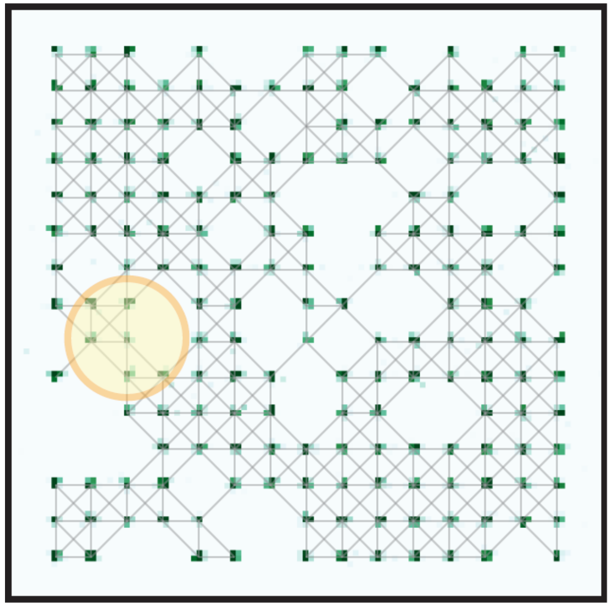
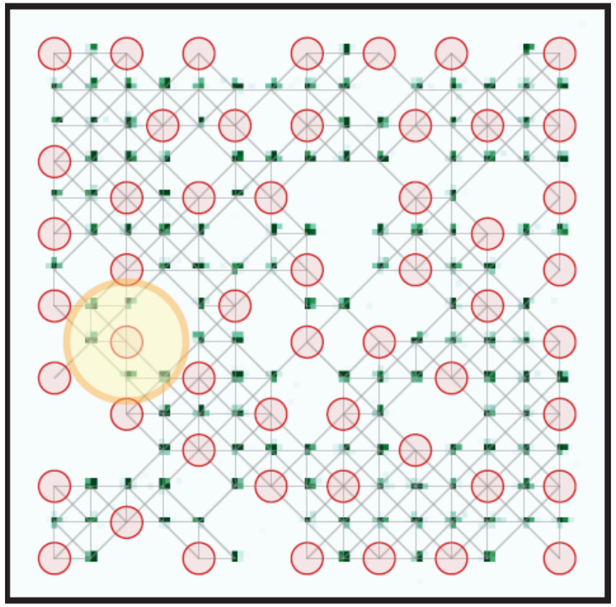
Quantum speedup for combinatorial optimization with flat energy landscapes



Madelyn Cain, Lukin group

Harvard University

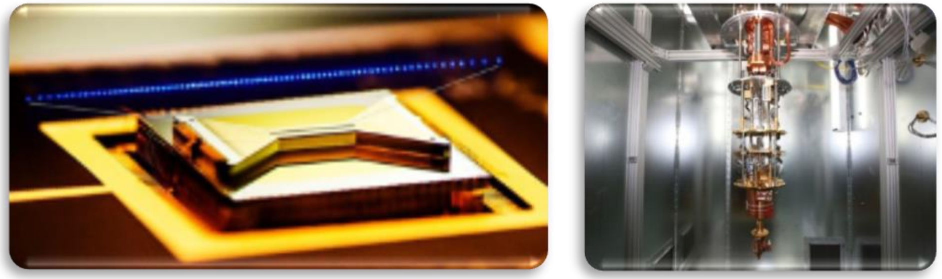
DOE CSGF Program Review



Quantum computing with microscopic systems

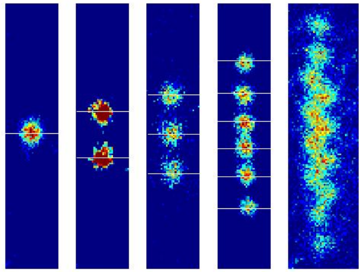
Quantum computing is a type of computation that harnesses the unique properties of quantum mechanics at microscopic scales to perform calculations.

Current challenge is to develop platforms that can control and manipulate many quantum bits (*qubits*)



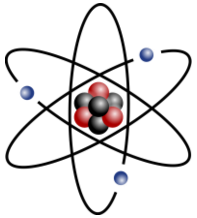
Superconducting qubits (Google, IBM, Rigetti, MIT, Caltech, ETH Zurich, Princeton, Yale, …)

Photonic qubits (USTC, PsiQuantum, …)

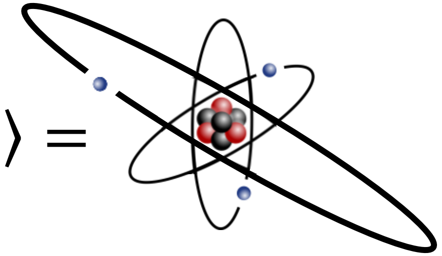


Ions (Maryland, IonQ, Innsbruk, Quantinuum, Honeywell …)

Today: neutral atom quantum computers

$|0\rangle =$ 

electronic ground state

$|1\rangle =$ 

electronic excited *Rydberg* state

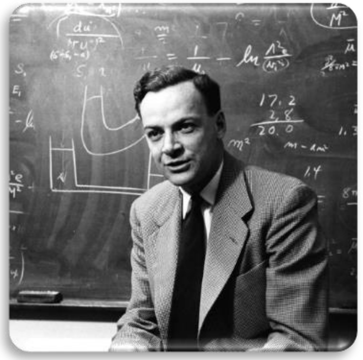
Two electronic states in Rubidium atom form a qubit

1. **Superposition:** atom simultaneously in both electronic states
2. **Entanglement:** non-classical correlations between qubits enabled by superposition

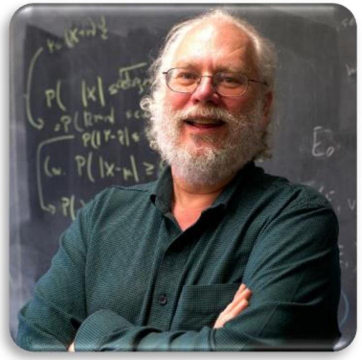
Operations performed with external laser excitation

Harnessing quantum mechanics for computational speedup

Many quantum algorithms yield a **quantum speedup** over conventional “classical” computers

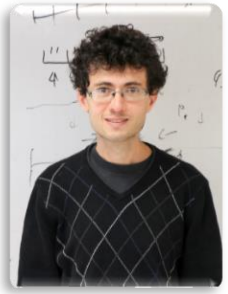


Quantum simulation (1980s)



Factoring/Fourier transform (1994)

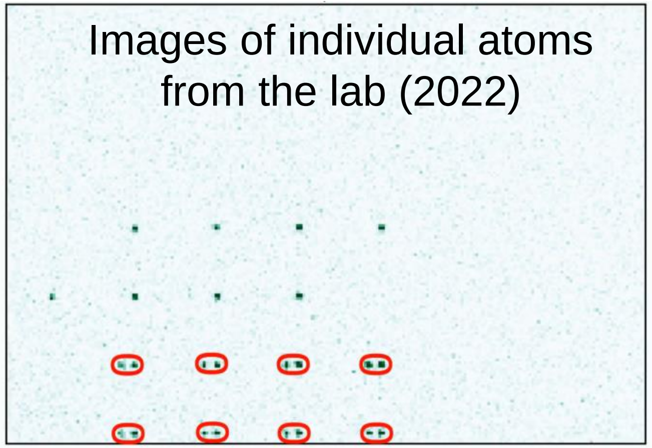
Database search (1996)



Solving linear systems of equations (2008)

But, running these algorithms requires fine control over microscopic quantum systems.

