Weighting God's Dice: Exploiting Symmetry in Randomized Measurement Protocols

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Randomized measurement protocols for lattice gauge theories

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Randomized measurement protocols, including classical shadows, entanglement tomography, and randomized benchmarking are powerful techniques to estimate observables, perform state tomography, or extract the entanglement properties of quantum states. While unraveling the intricate structure of quantum states is generally difficult and resource-intensive, quantum systems in nature are often tightly constrained by symmetries. This can be leveraged by the symmetry-conscious randomized measurement schemes we propose, yielding clear advantages over symmetry-blind randomization such as reducing measurement costs, enabling symmetry-based error mitigation in experiments, allowing differentiated measurement of (lattice) gauge theory entanglement structure, and, potentially, the verification of topologically ordered states in existing and near-term experiments.

Jonathan Kunjummen

Niklas Mueller

arxiv:2303.15519

The Law of Leaky Abstractions

> All non-trivial abstractions are, to some degree, leaky.
>
> —Joel Spolsky
> *Stack Overflow, co-founder*

Example

Abstraction: All memory is equally fast to access.

Leak: Time to iterate over a large array (horizontally vs. vertically)

Avoiding costly abstractions leads to faster hardware/software.

This Talk
Apply this approach to a particular task in quantum computing

Quantum Computing 101

	Classical Computer	Quantum Computer
Building Block	bits	qubits (two level quantum system)
Physical Instantiation	punch cards, capacitors, magnetic disks, ...	spin-1/2 particles, atomic energy levels, Josephson junctions...
States	bit strings, $b \in \{0, 1\}^n$	$2^n \times 2^n$, trace 1, positive-semidefinite Hermitian matrices, ρ
Operations	logic gates	unitary matrices, U
Output	bit strings, $b \in \{0, 1\}^n$	bit strings, $b \in \{0, 1\}^n$

Quantum states as generalized probability distributions

Consider a probability distribution over $b \in \{0,1\}^n$.

Express as a diagonal $2^n \times 2^n$ matrix:

$$D = \begin{pmatrix} p_{0\ldots00} & & & \\ & p_{0\ldots01} & & \\ & & \ddots & \\ & & & p_{1\ldots11} \end{pmatrix}$$

Properties

- $\mathrm{Tr}(\rho) = \mathrm{Tr}(D) = 1$
- $\rho \geq 0$
- $\rho^\dagger = (U^\dagger \rho U)^\dagger = \rho$

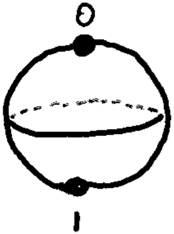
Quantum states are "rotated" probability distributions:

$$\rho = U^\dagger D U$$

Intuition: Not all properties encoded in a quantum state are compatible

State Space of a Qubit

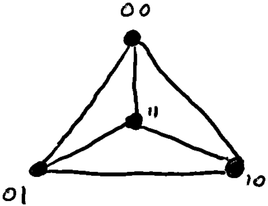
From classical probability distributions...

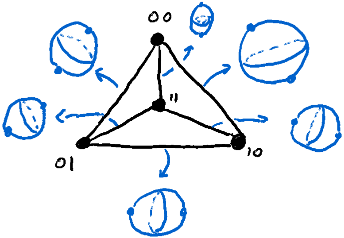


...to quantum states.

State Space of 2 Qubits

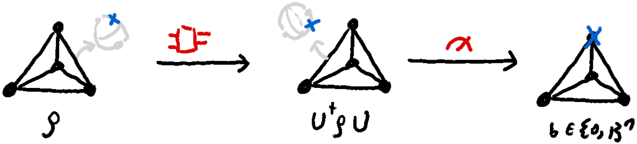
From classical probability distributions...





...to quantum states.

