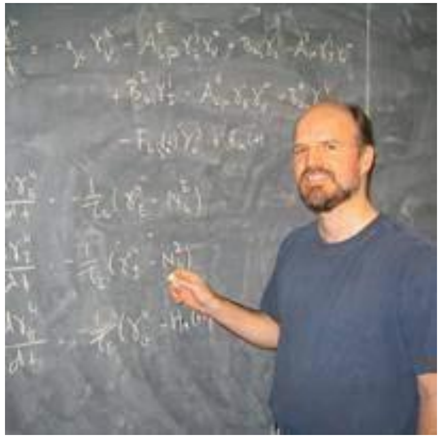
USC University of Southern California

A variational approach to quantum tomography

Nic Ezzell et. al. (next slides)

19 July 2023

The LANL practicum team acknowledgement



Andrew T. Sornborger
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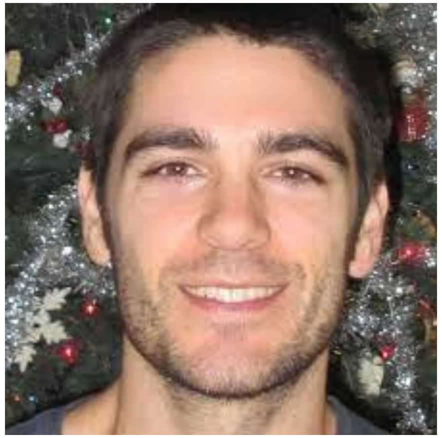
Zoë Holmes
LANL, EPFL



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Aliza U. Siddiqui
Louisiana State (LSU)

Mark M. Wilde
LSU, Cornell

Joe Gibbs
Surrey

Robert Huang
Caltech

Matthias C. Caro
Caltech

Works we will discuss

	State tomography papers	Process tomography papers
Theory	**The quantum low-rank approximation problem** N Ezzell, Z Holmes, PJ Coles arXiv preprint arXiv:2203.00811	Article | Open Access | Published: 05 July 2023 **Out-of-distribution generalization for learning quantum dynamics** Matthias C. Caro, Hsin-Yuan Huang, Nicholas Ezzell, Joe Gibbs, Andrew T. Sornborger, Lukasz Cincio, Patrick J. Coles & Zoë Holmes *Nature Communications* **14**, Article number: 3751 (2023) | Cite this article
Experiment	PAPER • OPEN ACCESS **Quantum mixed state compiling** Nic Ezzell[10,1,2], Elliott M Ball[3], Aliza U Siddiqui[4,5], Mark M Wilde[4,6], Andrew T Sornborger[2,7], Patrick J Coles[7,8] and Zoë Holmes[2,9] Published 4 April 2023 • © 2023 The Author(s). Published by IOP Publishing Ltd Quantum Science and Technology, Volume 8, Number 3 **Citation** Nic Ezzell *et al* 2023 *Quantum Sci. Technol.* **8** 035001 **DOI** 10.1088/2058-9565/acc4e3	**Dynamical simulation via quantum machine learning with provable generalization** J Gibbs, Z Holmes, MC Caro, N Ezzell, HY Huang, L Cincio, ... arXiv preprint arXiv:2204.10269

Overview

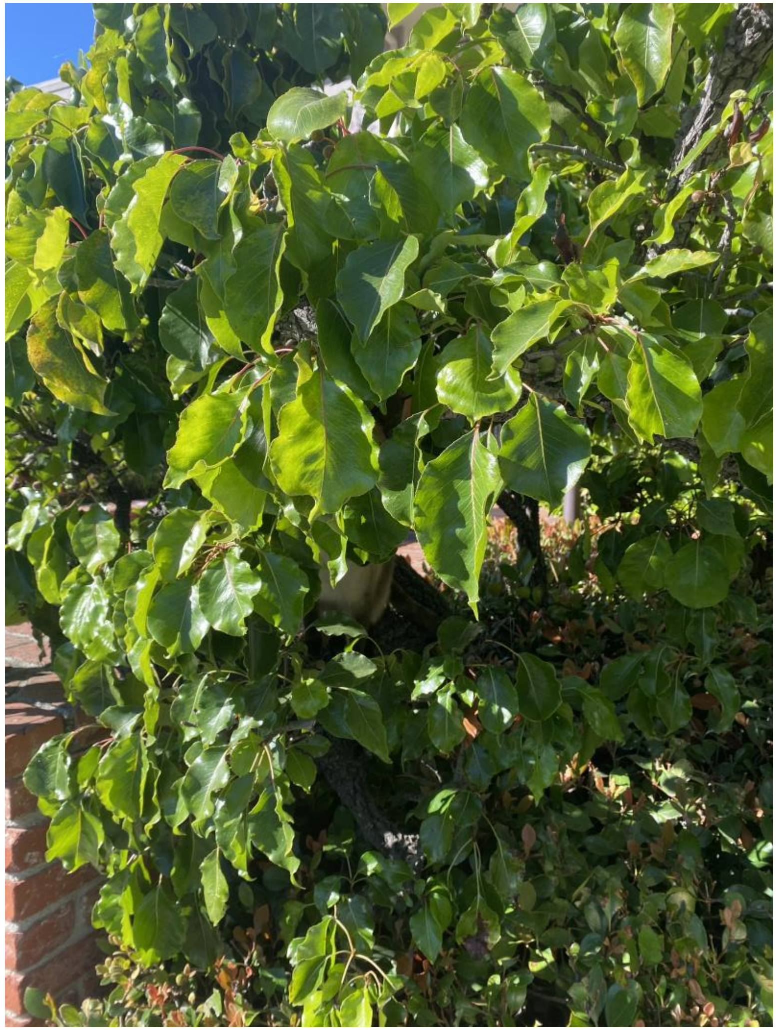
- Define “quantum tomography” and “variational” informally
- Define them formally
- Explain why we take a variational approach
- Show results
- Comment on the details
- Conclude

