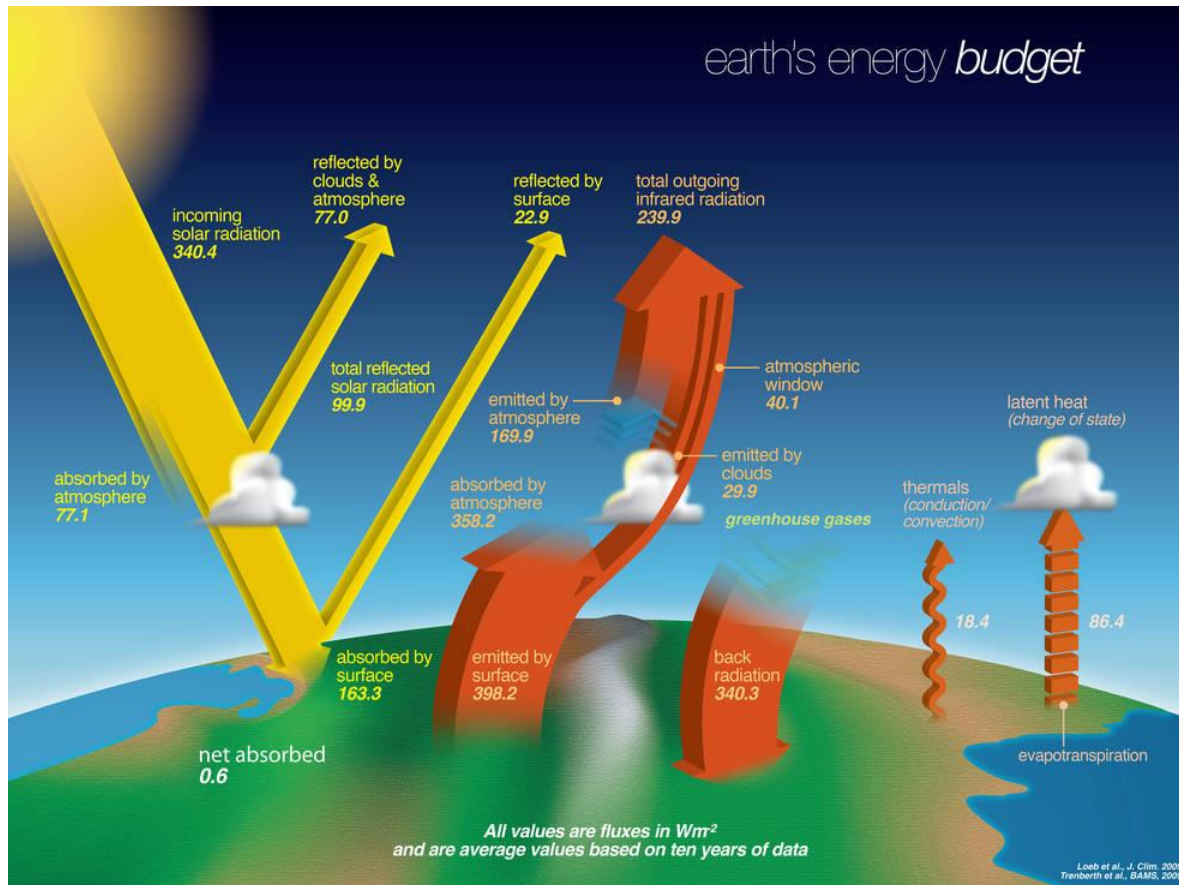# Angular *hp*-Adaptivity for Radiative Transfer

Jason Torchinsky

University of Wisconsin-Madison

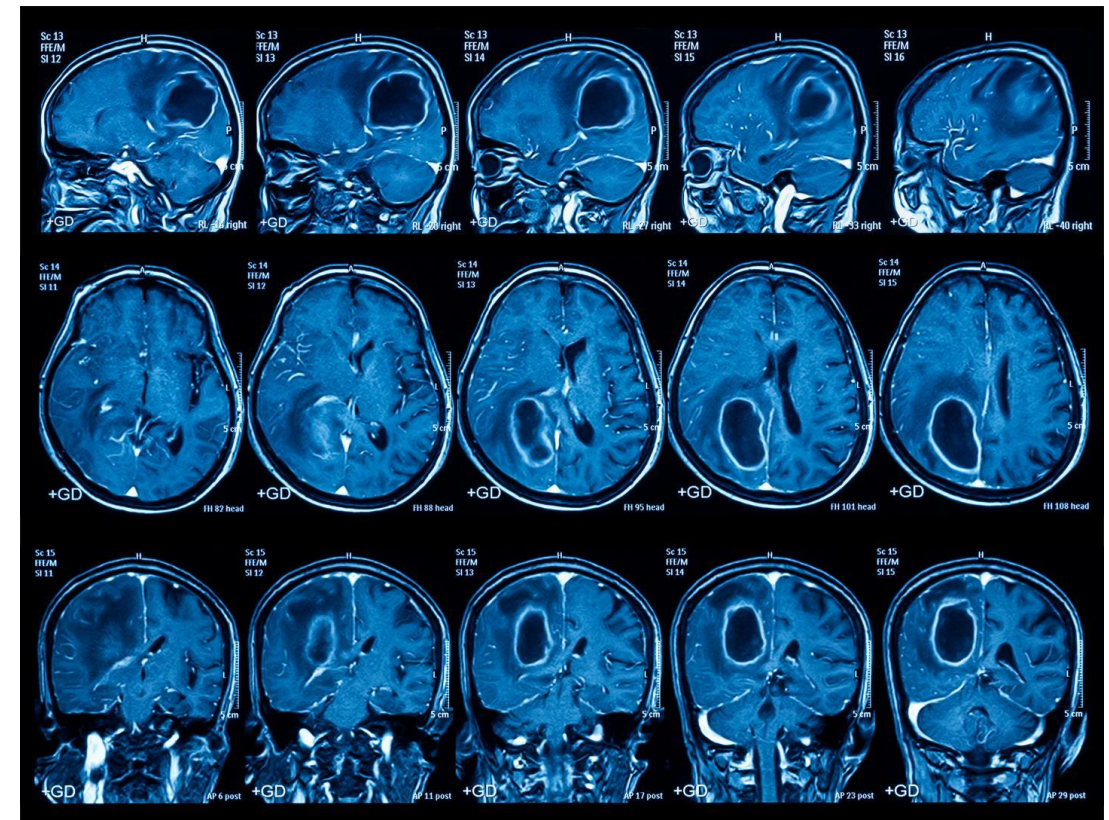**Department of Mathematics**

July 17th, 2023

What is radiative transfer?

