THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

DOE CSGF

Complex Langevin in Nonrelativistic Rotating Bosonic Systems

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DOE CSGF Annual Program Review

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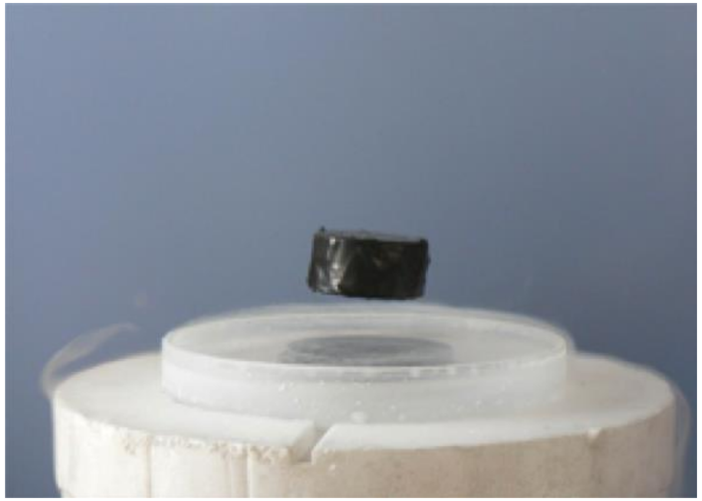
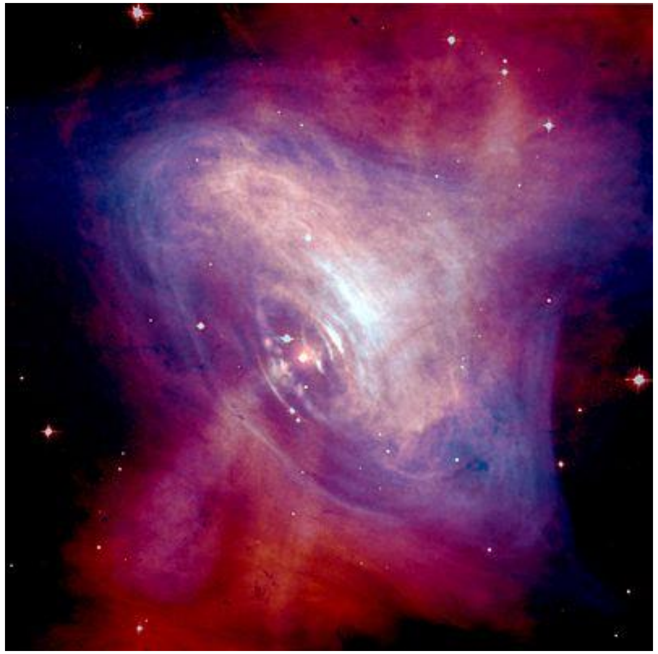
Rotating Bosonic Systems

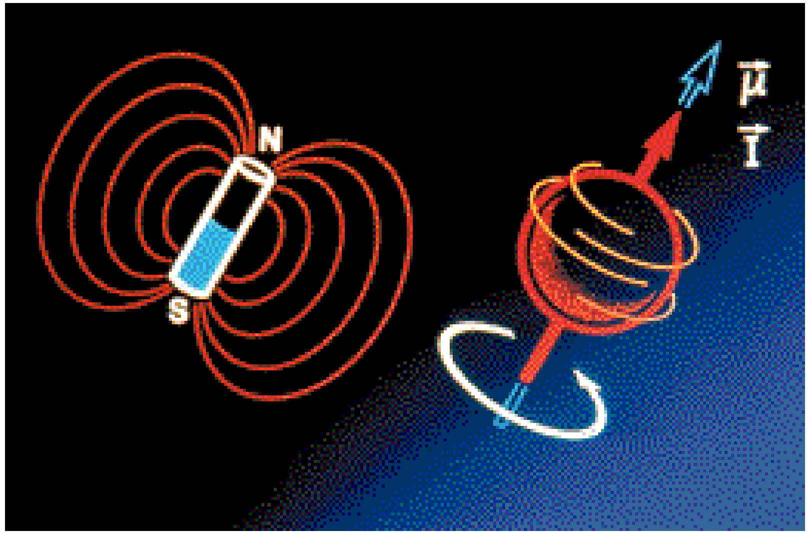
- What are nonrelativistic rotating bosons?
  - Integer-spin particles
  - Not moving very fast
  - Held in a region by an external trap
  - Interacting with each other
  - Rotating

