GEOMETRIC VOLUME OF FLUID METHOD

SIMULATION OF TWO-PHASE FLOW ON UNSTRUCTURED MESHES

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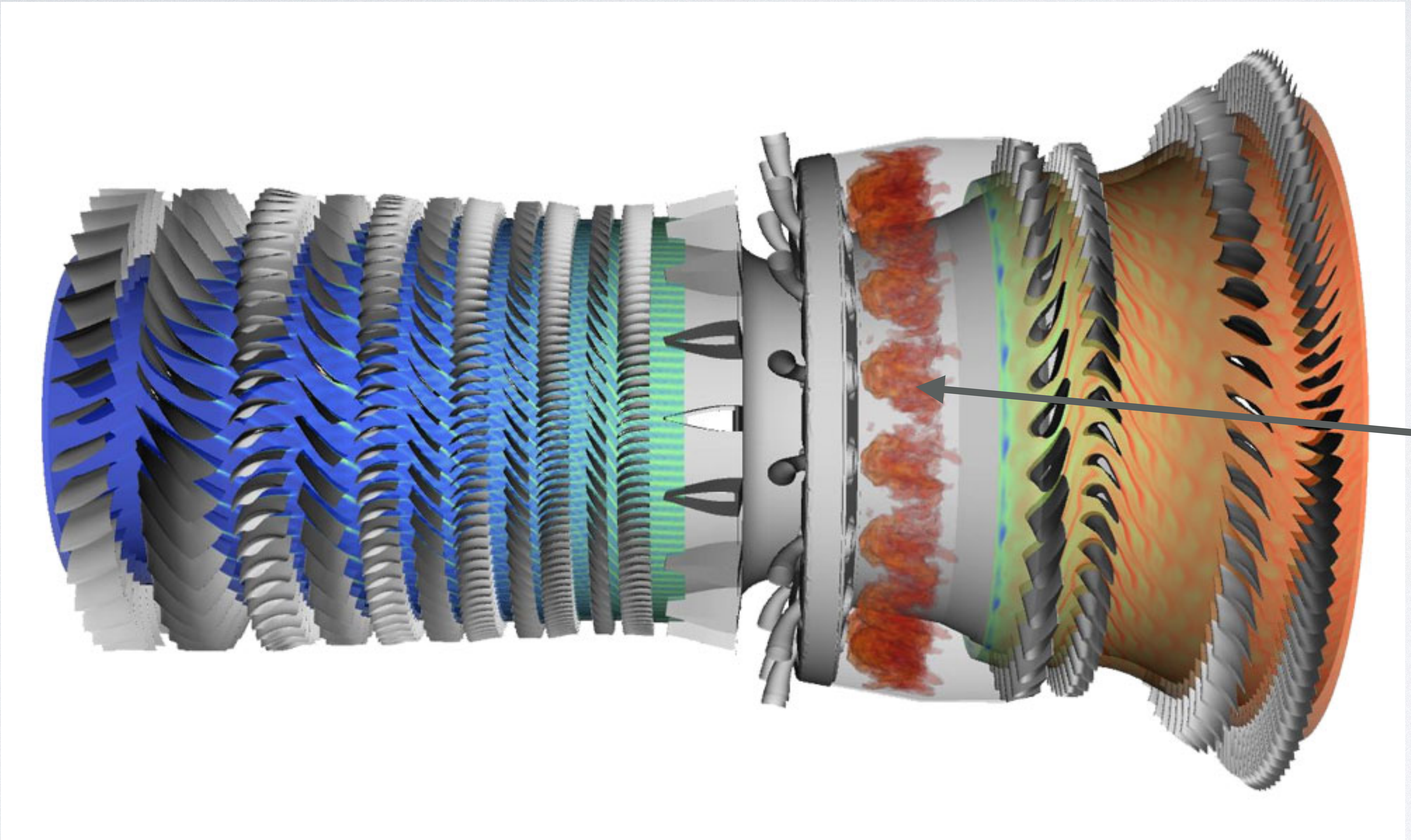
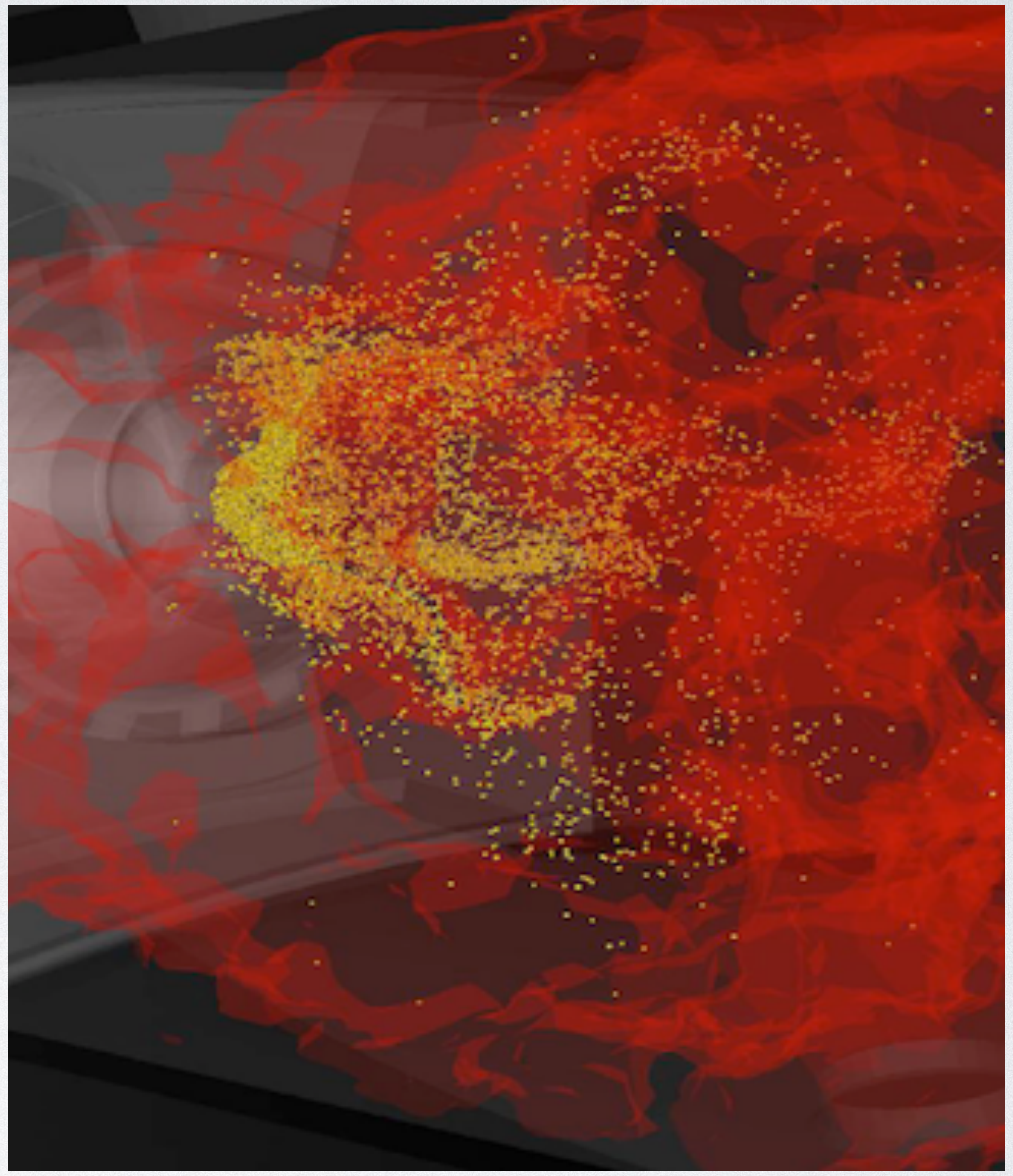
MOTIVATION