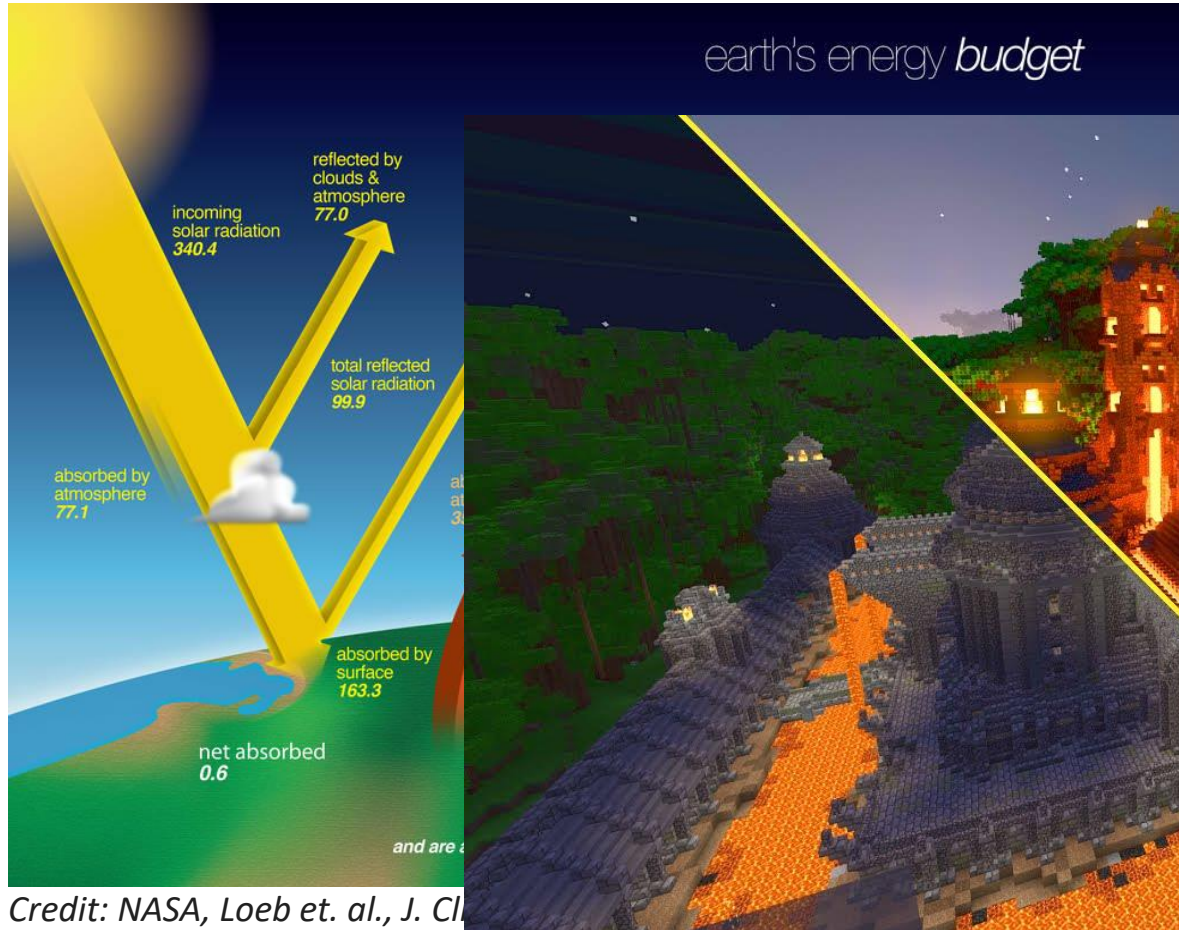The transmission of electromagnetic radiation through a medium

Credit: NASA, Loeb et. al., J. Clim., 2009, Trenberth et. al., BAMS, 2009

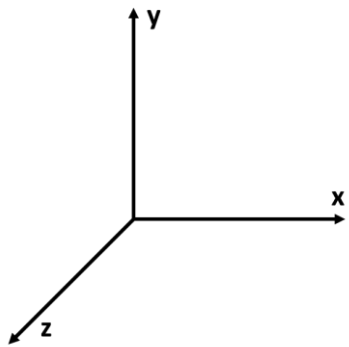Credit: CERN

What is radiative transfer?

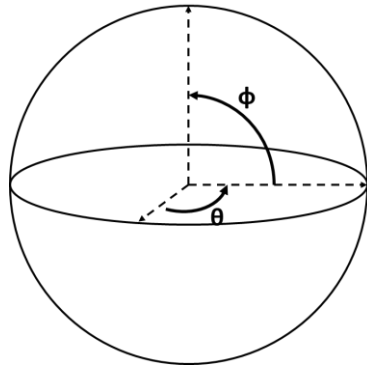### The transmission of electromagnetic radiation through a medium

Credit: NASA, Loeb et. al., J. Cl...

Credit: 'Ray Tracing Revolution', 2021

Why is radiative transfer difficult?

