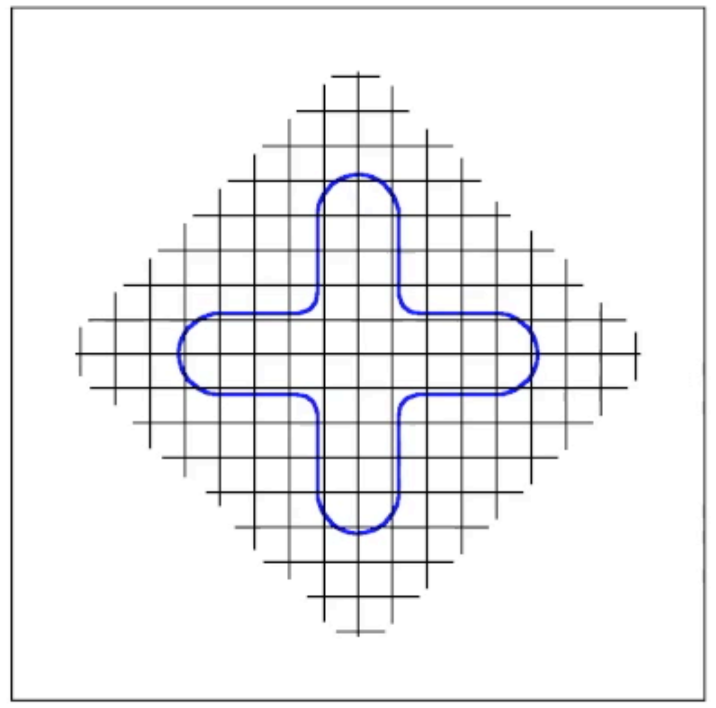
Reference Map Technique

A fully Eulerian method for
fluid-structure interactions in 3D



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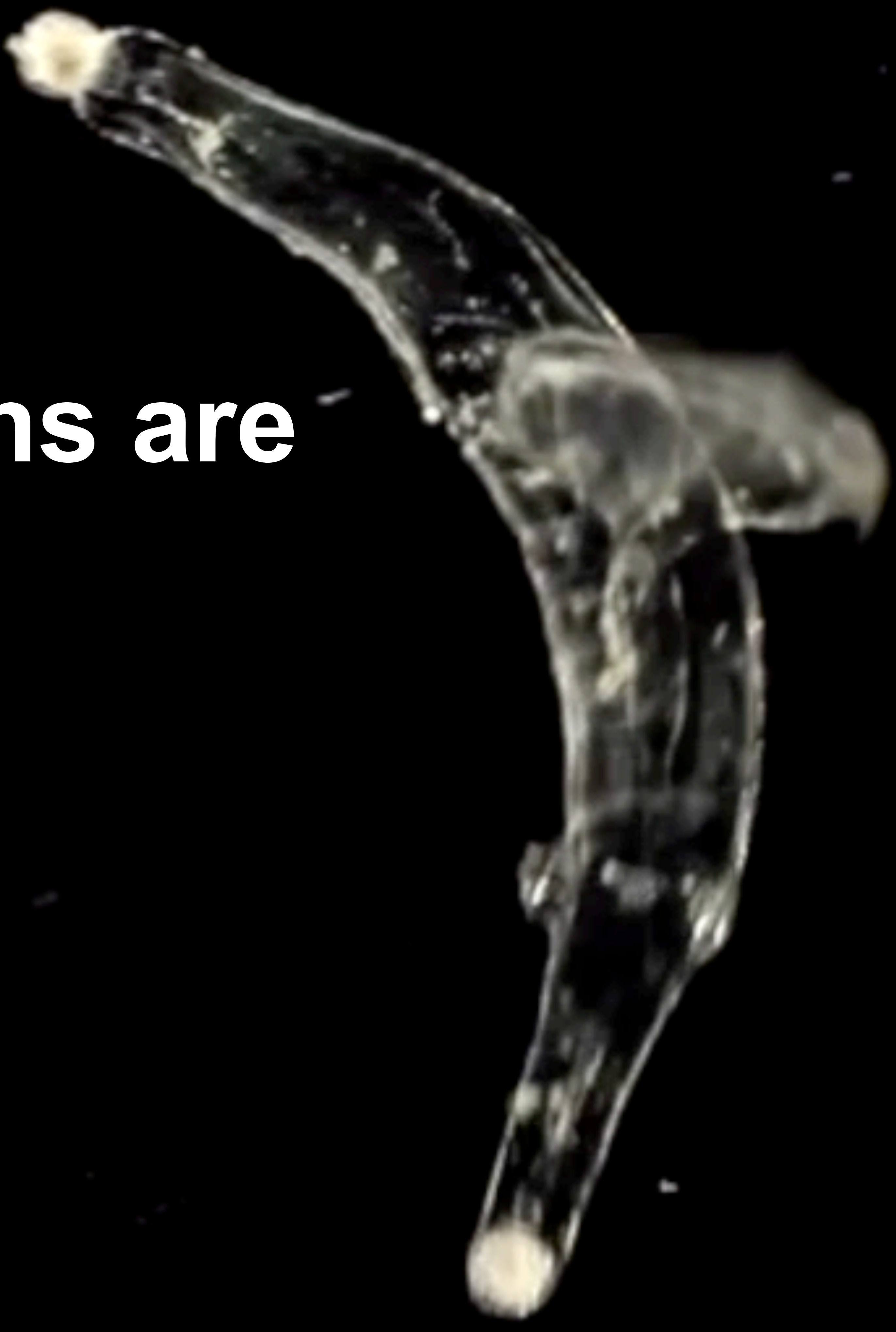
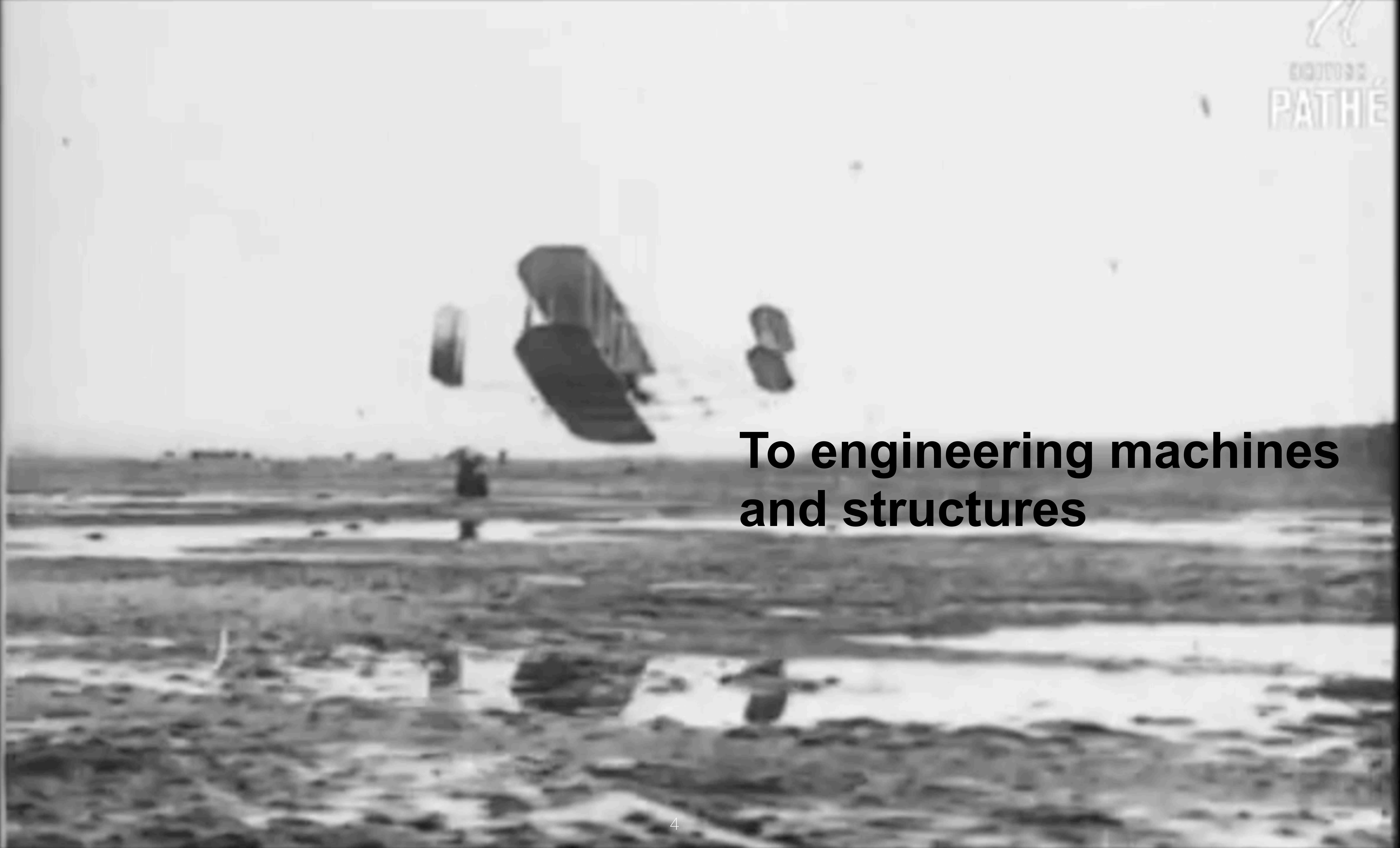
Fluid-solid interactions are as ubiquitous as life