Informal definitions

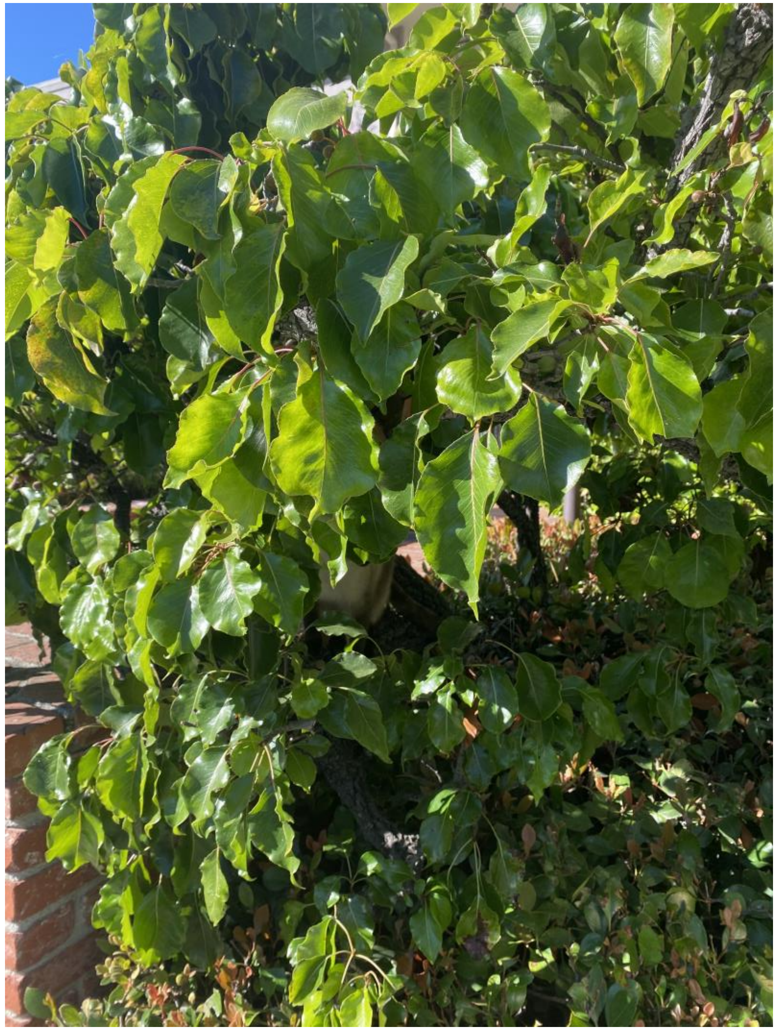
Tomography by etymology

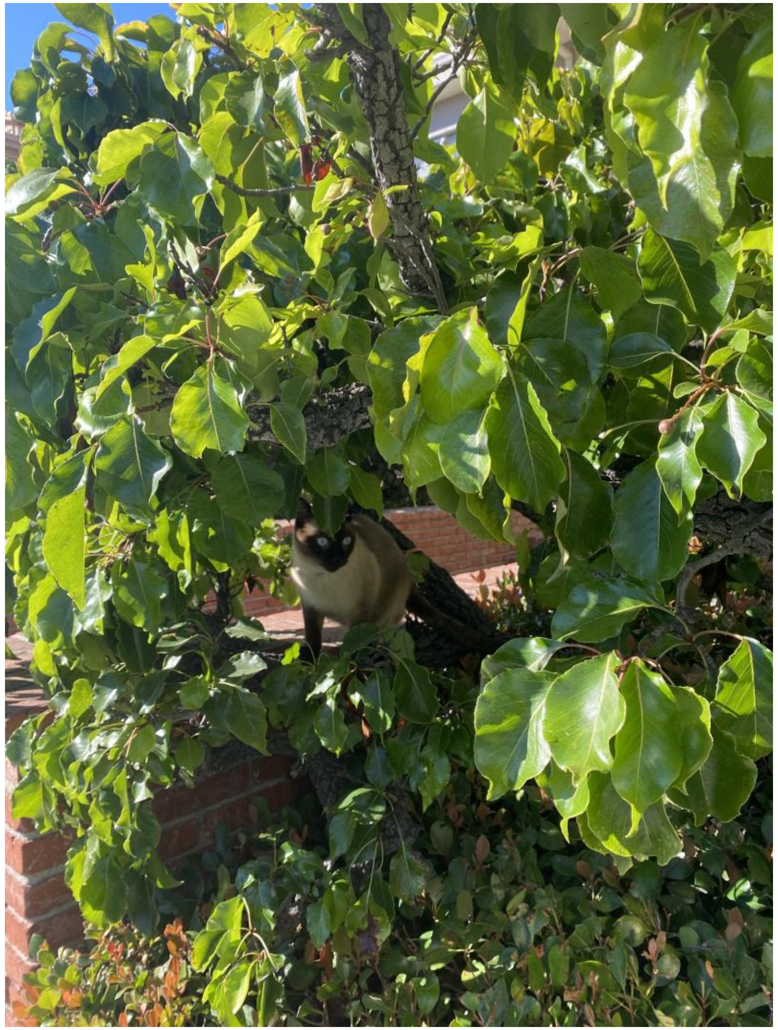
- tomos – slice, section (Greek, τόμος)
- *graphō* – to write, describe (Greek, γράφω)