- High dimensional

Three in space…

Two in angle…

Relative Cost of Physics Packages in SCREAM

Clouds and Moist Processes

Radiation

And one in wavelength!
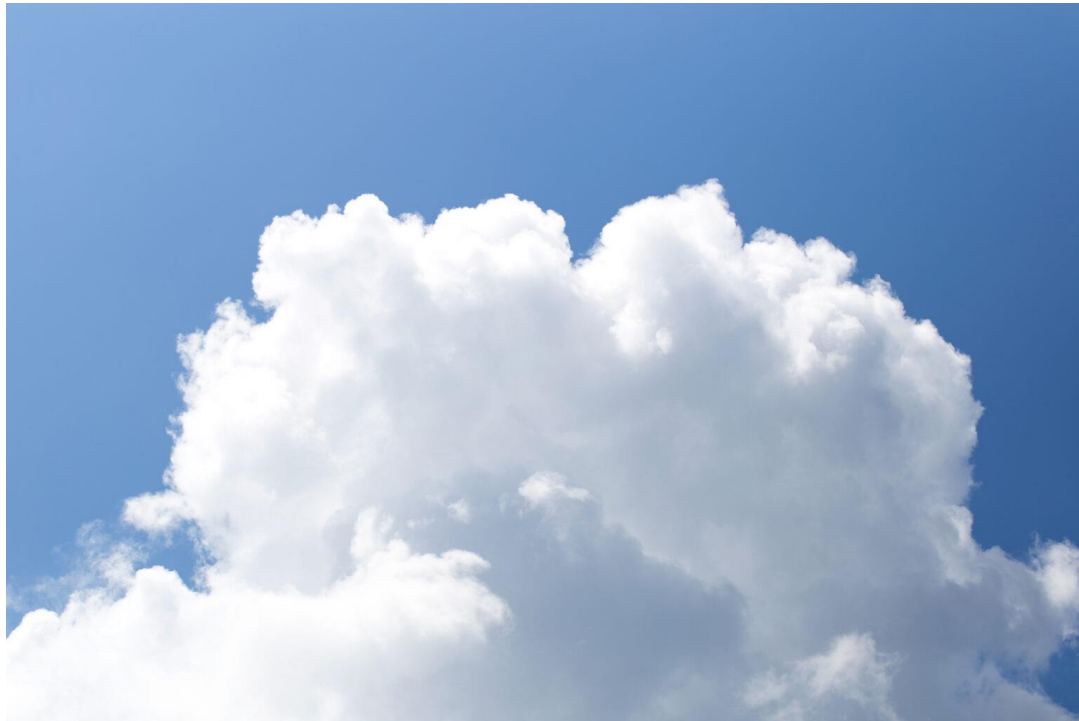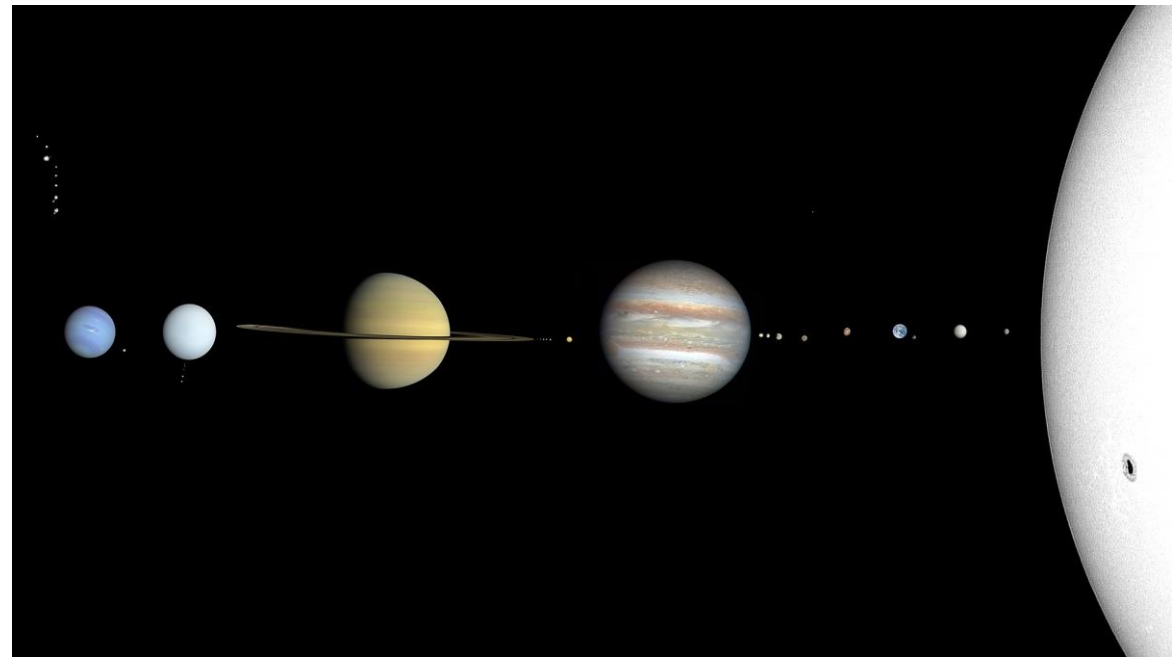
Other
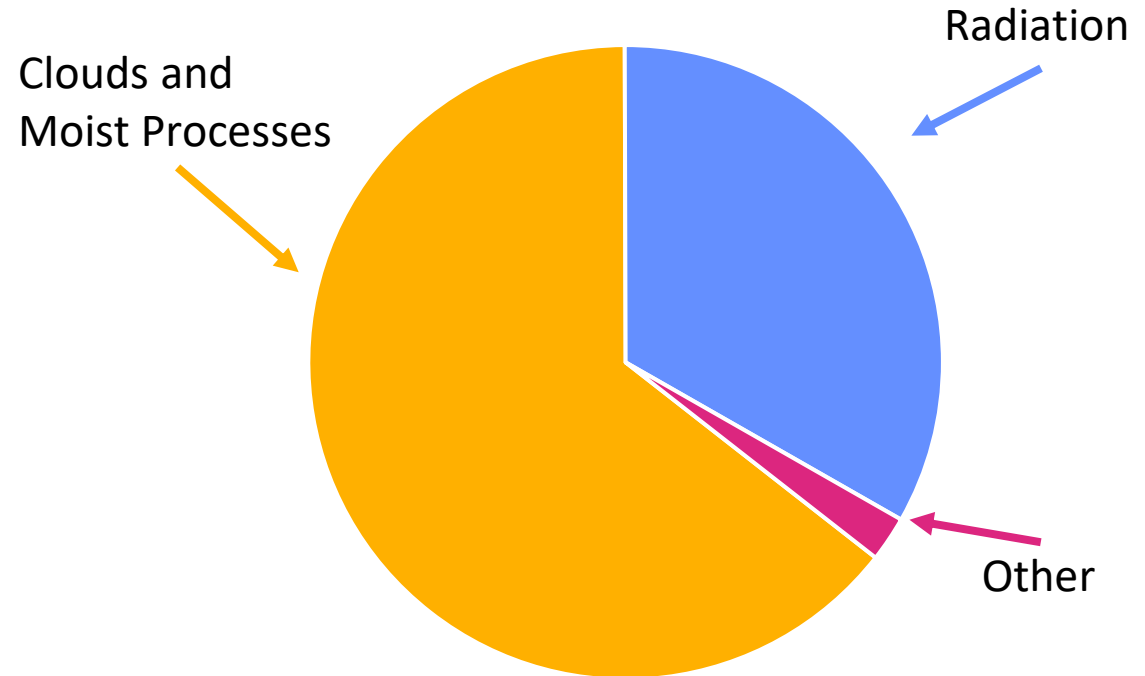
Credit: B. Hillman, Reducing the cost of radiative transfer in multi-scale atmosphere models. Electronically, 2019.

Why is radiative transfer difficult?

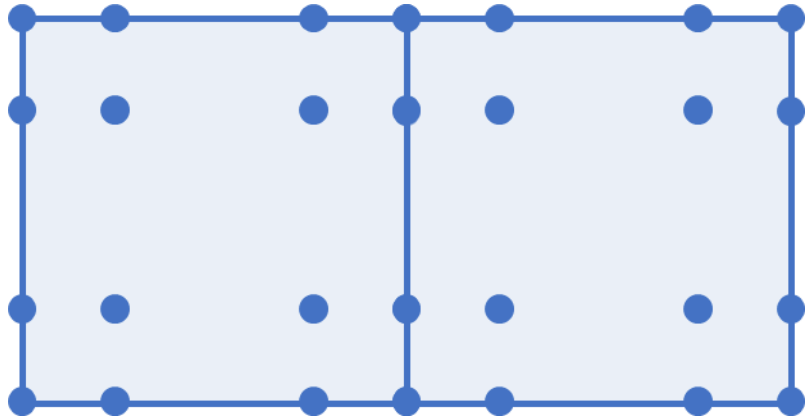- High dimensional
- Sharp gradients

In space…
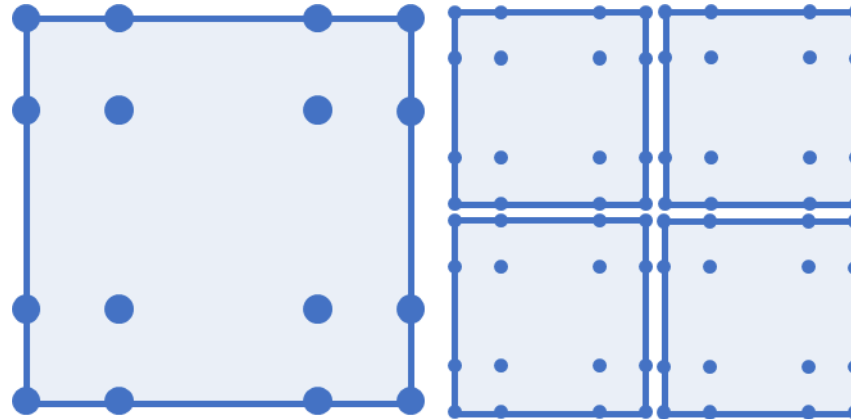

And angle!