Substantial progress in quantum hardware over the past few years



Universal, low error operations

Evered, Bluvstein, Kalinowski et al. (2023) arXiv:2304.05420

All-to-all connectivity between 100s of qubits

Bluvstein et al. (2022) *Nature* 604 (7906), 451-456

Current devices cannot yet run large-scale quantum algorithms.

Can we still observe quantum speedup in the near-term?

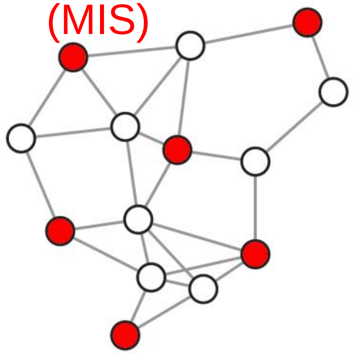
See also Wisconsin, LKB, Princeton, Boulder, Caltech groups

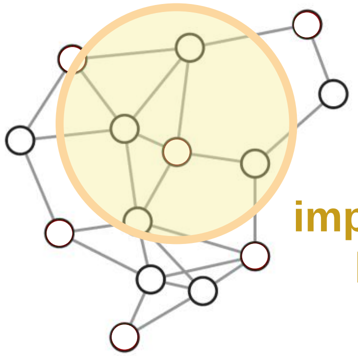
Hardware-efficient optimization of Maximum Independent Set (MIS)

Key idea: focus on algorithms with a natural, hardware-efficient implementation

Maximum Independent Set: representative
combinatorial optimization problem

Maximum independent set (MIS)





Naturally implemented in hardware

Maximize: size of set of vertices
Independent set constraint: no vertices in set are connected by an edge

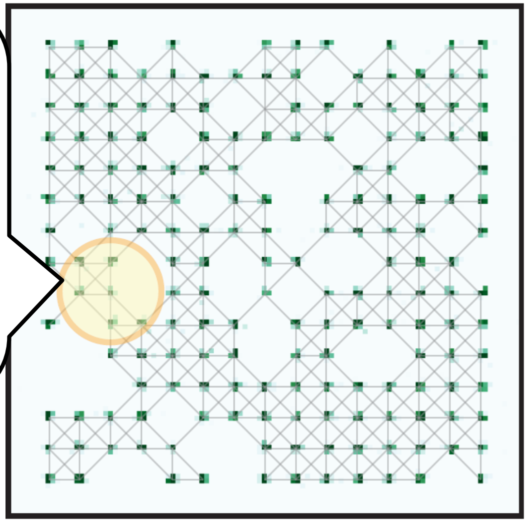
Unit disk graphs: edge between vertices within a unit radius

Arrange up to 289 Rb atoms (qubits) deterministically in 2D
Each qubit represents a vertex in the graph

Excited Rydberg state: in independent set

Ground state: out of independent set

Rydberg interaction enforces independent set constraint

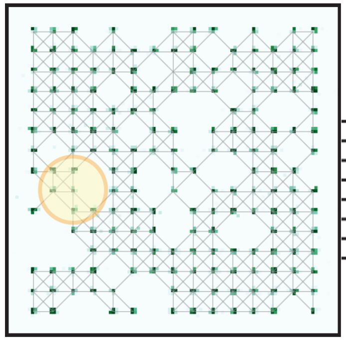


Prepare system ground state via slow (adiabatic) evolution: maximize vertices in independent set

Ebadi*, Keesling*, Cain*, et al., *Science* 376, 6598 (2022)