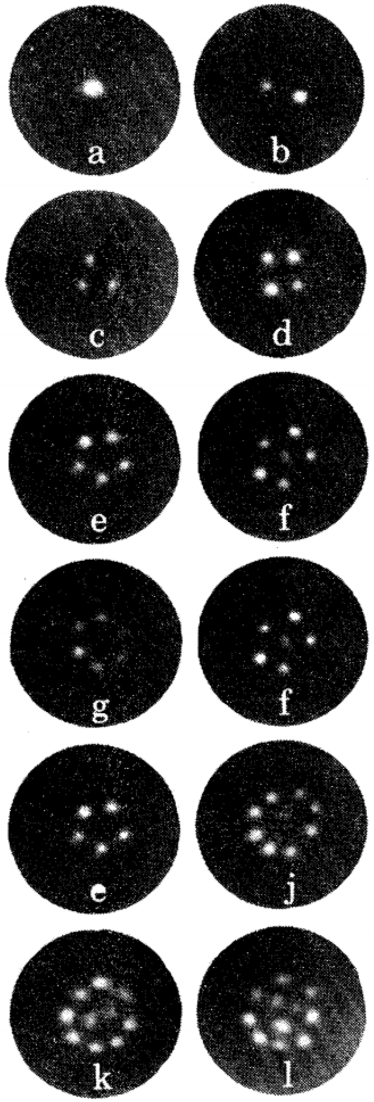
Rotating Bosonic Systems

- Relevant physical systems
  - Superconductor in a magnetic field
  - Pulsars
  - Rotating nuclei
  - Rotating superfluid
  - Rotating dilute atomic gas







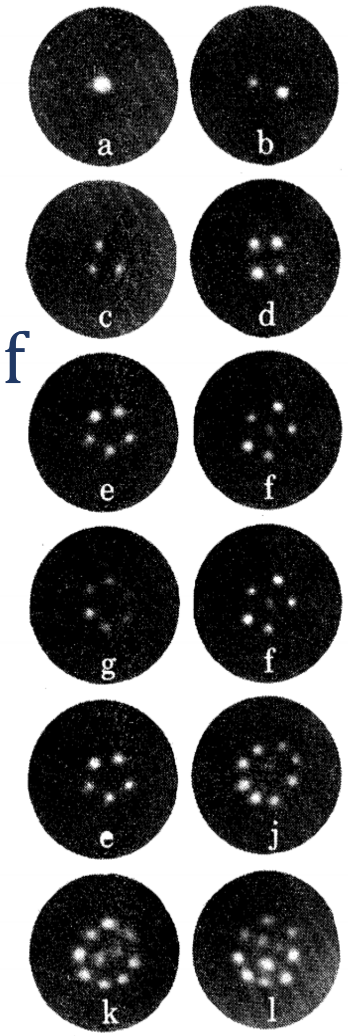


What work has been done so far on rotating bosons?

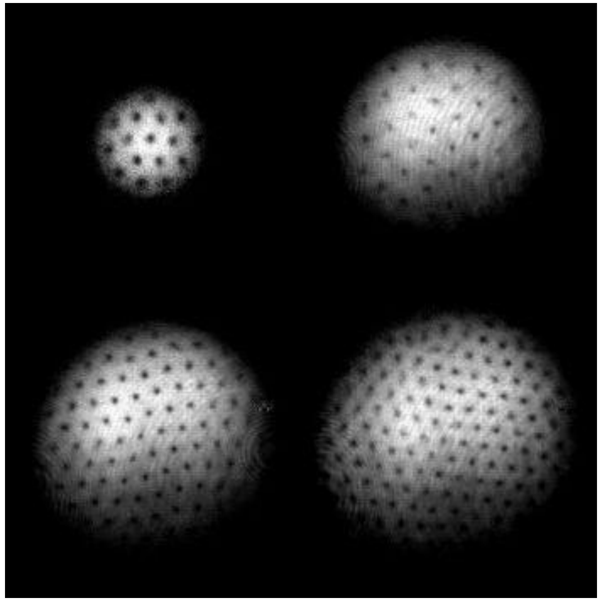
Rotating Bose-Einstein condensates

1949: Onsager predicts rotating superfluids will form vortices

1979: First observation of vortices in rotating ^4He



Phys. Rev. Lett. **4** 14 (1979)



Science **292** 5516 (2001)

1990s-2000s: rotating BECs in ^4He and dilute atomic gases

Advances in theory

Rotating Bose-Einstein condensates

Theoretical advancements in study of rotating superfluids since 1950s

?

Theoretical progress

- Why are we stuck?
- Many-body quantum systems → Quantum Monte Carlo (QMC)
- Path-integral formulation for QM allows us to compute observables

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-S[\phi]}$$

- S encodes the dynamics of the system

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \; e^{-S[\phi]} \mathcal{O}[\phi] = \int \mathcal{D}\phi \mathcal{P}[\phi] \mathcal{O}[\phi]$$

- QMC lets us evaluate stochastically, with $\mathcal{P}[\phi] = \frac{e^{-S[\phi]}}{\mathcal{Z}}$