Pratt & Whitney 6000 turbofan engine
Stanford ASC program



zoom of combustor sector with spray
Cascade Technologies, Inc.

VOLUME OF FLUID (VOF) METHOD

$$\nabla \cdot \vec{u} = 0$$

$$\boxed{\frac{df}{dt} + \nabla \cdot (f\vec{u}) = 0}$$

$$\frac{d\left(\rho\vec{u}\right)}{dt} + \nabla \cdot (\rho\vec{u}) = -\nabla P + \nabla \cdot \left(\mu\left(\nabla\vec{u} + \nabla\vec{u}^{\top}\right)\right) - \boxed{\sigma\kappa\hat{n}\delta_S}$$

$$\{\rho, \mu\} = f\{\rho_1, \mu_1\} + (1-f)\{\rho_2, \mu_2\}$$

$$\boxed{\kappa = \nabla \cdot \hat{n}}$$

- advection of the marker function must be conservative and bounded
- estimates of the normal and curvature must come from the marker function field

HEIGHT FUNCTIONS

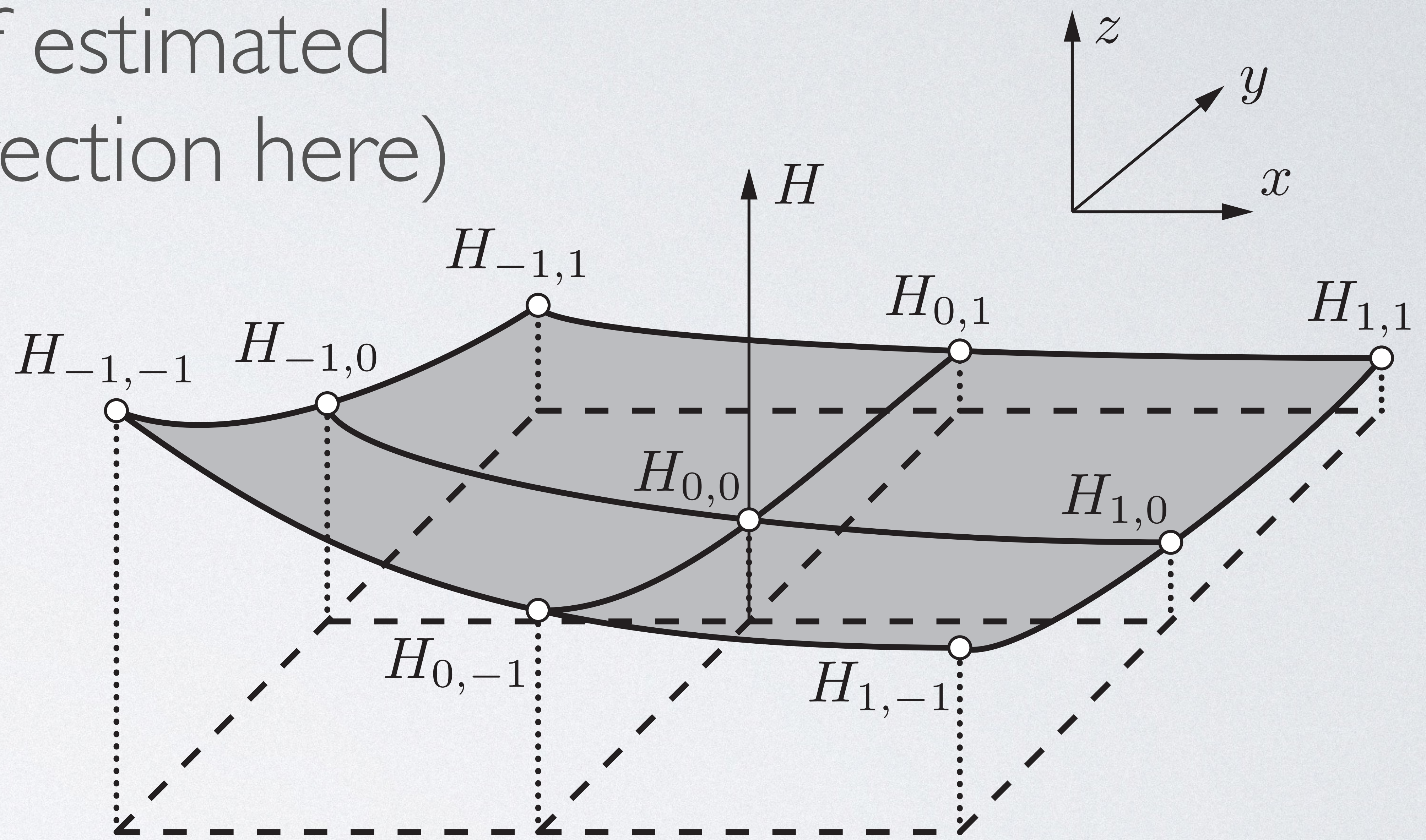
height function in direction of max component of estimated surface normal (z direction here)