From animals swimming and squirming


To engineering machines
and structures
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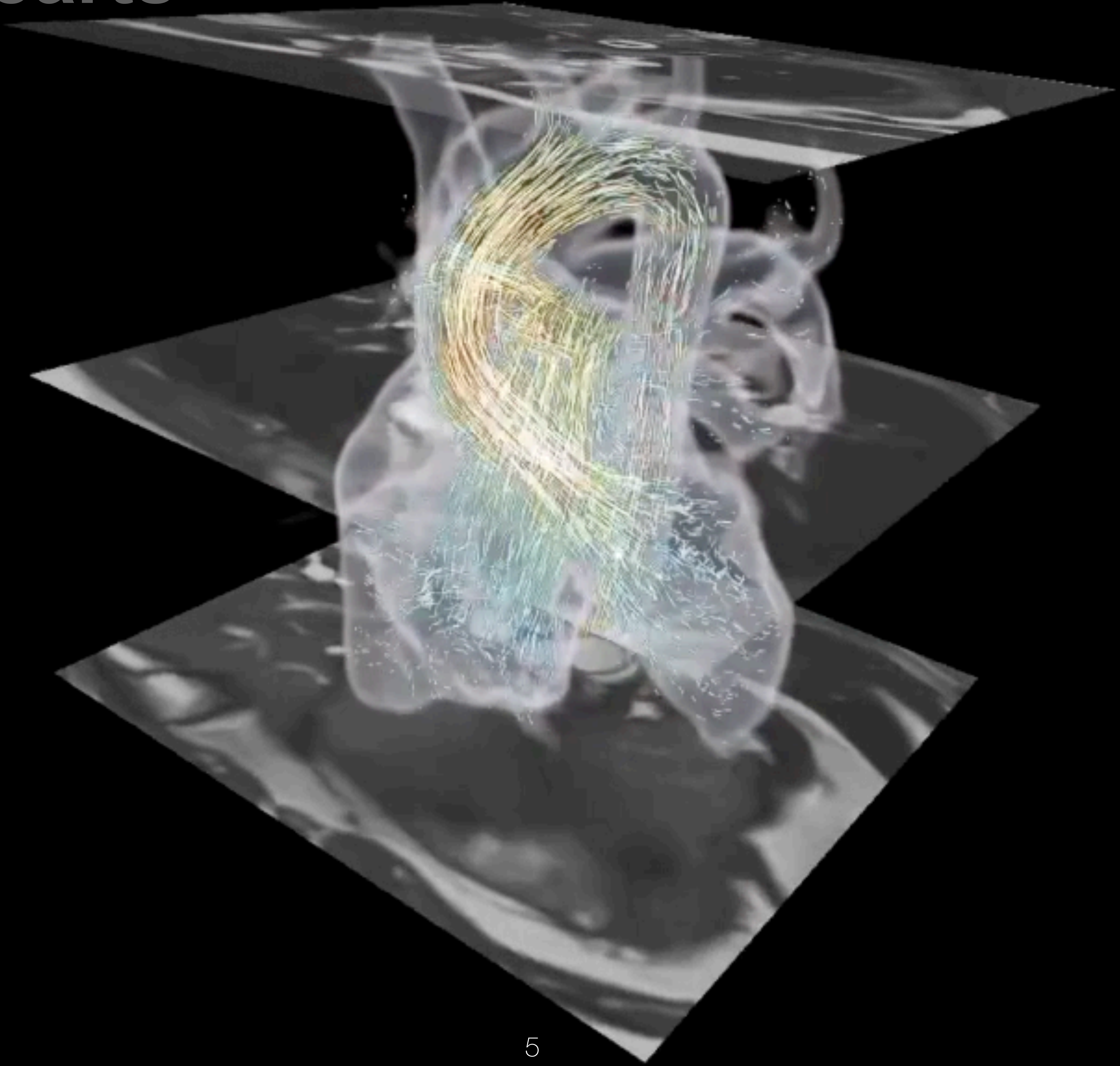
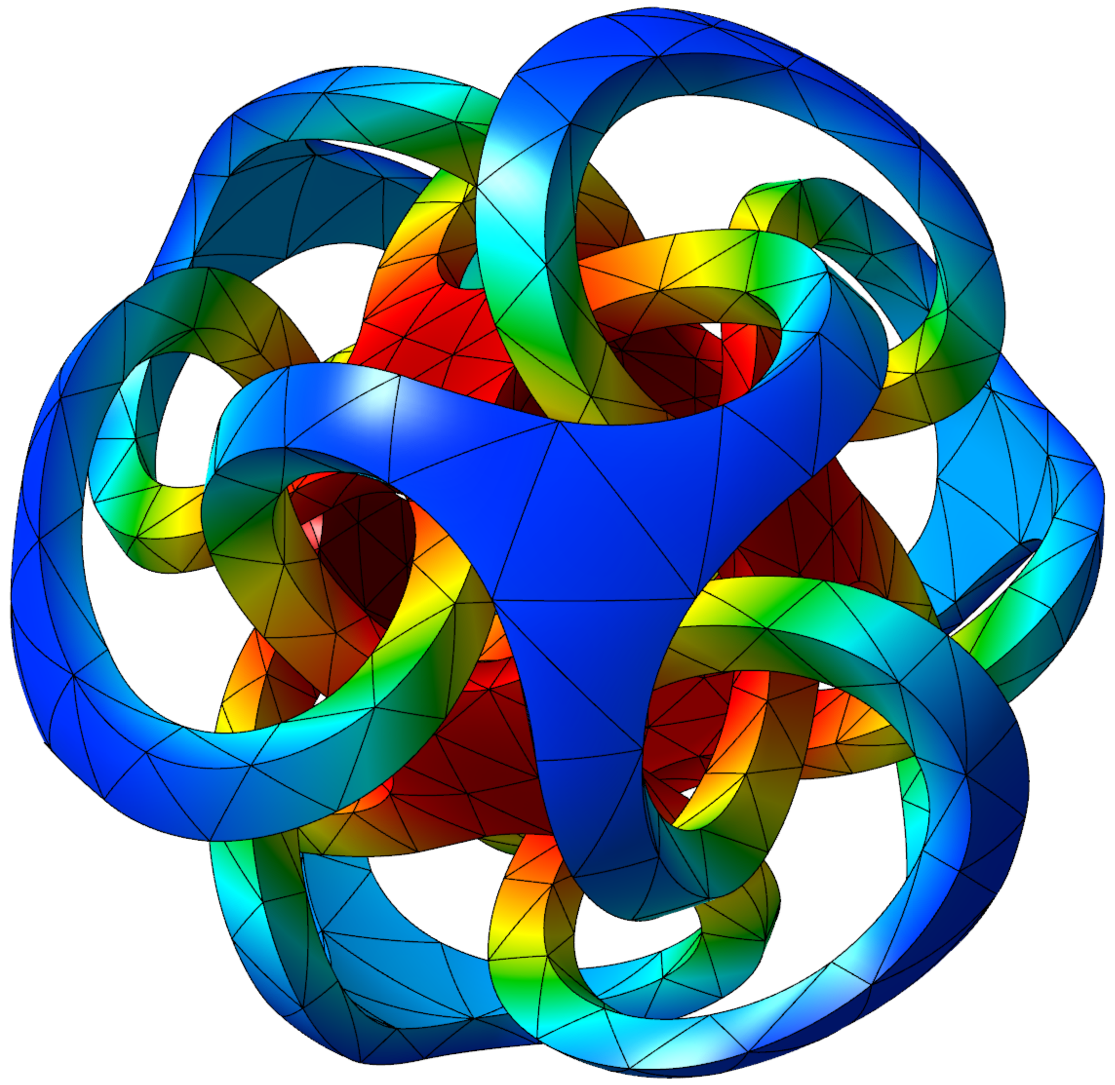
To blood flowing through our hearts