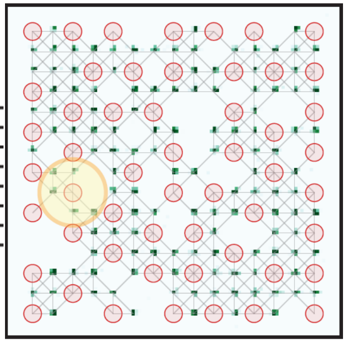
Exploring quantum performance

1. Arrange atoms to chosen graph

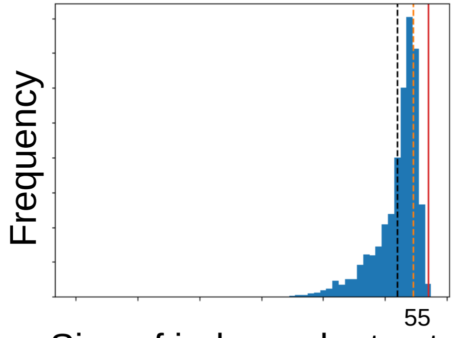


2. Optimized quantum evolution

3. Prepare large independent set



Repeat to compute probability of finding MIS

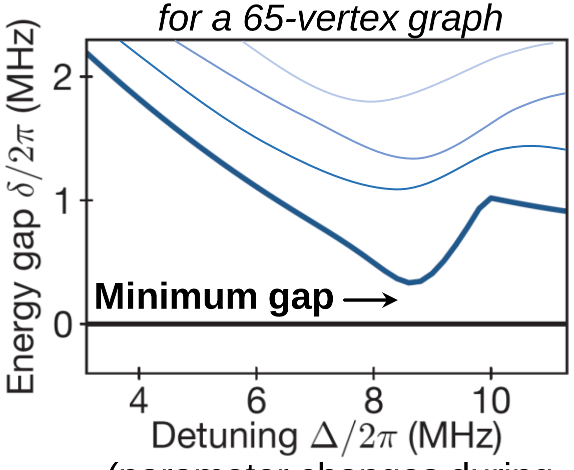


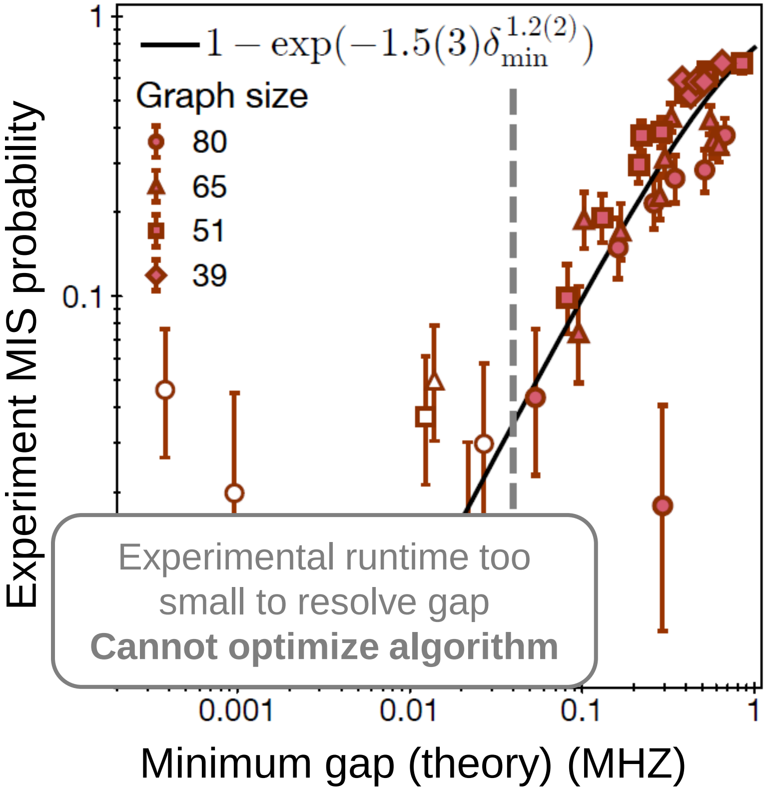
Study over 100 graph instances

What makes a graph hard or easy for the quantum algorithm to solve?

Adiabatic theorem:
MIS probability determined by minimum gap between ground and first excited state

Numerically computed for a 65-vertex graph





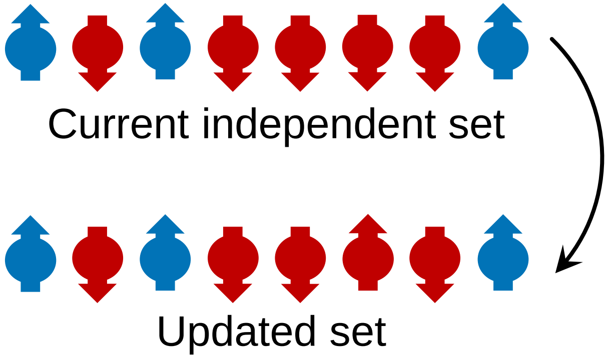
Ebadi*, Keesling*, Cain*, et al., *Science* 376, 6598 (2022)

Benchmark quantum-classical performance on fundamentally hard graph instances

Focus on simulated annealing: “classical analogue” of the quantum adiabatic algorithm

Stochastically update a candidate spin configuration (spin ↔ vertex)

1. Proposal (symmetric)

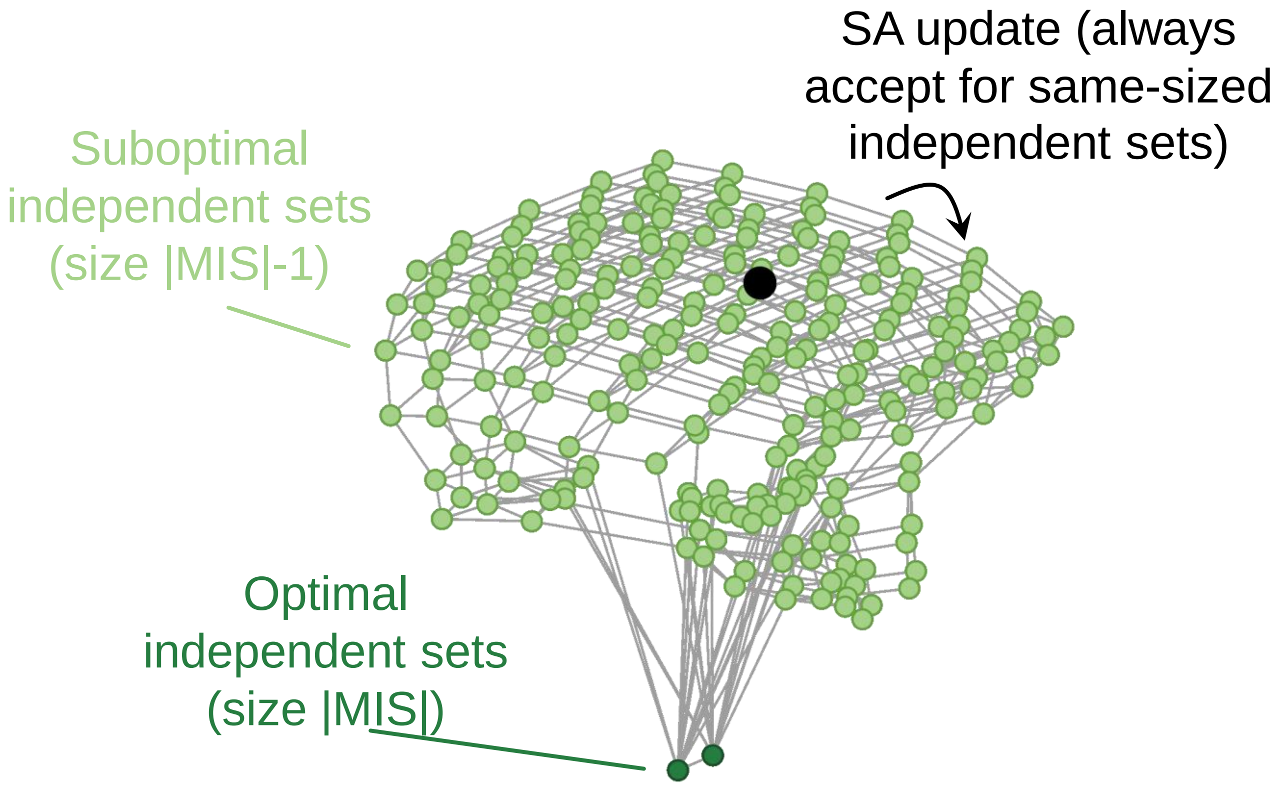


2. Accept with probability

$$\min\left(1, e^{-\beta\,\cdot\,\text{change in independent set size}}\right)$$

Prepares MIS at low temperature after many updates

What controls SA time to find the MIS?