The discontinuous Galerkin spectral element method with *hp*-adaptivity

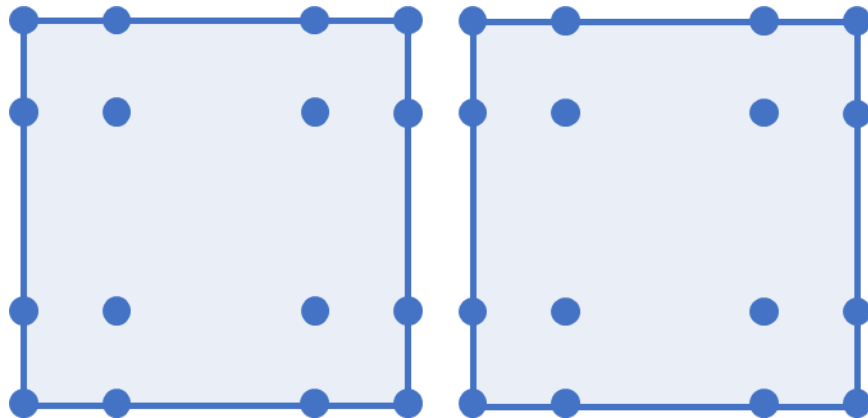Continuous Galerkin Spectral Element (SE)

Discontinuous Galerkin (DG) SE

DG SE with *h*-Adaptivity

DG SE with *hp*-Adaptivity

Local features resolved by *h*-refinement

High-order convergence by *p*-refinement
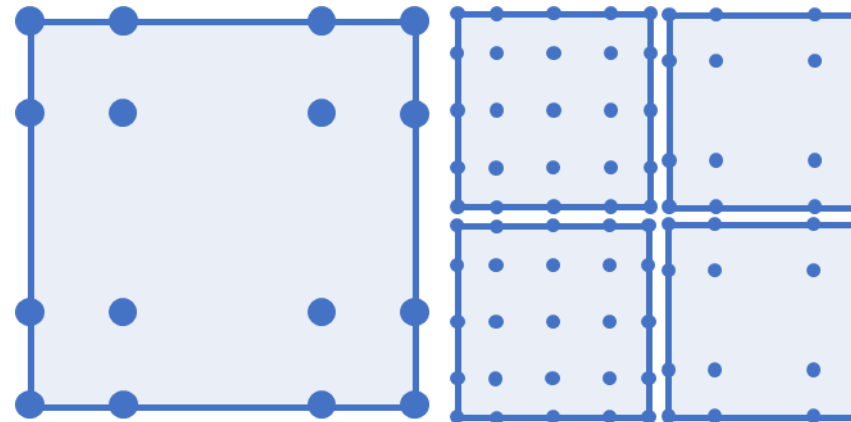
$$Error \sim DOFs^{-p/d}$$

Highly parallelizable

The DGSEM for Radiative Transfer

$$\hat{s} \cdot \nabla_{\vec{x}} u(\vec{x}, \hat{s}) + \beta_e(\vec{x}) \, u(\vec{x}, \hat{s}) - \beta_e(\vec{x}) \, \widetilde{\omega}(\vec{x}) \int_{\mathcal{S}} \Phi(\hat{s}, \hat{s}') \, u(\vec{x}, \hat{s}') d\hat{s}' = \beta_e(\vec{x}) \left(1 - \widetilde{\omega}(\vec{x})\right) B(\vec{x}, \hat{s})$$

The DGSEM for Radiative Transfer

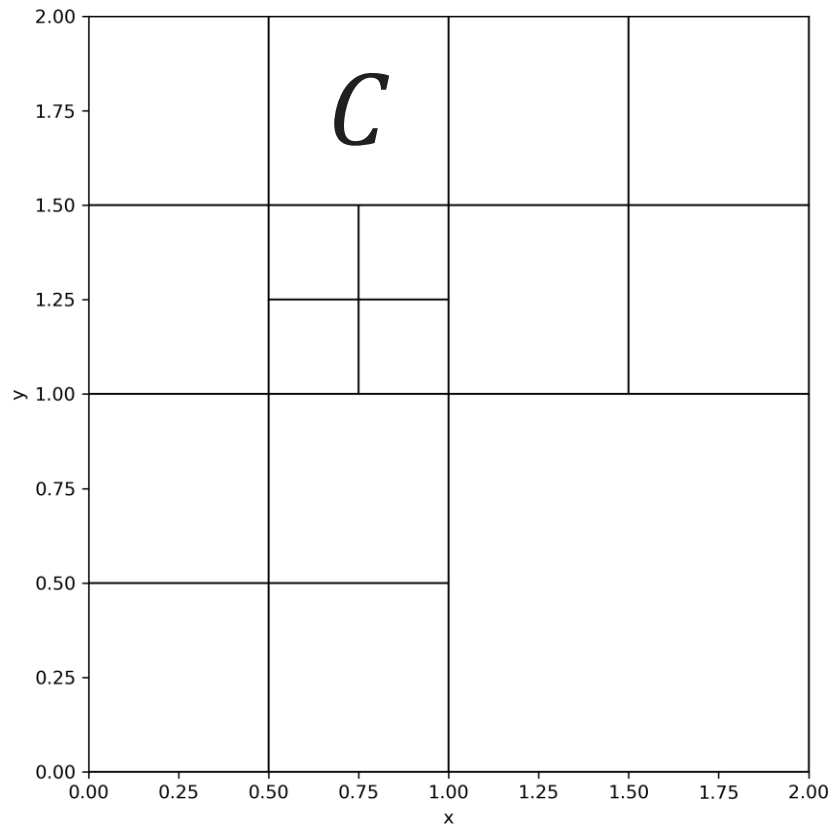$$\overbrace{\hat{s} \cdot \nabla_{\vec{x}} u(\vec{x}, \hat{s})}^{Propagation} + \overbrace{\beta_e(\vec{x})\, u(\vec{x}, \hat{s})}^{Extinction} - \overbrace{\beta_e(\vec{x})\, \widetilde{\omega}(\vec{x}) \int_{\mathcal{S}} \Phi(\hat{s}, \hat{s}')\, u(\vec{x}, \hat{s}') d\hat{s}'}^{Scattering} = \overbrace{\beta_e(\vec{x}) \left(1 - \widetilde{\omega}(\vec{x})\right) B(\vec{x}, \hat{s})}^{Blackbody\ Radiation}$$

The DGSEM for Radiative Transfer

$$\overbrace{\hat{s} \cdot \nabla_{\vec{x}} u(\vec{x}, \hat{s})}^{Propagation} + \overbrace{\kappa(\vec{x})\, u(\vec{x}, \hat{s})}^{Extinction} - \sigma(\vec{x}) \overbrace{\int_{\mathcal{S}} \Phi(\hat{s}, \hat{s}')\, u(\vec{x}, \hat{s}')d\hat{s}'}^{Scattering} = \overbrace{f(\vec{x}, \hat{s})}^{Forcing}$$

Spatial Basis

$$In\ x:\ \phi_i^C(x), \qquad i = 0, \dots, n_x^C - 1$$
$$In\ y:\ \psi_j^C(y), \qquad j = 0, \dots, n_y^C - 1$$