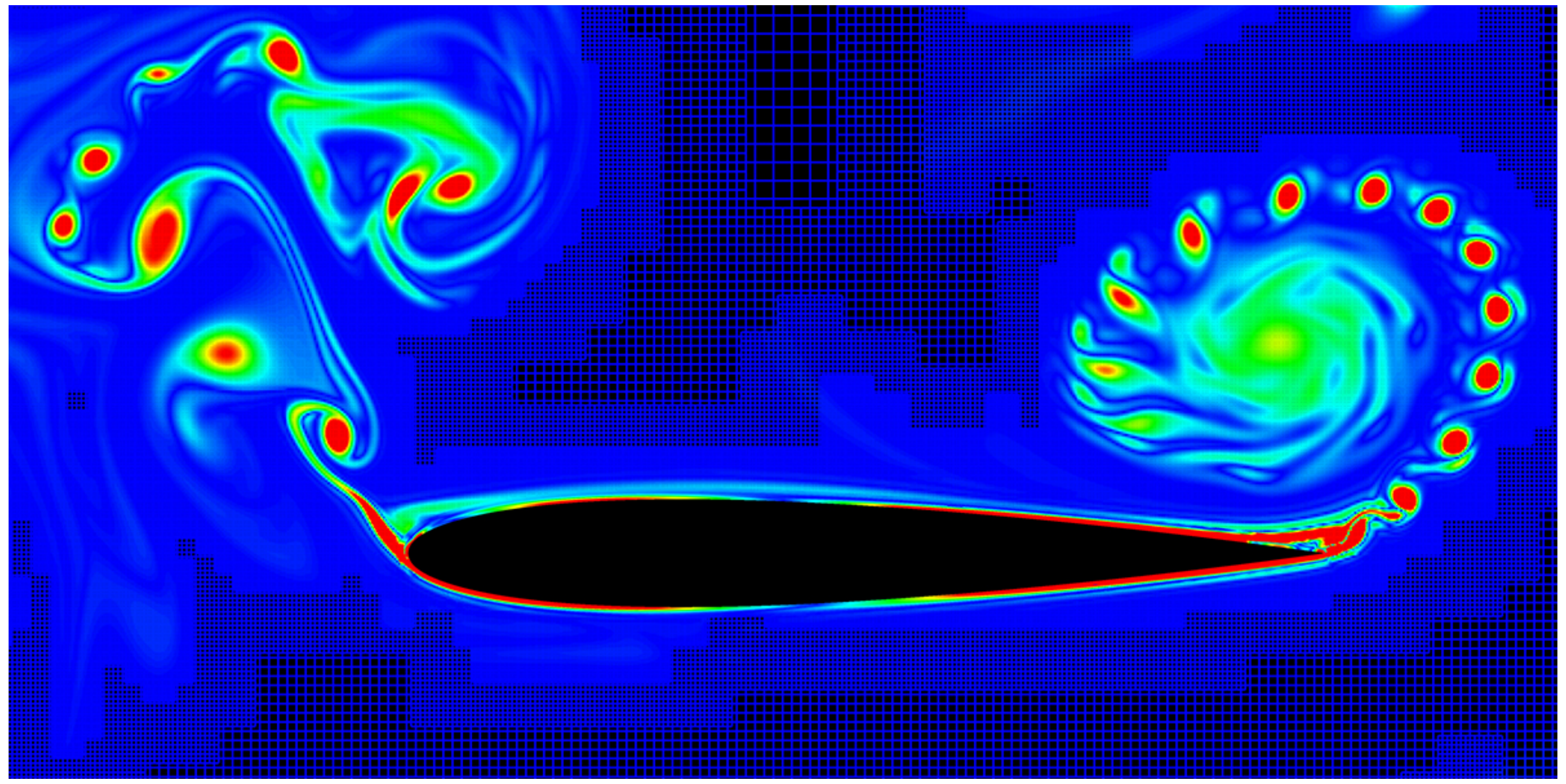


To experiments on droplets in a microfluidic channel

Numerical methods for solid or fluid alone exploit their respective stress-strain responses



LLNL GLVis logo, glvis.org



Neal M. Chaderjian, NASA/Ames

Solid deformation:

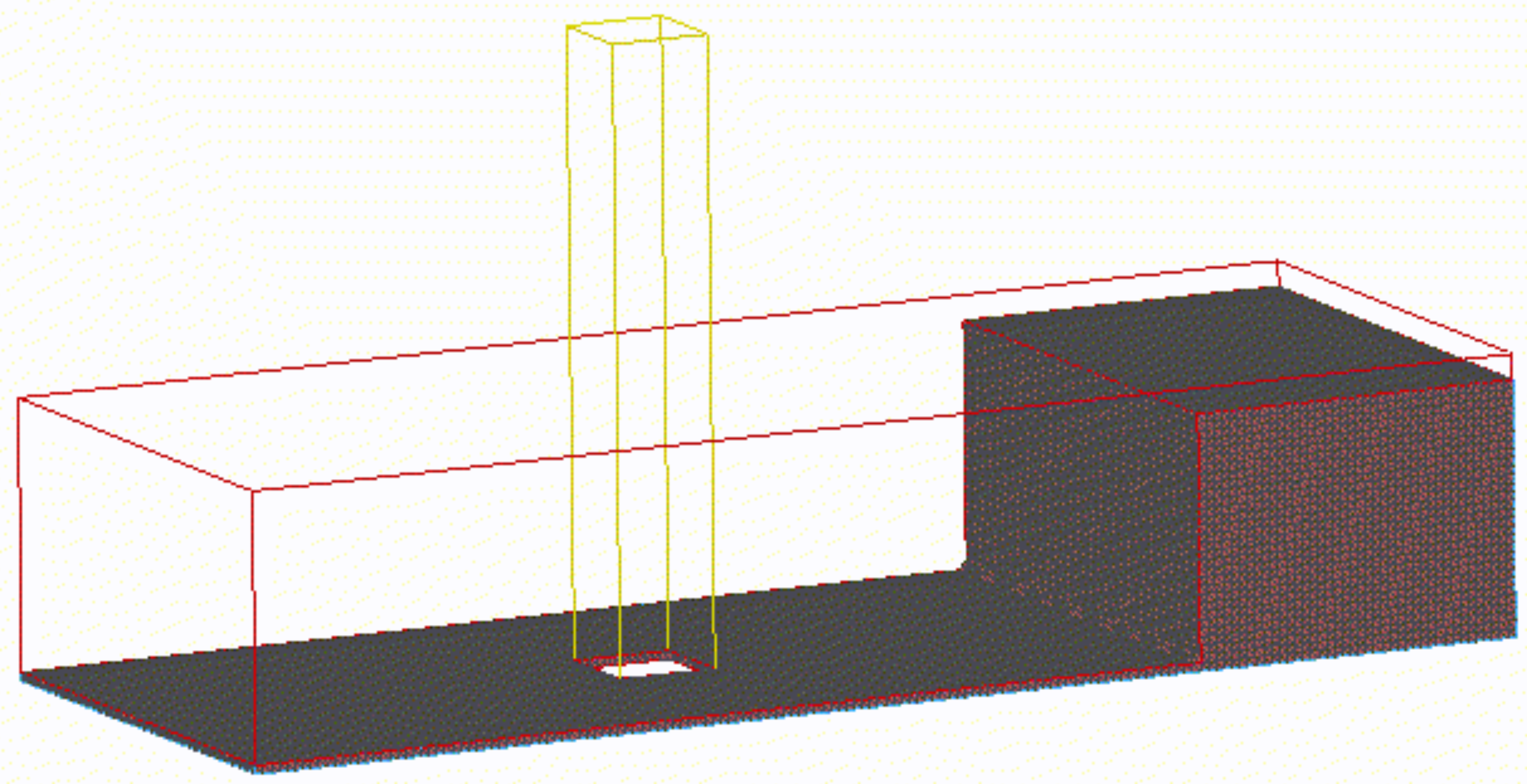
- Stress arises from deformation
- Lagrangian mesh
- Finite element analysis, material points, etc

Fluid flow:

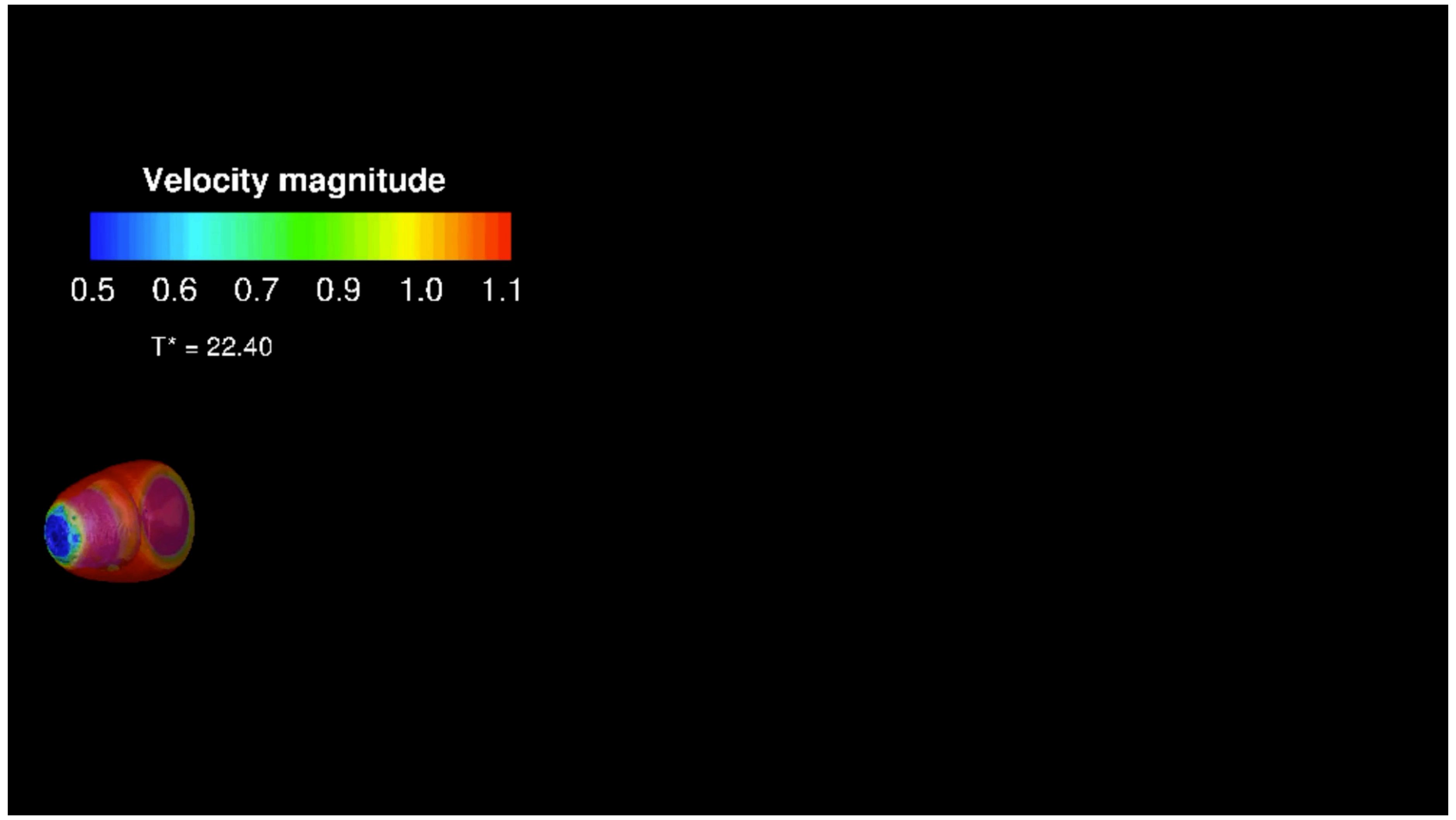
- Stress arises from deformation rate
- Eulerian mesh
- Finite difference, finite volume, etc