Quantum Monte Carlo

- Simple example:

$$S[\phi] = a\phi^2$$

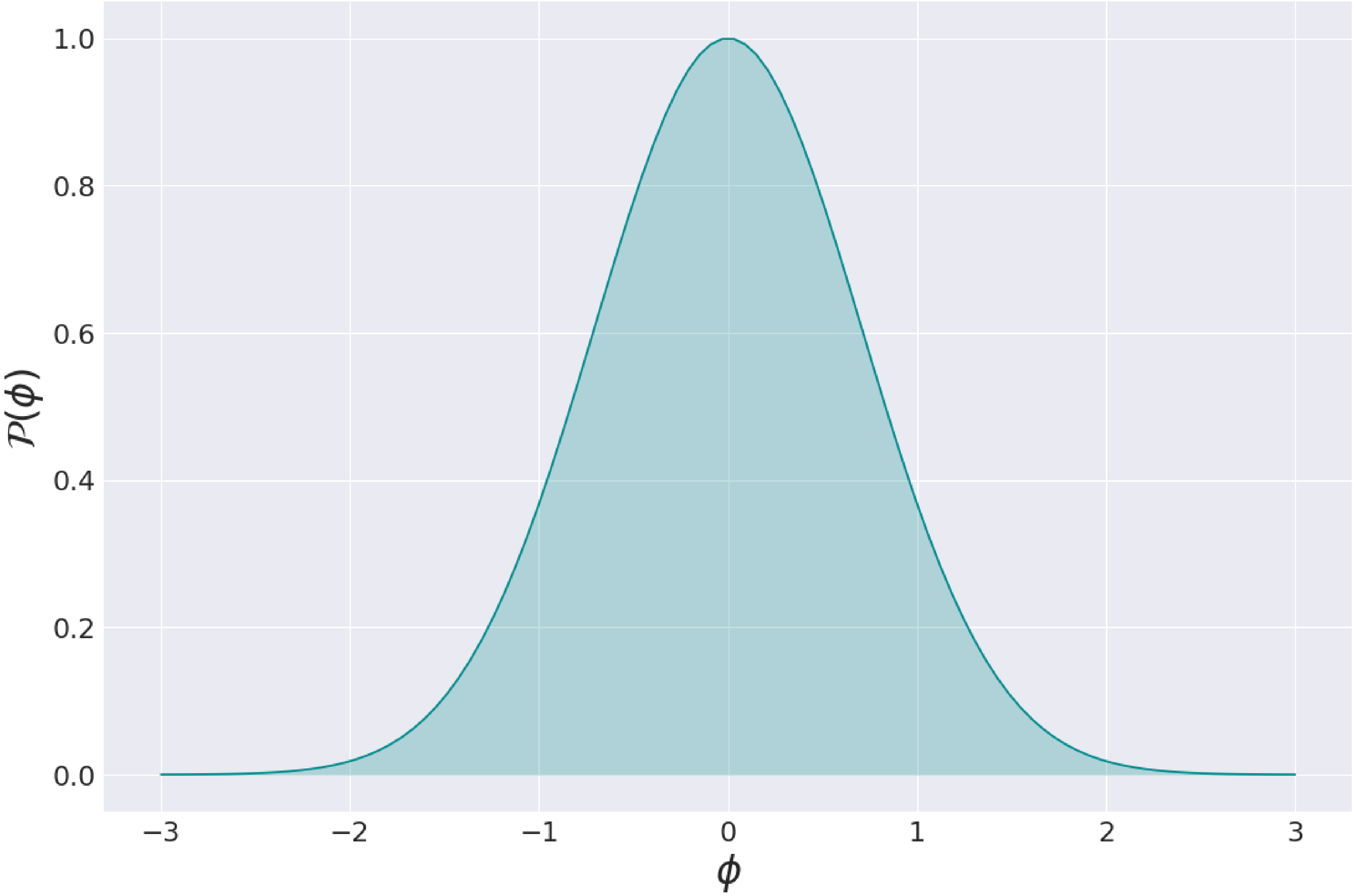
- Probability distribution:

$$\mathcal{P}[\phi] = e^{-a\phi^2}$$

- Calculating the energy:

$$E[\phi] = a\phi^2 \rightarrow \langle E \rangle = \sum E[\phi]P[\phi]$$

Quantum Monte Carlo



The sign problem

- Problem: what if our probability distribution is not well-defined?

