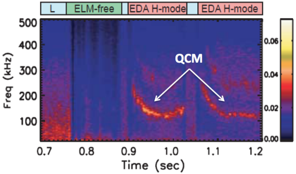

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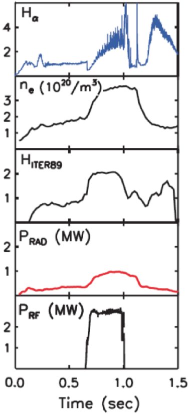
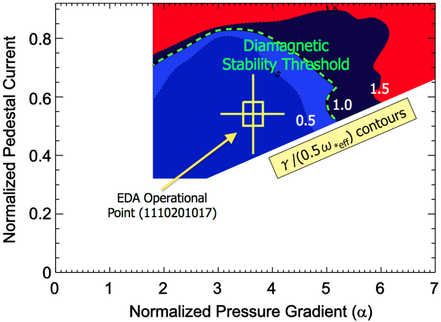


[10]

EDA is *stable* to PB modes!



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Nonideal reduced MHD equations include several effects important for modeling C-Mod's collisional EDA plasmas

Vorticity: $$\frac{\partial \omega}{\partial t} + \mathbf{V}_\perp \cdot \nabla \omega = B^2 \nabla_{||} \left(\frac{J_{||}}{B} \right) + 2\hat{\mathbf{b}} \times \boldsymbol{\kappa} \cdot \nabla P$$

Pressure: $$\frac{\partial P}{\partial t} + \mathbf{V}_\perp \cdot \nabla P = 0$$

Ohm's: $$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}\Phi - \eta J_{||} + \frac{1}{2en}\nabla_{||} P$$

Nonideal Physics[11]
Resistivity
Hall physics
Diamagnetism

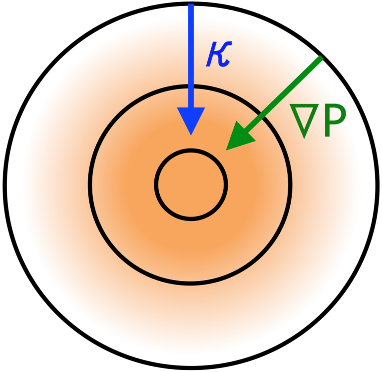
Definitions

$$\omega = \frac{nm_i}{B}\left(\nabla_\perp^2 \Phi + \frac{1}{2en}\nabla_\perp^2 P \right), \qquad \mathbf{V}_\perp = \frac{1}{B}\hat{\mathbf{b}} \times \nabla_\perp \left(\Phi + \frac{P}{2en} \right)$$

$$J_{||} = J_{||0} - \frac{1}{\mu_0}\nabla_\perp^2 A_{||}$$

[11] Hazeltine & Meiss, *Plasma Confinement* '03

The ballooning drive is closely related to the magnetic field curvature $\boldsymbol{\kappa}$



$$\boldsymbol{\kappa} \cdot \nabla P > 0$$

$\Rightarrow$ Bad Curvature

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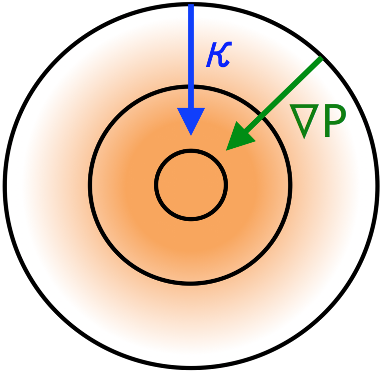