Tomography by example



Tomography by example





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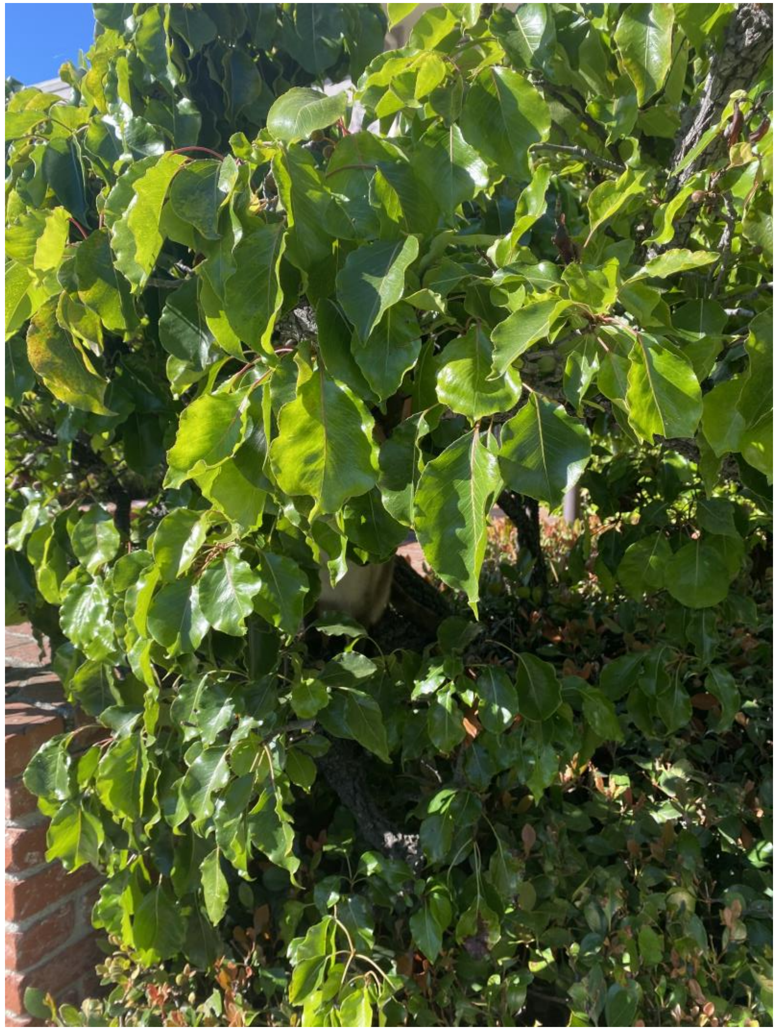
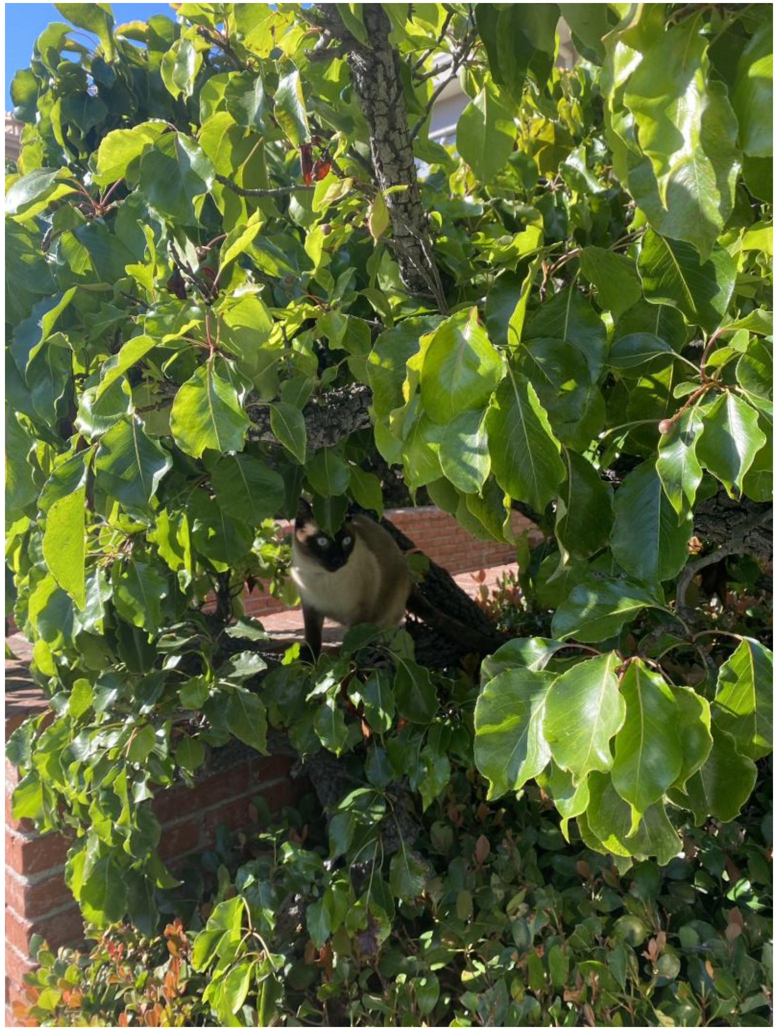
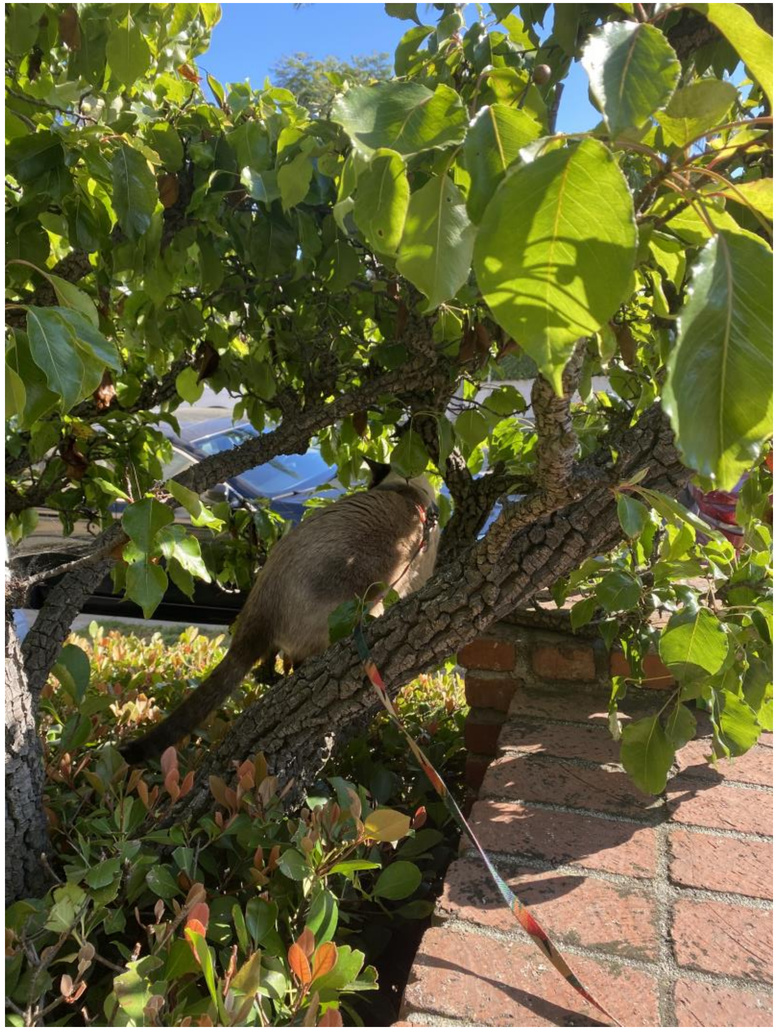

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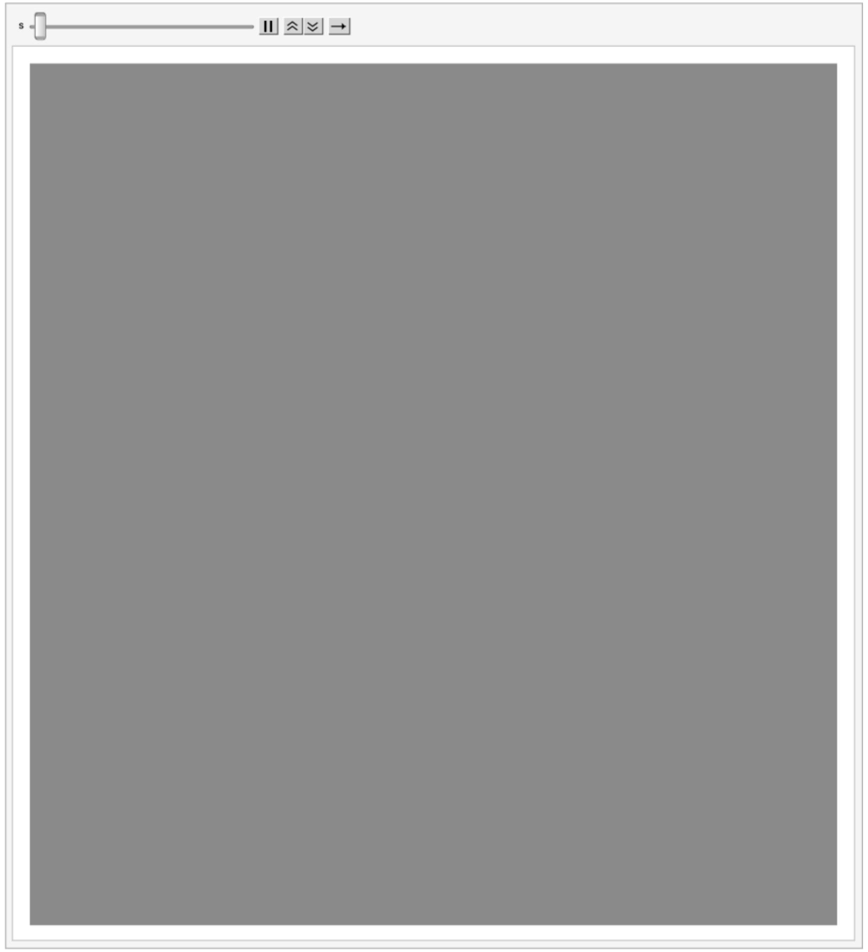
Variational approach to identity

Variational approach to identity

- Goal: Have an image of Junebug →

Junie presiding over his kingdom

The variational algorithm in action



- Ansatz: 718 x 766 gray rectangle
- Cost: pixel difference
- Update rule: interpolation (cheating)
- Stopping criteria: cuteness