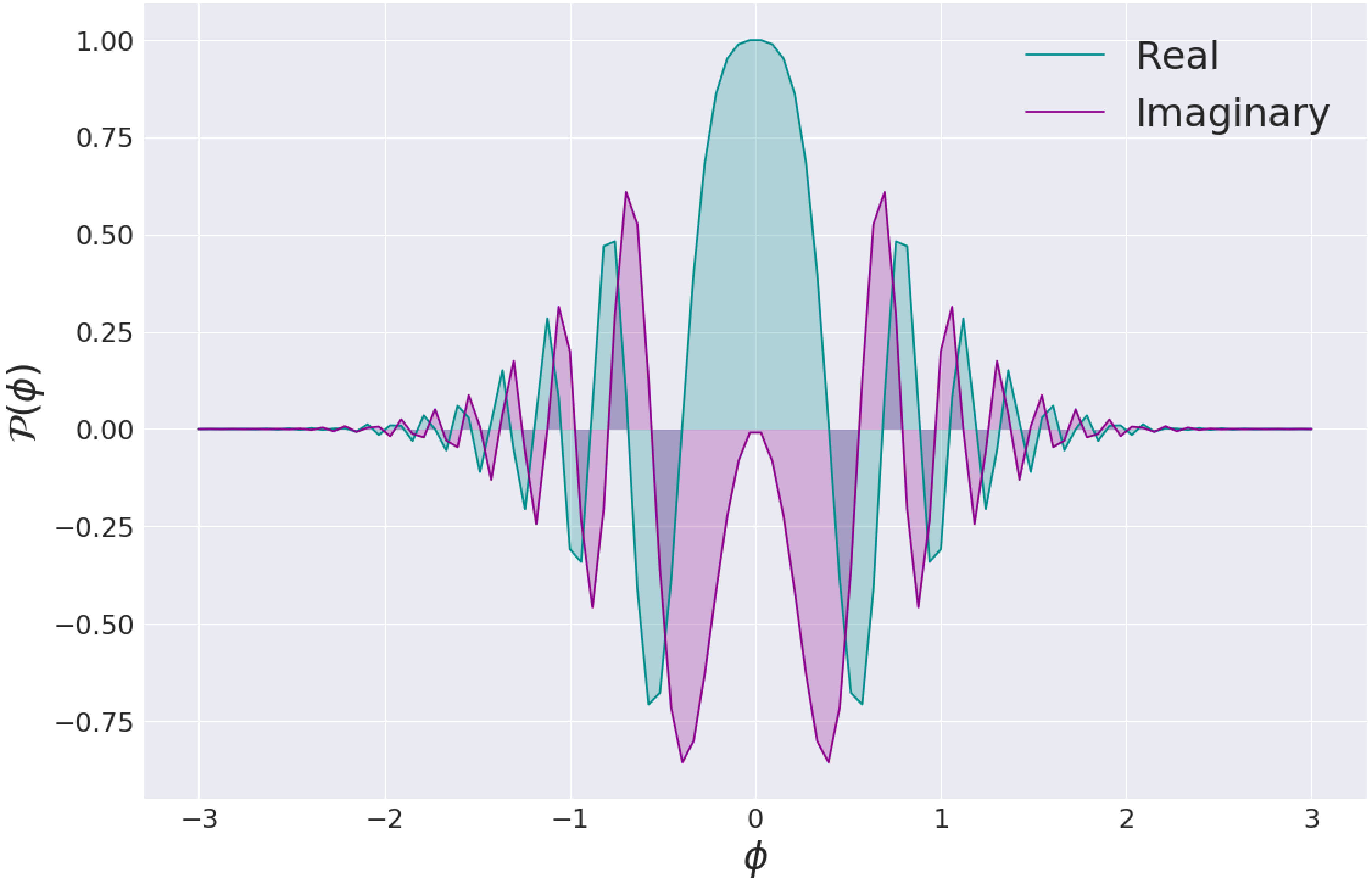
- Simple example:

$$S[\phi] = a\phi^2 + ib\phi^2$$

- Probability distribution:

$$\mathcal{P}[\phi] = e^{-a\phi^2 - ib\phi^2} = e^{-a\phi^2}\left(\cos(b\phi^2) + i\sin(b\phi^2)\right)$$

The sign problem



Complex Langevin…

The Langevin Method (Stochastic Quantization)

- Goal: compute observables

$$\langle \mathcal{O}[\phi] \rangle = \int \mathcal{D}\phi \mathcal{P}[\phi] \mathcal{O}[\phi]$$

- The Langevin equation evolves the fields in "fictitious" time

$$d\phi = -S'[\phi]dt + dW(t)$$

$$\frac{d\mathcal{P}(\phi;t)}{dt} = \left[\partial_\phi^2 S[\phi] + \partial_\phi^2\right] \mathcal{P}(\phi;t)$$

- Long time evolution produces sets of solutions distributed as e^{-S}

$$\langle \mathcal{O}[\phi] \rangle \approx \frac{1}{T} \int_{t_{\rm th}}^{t_{\rm th}+T} d\tau \mathcal{O}[\phi(\tau)]$$