Benchmark quantum-classical performance on fundamentally hard graph instances

SA dynamics are *unstructured search* for MIS

Prove that SA runtime $\gtrsim \frac{\#\ \text{suboptimal}\ |\text{MIS}|-1\ \text{sets}}{\#\ \text{optimal}\ |\text{MIS}|\ \text{sets}}$

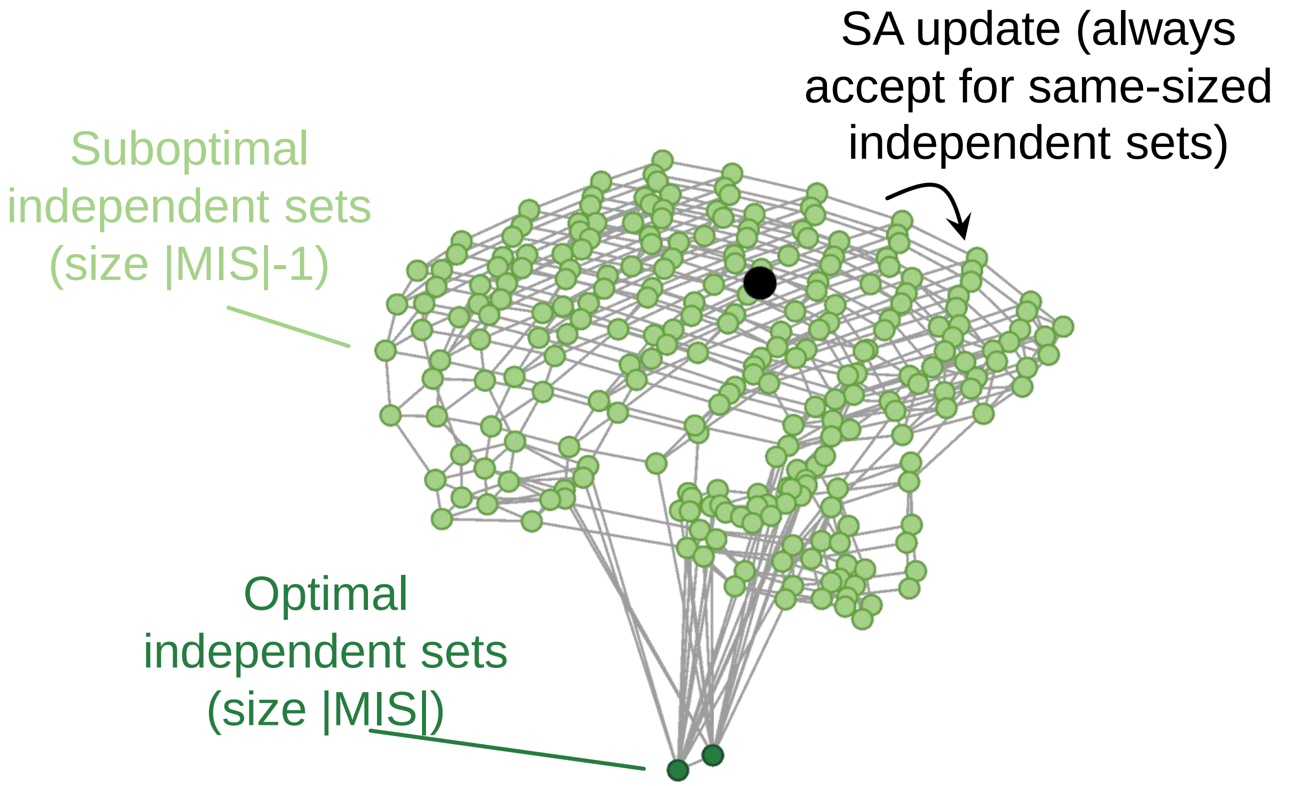
$\equiv$ SA hardness parameter $\mathcal{HP}$

(SA runtime = time to reach steady-state with error < ½)

Prove that similar obstructions hold for classical parallel tempering and quantum Monte Carlo algorithms

Cain, Chattopadhyay, Liu, Samajdar, Pichler, Lukin (2023) arXiv:2306.13123

What controls SA time to find the MIS?



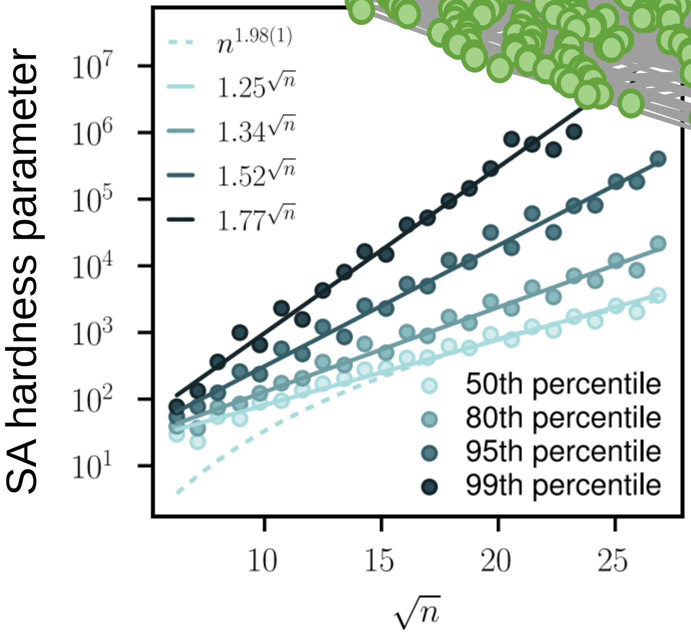
Utilize tensor-network computational methods to identify hard graph instances