And now, a formal introduction

Quantum state tomography with a single qubit

A single qubit state can be represented by a 2×2 matrix with three parameters,

$$\rho(v_x, v_y, v_z) = \frac{1}{2}\begin{pmatrix} 1 + v_z & v_x - i v_y \\ v_x + i v_y & 1 - v_z \end{pmatrix}$$

that satisfy $||\vec{v}|| \leq 1$ (i.e. $v_x^2 + v_y^2 + v_z^2 \leq 1$)

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Visualization on Bloch sphere

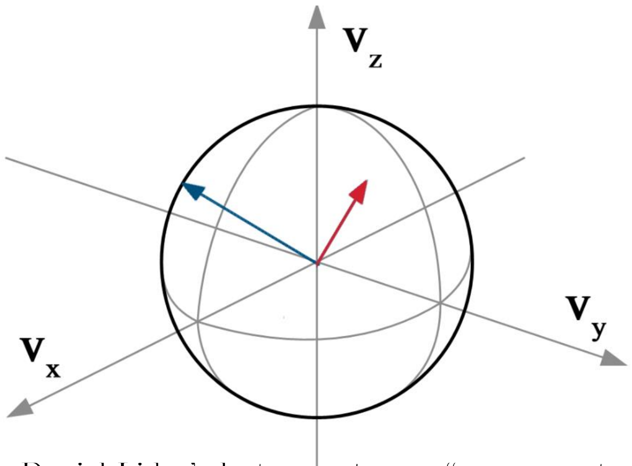

FIG 1 in Daniel Lidar's lecture notes on "open quantum systems"

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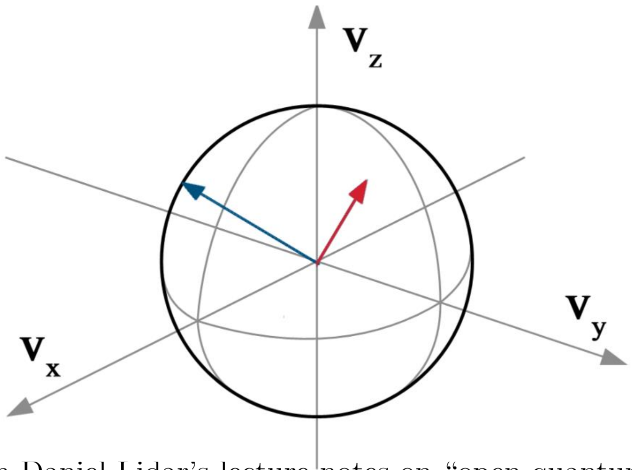
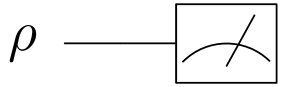


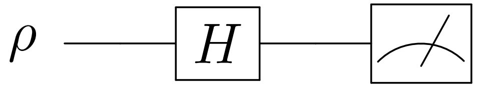
FIG 1 in Daniel Lidar's lecture notes on "open quantum systems"

Quantum state tomography estimates $(\hat{v}_x, \hat{v}_y, \hat{v}_z)$

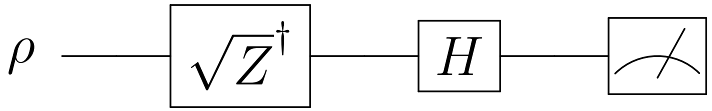
Is v_z positve?



Is v_x positve?



Is v_y positve?



Quantum state tomography scaling

An n qubit state has dimension $d = 2^n$ and is defined by $d^2 - 1$ values of v_j

$$\rho = \frac{I}{2^n} + \sum_{i=1}^{4^n - 1} v_j F_j \tag{1}$$

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and downstream manipulations such as computing expectation values, $\langle A \rangle = \mathrm{Tr}[A\rho]$ require $O(d^{2.37})$ time

The variational approach: A new identity for quantum states

- Let us instead define a quantum state constructively

The variational approach: A new identity for quantum states

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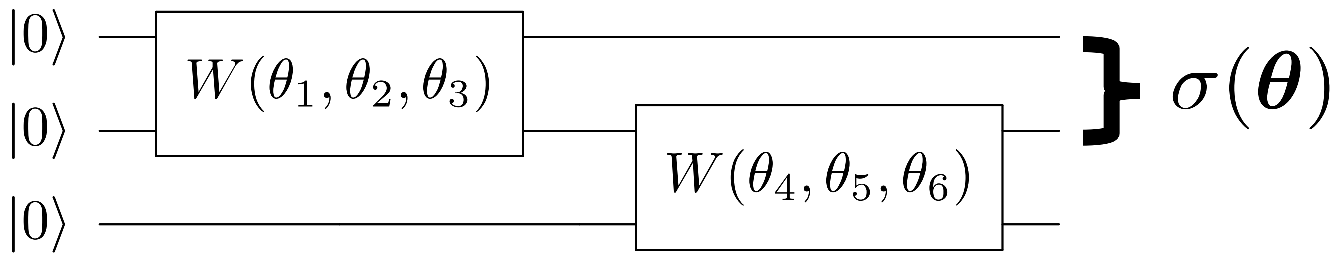
- A quantum state is defined by a (low-depth) quantum circuit which we can varitionally learn



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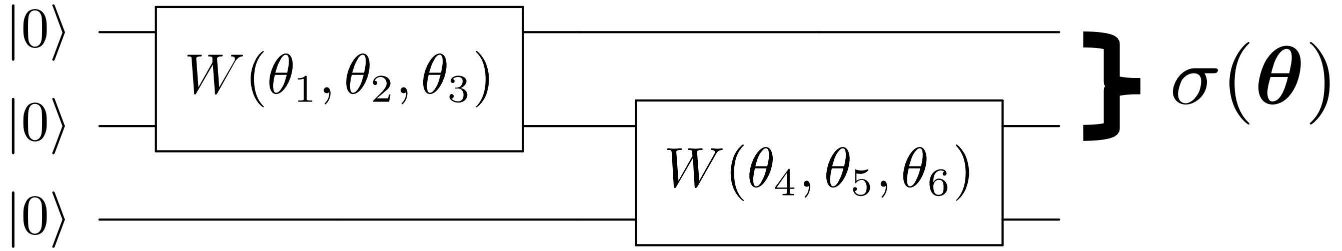
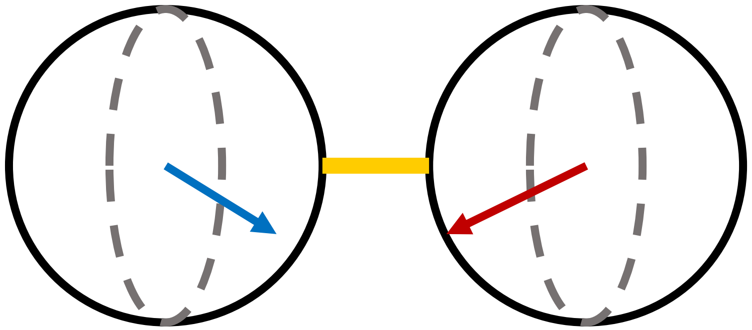


- By construction, low-depth quantum circuits are efficient to store classically and efficient to prepare quantumly