The DGSEM for Radiative Transfer

$$\overbrace{\hat{s} \cdot \nabla_{\vec{x}} u(\vec{x}, \hat{s})}^{Propagation} + \overbrace{\kappa(\vec{x}) \, u(\vec{x}, \hat{s})}^{Extinction} - \overbrace{\sigma(\vec{x}) \int_{\mathcal{S}} \Phi(\hat{s}, \hat{s}') \, u(\vec{x}, \hat{s}') d\hat{s}'}^{Scattering} = \overbrace{f(\vec{x}, \hat{s})}^{Forcing}$$
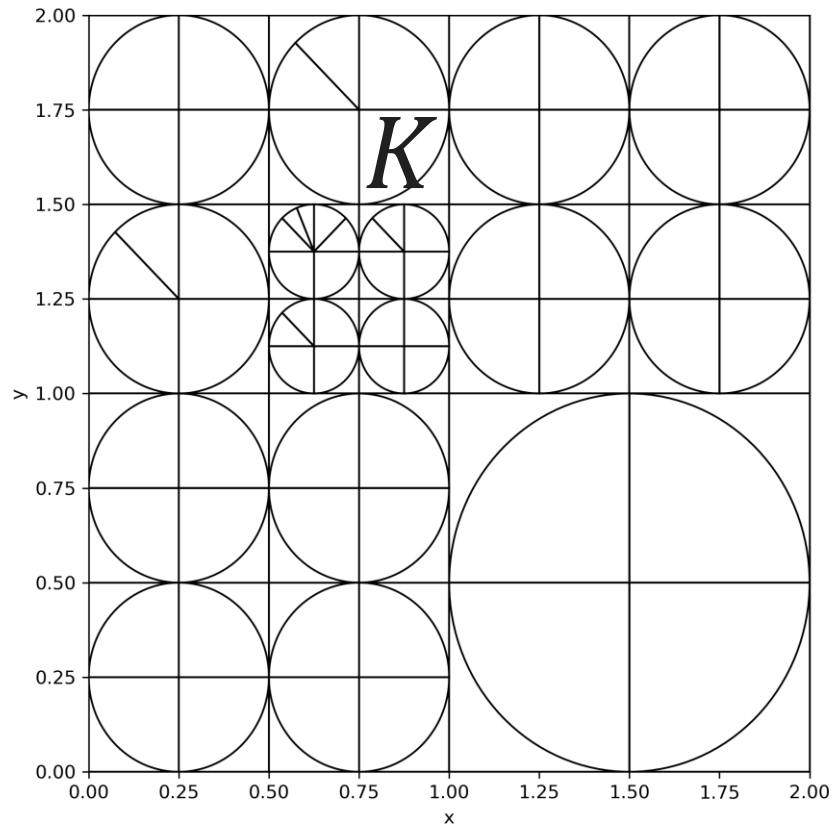

Spatial Basis

$In \; x: \; \phi_i^C(x), \qquad i = 0, \dots, n_x^C - 1$

$In \; y: \; \psi_j^C(y), \qquad j = 0, \dots, n_y^C - 1$

Angular Basis
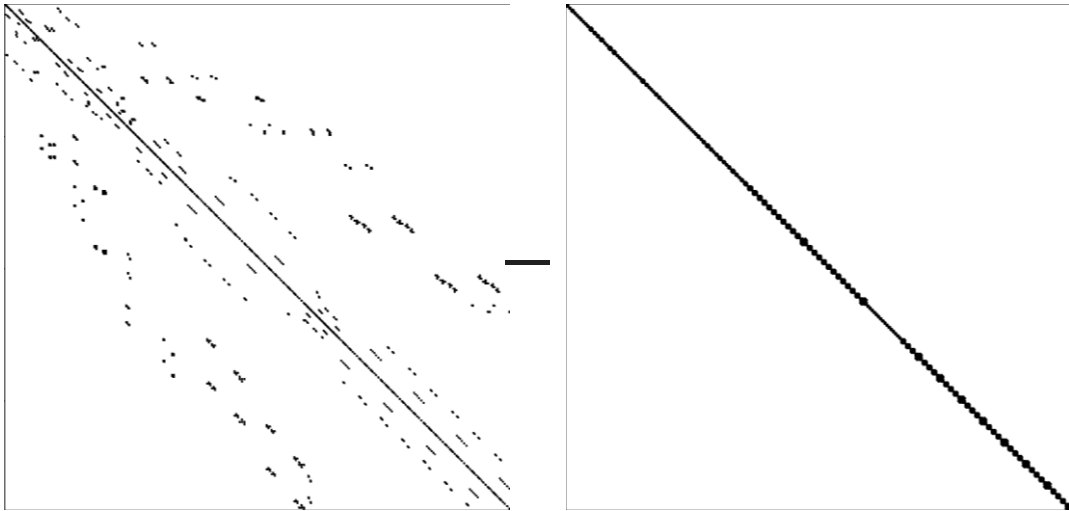
$\xi_a^K(\theta), \qquad a = 0, \dots, n_\theta^K - 1$

$$u_{hp}^{(C,K)}(\vec{x}, \hat{s}) = \sum_{a=0}^{n_\theta^K - 1} \sum_{j=0}^{n_y^C - 1} \sum_{i=0}^{n_x^C - 1} u_{ija}^{(C,K)} \, \phi_i^C(x) \, \psi_j^C(y) \, \xi_a^K(\theta)$$

The DGSEM for Radiative Transfer

$$\overbrace{\hat{s} \cdot \nabla_{\vec{x}} u(\vec{x}, \hat{s})}^{Propagation} \quad \overbrace{+ \kappa(\vec{x}) \, u(\vec{x}, \hat{s})}^{Extinction} \quad \overbrace{- \sigma(\vec{x}) \int_{\mathcal{S}} \Phi(\hat{s}, \hat{s}') \, u(\vec{x}, \hat{s}') d\hat{s}'}^{Scattering}$$

The DGSEM for Radiative Transfer

$$\overbrace{\qquad\qquad Propagation \qquad\qquad}$$

$$- \qquad\qquad + \overbrace{\kappa(\vec{x})\,u(\vec{x},\hat{s})}^{Extinction} \qquad - \sigma(\vec{x})\overbrace{\int_{\mathcal{S}}\Phi(\hat{s},\hat{s}')\,u(\vec{x},\hat{s}')d\hat{s}'}^{Scattering}$$

The DGSEM for Radiative Transfer

$$\overbrace{\qquad\qquad}^{\textit{Propagation}} \qquad \overbrace{\qquad\qquad}^{\textit{Extinction}}$$

$$- \;\; \qquad + \qquad \overbrace{-\sigma(\vec{x}) \int_{\mathcal{S}} \Phi(\hat{s}, \hat{s}')\, u(\vec{x}, \hat{s}') d\hat{s}'}^{\textit{Scattering}}$$

The DGSEM for Radiative Transfer

$$\overbrace{\qquad}^{Propagation} - \qquad + \overbrace{\qquad}^{Extinction} - \overbrace{\qquad}^{Scattering}$$

The DGSEM for Radiative Transfer

$$\underbrace{\qquad\qquad}_{} \;-\; \underbrace{\qquad\qquad}_{} \;+\; \underbrace{\qquad\qquad}_{} \;-\; \underbrace{\qquad\qquad}_{}$$