$$F = \int_{\Omega} f dV$$

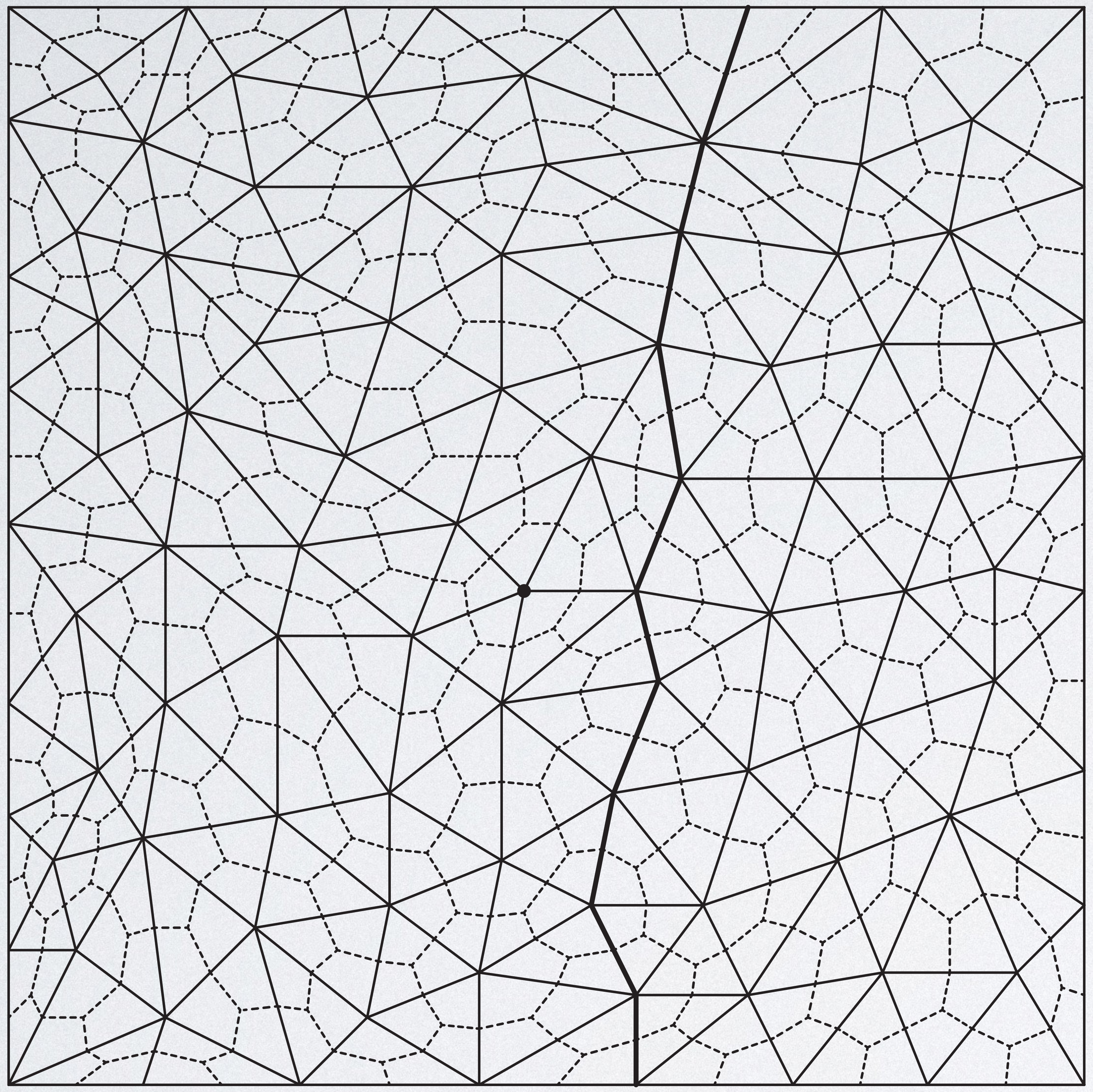
$$\boxed{H_{r,s} \approx \sum_{t=-t_{down}}^{t_{up}} F^*_{i+r,j+s,k+t} \Delta z}$$

$$\hat{n} = -\frac{\langle H_x \;\; H_y \;\; -\operatorname{sign}(n_z) \rangle^\top}{\sqrt{1 + H_x^2 + H_y^2}}$$