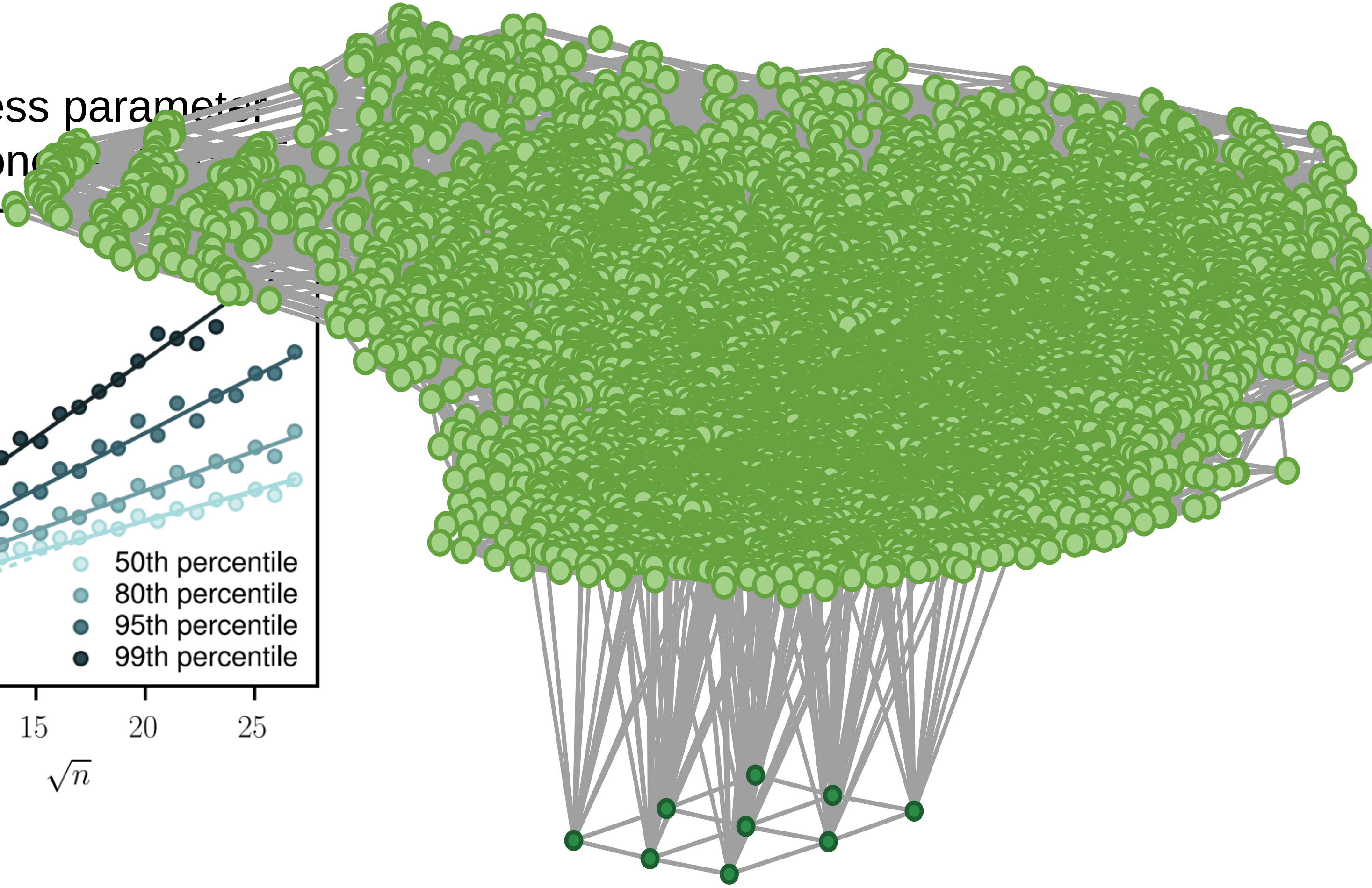
Tensor-network approach to computing solution space properties of combinatorial optimization problems

Liu, Gao, Cain, Lukin, Wang (2022) SIAM JoSC

- Compute MIS size, independence polynomial, enumerate MIS and |MIS|-1 solutions

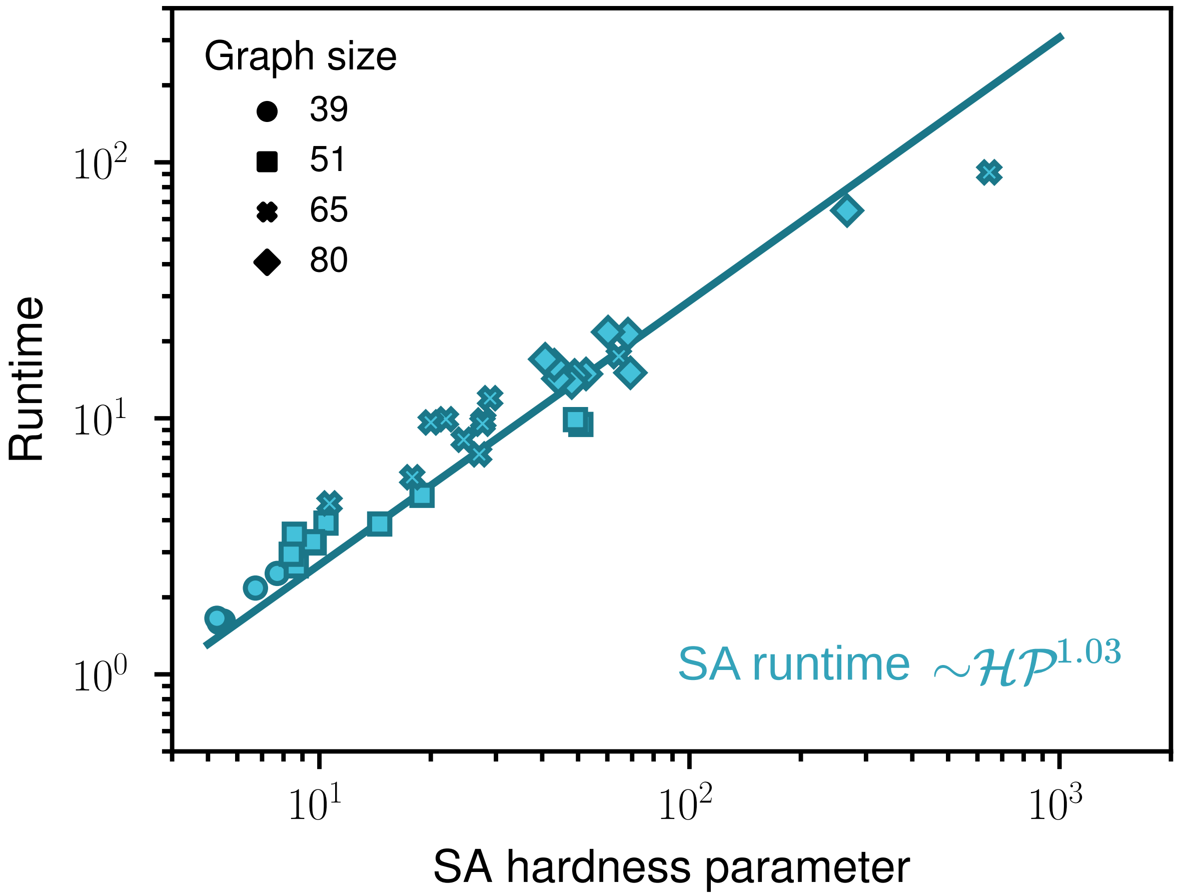
SA hardness parameter grows expon





Benchmark quantum-classical performance on fundamentally hard graph instances

Study top 2% hardest instances maximizing SA hardness parameter



SA performance determined by hardness parameter

$$\mathcal{HP} = \frac{\#\ \text{suboptimal}\ |\text{MIS}|-1\ \text{sets}}{\#\ \text{optimal}\ |\text{MIS}|\ \text{sets}}$$

Ebadi*, Keesling*, Cain*, et al., *Science* 376, 6598 (2022)