Complex Langevin

- Complex Langevin extends this to a complex field

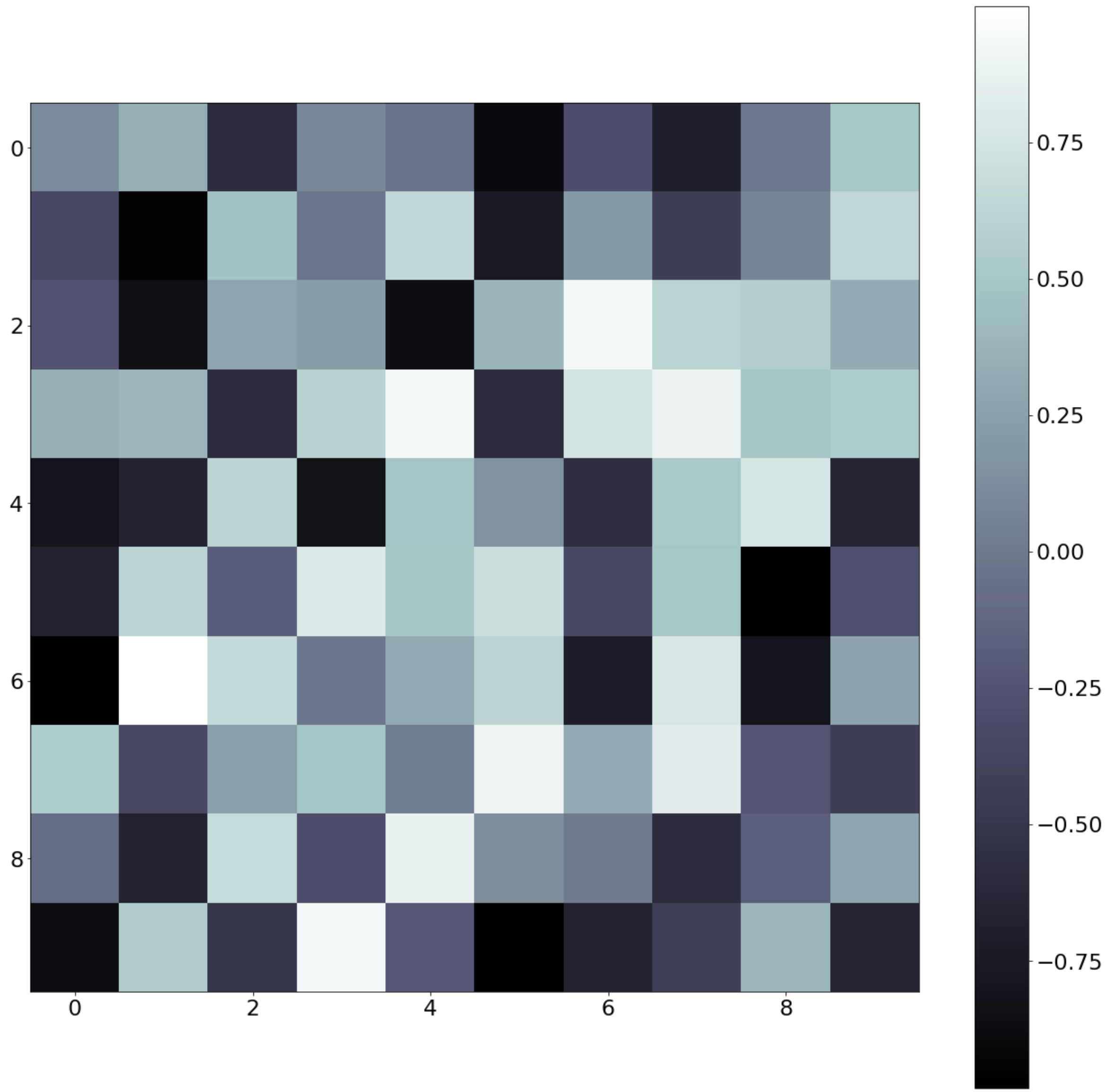
$$\phi \rightarrow \phi^R + i\phi^I$$
$$S[\phi] \rightarrow S[\phi^R + i\phi^I] = u + iv$$

$$d\phi^R = \mathrm{Re}\left[\frac{\partial S[\phi]}{\partial t}\right] dt + \eta^R$$
$$d\phi^I = \mathrm{Im}\left[\frac{\partial S[\phi]}{\partial t}\right] dt + \eta^I$$