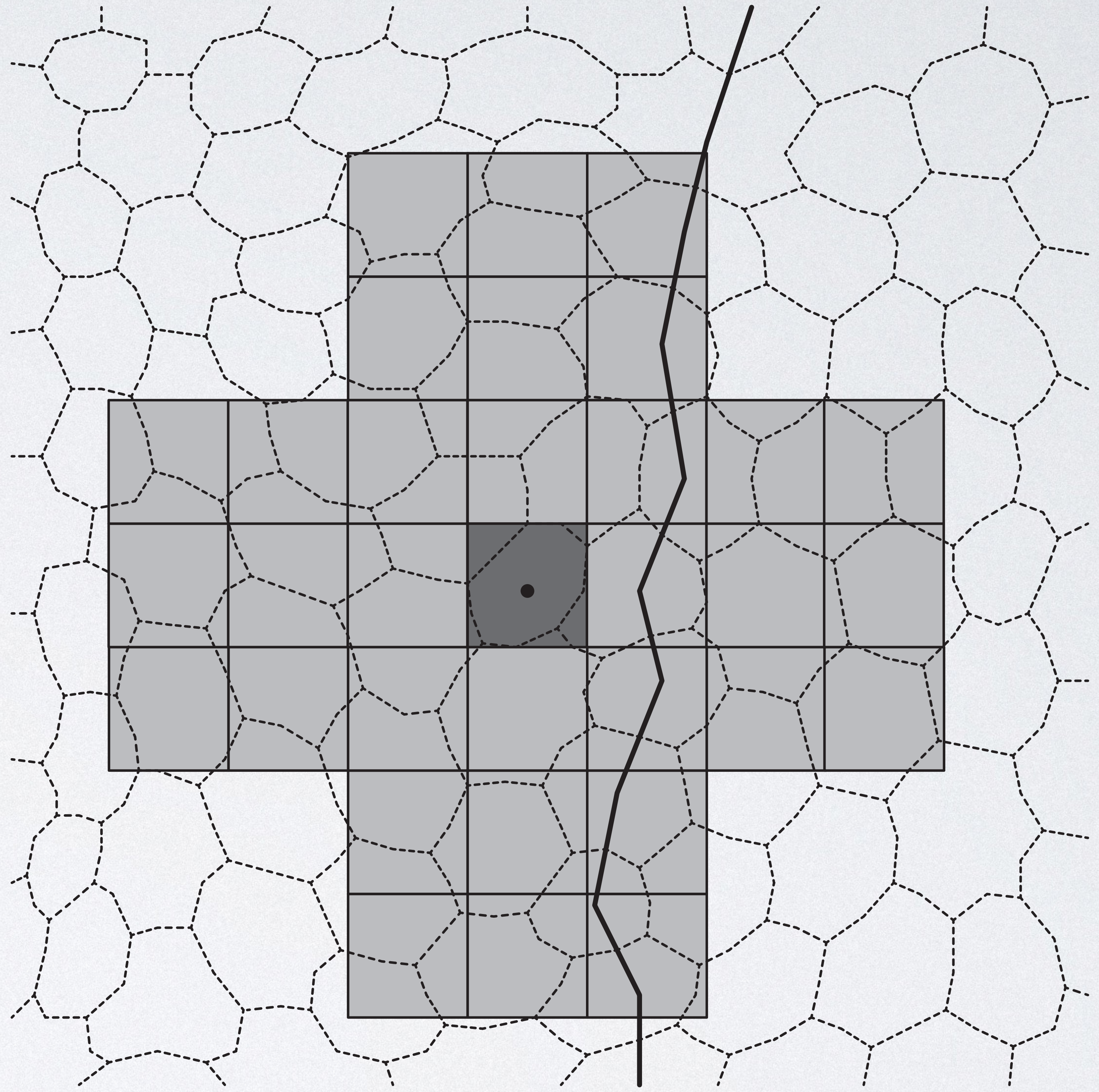
$$\kappa = -\frac{H_{xx}\left(1 + H_y^2\right) + H_{yy}\left(1 + H_y^2\right) - 2H_{xy}H_xH_y}{\left(1 + H_x^2 + H_y^2\right)^{3/2}}$$



EMBEDDED HEIGHT FUNCTIONS (EHF)