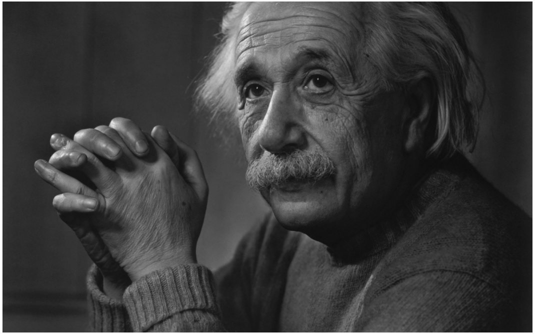
Dynamics of a Quantum State



Evolution: state is rotated via unitary dynamics

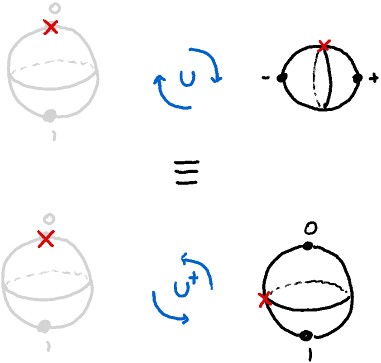
Measurement (Computational Basis): state jumps (probabilistically) to a bit string.

"God does not play dice with the universe."



Measurement Beyond the Computational Basis

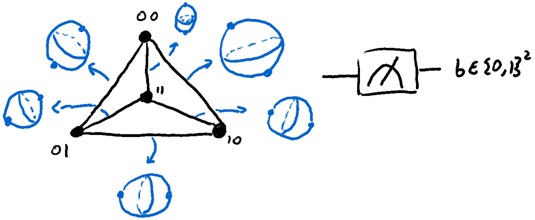
Can also measure with respect to a "rotated" basis.



general measurement = rotate state + measure in computational basis

Curse of Dimensionality

A single measurement in quantum mechanics yields limited information.



$\implies$ learning an unknown quantum state requires $\exp(n)$ measurements.

Breaking the Curse

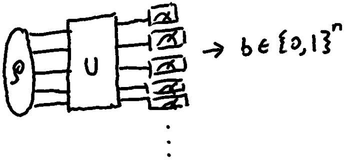
Often only care about a limited set of quantities.

e.g. expectation values of physical observables, correlation functions, entropies

Randomized measurement

Step 1

random basis measurements = randomly rotate + computational basis measurement



Step 2

Use properties of random ensemble of unitaries + measurement outcomes to compute desired properties of state

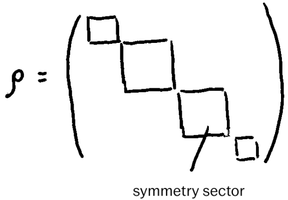
The Leaky Abstraction of Randomized Measurement

Typically know a lot about our state (e.g. symmetries).

Symmetries $\longleftrightarrow$ Conserved Quantities

time translation symmetry $\longleftrightarrow$ conservation of energy

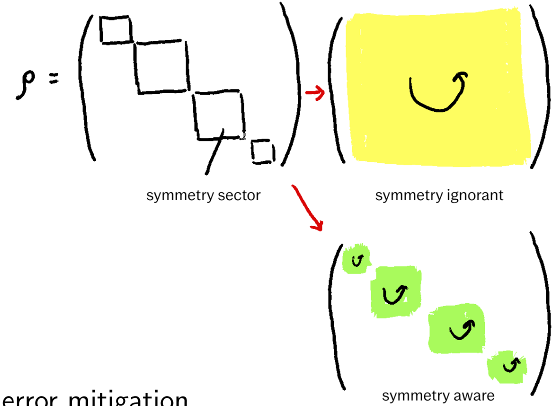
Mathematically, states with symmetries are block diagonal:



The Leaky Abstraction of Randomized Measurement

Typically know a lot about our state (e.g. symmetries).

Symmetry aware randomized measurement

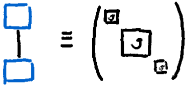


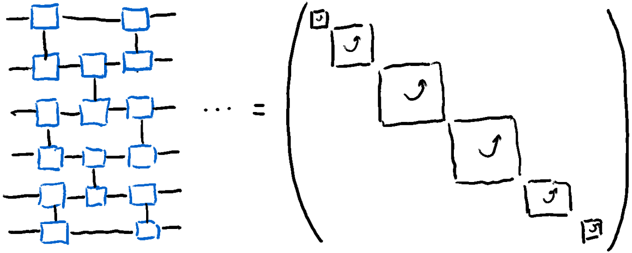
Advantages:

- cost reduction
- symmetry-based error mitigation
- access to symmetry-resolved quantities

Blueprint for Symmetry Aware Randomized Measurement

Main Idea
Use random, local symmetry-respecting unitary operations (gates) to build up random, global symmetry-respecting unitary operations.