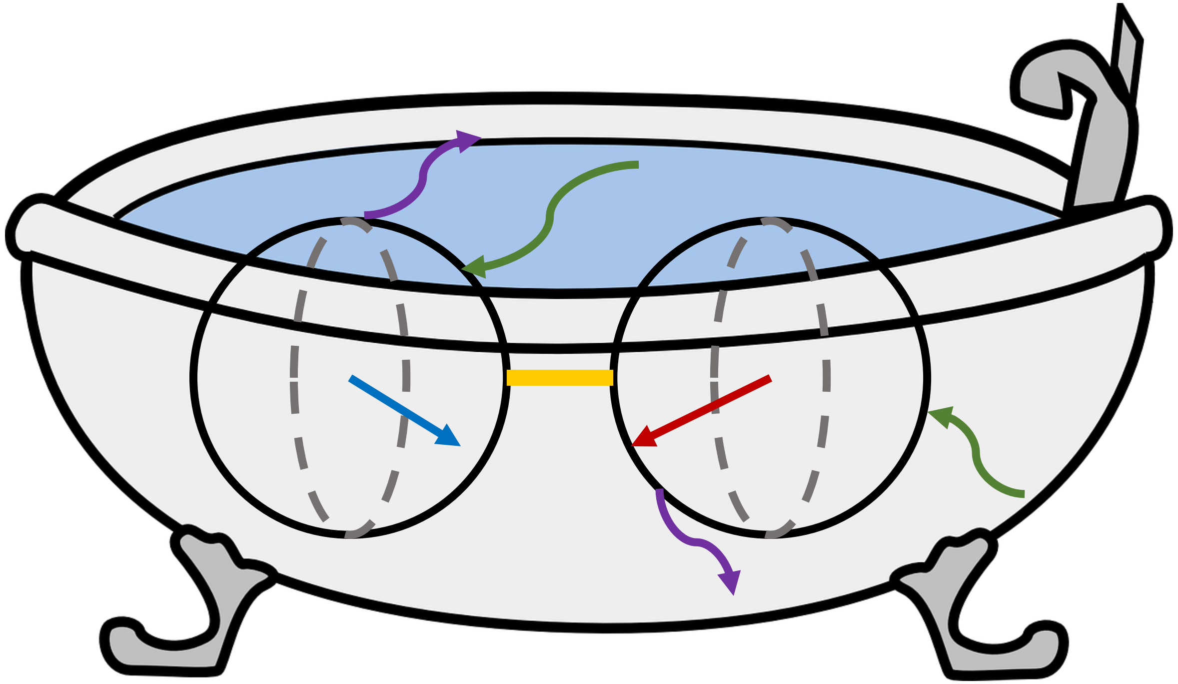
Variational quantum state tomography example

A two qubit system



Variational quantum state tomography example

A two qubit system in thermal equilibrium with a bath



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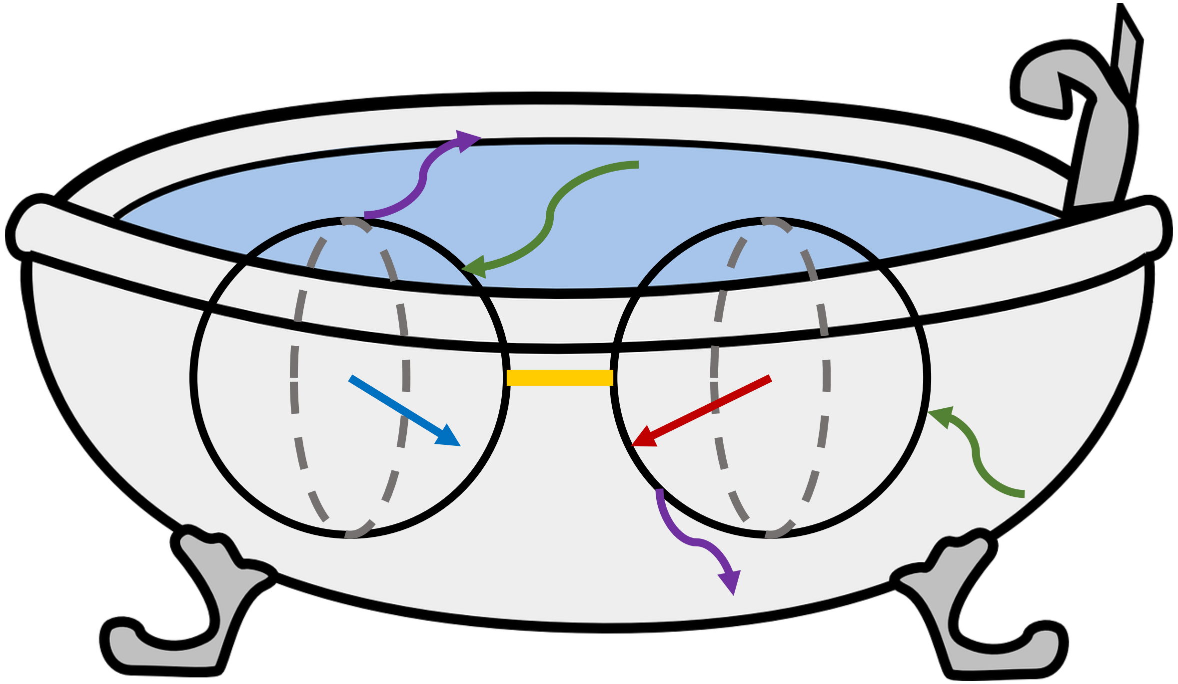

Abstract Gibbs representation

$$\rho = \frac{e^{-\beta H}}{\mathrm{Tr}\left[e^{-\beta H}\right]}$$

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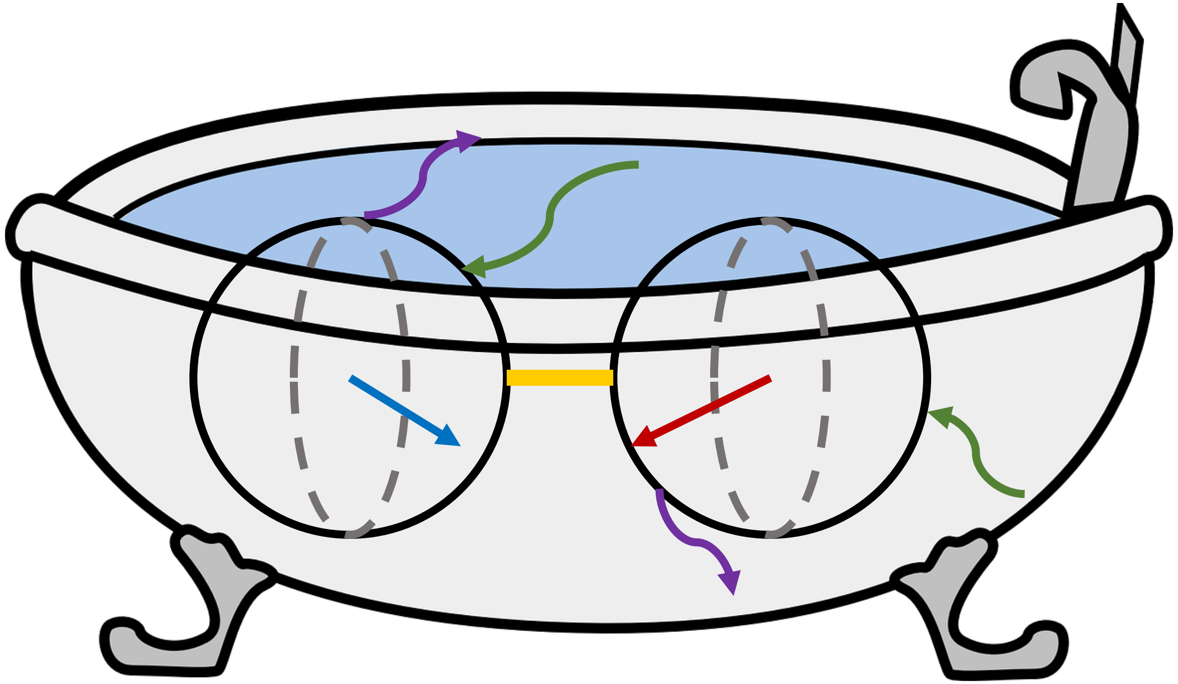