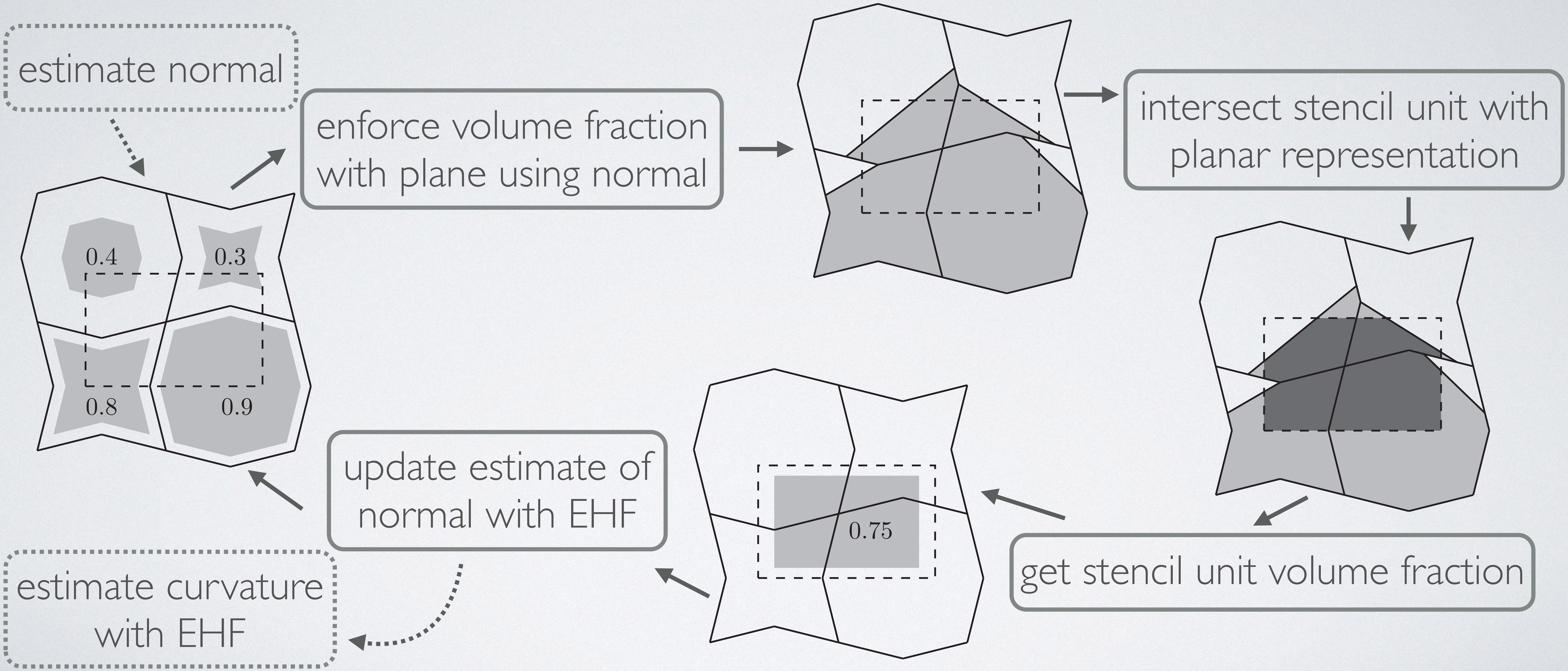


primal (solid line) and median-dual (hashed line) meshes with partitioning (thick line)



embedded cartesian stencil (light gray region) for bounding box (dark gray region) of median-dual mesh element