Existing FSI methods often need to consider both Eulerian and Lagrangian coordinates

- Arbitrary Lagrangian-Eulerian[1]
  - Intermediate mesh between fluid & solid
  - Re-mesh to avoid excessive distortion



- Immersed boundary methods[2]
  - Lagrangian points represent solid boundaries
  - Use discrete Delta function to couple fluid & solid

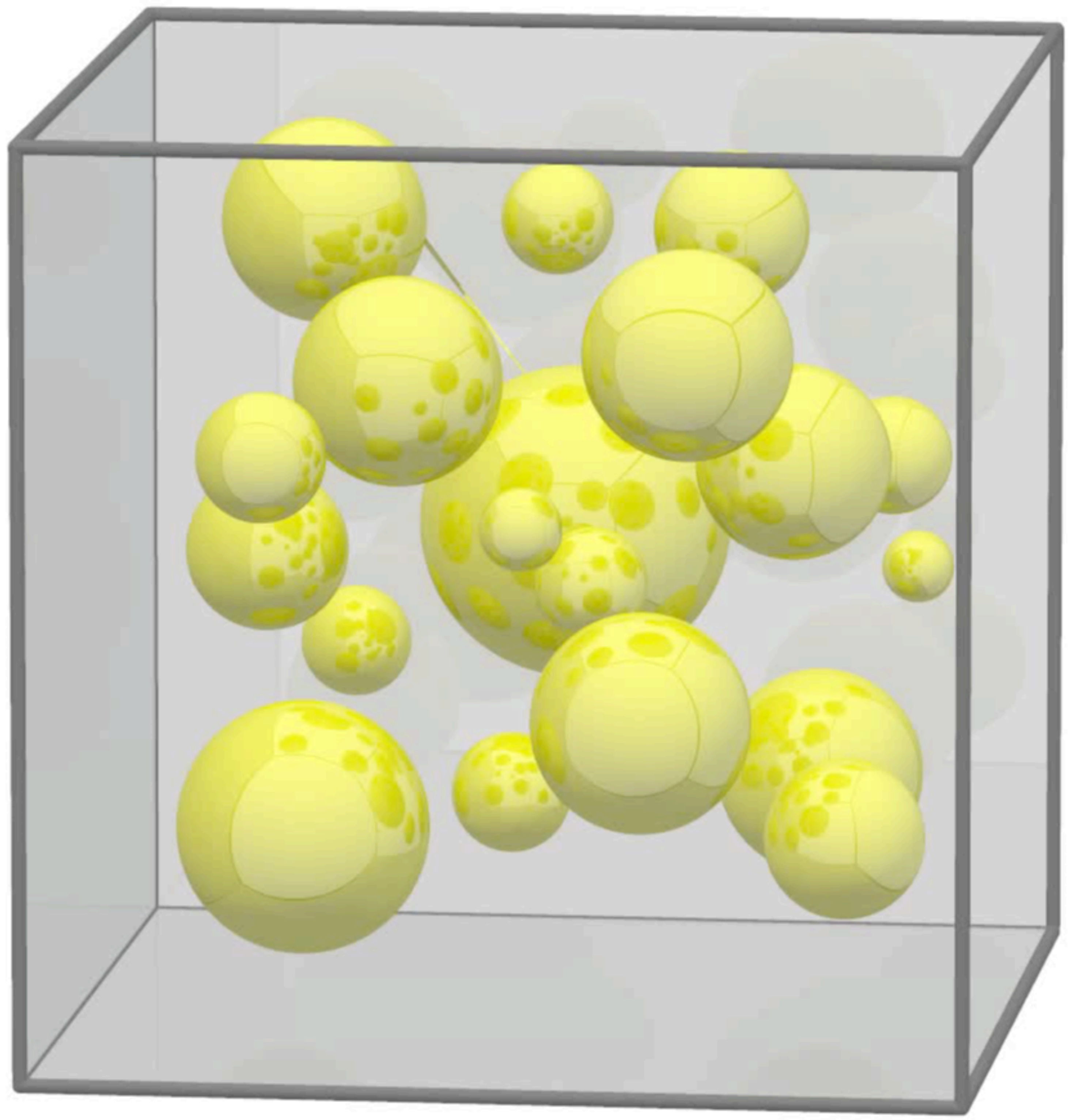


[1]Dynaexamples.com, wave-structure interaction. [2]Angelidis, D. SAFL, UMN

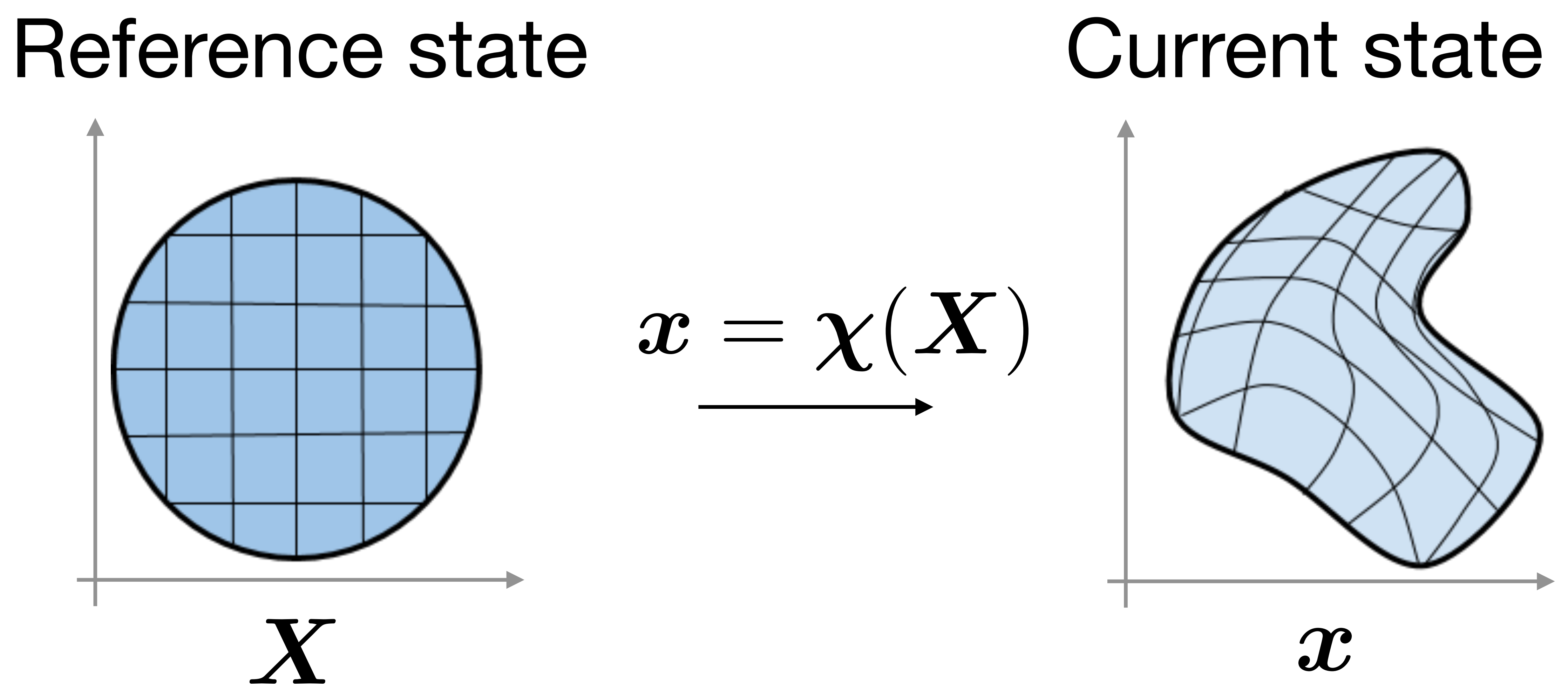
Reference map technique (RMT) is fully Eulerian, i.e. it uses a single, Eulerian grid for both solid & fluid

RMT comes with several advantages:

- Avoid meshing difficulties
- Relatively easy to parallelize
- Easy representation of physics
- *Bonus: large deformation in solids*



