Complex Langevin in Nonrelativistic Rotating Bosonic Systems

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• What are nonrelativistic rotating bosons?
  • Integer-spin particles
  • Not moving very fast
  • Held in a region by an external trap
  • Interacting with each other
  • Rotating
Rotating Bosonic Systems

- Relevant physical systems
  - Superconductor in a magnetic field
  - Pulsars
  - Rotating nuclei
  - Rotating superfluid
  - Rotating dilute atomic gas
What work has been done so far on rotating bosons?
1949: Onsager predicts rotating superfluids will form vortices

1979: First observation of vortices in rotating $^4$He

1990s-2000s: rotating BECs in $^4$He and dilute atomic gases

Advances in theory

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Rotating Bose-Einstein condensates

Theoretical advancements in study of rotating superfluids since 1950s
Theoretical progress

• Why are we stuck?
• Many-body quantum systems → Quantum Monte Carlo (QMC)
• Path-integral formulation for QM allows us to compute observables

\[ \mathcal{Z} = \int \mathcal{D}\phi e^{-S[\phi]} \]

• S encodes the dynamics of the system

\[ \langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \ e^{-S[\phi]} \mathcal{O}[\phi] = \int \mathcal{D}\phi \mathcal{P}[\phi] \mathcal{O}[\phi] \]

• QMC lets us evaluate stochastically, with \( \mathcal{P}[\phi] = \frac{e^{-S[\phi]}}{\mathcal{Z}} \)
Quantum Monte Carlo

• Simple example:

\[ S[\phi] = a\phi^2 \]

• Probability distribution:

\[ P[\phi] = e^{-a\phi^2} \]

• Calculating the energy:

\[ E[\phi] = a\phi^2 \rightarrow \langle E \rangle = \sum E[\phi]P[\phi] \]
Quantum Monte Carlo
The sign problem

• Problem: what if our probability distribution is not well-defined?

• Simple example:

\[ S[\phi] = a\phi^2 + ib\phi^2 \]

• Probability distribution:

\[ P[\phi] = e^{-a\phi^2 - ib\phi^2} = e^{-a\phi^2} \left( \cos(b\phi^2) + i \sin(b\phi^2) \right) \]
The sign problem

\[ P(\phi) \]

-3 -2 -1 0 1 2 3

Real

Imaginary
Complex Langevin...
The Langevin Method (Stochastic Quantization)

- Goal: compute observables
  \[
  \langle \mathcal{O}[\phi] \rangle = \int \mathcal{D}\phi \mathcal{P}[\phi] \mathcal{O}[\phi]
  \]

- The Langevin equation evolves the fields in “fictitious” time
  \[
  d\phi = -S'[\phi]dt + dW(t)
  \]

- Long time evolution produces sets of solutions distributed as $e^{-S}$
  \[
  \langle \mathcal{O}[\phi] \rangle \approx \frac{1}{T} \int_{t_{th}}^{t_{th}+T} d\tau \mathcal{O}[\phi(\tau)]
  \]
• Complex Langevin extends this to a complex field

\[ \phi \rightarrow \phi^R + i\phi^I \]

\[ S[\phi] \rightarrow S[\phi^R + i\phi^I] = u + iv \]

\[ d\phi^R = \text{Re} \left[ \frac{\partial S[\phi]}{\partial t} \right] dt + \eta^R \]

\[ d\phi^I = \text{Im} \left[ \frac{\partial S[\phi]}{\partial t} \right] dt + \eta^I \]
Complex Langevin in action...
Putting the problem in discrete space

- Represent space as a finite lattice
- Evolve the fields at each site on the lattice
- Calculate observables from field values at each step
- Repeat until we have taken many samples (long time evolution)
• Calculating observables: time average of the samples
Checking our work...
• Exactly solvable: no interaction, rotation, or external potentials
Preliminary results...
Density versus chemical potential

\[ \beta \omega_z = 0.0 \]
\[ \beta \omega_z = 0.2 \]
\[ \beta \omega_z = 0.4 \]
Angular momentum versus rotation frequency

\[ \beta \omega_z \]

\[ \text{Re}(L_z) \]

- \( \beta \mu = 0.0 \)
- \( \beta \mu = -0.24 \)
- \( \beta \mu = -0.8 \)
- \( \beta \mu = -1.6 \)
Future work

• Look for vortex formation in larger lattices

• Look for triangular lattice structure of vortices

• Calculate circulation around vortices

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“Complex Langevin and other approaches to the sign problem in quantum many-body physics”

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