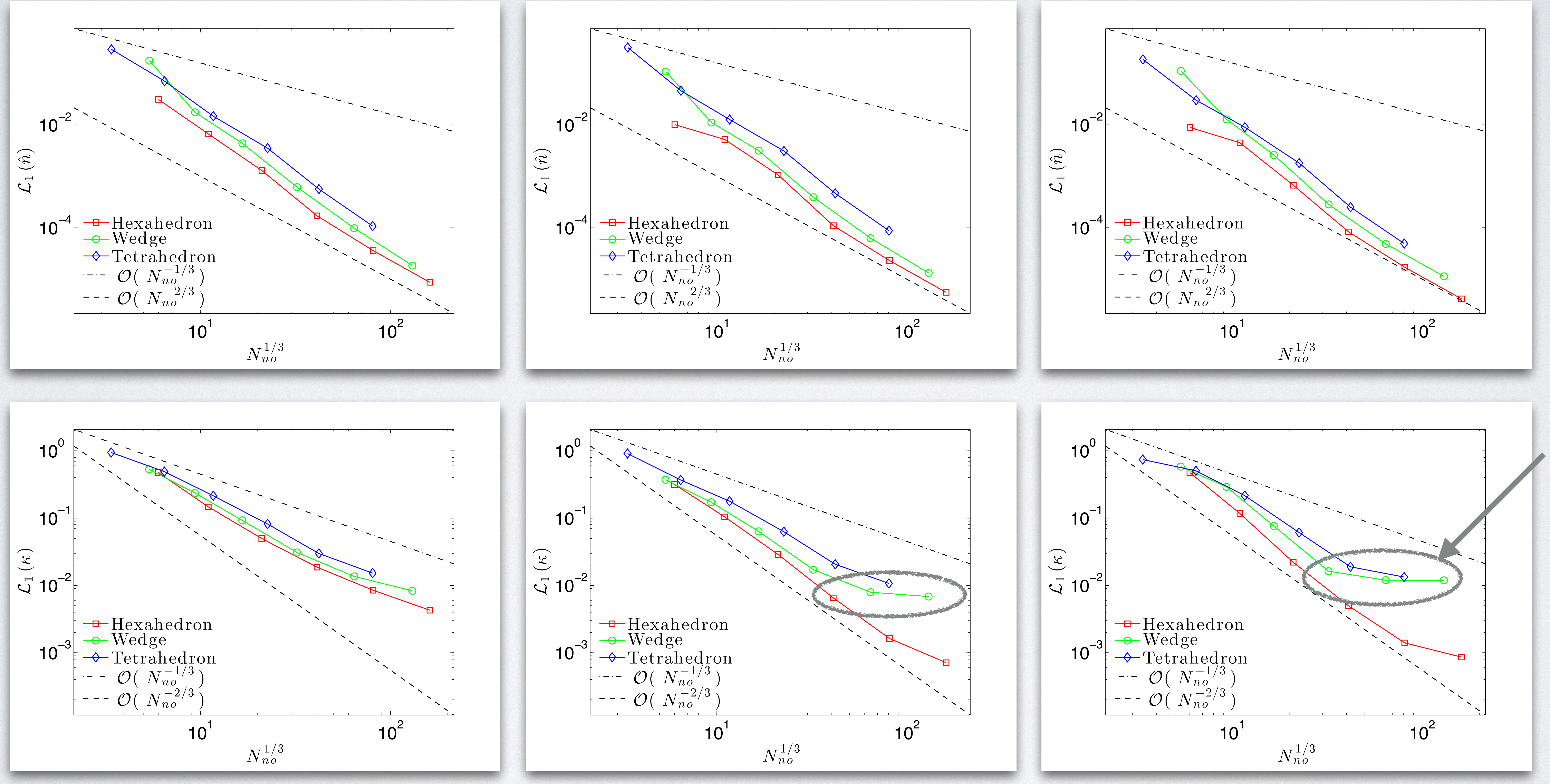
PROJECTION TO EHF STENCIL



NORMAL AND CURVATURE TESTS

normal

curvature

