

# Lattice QCD Approach to Radiative Leptonic Decays

Speaker: Christopher Kane<sup>1</sup>

Advisor: Stefan Meinel<sup>1</sup>

Colaborators: Davide Giusti<sup>2</sup>, Christoph Lehner<sup>2</sup>, Amarjit Soni<sup>3</sup>

CSGF Program Review 2023

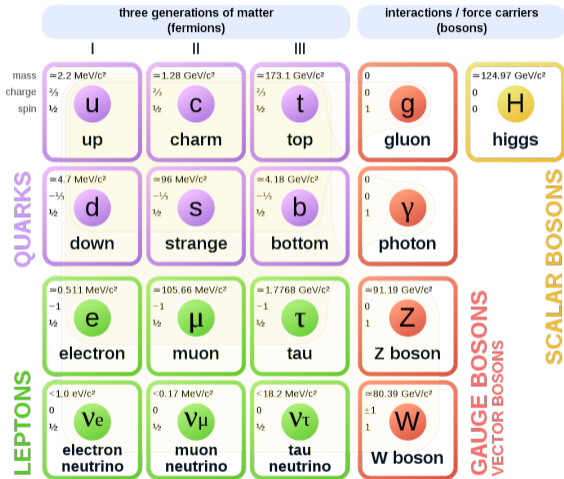
Date: July 19, 2023

<sup>1</sup>University of Arizona

<sup>2</sup>University of Regensburg, Germany

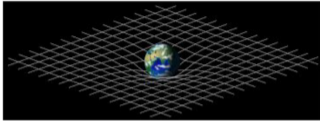
<sup>3</sup>Brookhaven National Lab

# Standard Model of Elementary Particles

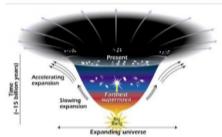


# Unsolved problems in particle physics

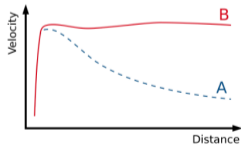
## Quantum Gravity?



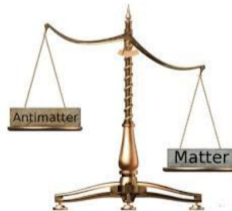
## Dark Energy



## Dark Matter



## Matter anti-matter asymmetry



# Indirect detection methods

Calculate Decay Rate Using Standard Model

**Standard Model of Elementary Particles**

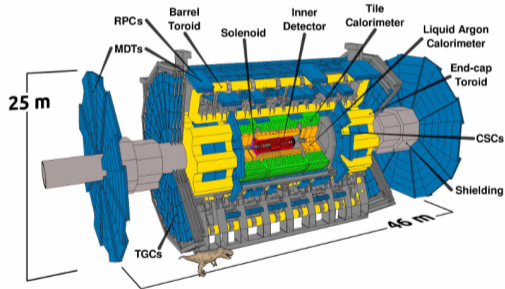
	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass	$\sim 2.2 \text{ MeV}/c^2$	$\sim 1.28 \text{ GeV}/c^2$	$\sim 173.1 \text{ GeV}/c^2$	0	$\sim 124.37 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> higgs
QUARKS	$\sim 4.7 \text{ MeV}/c^2$	$\sim 96 \text{ MeV}/c^2$	$\sim 4.18 \text{ GeV}/c^2$	0	
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
LEPTONS	$\sim 0.511 \text{ MeV}/c^2$	$\sim 106.06 \text{ MeV}/c^2$	$\sim 1.7769 \text{ GeV}/c^2$	0	
	-1	-1	-1	0	
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	
	$\sim 1.0 \text{ eV}/c^2$	$\sim 0.17 \text{ MeV}/c^2$	$\sim 18.2 \text{ MeV}/c^2$	$\sim 80.39 \text{ GeV}/c^2$	
	0	0	0	1	
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	

SCALAR BOSONS (H, Higgs)  
VECTOR BOSONS (g, photon, Z, W)

?