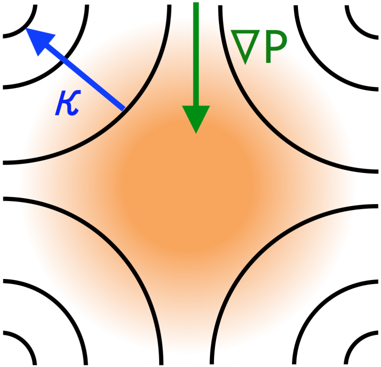

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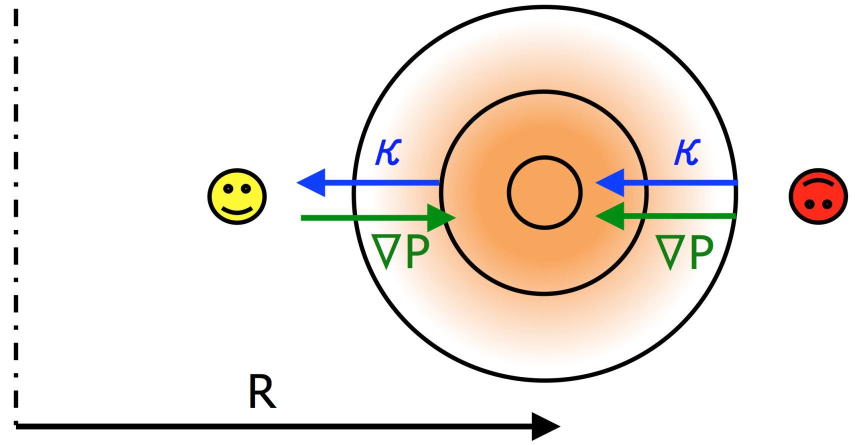
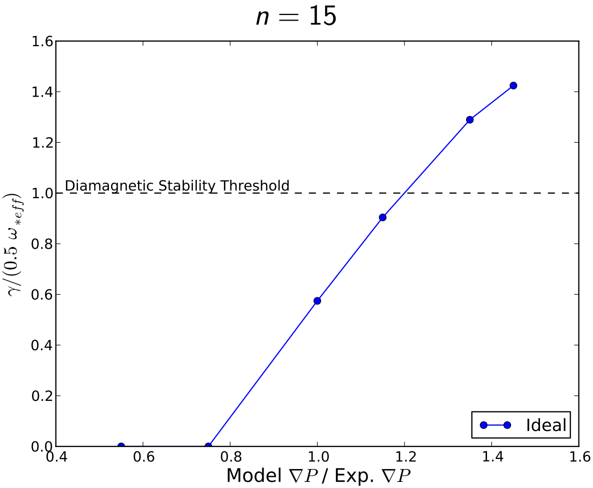
Tokamak Cross Section



Resistive Ballooning Modes (RBMs) may be responsible for C-Mod's QCM



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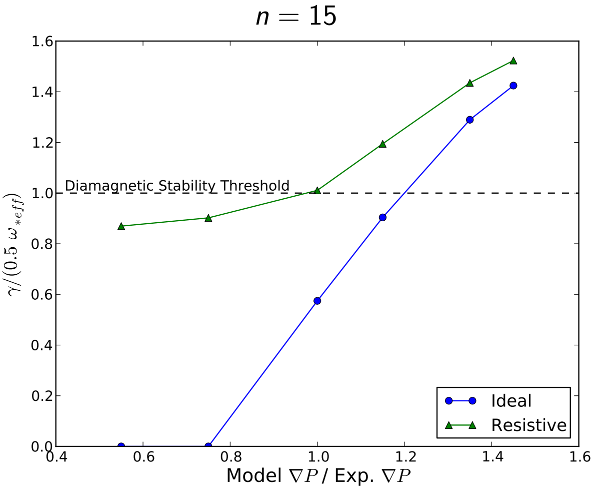

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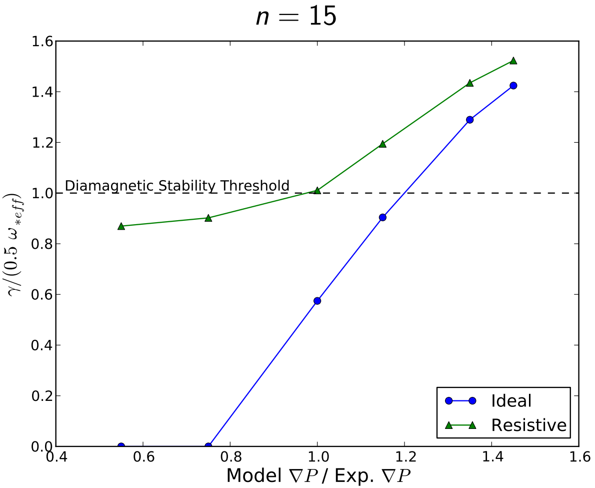
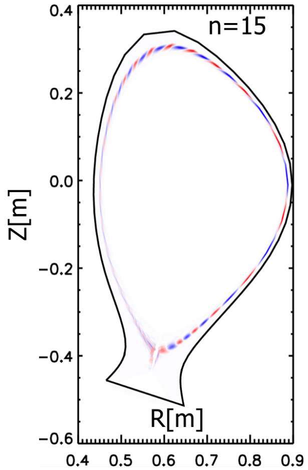
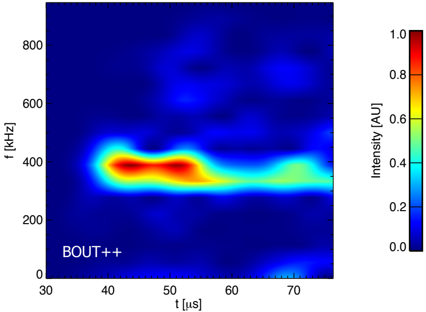




Nonlinear simulations produce a feature similar to EDA's QCM



Propagates in electron diamagnetic direction!