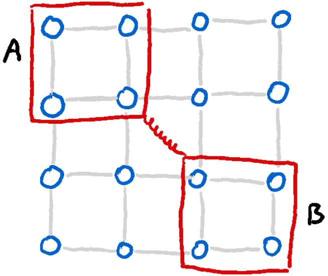
Warning!
Not guaranteed to work
[Marvian, Nat. Phys. (2022)]

Key Example

Entanglement Structure of Lattice Gauge Theory

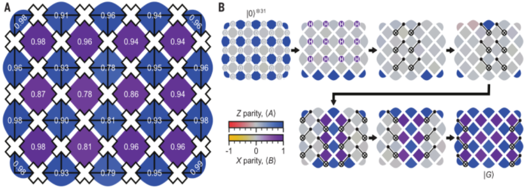
the structure of a certain type of non-classical correlations in the measurement statistics of a quantum state

a physical theory defined on a lattice with local symmetries

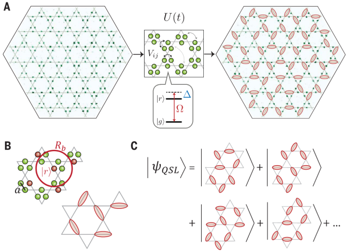


Topologically Ordered States

Experimental realizations



[Satzinger et. al., Pollmann, Roushan, Science (2021)]



[Semeghini et al, Science (2021)]

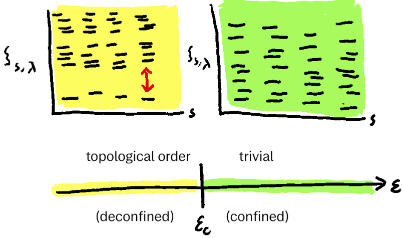
See also:

Chiral spin liquid with cold atoms (proposal) [Sun, Goldman, Aidelsburger, Bukov, PRX Quantum (2023)]

$\mathbb{Z}_2$ Lattice Gauge Theory in (2+1)D

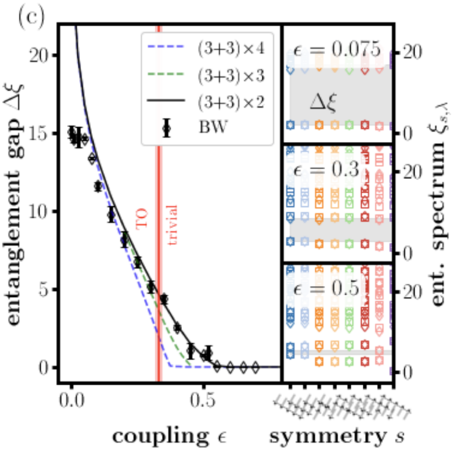
Simple gauge theory with phase transition in entanglement structure of its ground state

Signature: "entanglement gap"



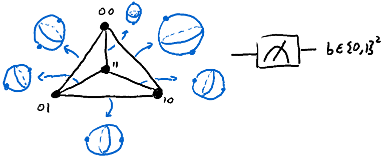
$\xi_{s,\lambda}$ spectrum of $\equiv -\log \rho_A$

[Li Haldane, PRL (2008)]
[Mueller, Zache, Ott, PRL (2022)]

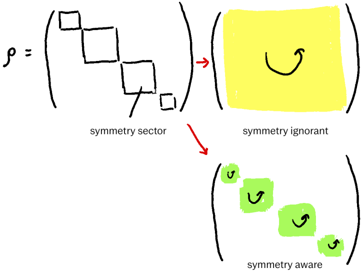


Summary

- "Law of Leaky Abstractions" applies equally well to quantum algorithms/hardware as HPC
- Quantum state space is big, but extracting information is expensive



- Randomized measurement protocols can help...



...and physical insight (e.g. symmetries) can make them even better.

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