Propagation   *Extinction*   *Scattering*

Block Diagonal   *Diagonal*   *Block Diagonal*
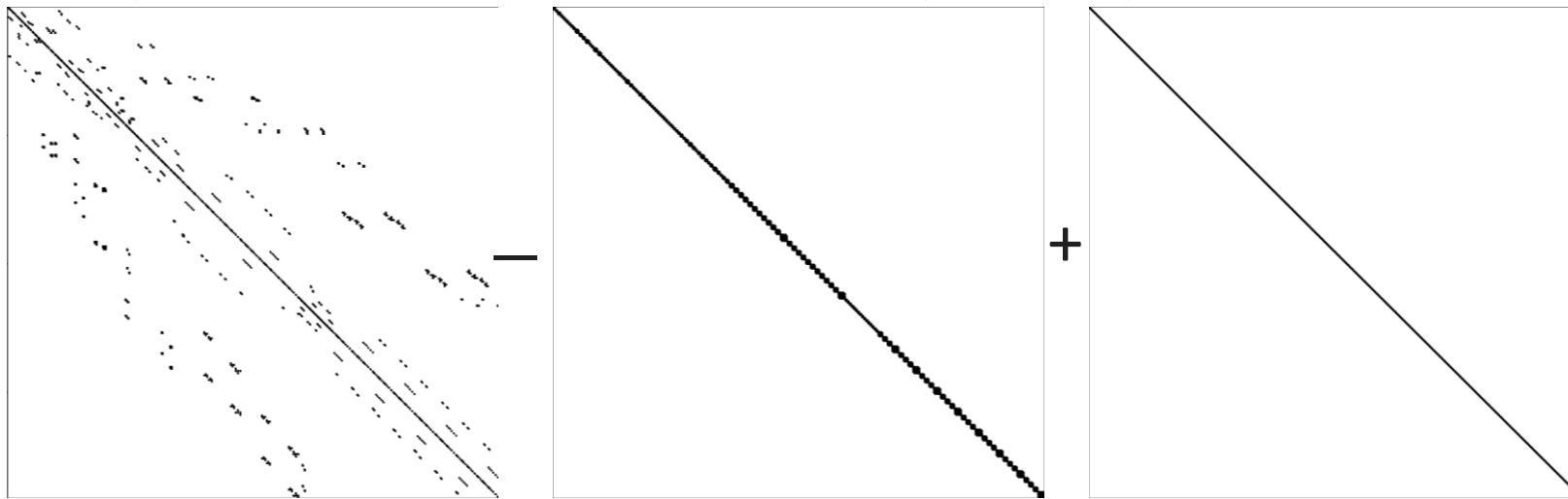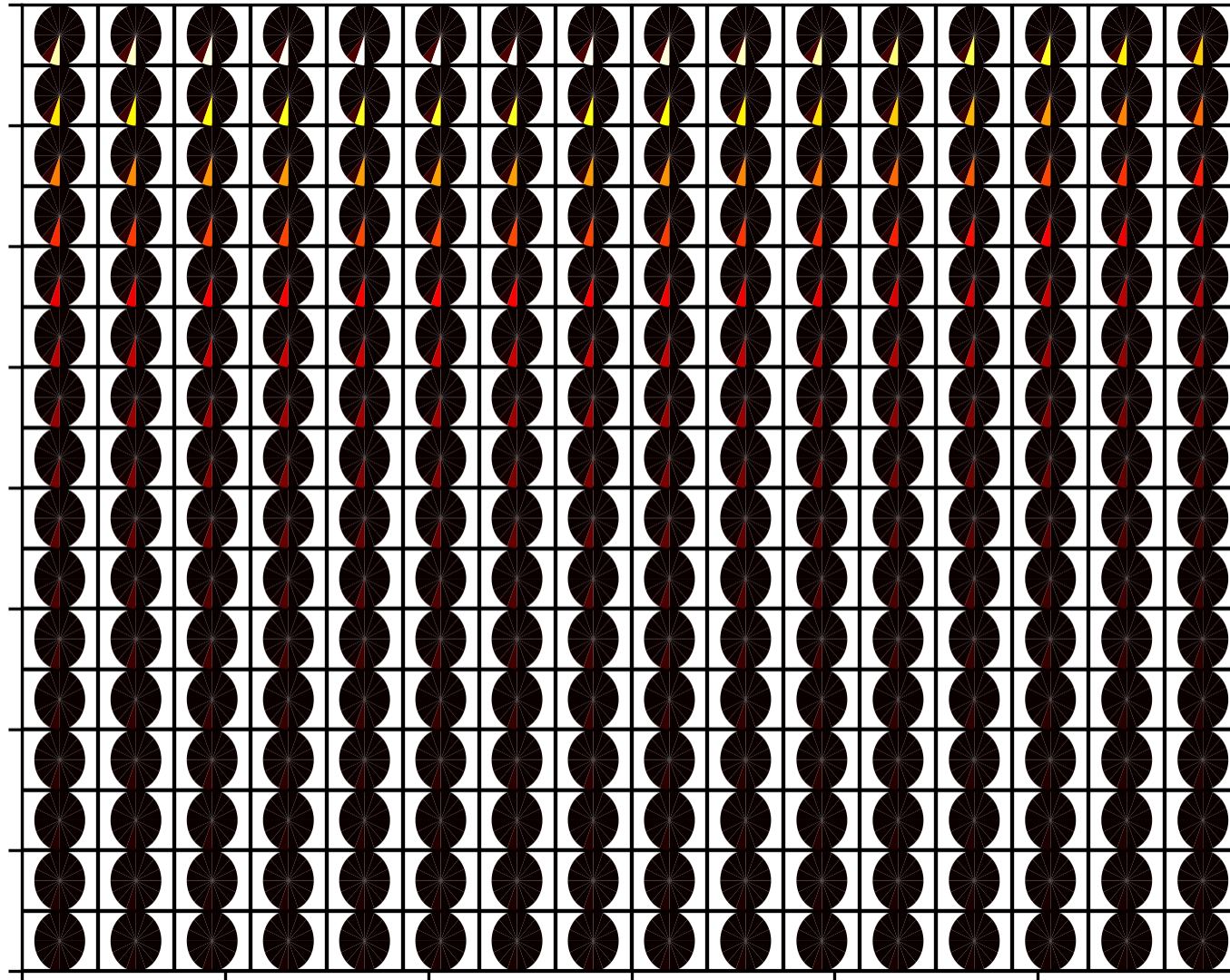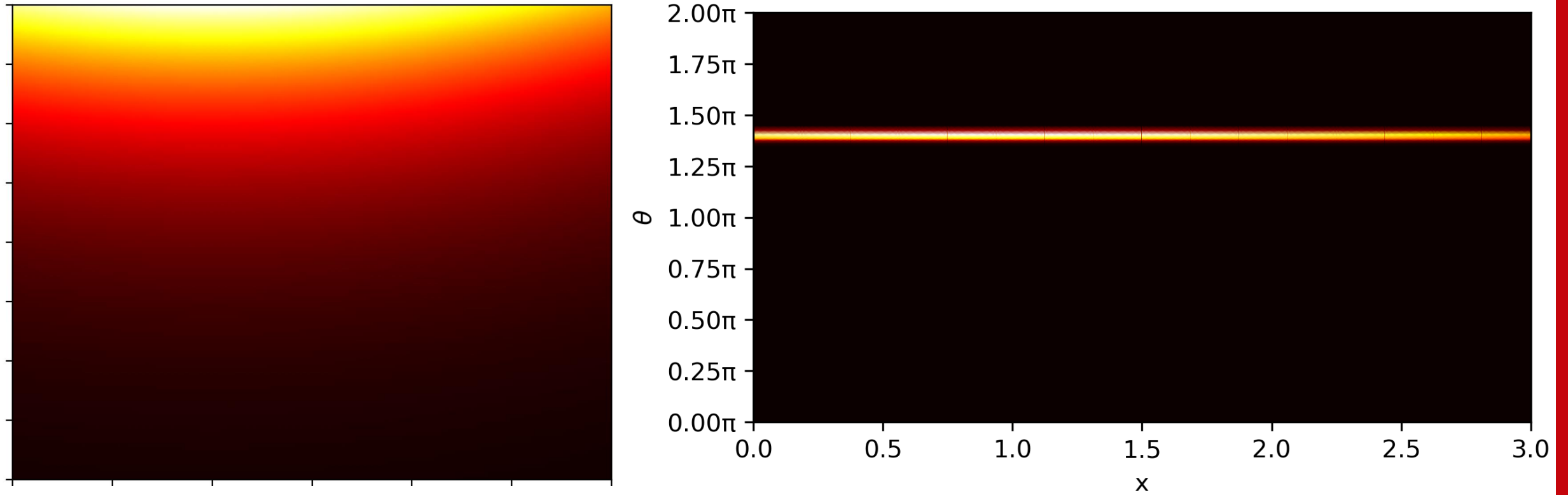
Each block can be computed simultaneously