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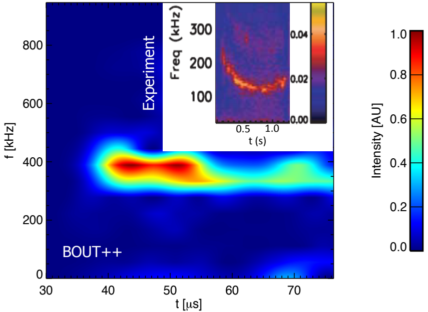

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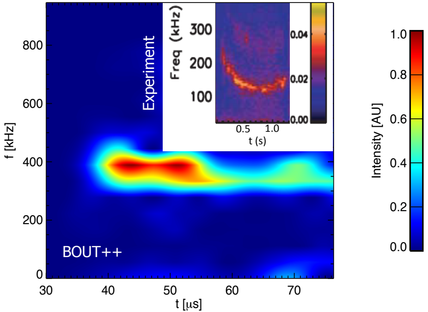

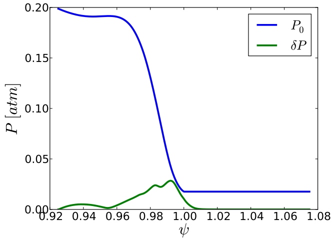

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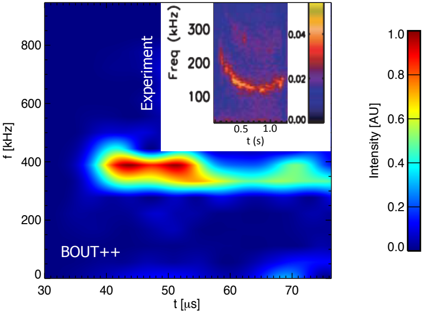

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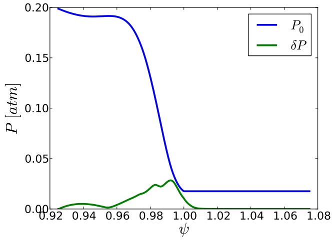

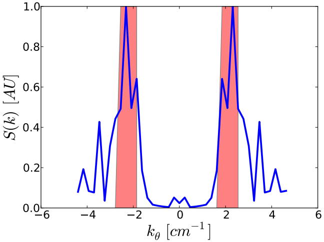

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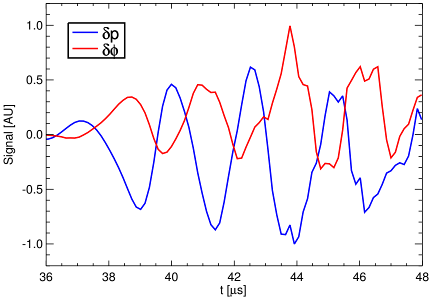

$\Rightarrow$ **Ballooning mode**

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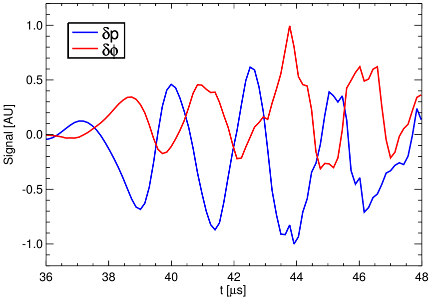

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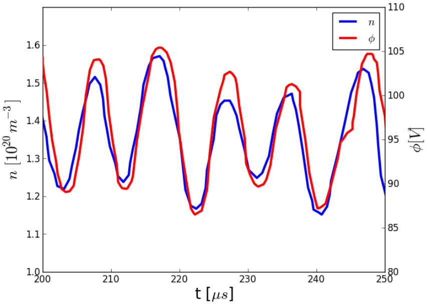
Recent Measurements[12]:



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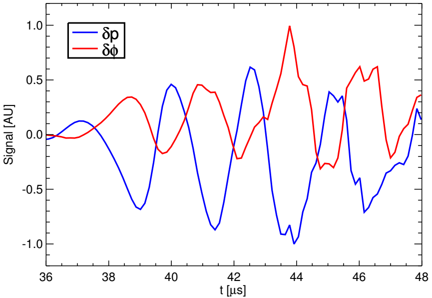
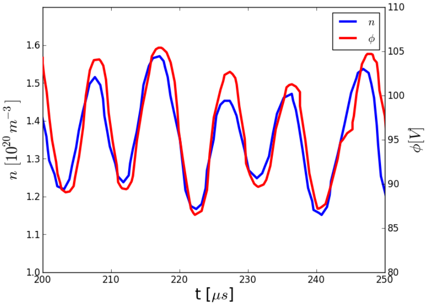