RMT is based on large deformation theory, going beyond linear elasticity

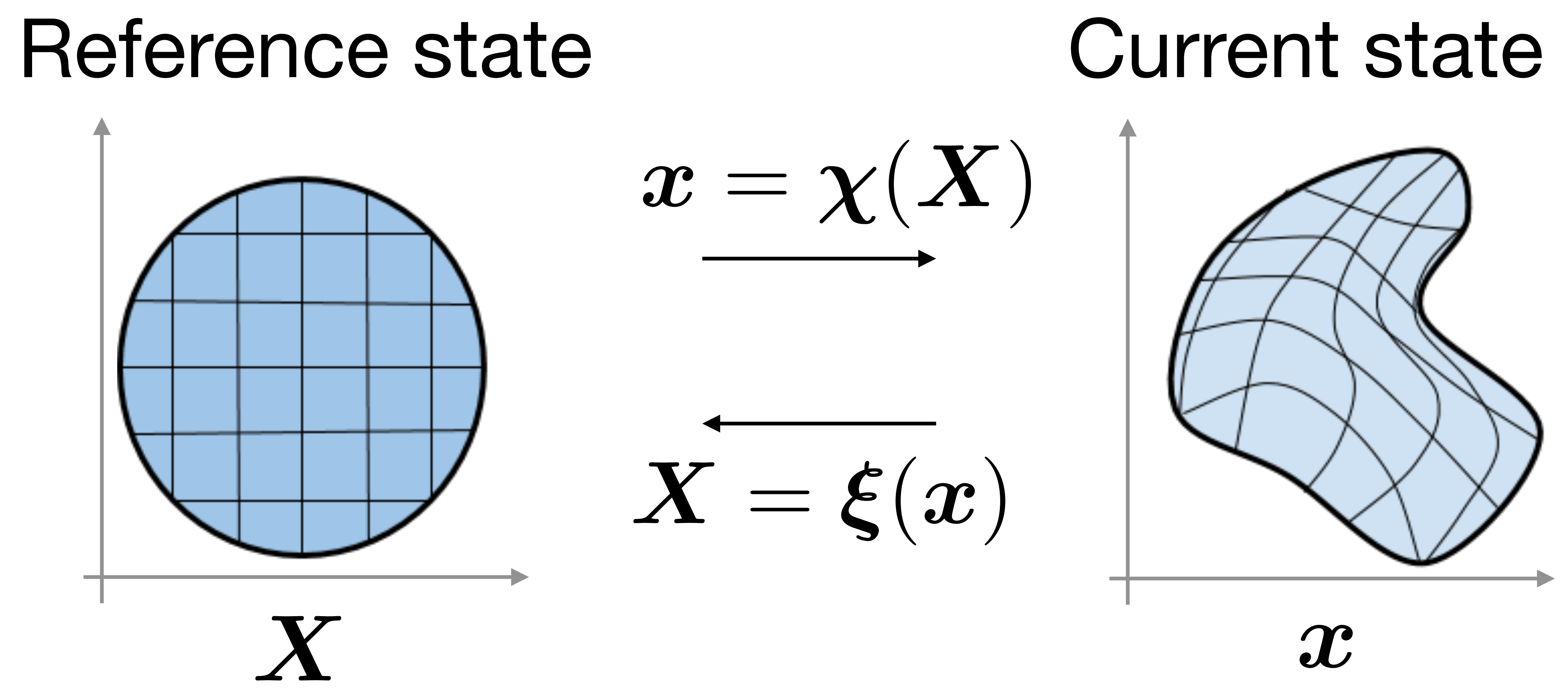


Deformation gradient: $\boldsymbol{F} = \frac{\partial \boldsymbol{\chi}(\boldsymbol{X})}{\partial \boldsymbol{X}} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}$

Solid stress*: $\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\boldsymbol{F})$

* ^ denotes a constitutive function.

Reference map is the inverse motion function, always leads back to the reference state



Deformation gradient:

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}} = (\nabla \boldsymbol{\xi})^{-1}$$

Solid Cauchy stress*: $\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\boldsymbol{F})$

Time evolution:

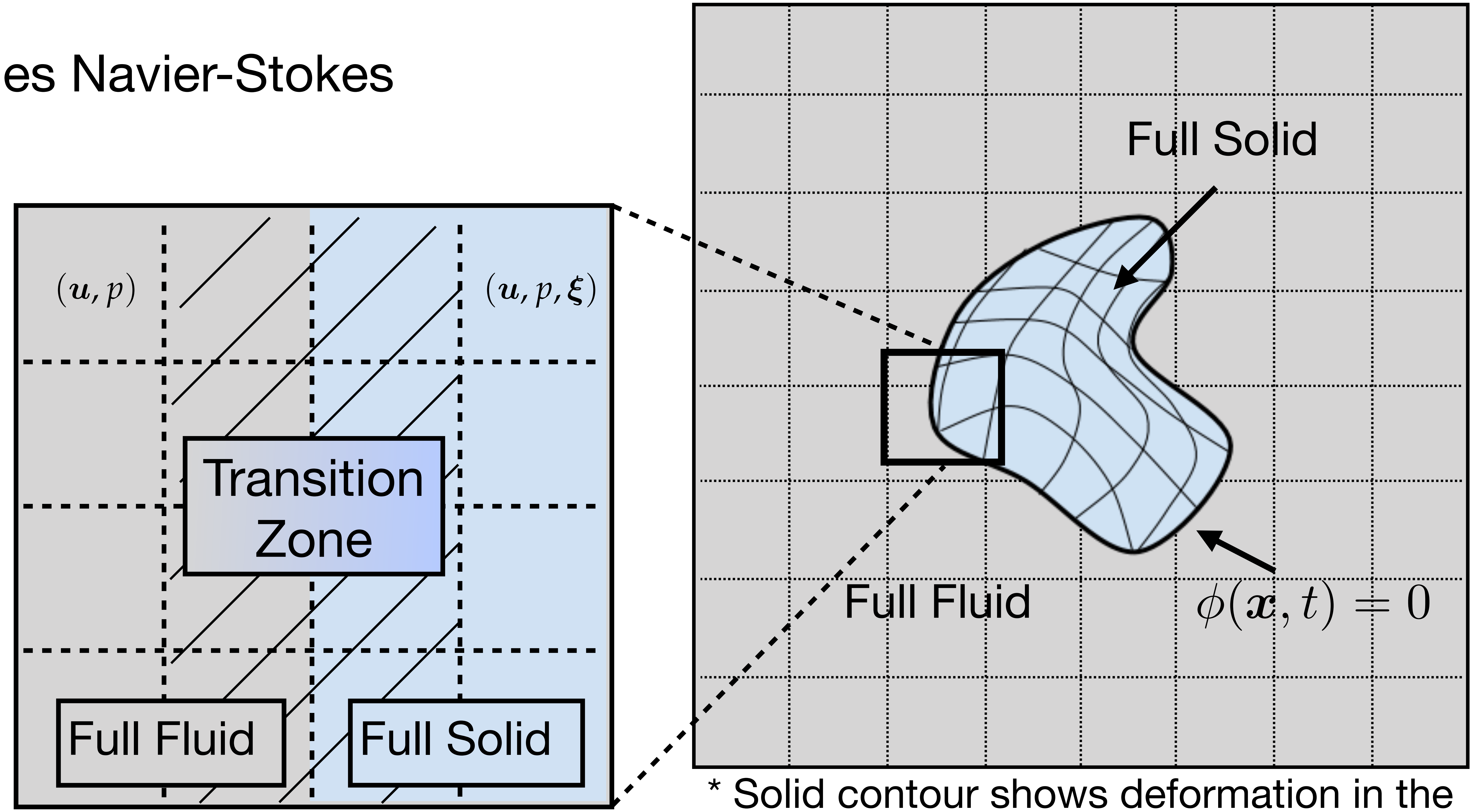
$$\frac{\partial \boldsymbol{\xi}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\xi} = \boldsymbol{0}$$

Velocity: $\boldsymbol{u}$

* ^ denotes a constitutive function.

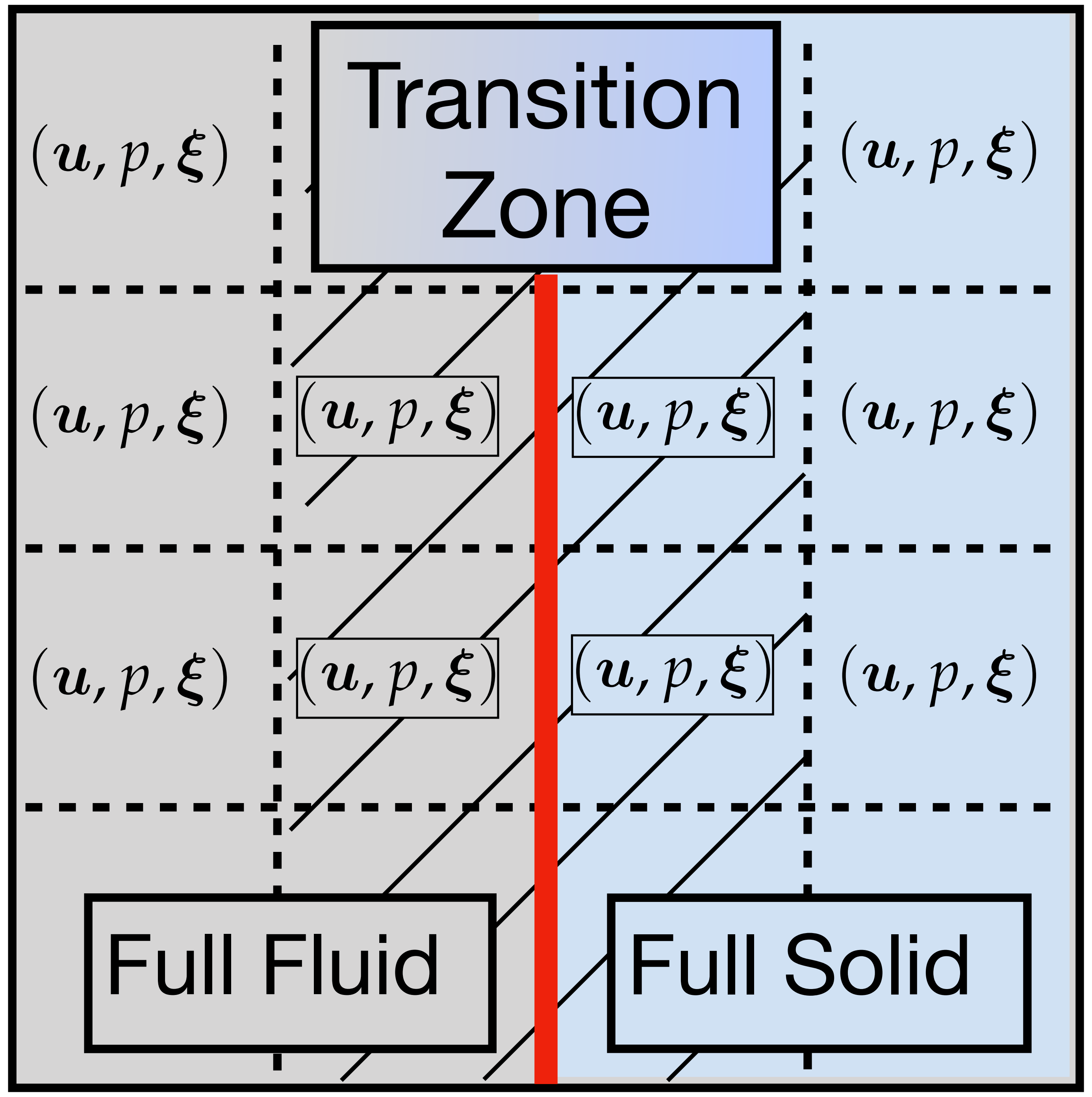
Single system of governing equations for both phases; shared velocity and pressure fields