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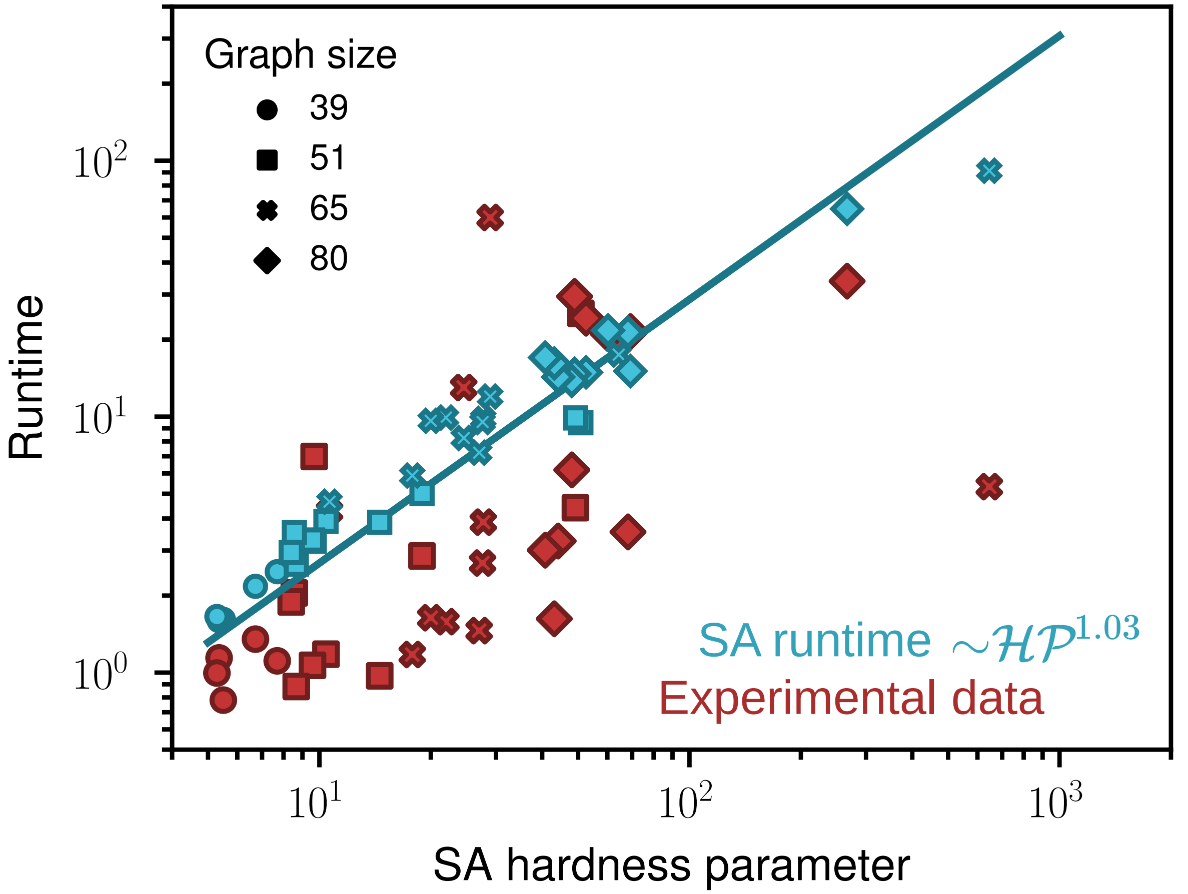

Quantum hardness determined by minimum gap, not SA hardness parameter!

Ebadi*, Keesling*, Cain*, et al., *Science* 376, 6598 (2022)

Benchmark quantum-classical performance on fundamentally hard graph instances

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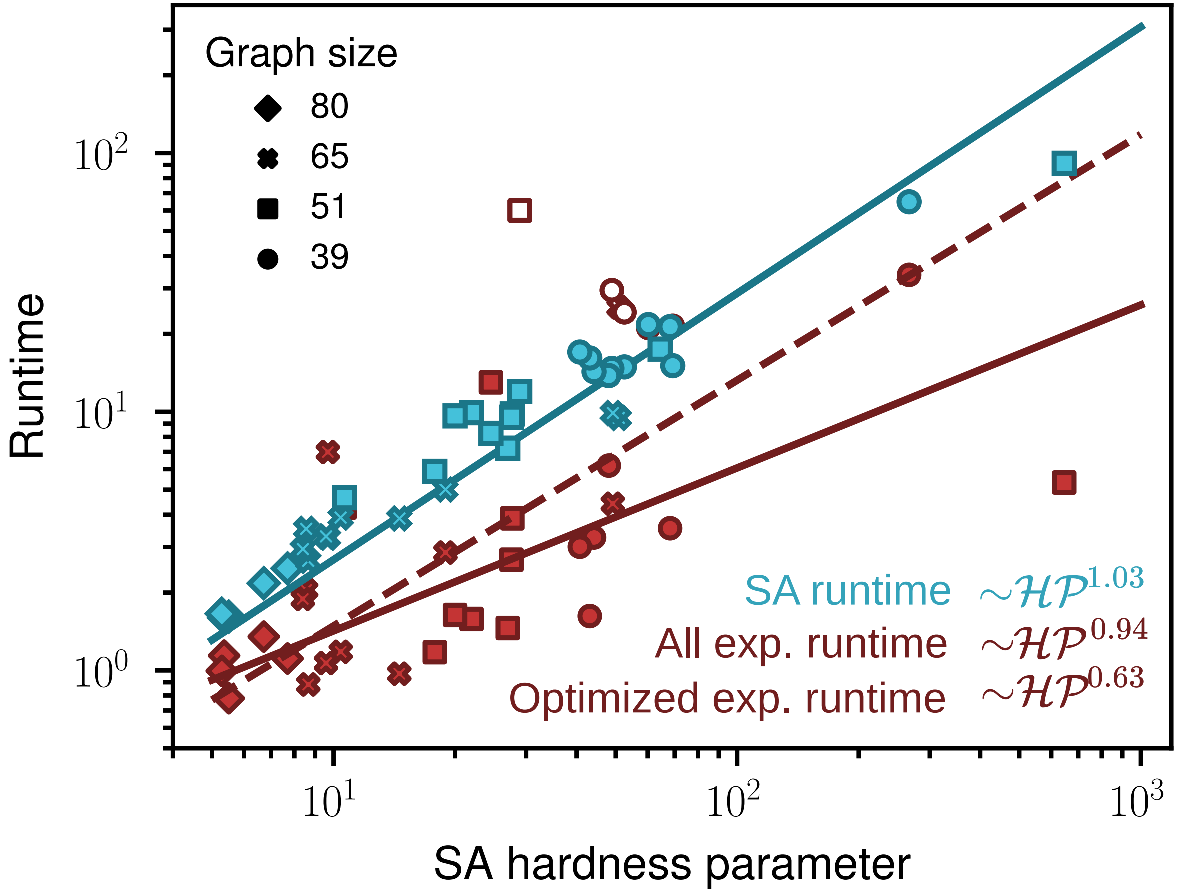

Near-quadratic speedup on instances in deep circuit regime: minimum gap large enough to optimize the quantum algorithm's evolution

What controls the minimum gap, and therefore the quantum performance?