$\Rightarrow$ **Ballooning mode**

Recent Measurements[12]:

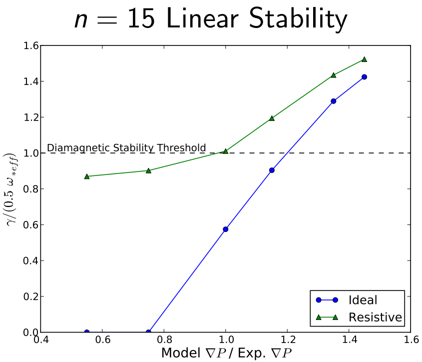


$\Rightarrow$ **Drift wave!**

Hall term may help: $$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}\Phi - \eta J_{||} + \boxed{\frac{1}{2en}\nabla_{||}P}$$

[12] LaBombard et al. PoP '14

The Hall term negligibly influences linear stability, but it drastically changes the nonlinear evolution



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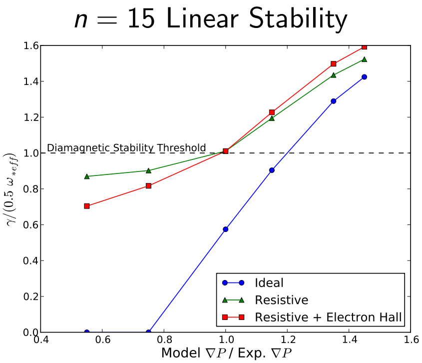

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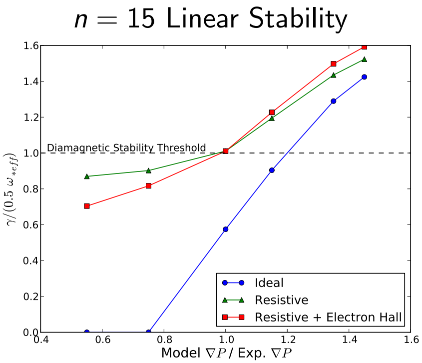
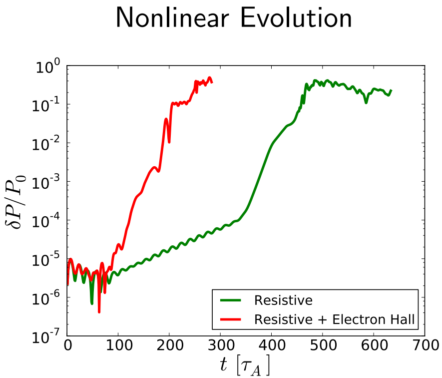
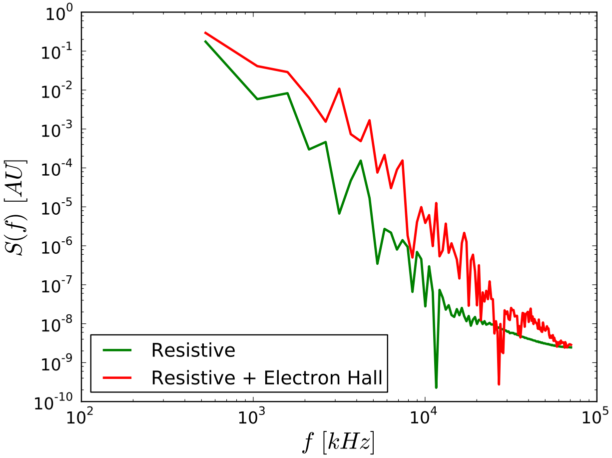




The Hall term drives more high frequency turbulence, producing more efficient cascades and potentially explaining differing time series



Conclusions and future work

- Tokamaks are the leading concept for a magnetic confinement fusion reactor
- Tokamak performance is crucially determined by parameters in the device edge
- The quasicoherent mode (QCM) that continuously exhausts impurities in C-Mod's EDA H-mode is *not* predicted by ideal MHD, but linear stability calculations indicate it may be a resistive ballooning mode (RBM)
- RBMs drive a nonlinear feature that is macroscopically similar to the QCM
  - However, recent measurements indicate that the QCM is a drift wave, not a RBM
- Attempts to excite the experimentally observed drift wave response in the model have *not* yet been successful
  - May need to move to a full 2-fluid model

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