Abstract Gibbs representation

$$\rho = \frac{e^{-\beta H}}{\mathrm{Tr}\left[e^{-\beta H}\right]}$$

Matrix representation

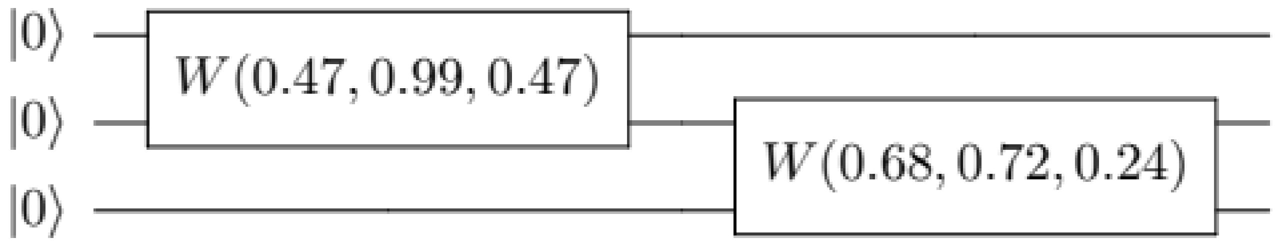
$$\begin{pmatrix} 0.031 & 0 & 0 & -0.031 \\ 0 & 0.469 & -0.469 & 0 \\ 0 & -0.469 & 0.469 & 0 \\ -0.031 & 0 & 0 & 0.031 \end{pmatrix}$$

Variational quantum state tomography example

Matrix representation

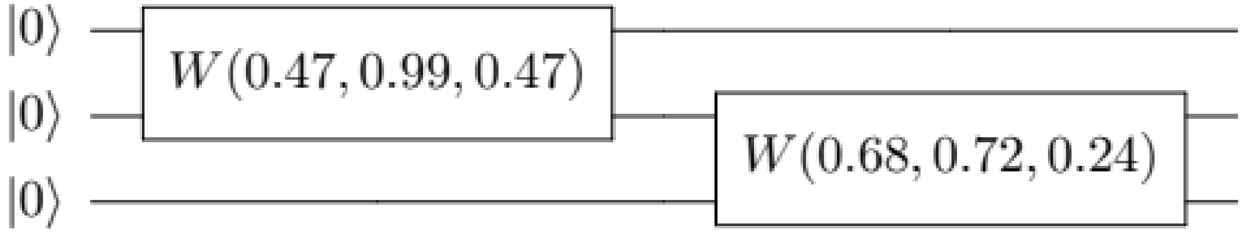
$$\begin{pmatrix} 0.031 & 0 & 0 & -0.031 \\ 0 & 0.469 & -0.469 & 0 \\ 0 & -0.469 & 0.469 & 0 \\ -0.031 & 0 & 0 & 0.031 \end{pmatrix}$$

Variational quantum circuit representation (ansatz)



$$\begin{pmatrix} 0.058 & 0. & 0. & -0.226 \\ 0. & 0.007 & 0.02 & 0. \\ 0. & 0.02 & 0.053 & 0. \\ -0.226 & 0. & 0. & 0.882 \end{pmatrix}$$

Variational quantum state tomography example



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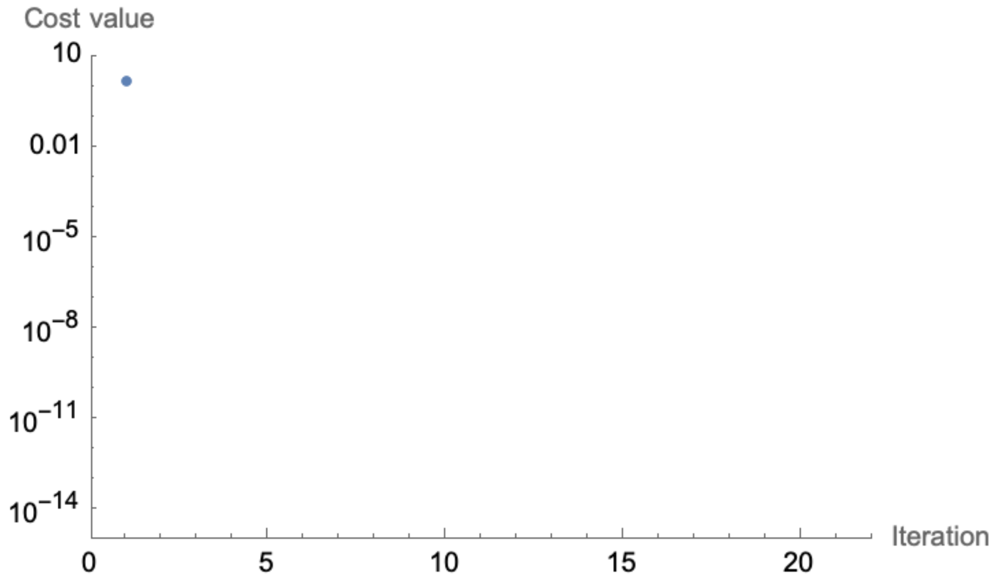