Ebadi*, Keesling*, Cain*, et al., *Science* 376, 6598 (2022)

Hardware-efficient way to observe a *Grover speedup?*

Grover's search: quadratic speedup over all classical algorithms in unstructured search for a marked item

⋮

|MIS|-1

|MIS|-1

|MIS|-1

|MIS|

|MIS|-1

|MIS|-1

⋮

Best classical strategy

Random guessing, $O(\text{database size})$

Grover's search

$O(\sqrt{\text{database size}})$

Uniform, *delocalized* superposition → marked state

Hardware-efficient way to observe a *Grover speedup?*

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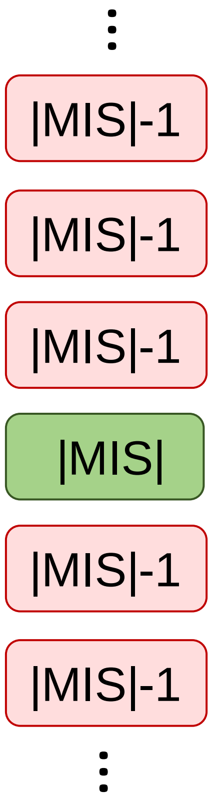
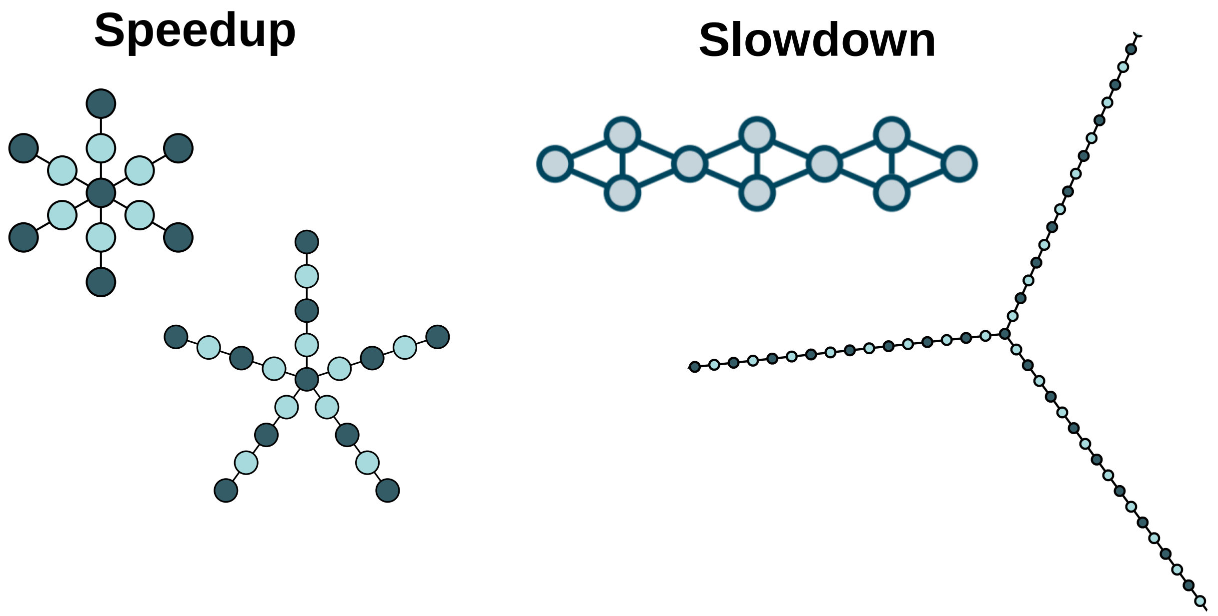


Standard approach requires complex circuits

Do we have a *natural* Grover-type speedup?

An instance has a Grover-type speedup when its low-energy states are *delocalized*

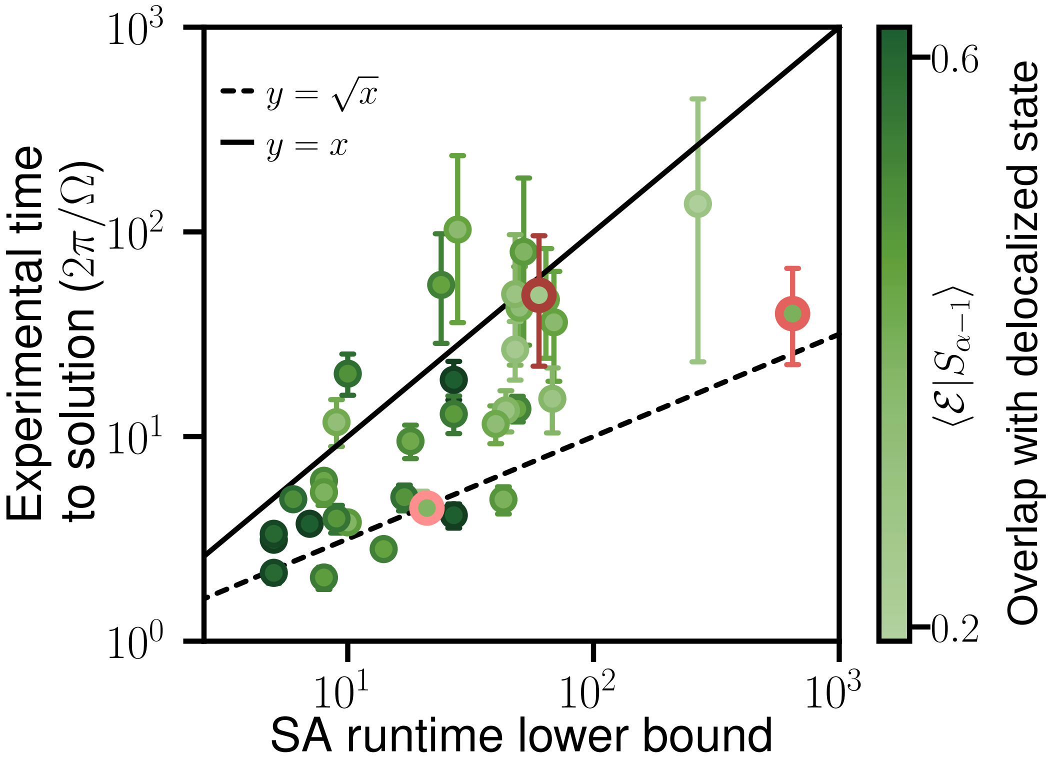


Localized instances can have a speedup or slowdown!