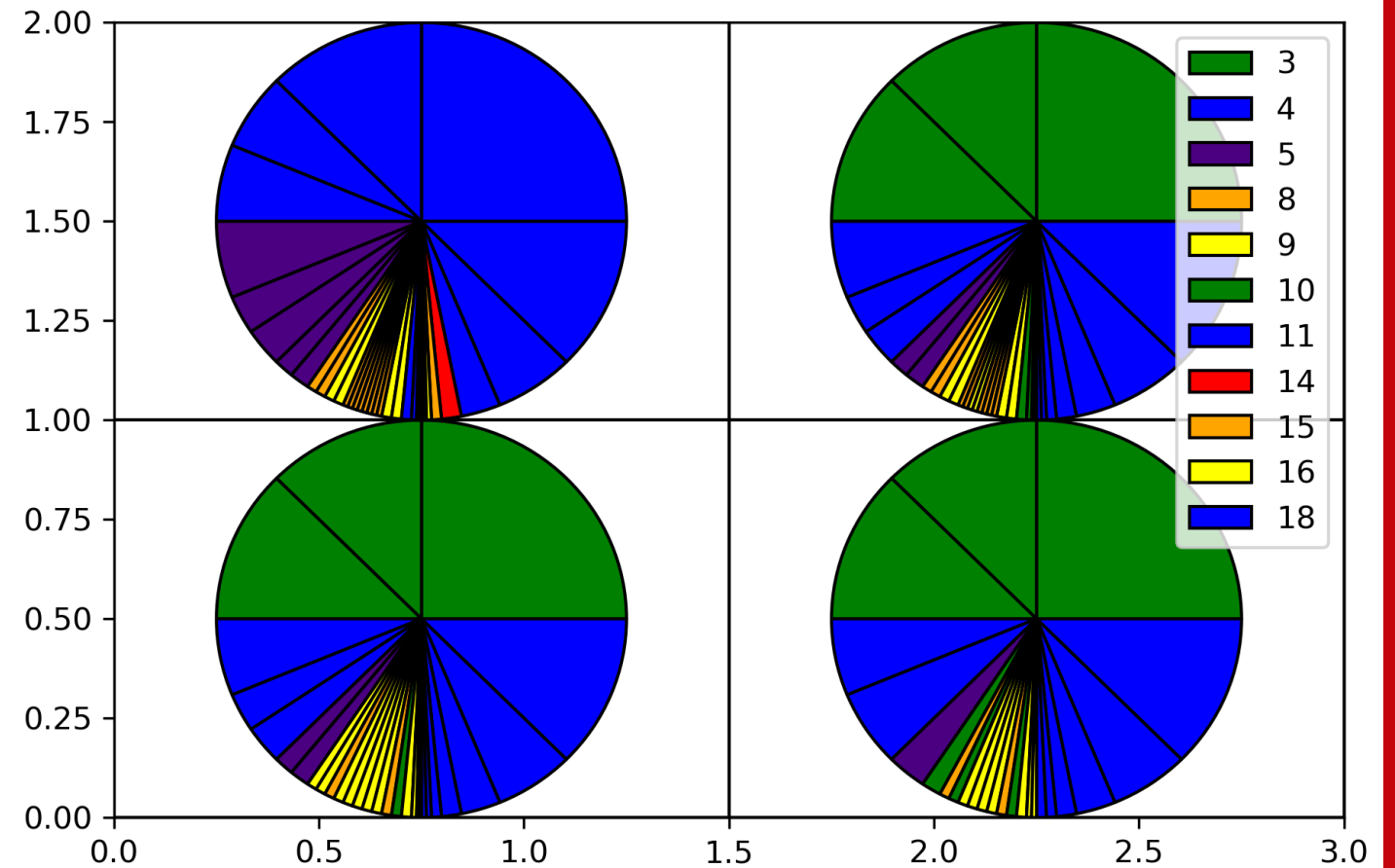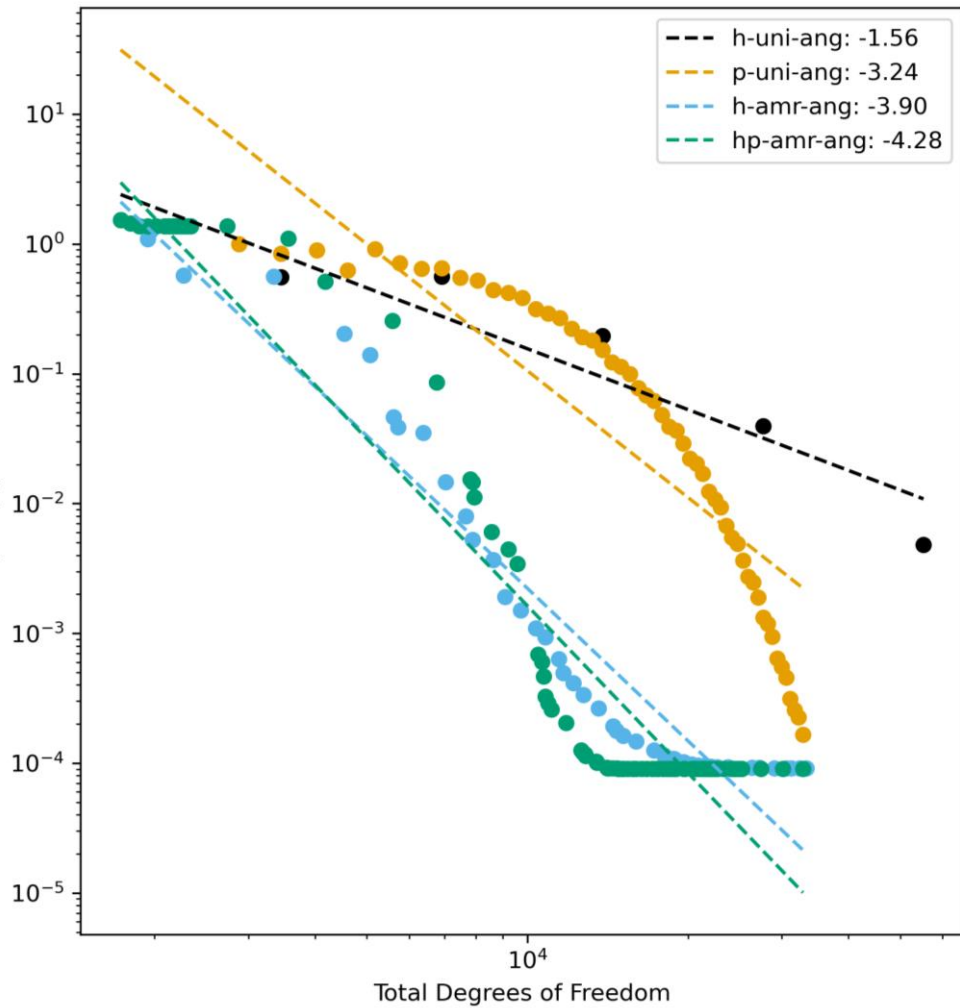Many blocks can be reused between refinements