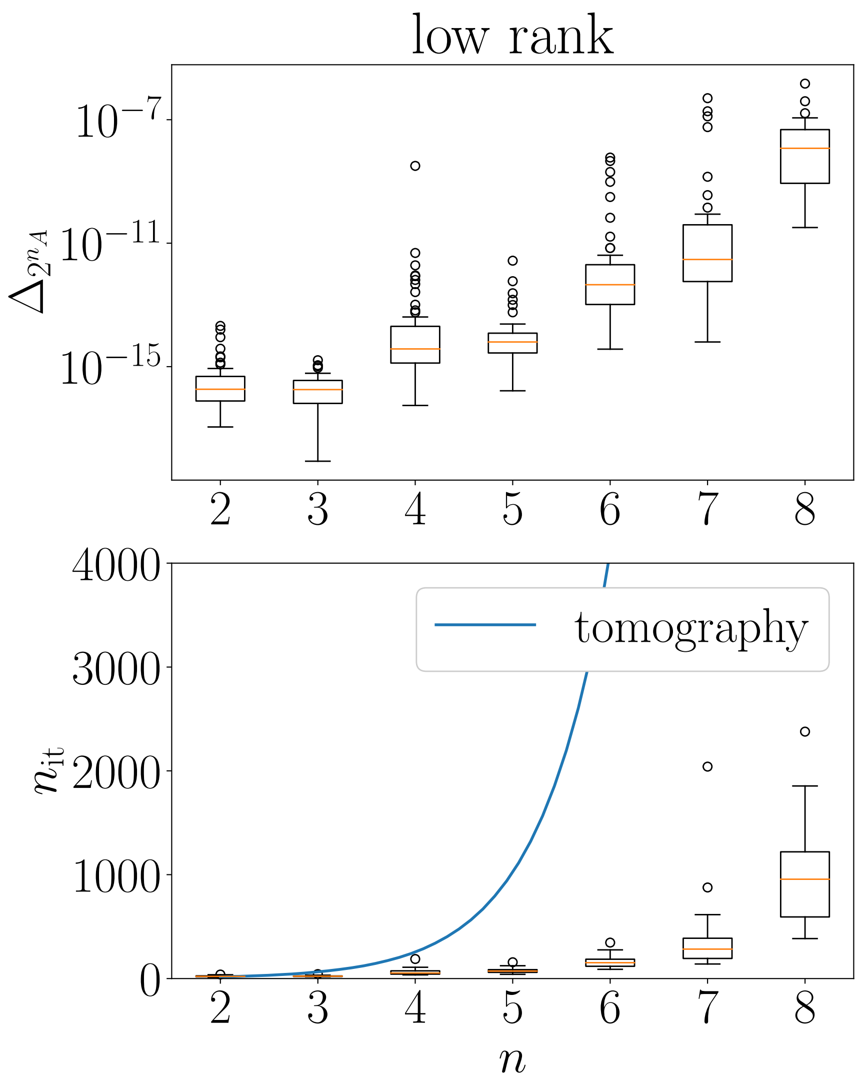
A couple algorithm results

A snapshot of numerical results

$$\rho_n^{(\mathrm{XY})} \equiv \frac{e^{-\beta H_{\mathrm{XY}}}}{\mathrm{Tr}[e^{-\beta H_{\mathrm{XY}}}]},$$

$$H_{\mathrm{XY}} \equiv \sum_{i=1}^{n-1} J_i X_i X_{i+1} + K_i Y_i Y_{i+1},$$

$$J_i, K_i \sim \mathcal{N}(0,1)$$



Ezzell N 2022 naezzell/qmsc: qmsc arXiv v1.0.1 release (Zenodo) (https://doi.org/10.5281

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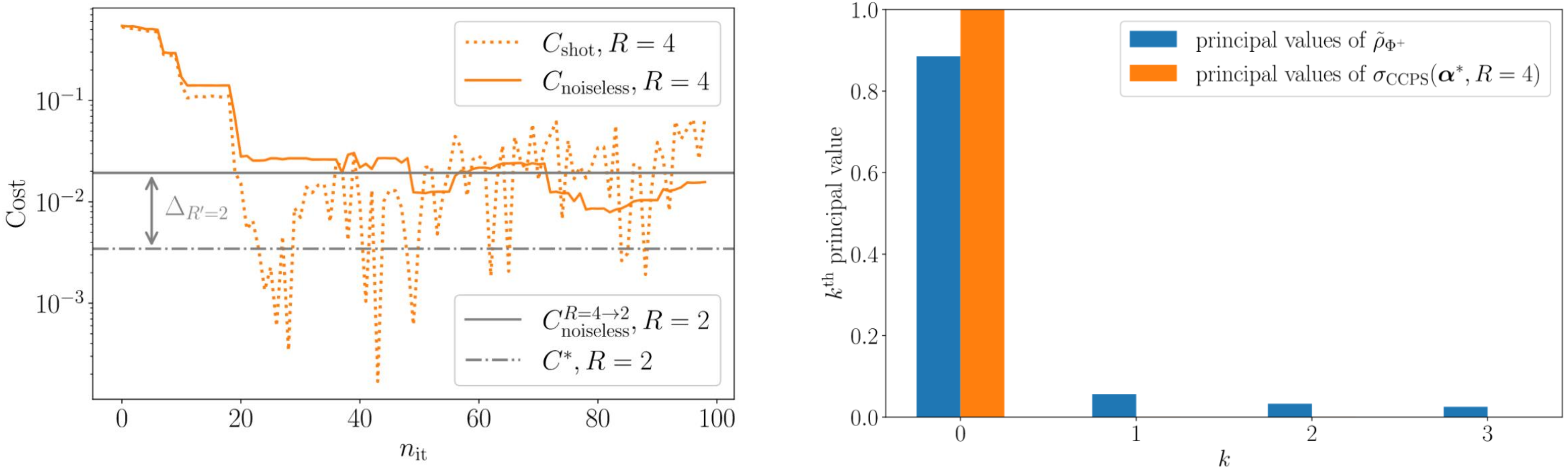
A snapshot of quantum hardware results



A few comments on theory

The quantum low-rank approximation problem

Given $\rho = \sum_{i=1}^{r} \lambda_i \, |\lambda_i\rangle\langle\lambda_i| \quad \lambda_1 > \lambda_2 > \ldots > \lambda_r > 0$, our task is to solve

$$\sigma^*(R) \equiv \underset{\sigma \geq 0, \mathrm{Tr}[\sigma]=1, \mathrm{rank}(\sigma) \leq R}{\mathrm{argmin}} ||\rho - \sigma||_p \tag{1}$$

Solution: The state

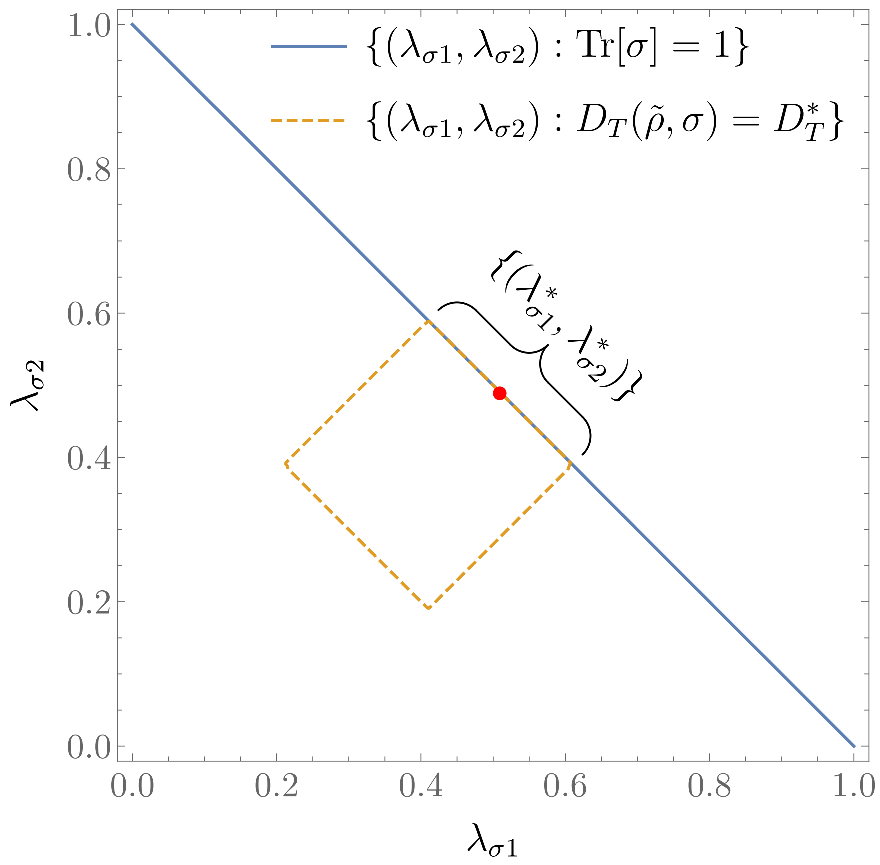
$$\sigma^*(R) = \tau_R + \frac{1 - \mathrm{Tr}[\tau_R]}{R} \Pi_R \tag{2}$$