Complex Langevin in action…

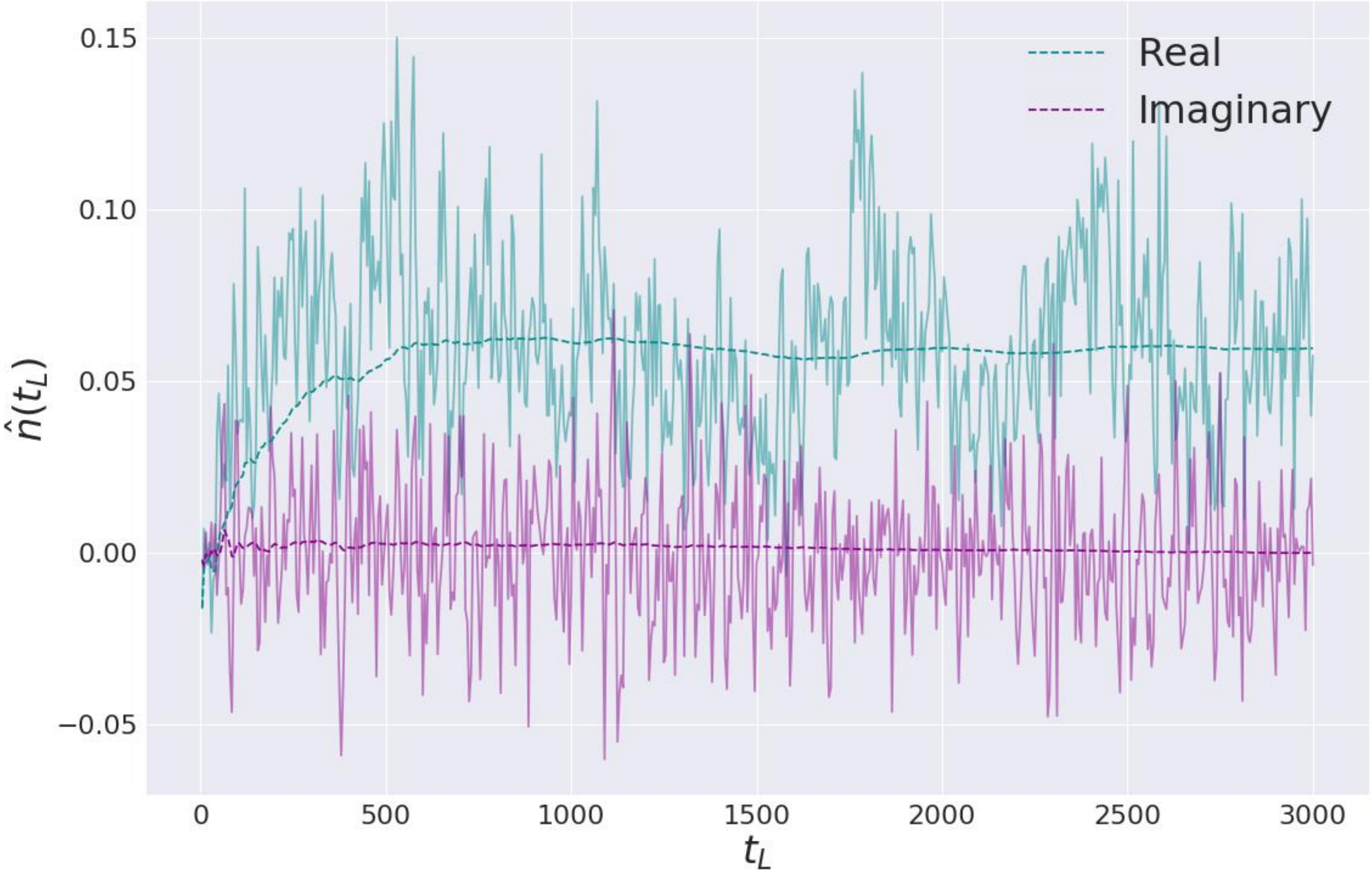
Putting the problem in discrete space

- Represent space as a finite lattice
- Evolve the fields at each site on the lattice
- Calculate observables from field values at each step
- Repeat until we have taken many samples (long time evolution)



Langevin time evolution of observables

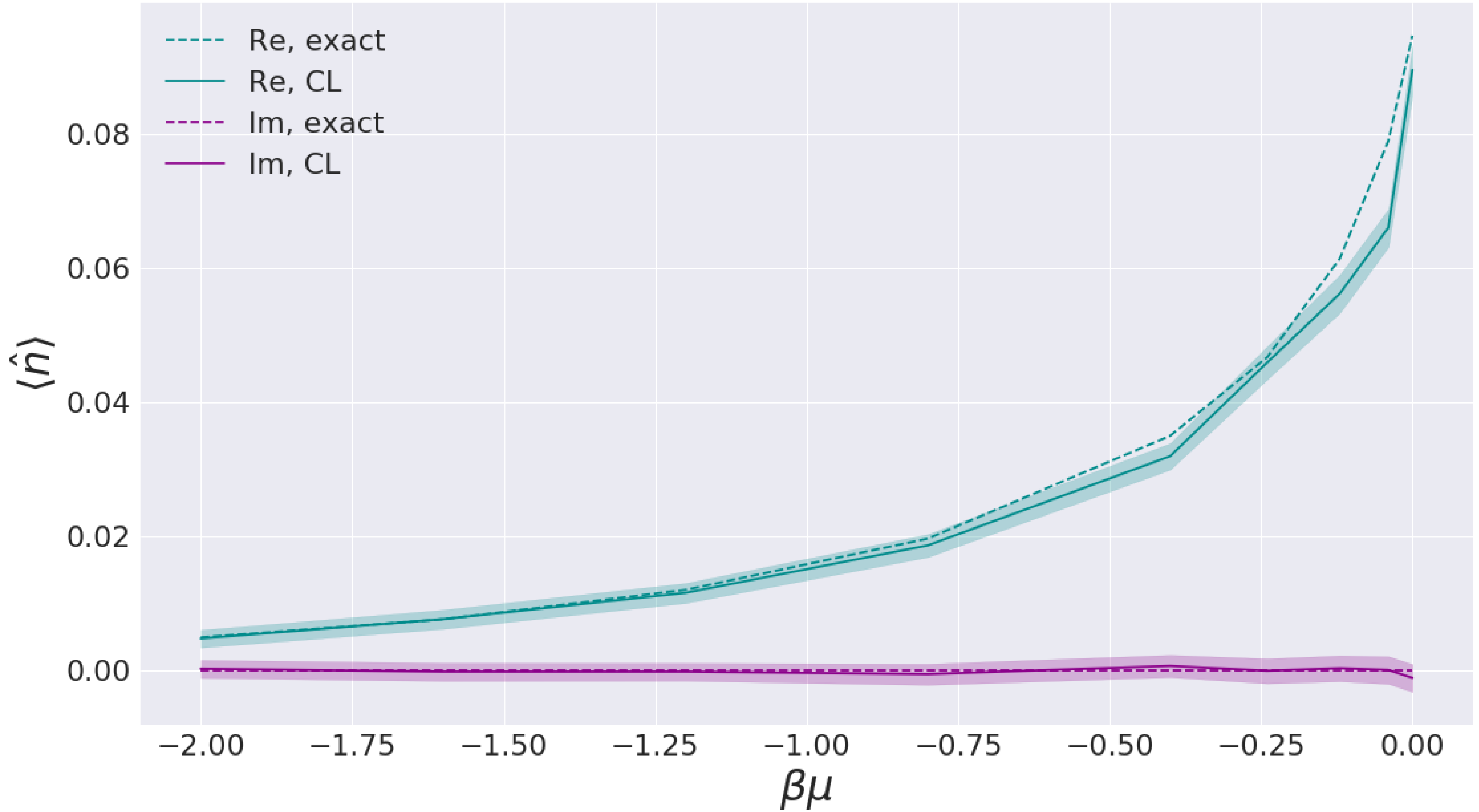
- Calculating observables: time average of the samples