- Solid equation reassembles Navier-Stokes
- Level-set, $\phi(\boldsymbol{x}, t)$, defines the interface
- Track reference map **only within solid**
- *Additional care at the interface*



* Solid contour shows deformation in the reference map filed, not in the grid

Extrapolation needed outside of solid
But common methods not suitable for RMT3D



- Need derivative at interface *(time-stepping & solid stress)*
- Ref. map exists in solid, but not fluid
- Need to extrapolate into fluid (within transition zone)

Common/Old ways

- PDE-based methods, or
- Level-set reinitialization + linear extrapolation

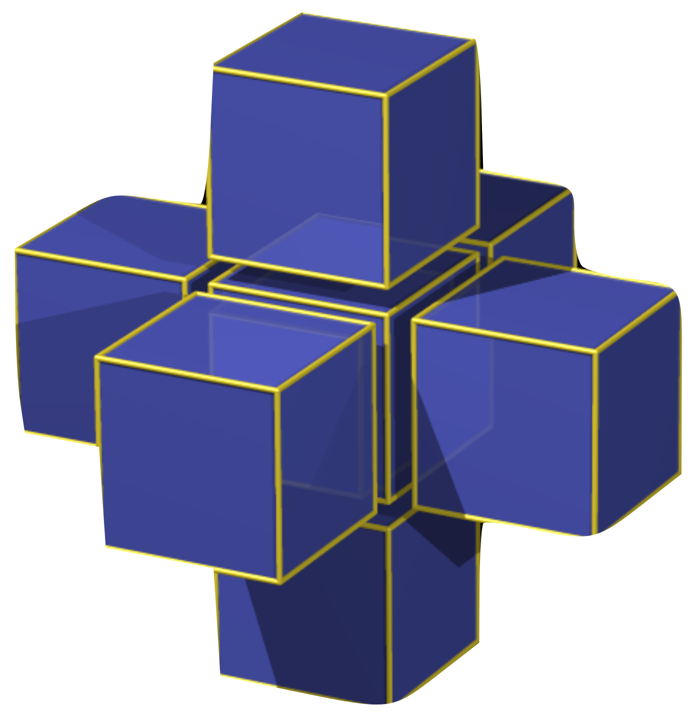
RMT3D way

"Onion" layers

+

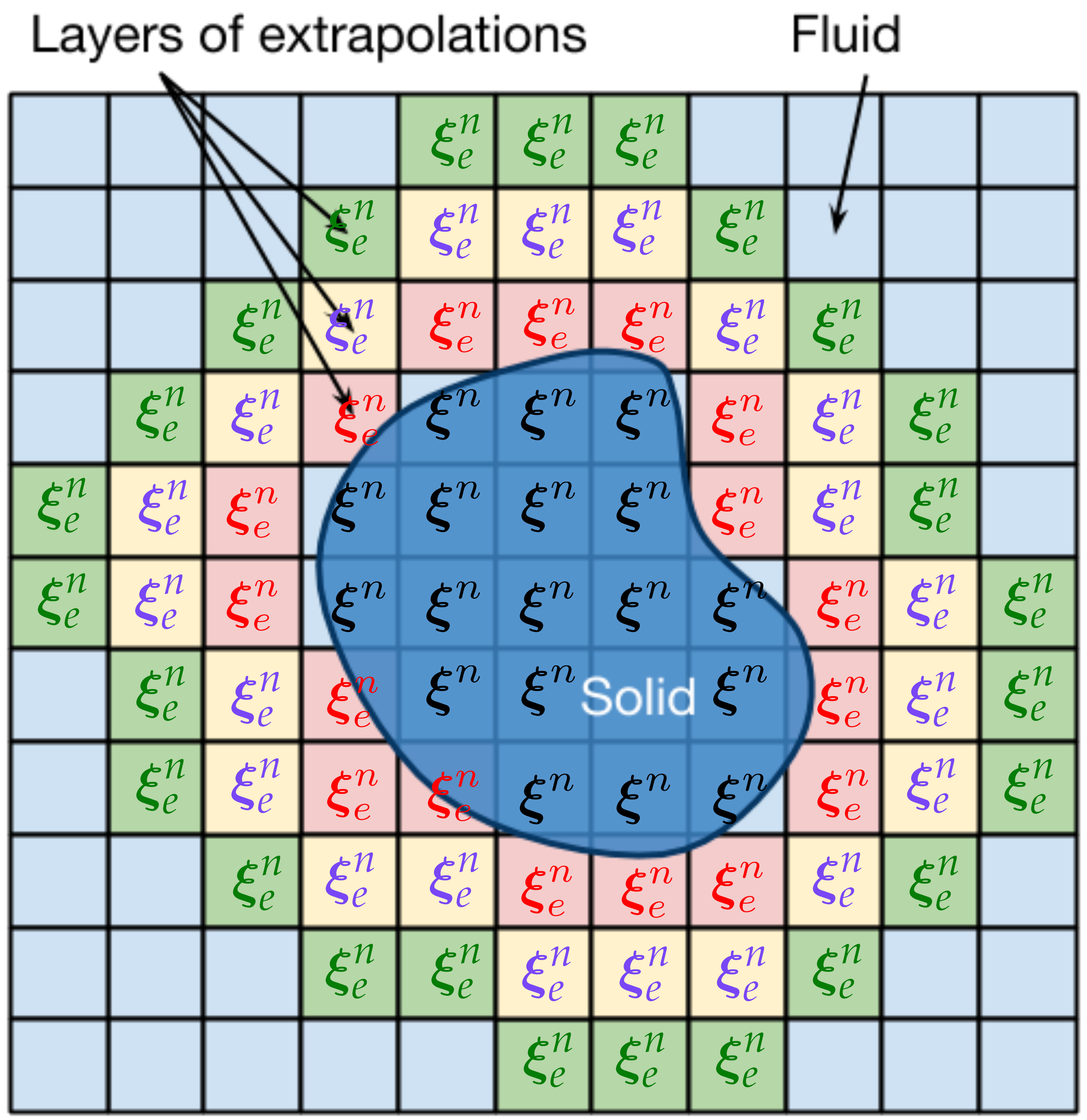
Linear regression

Linear regression-based extrapolation is effective, and easy to parallelize



Find next layer:

- Orthogonal neighbors
- Don’t have ref. map yet

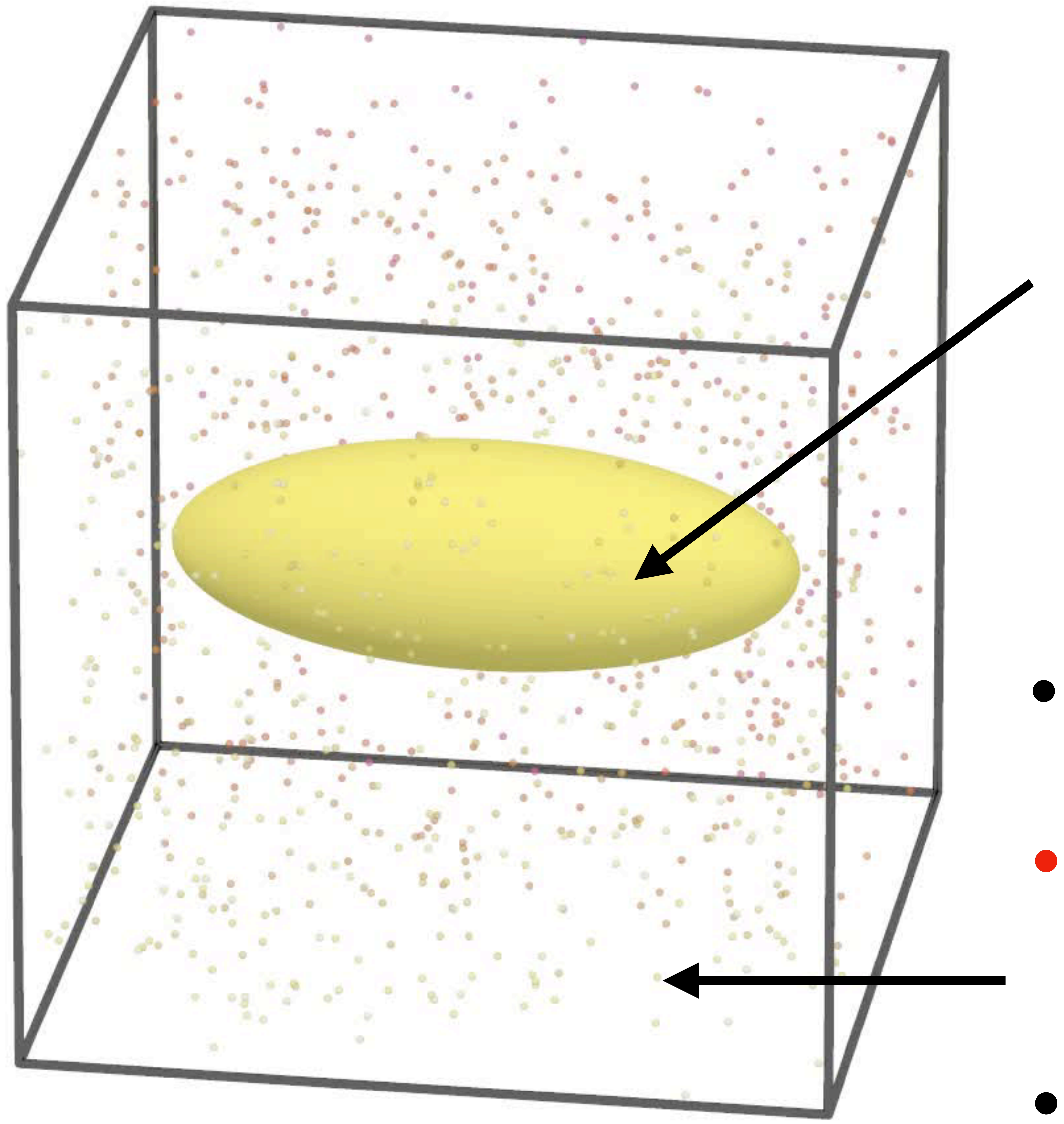


- First layer touches the solid
- Later layers use pre-existing data
- Adapted for domain decomposition (via MPI)
- Addresses the issues of PDE-based methods