$$= \Pi_R \rho \Pi_R + \frac{1 - \mathrm{Tr}[\Pi_R \rho]}{R} \Pi_R \tag{3}$$

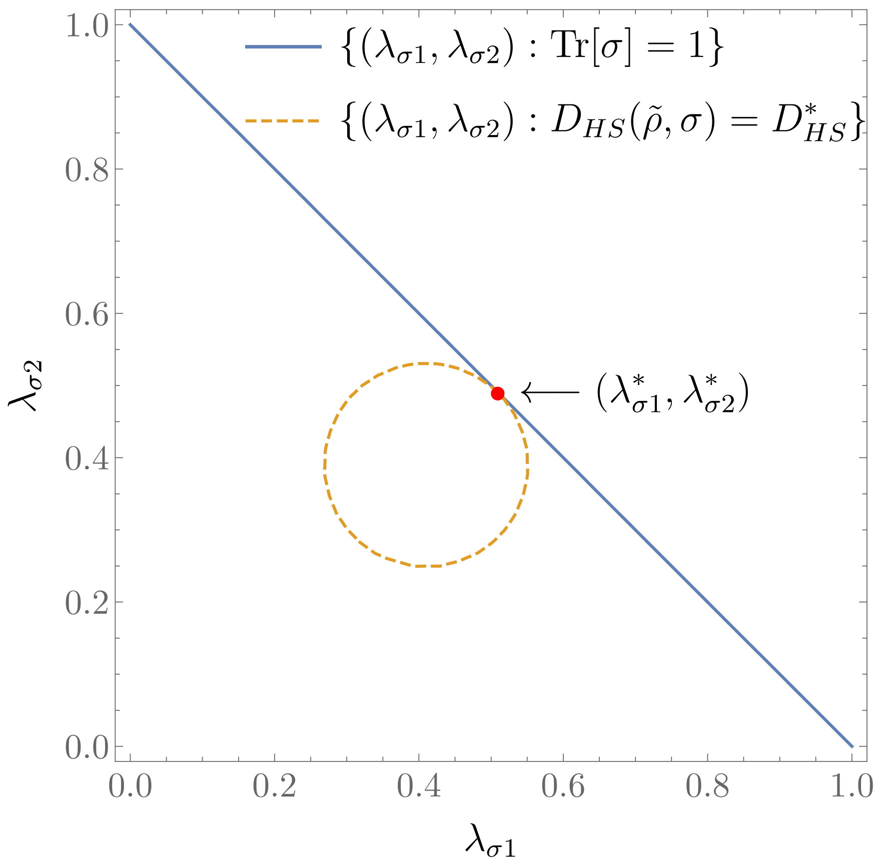
is the unique solution for $p > 1$ and one of many highly degenerate solutions for $p = 1$.

The quantum low-rank approximation problem

Trace distance, not unique



Hilbert-Schmidt distance, unique



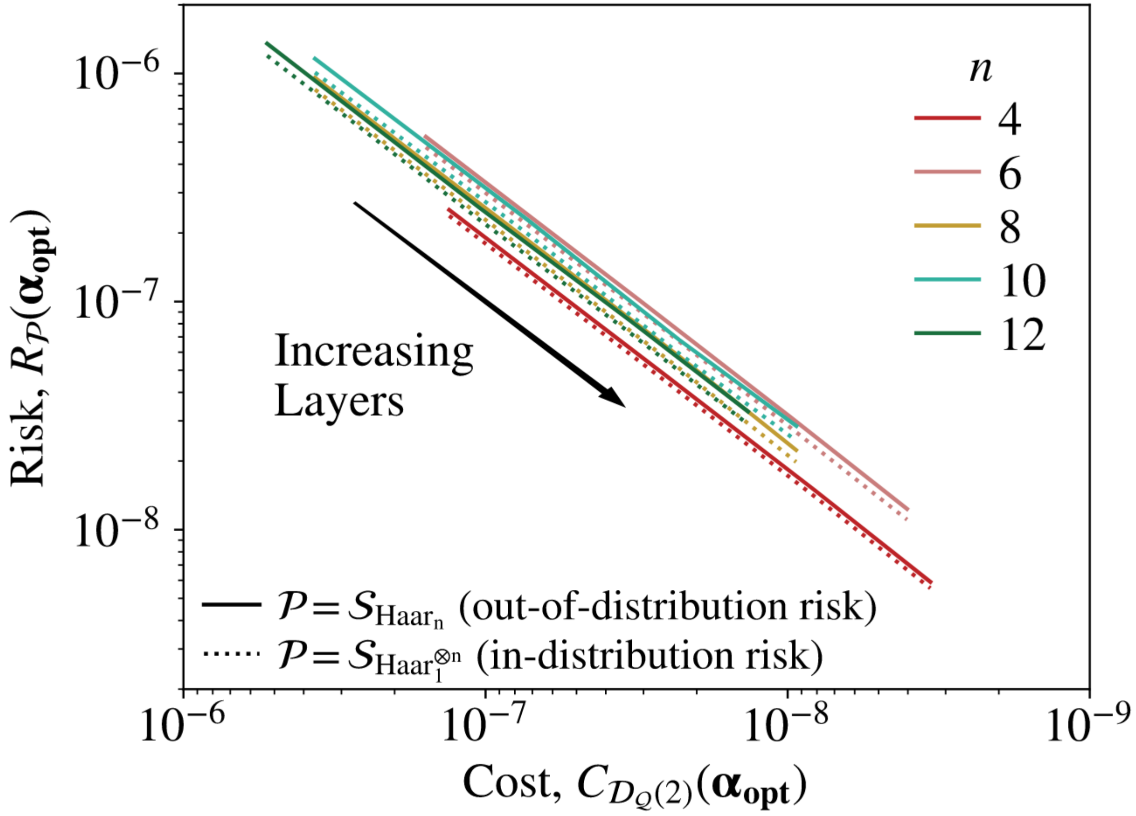
Some comments on the cost function

We use Hilbert-Schmidt distnce $C(\boldsymbol{\theta}) = ||\rho - \sigma(\boldsymbol{\theta})||_2^2$ as the cost function

- We prove it is classically hard to evaluate (under conjectures)
- We show it is quantumly easy
- Has a strong relationship to trace distance for low-rank states (avoids usual pitfall of HS distance)

A comment on sample complexity of learning unitaries

In the unitary learning paper, we show that we can learn train a variational gate $W(\theta_1, \ldots, \theta_k)$ to implement an unknown unitary U using only poly(k) product states!!!



Questions? (references on slide… for reference)

[1]
N. Ezzell *et al.*, "Quantum mixed state compiling," *Quantum Sci. Technol.*, vol. 8, no. 3, p. 035001, Apr. 2023, doi: 10.1088/2058-9565/acc4e3.

[2]
M. C. Caro *et al.*, "Out-of-distribution generalization for learning quantum dynamics," *Nat Commun*, vol. 14, no. 1, p. 3751, Jul. 2023, doi: 10.1038/s41467-023-39381-w.

[3]
N. Ezzell, Z. Holmes, and P. J. Coles, "The quantum low-rank approximation problem." arXiv, Mar. 31, 2022. doi: 10.48550/arXiv.2203.00811.

[4]
J. Gibbs *et al.*, "Dynamical simulation via quantum machine learning with provable generalization." arXiv, Sep. 06, 2022. doi: 10.48550/arXiv.2204.10269.