Cain, Chattopadhyay, Liu, Samajdar, Pichler, Lukin (2023) arXiv:2306.13123
See also: Schiffer, Wild, Maskara, Cain, Lukin, Samajdar (2023) arXiv:2306.13131

Hardware-efficient way to observe a *Grover speedup?*

Experiment: many instances are delocalized, scatter comes from localized instances



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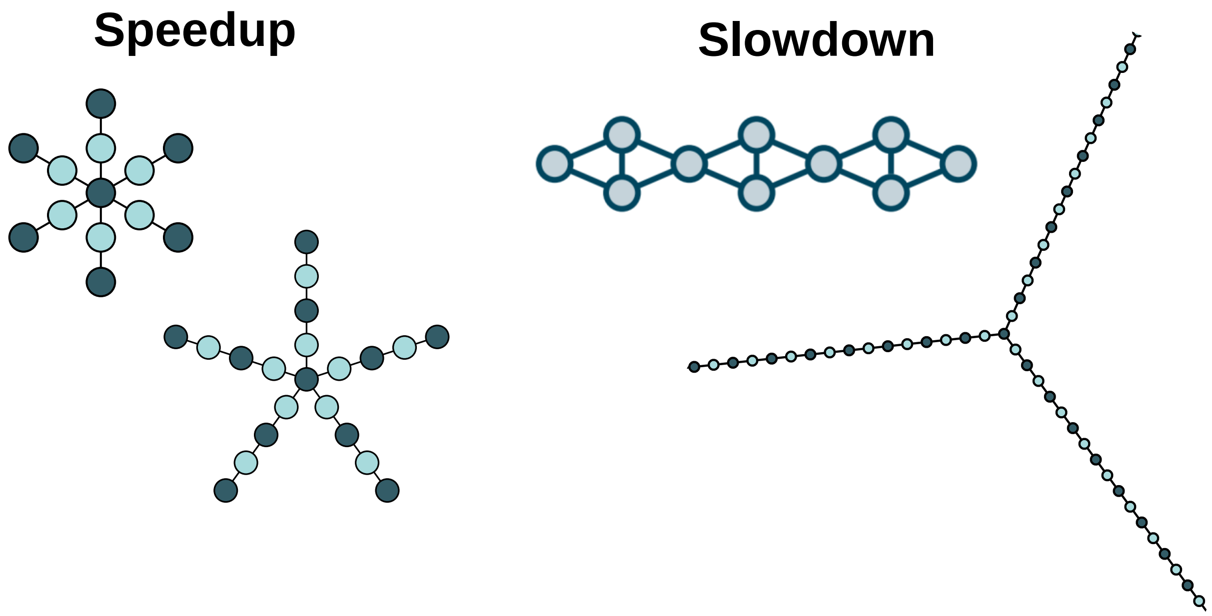

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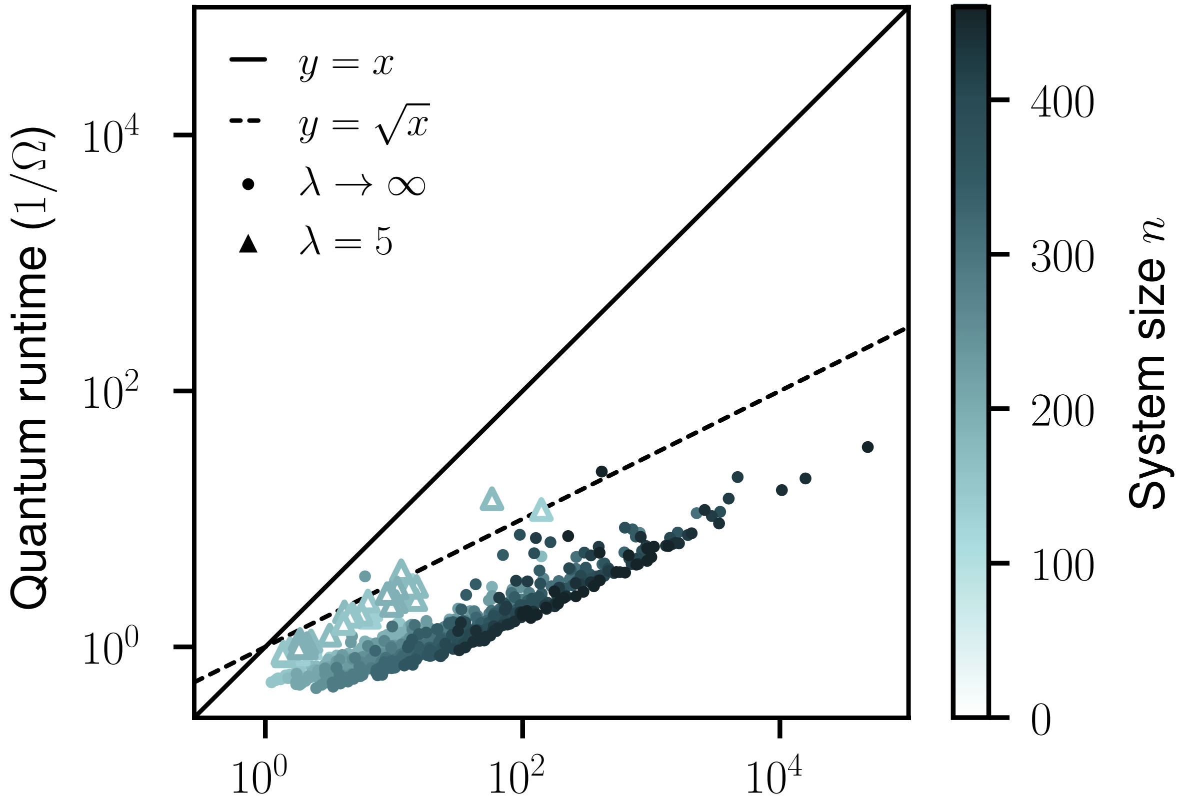
Hardware-efficient way to observe a *Grover speedup*

How can we obtain a Grover-type speedup on instances that have poor performance due to localization?

Develop a simple modification of the quantum adiabatic algorithm which obtains the Grover speedup in practice

Verify quantum speedup numerically via DMRG



Cain, Chattopadhyay, Liu, Samajdar, Pichler, Lukin (2023) arXiv:2306.13123

Acknowledgements

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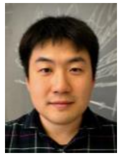

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