The devil is in the detail

General

- *Staggered arrangement of variables on cell center, corners, and faces*
- *Custom fluid tracers and IO functions for data visualization*



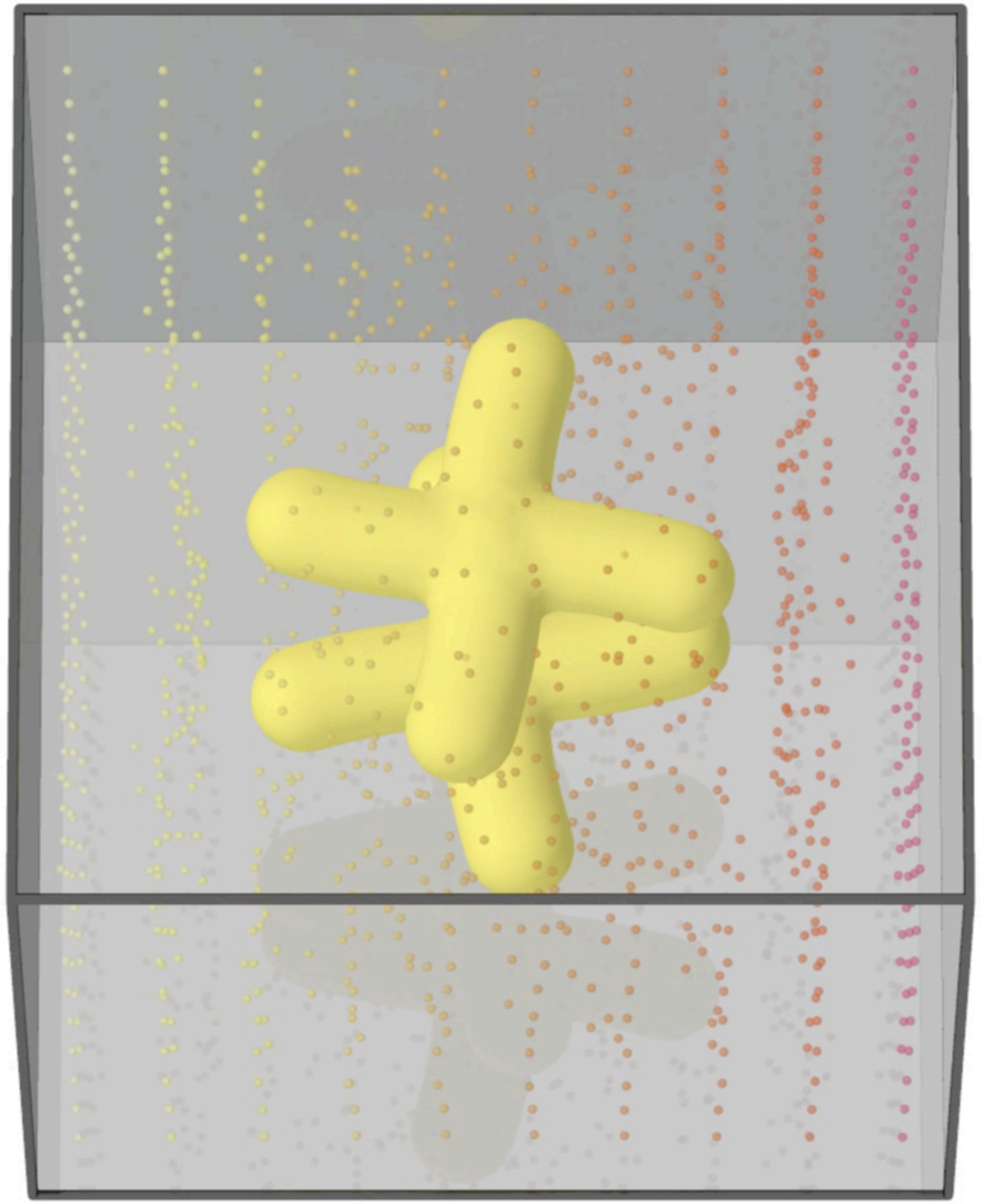
Solid

- *Specialized, efficient data structure to store extrapolations*
- *Solid behavior designs*

Fluid

- *Godunov-type upwinding method for hyperbolic parts of the Navier-Stokes*
- *Marker-and-Cell projection and approximate projection method to enforce incompressibility*
- *In-house Multigrid solver to solve large linear system*