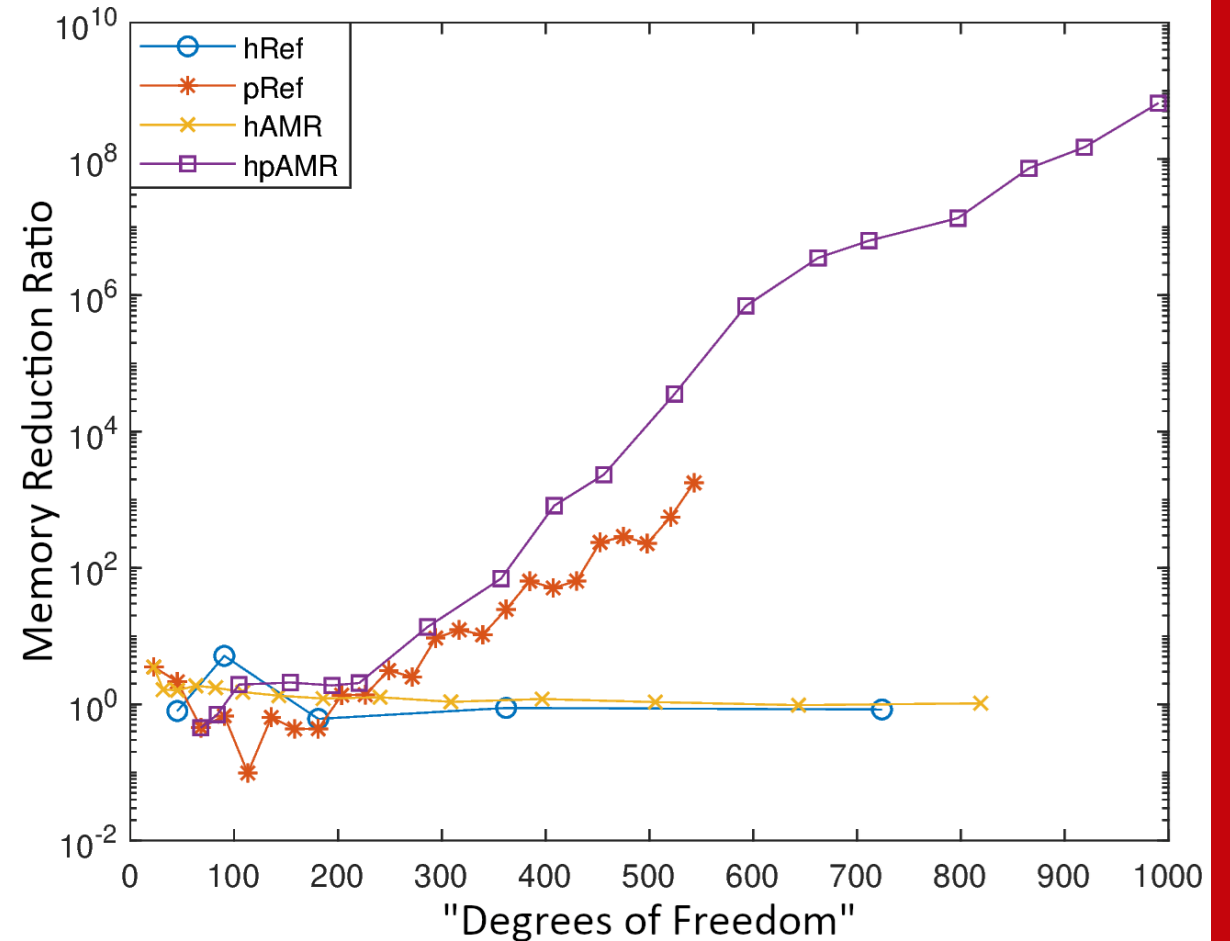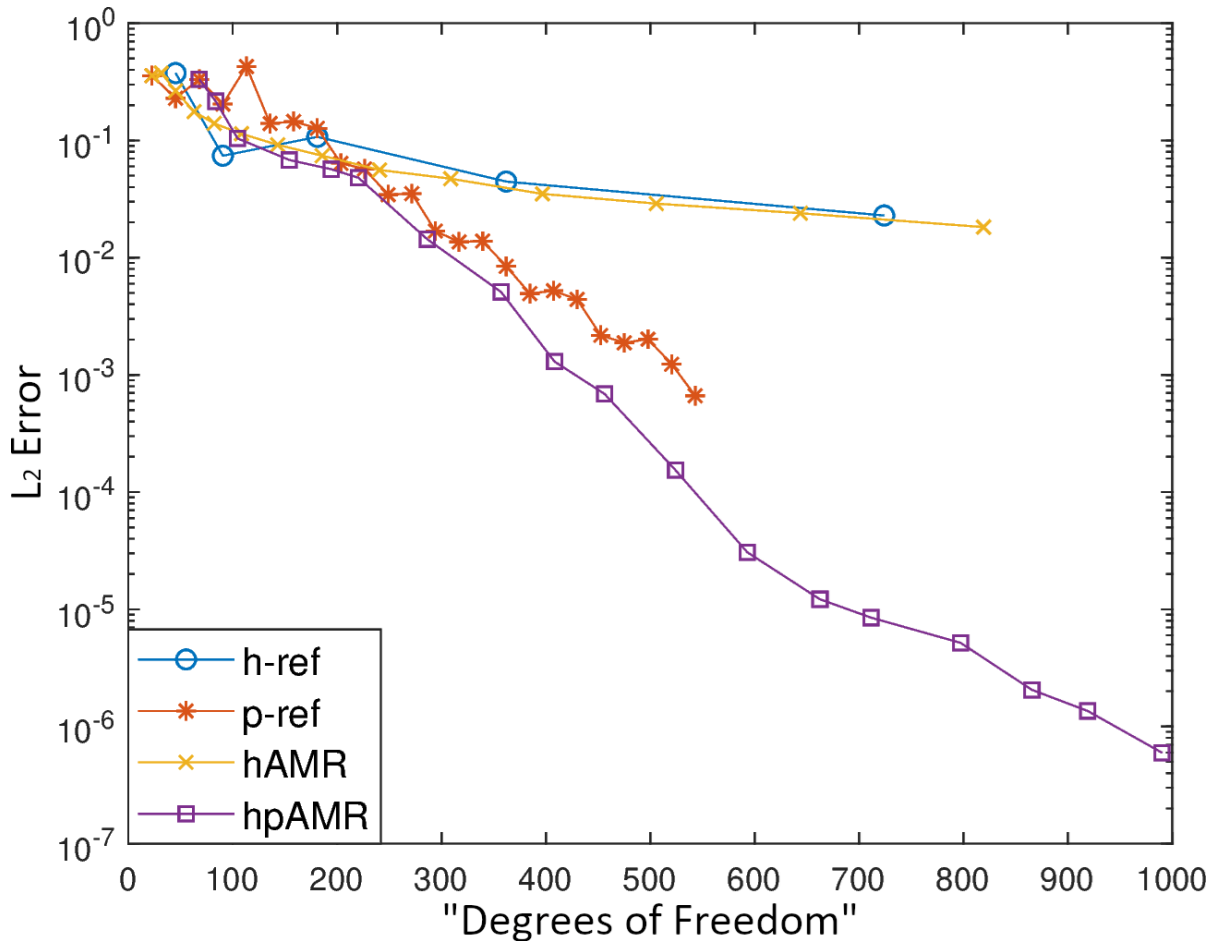# Angular *hp*-adaptivity for radiative transfer

Angular *hp*-adaptivity for radiative transfer

Angular *hp*-adaptivity for radiative transfer

Spatial *hp*-adaptivity for radiative transfer

Credit: S. Du, S. Stechmann, J. Comput. Phys., 2023

Spatio-angular *hp*-adaptivity for radiative transfer

Relatively memory-intensive and computationally-expensive

Enabled by PETSc and MPI

Results coming soon!

Acknowledgements

- Shukai Du, first author of the 'spatial *hp*-adaptivity for radiative transfer' work and advisor of this project.

- Sam Stechmann [CSGF '07], my dissertation advisor and PI of this project.

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