Checking our work…

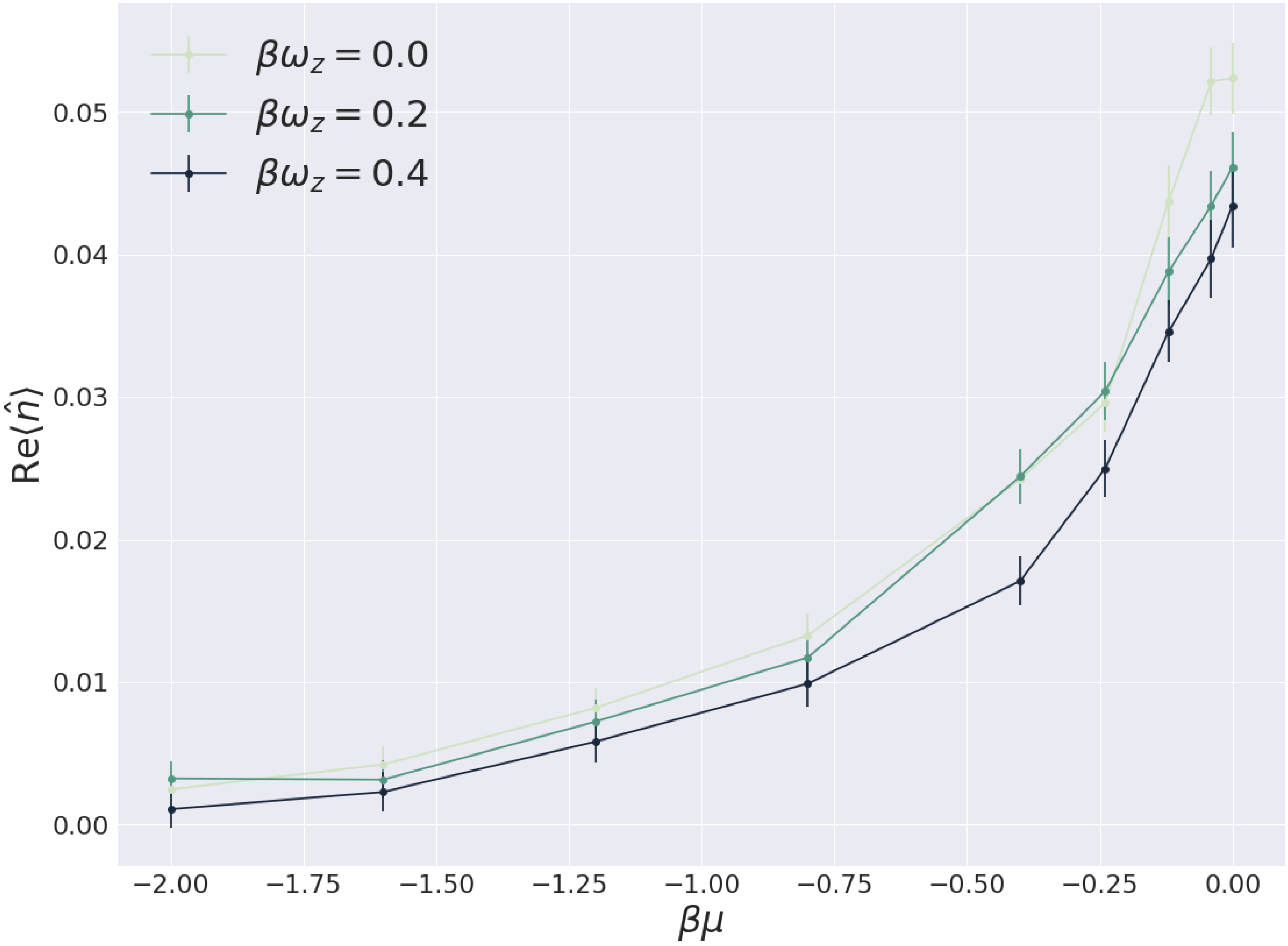
The free Bose gas

- Exactly solvable: no interaction, rotation, or external potentials

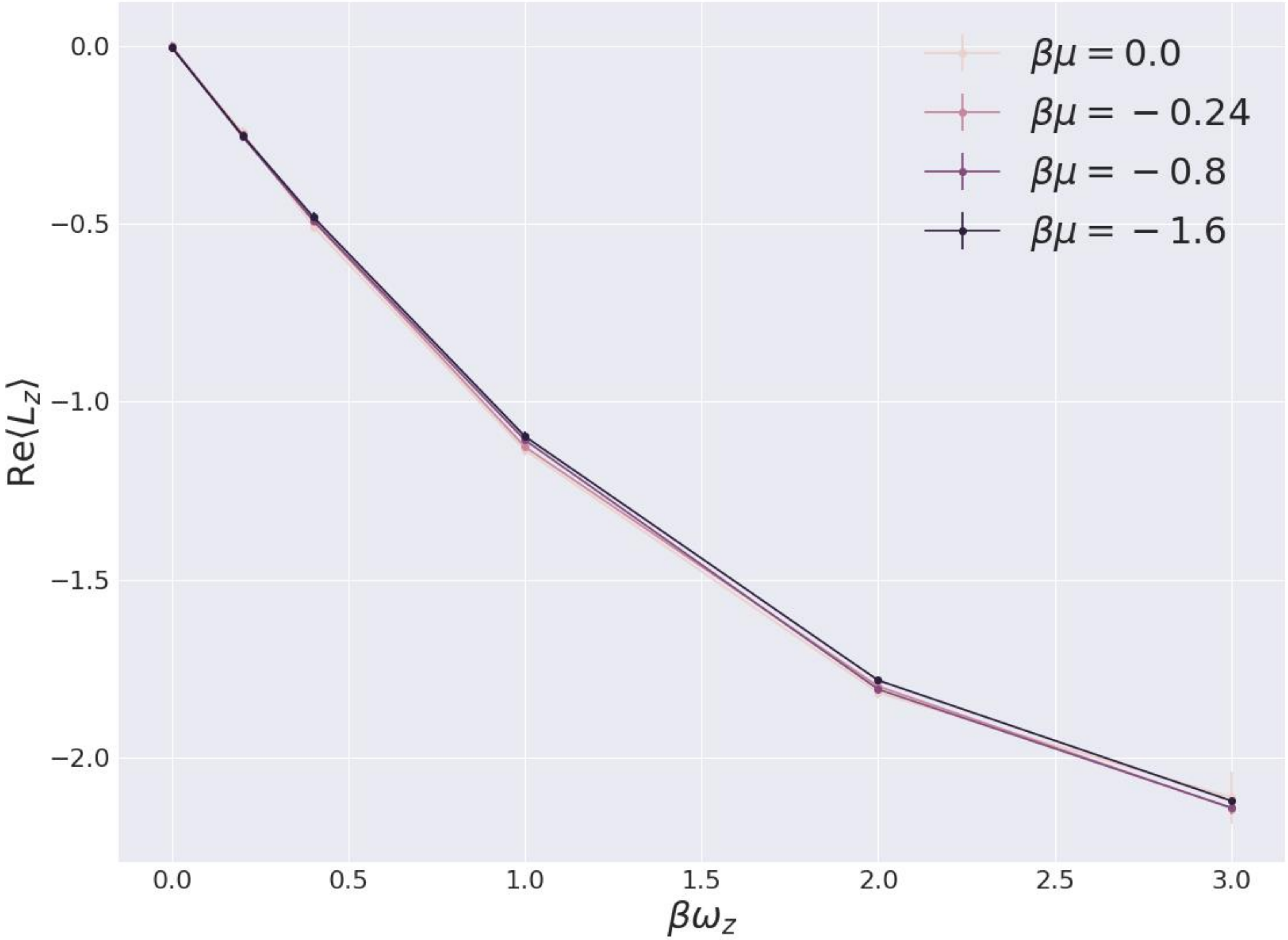


Preliminary results…

Density versus chemical potential

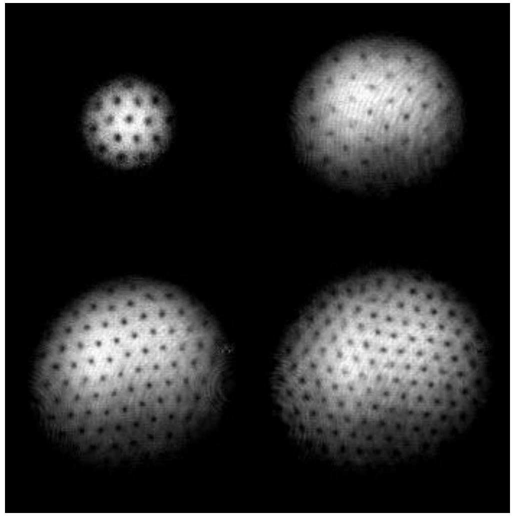


Angular momentum versus rotation frequency



Future work

- Future work
  - Look for vortex formation in larger lattices
  - Look for triangular lattice structure of vortices
  - Calculate circulation around vortices



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More about complex Langevin

“Complex Langevin and other approaches to the sign problem in quantum many-body physics”

C.E. Berger, L Rammelmüller, A.C. Loheac, F. Ehmann, J. Braun, J.E. Drut

Acknowledgments

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