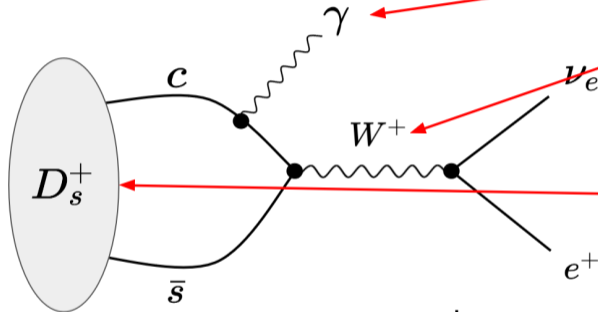
||

Measure Decay Rate experimentally



# Radiative leptonic decays

$$D_s^+ \rightarrow \gamma e^+ \nu_e$$



Electromagnetism: perturbation theory ✓

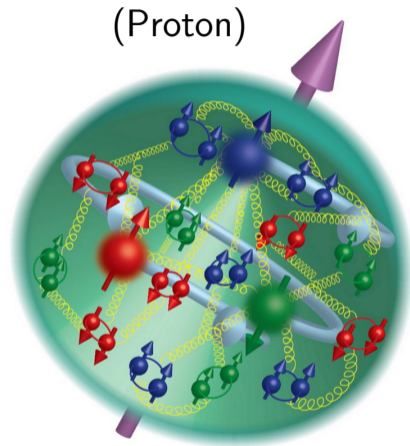
Weak Force: effective Fermi interaction ✓

Quantum Chromodynamics

$$\frac{d\Gamma}{dE_\gamma} = \underbrace{\alpha_{\text{em}} \frac{G_F^2 |V_{cs}|^2}{6\pi^2} m_{D_s} E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_{D_s}}\right)}_{\text{known}} \underbrace{(|F_V|^2 + |F_{A,SD}|^2)}_{\text{QCD}}$$

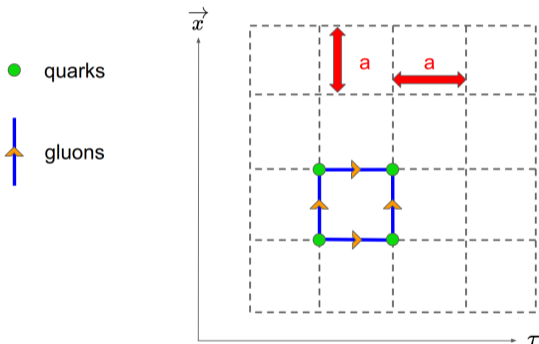
# How to calculate the QCD piece?

- Strength of QCD is larger at smaller energy scales
- Inside the  $D_s^+$  meson, quarks and gluons at low energy scales
- Perturbation theory not practical, expansion parameter is not small
- Need a non-perturbative method



# Intro to lattice QCD

Goal: Numerically solve the QCD path integral



- Discretize space and time on a finite lattice
- Perform integral over quarks analytically
- Perform Wick rotation to imaginary time  
→ replace  $t = -i\tau$  with  $\tau \in \mathbb{R}$

# Intro to lattice QCD

Re-writing the path integral (schematically):

$$\langle \text{Object to calculate} \rangle = \langle f(M^{-1}[U]) \rangle = \frac{\int \mathcal{D}[U] \rho[U] f(M^{-1}[U])}{\int \mathcal{D}[U] \rho[U]}$$

- Probability density  $\sim \rho[U] = \det(M[U]) e^{-S_G^E[U]}$
- Solve the integral over gluon fields  $U$  using Monte Carlo methods
- Calculate propagators  $M^{-1}[U]$ , plug into  $f(M^{-1}[U])$

What is the typical size of matrix  $M$ ?

$$\underbrace{32^3}_{\text{sites in } x,y,z \text{ directions}} \times \underbrace{64}_{\text{sites in } \tau \text{ direction}} \times \underbrace{3}_{\text{quark colors}} \times \underbrace{4}_{\text{Dirac fermion indices}} \approx 25 \times 10^6 \text{ rows/columns}$$



# High performance computing centers utilized

## Stampede2 Supercomputer



University of Texas

## Supermuc-NG

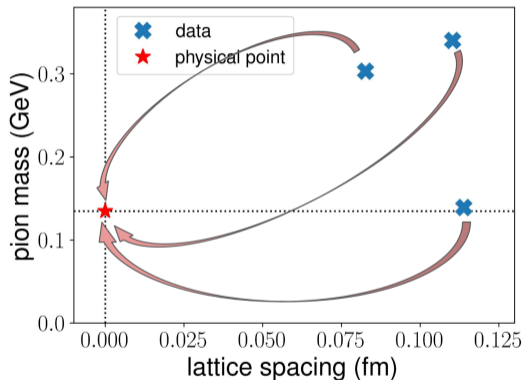


Leibniz Supercomputing Centre, Germany


# Lattice parameters

Outline of the calculation:

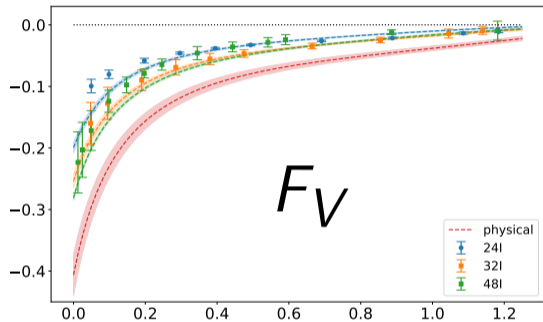
- 1 Perform calculation on multiple lattices
  - Different lattice spacing  $a$
  - Different pion mass  $m_\pi$  (quark masses)
- 2 Extrapolate to physical result
  - lattice spacing  $a \rightarrow 0$
  - pion mass  $m_\pi \rightarrow m_{\pi,\text{physical}}$



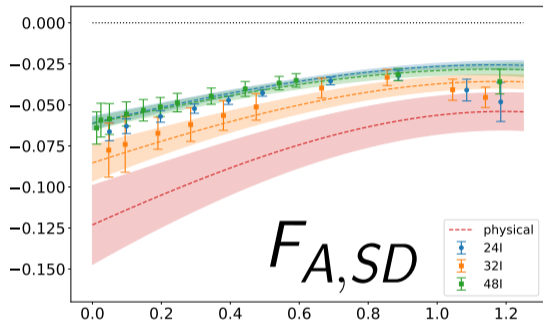
# DISCLAIMER

<p><b>PARENTS STRONGLY CAUTIONED</b></p>	<p>For showing plots that might still contain systematic errors</p>	
<p><b>PG-13</b></p>		
<p>SOME MATERIAL MAY BE INAPPROPRIATE FOR CHILDREN UNDER 13</p>		

# Lattice QCD results for $F_V$ and $F_{A,SD}$



$$x_\gamma = 2E_\gamma/m_{D_s}$$



$$x_\gamma = 2E_\gamma/m_{D_s}$$

Physical values:  $0 < x_\gamma \leq 1$

## Branching fraction prediction (preliminary)

- Branching fraction is fraction of  $D_s^+$  events that decay into a particular final state

$$\mathcal{B}(D_s^+ \rightarrow \gamma e^+ \nu_e) = \text{Fraction of } D_s^+ \text{ that decay into } \gamma e^+ \nu_e$$

$$\text{Our prediction: } \mathcal{B}(D_s^+ \rightarrow \gamma e^+ \nu_e) = 2.8(4) \times 10^{-6}$$

$$\text{Experimental upper bound: } \mathcal{B}(D_s^+ \rightarrow \gamma e^+ \nu_e) < 1.3 \times 10^{-4}$$

- Upcoming experiments can improve the upper bound and possibly quote results

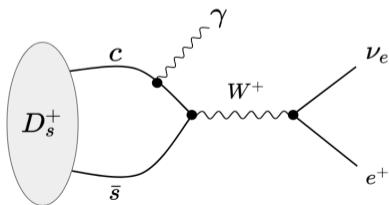
## Quick digression: practicums

- Two practicums at Lawrence Berkeley National Lab
- Working on methods for quantum computer simulations of high energy physics
- The CSGF has changed the trajectory of my career in a hugely positive way

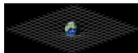
Thank you to everyone at the CSGF and Krell who has helped me along the way :)

# Summary

- Standard model is incomplete
- Generally, calculations of bound states of quarks, i.e. protons,  $D_s$  meson, often require non-perturbative lattice QCD
- Radiative leptonic decays are an interesting process, and we are calculating the decay rate



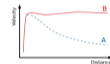
Quantum Gravity?



Dark Energy



Dark Matter



Matter anti-matter asymmetry



• quarks

• gluons