Two rotors stirring a box



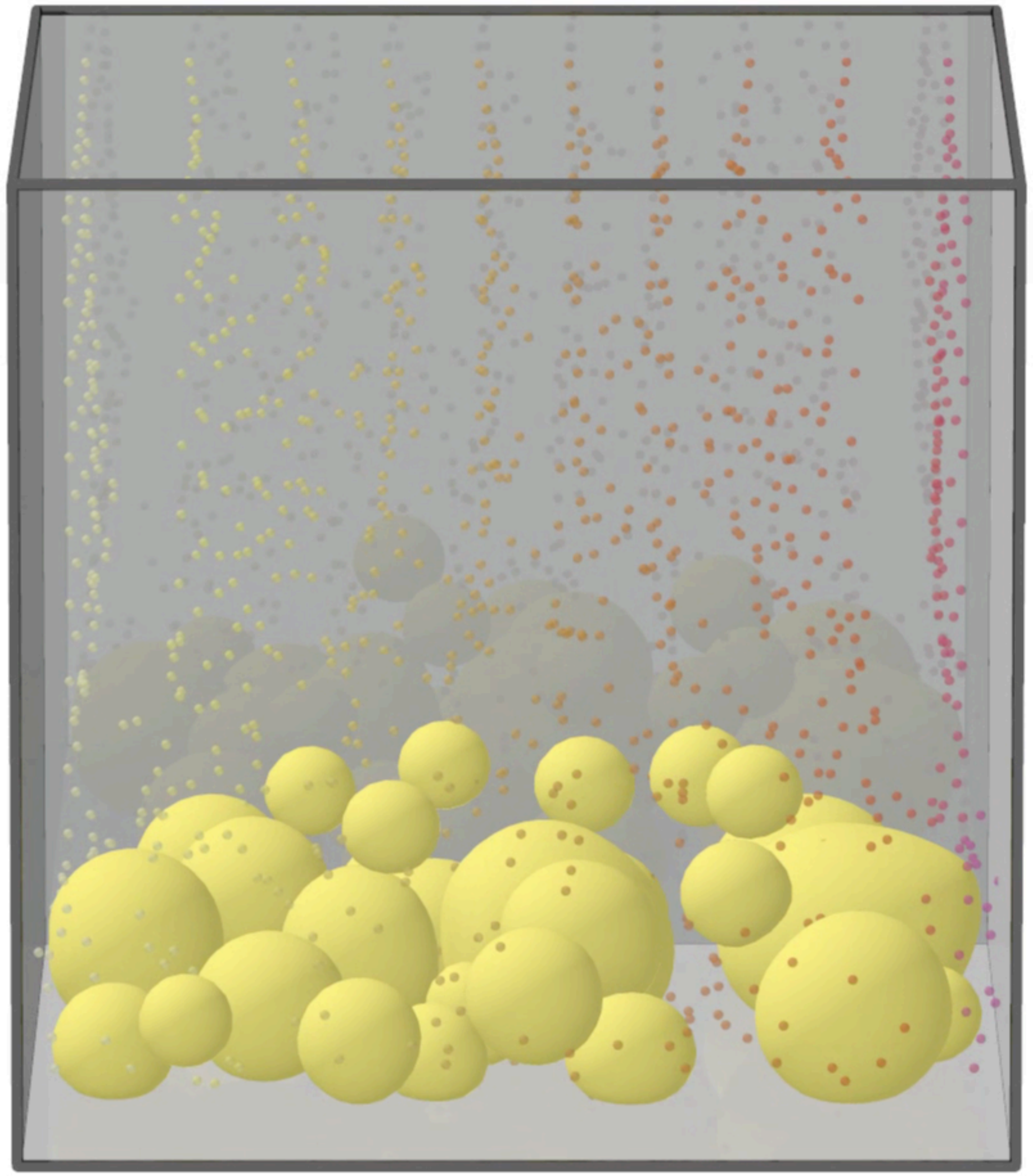
Parameters are dimensionless

Fluid kinematic viscosity: $\mu = 10^{-3}$

Solid shear modulus: $\mathrm{G} = 1.25$

Rotor period: $2\pi^2$

Sedimentation of multiple objects



Parameters are dimensionless

Fluid kinematic viscosity: $\mu = 6.7 \times 10^{-3}$

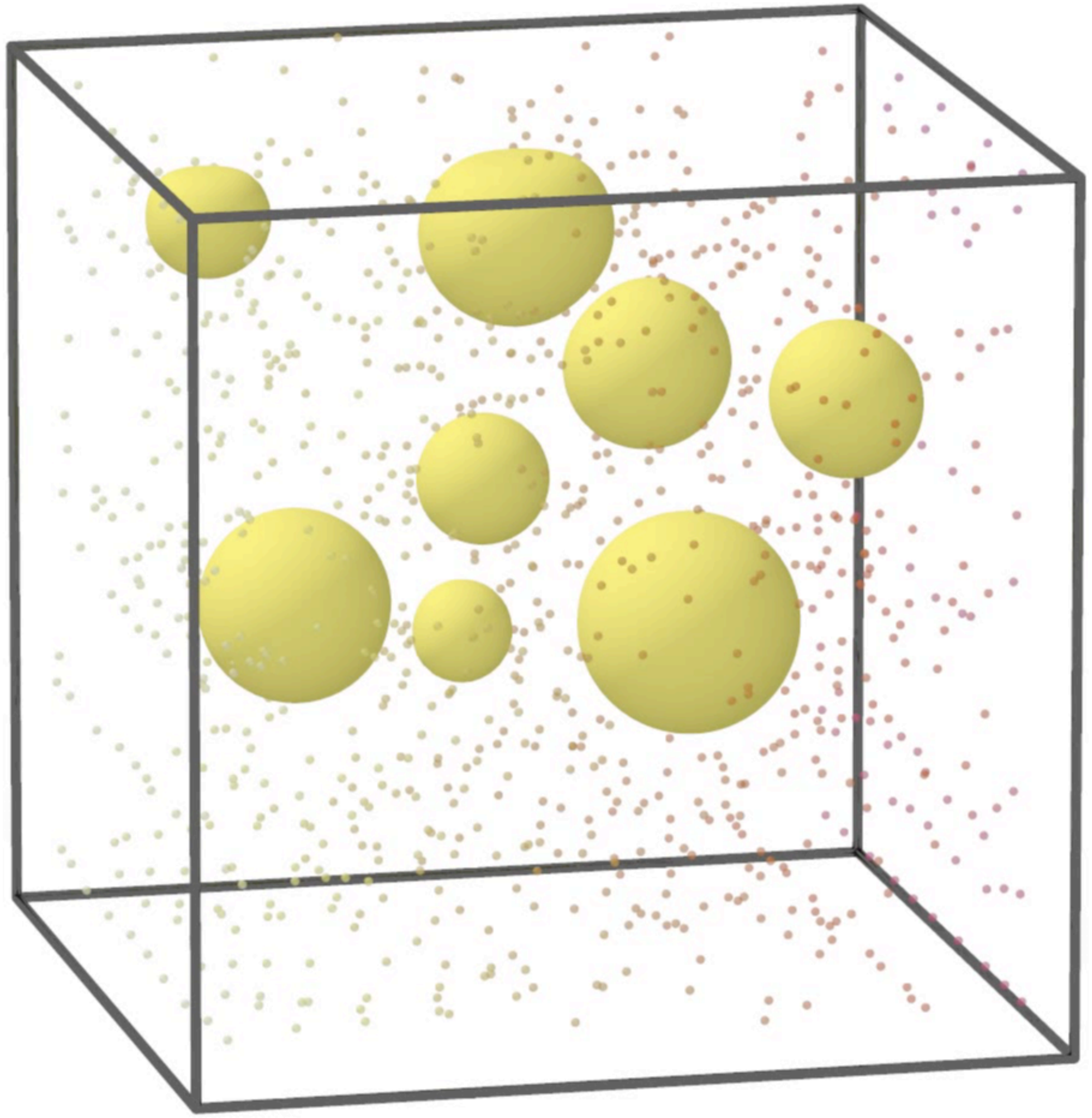
Solid shear modulus: $G = 0.5$

Density difference: $\Delta\rho = 0.1$

Number of objects: 30

Total time: 12

Lid driven cavity with soft, neutrally buoyant objects



Parameters are dimensionless

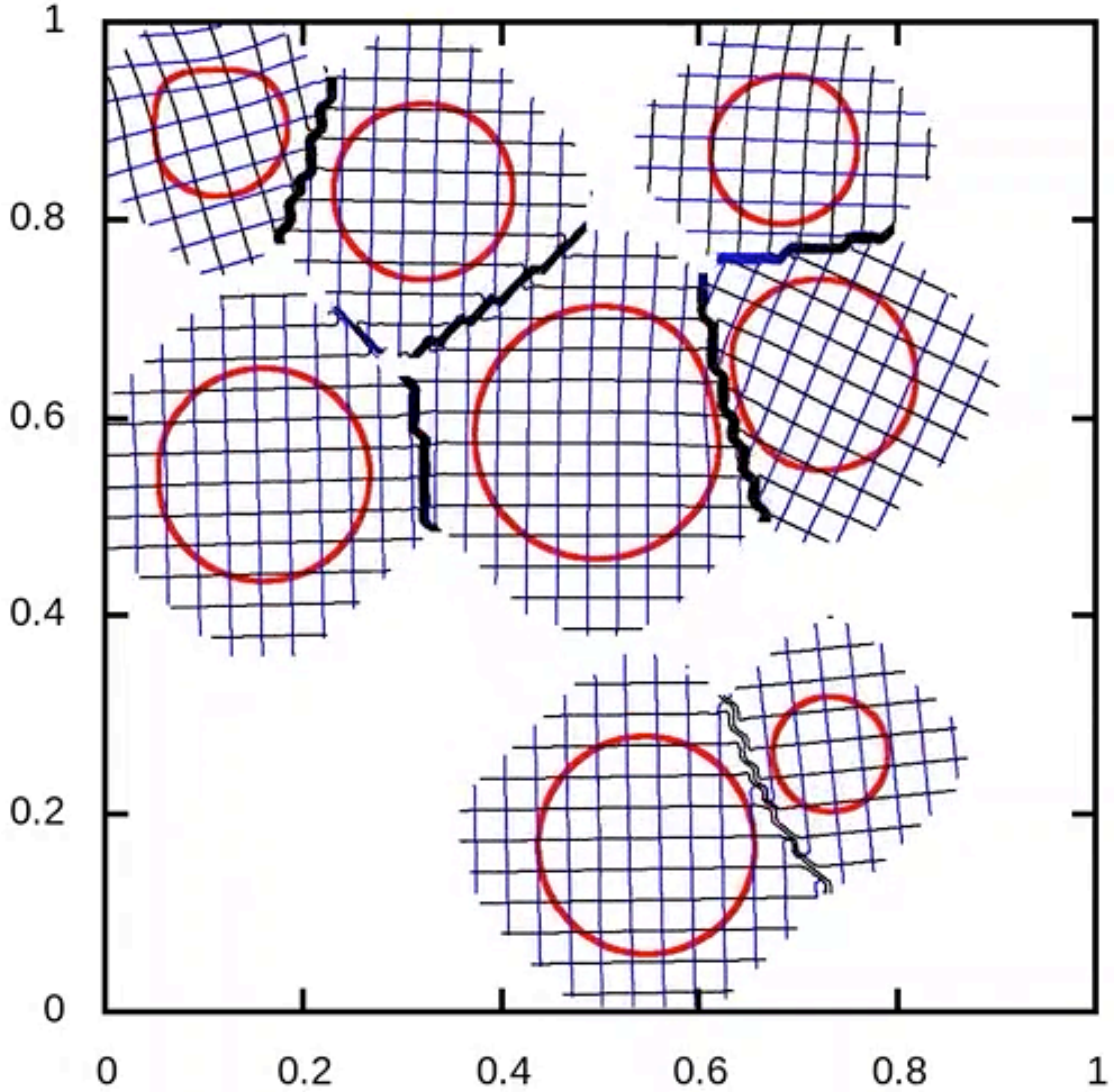
Fluid kinematic viscosity: $\mu = 10^{-3}$

Solid shear modulus: $\mathrm{G} = 0.25$

Top lid velocity: $v_x = 1$

Number of objects: 8

Lid driven cavity with soft, neutrally buoyant objects



Deformation in the reference map

Parameters are dimensionless

Fluid kinematic viscosity: $\mu = 10^{-3}$

Solid shear modulus: $G = 0.25$

Top lid velocity: $v_x = 1$

Number of objects: 8

Finally…

- RMT3D is very suitable for FSI problems with many, very deformable solids, think active matter, poro-elastic materials.
- Continue work started in practicum at LBL (RMT3D with AMR to model fibers, membranes, and other multi-scale FSIs)
- I want to thank CSGF for the generous, continuous support over the last 4 years!

Selected references

Rycroft C. H. et al., *Reference map for Incompressible Fluid-Structure Interaction.* J. Fluid Mech. 2020 (in press).

Valkov B. et al.**, 2015,** *Eulerian Method for Multiphase Interactions of Soft Solid Bodies in Fluids.* ASME. J. Appl. Mech. **82**(4), 041011-041011-14.

Rycroft C. H., Gibou F., 2012, *Simulations of a Stretching Bar Using a Plasticity Model from the Shear Transformation Zone Theory.* J. Comput. Phys. **5**(1) 2155-79.

Kamrin K., et al.**, 2012,** *Reference Map Technique for Finite-Strain Elasticity and Fluid-Solid Interaction.* J. Mech. Phys. Solids, **60**(11), 1952-69.

Movies of Pteropods taken from the Plankton Chronicles Project (vimeo), trematodes from Rafael Martin-Ledo (Twitter), Wright Brothers' early flight from the British Pathé (Youtube), blood flow through heart from Fraunhofer MEVIS (Youtube), emulsion in microfluidics from Tang's group @ Stanford.