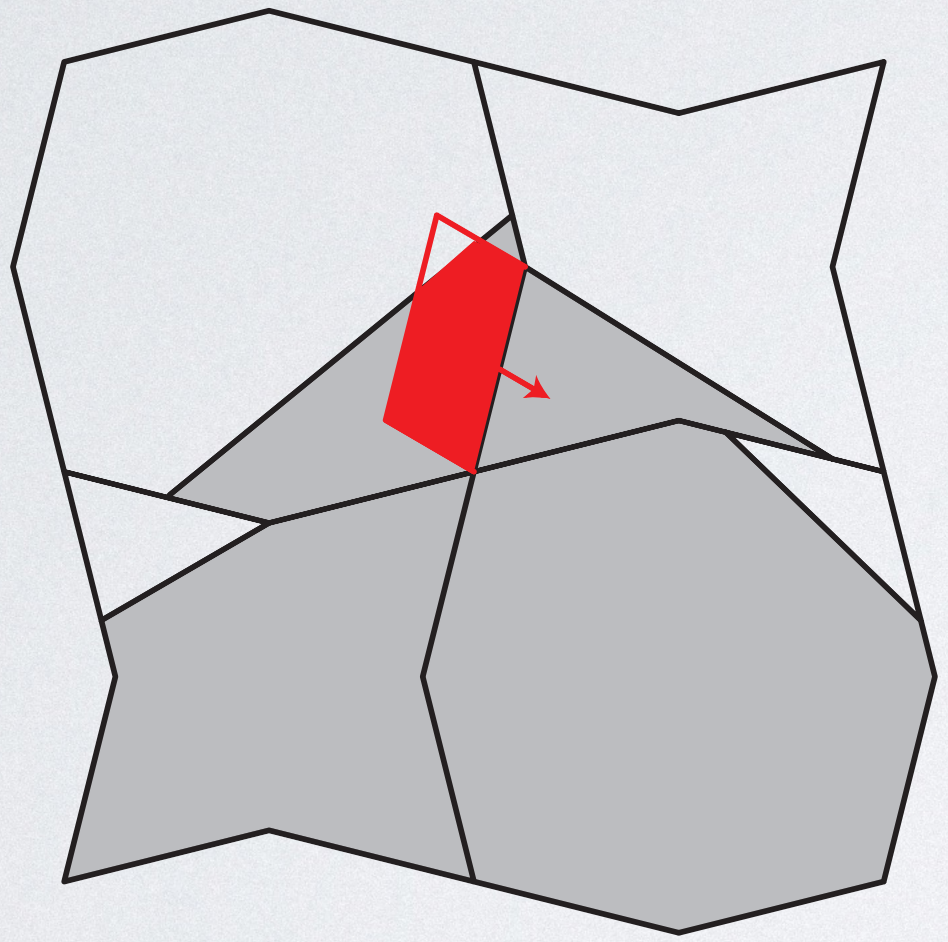


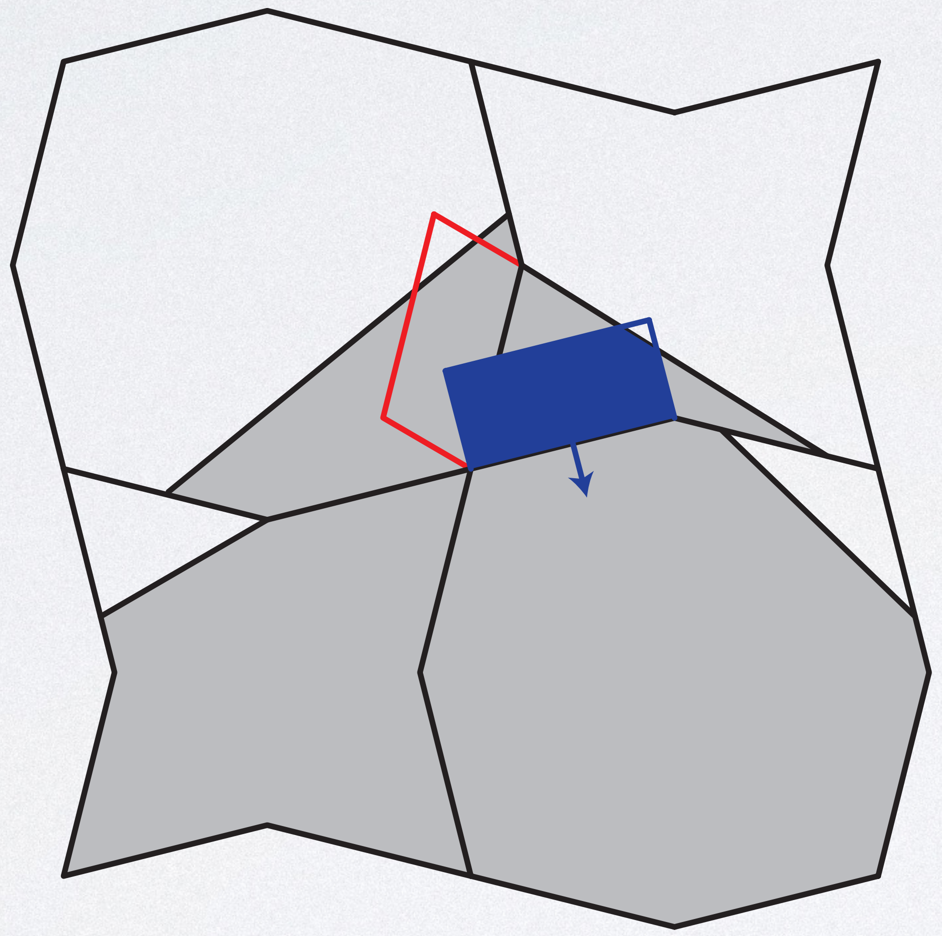
ellipsoid sphere cylinder

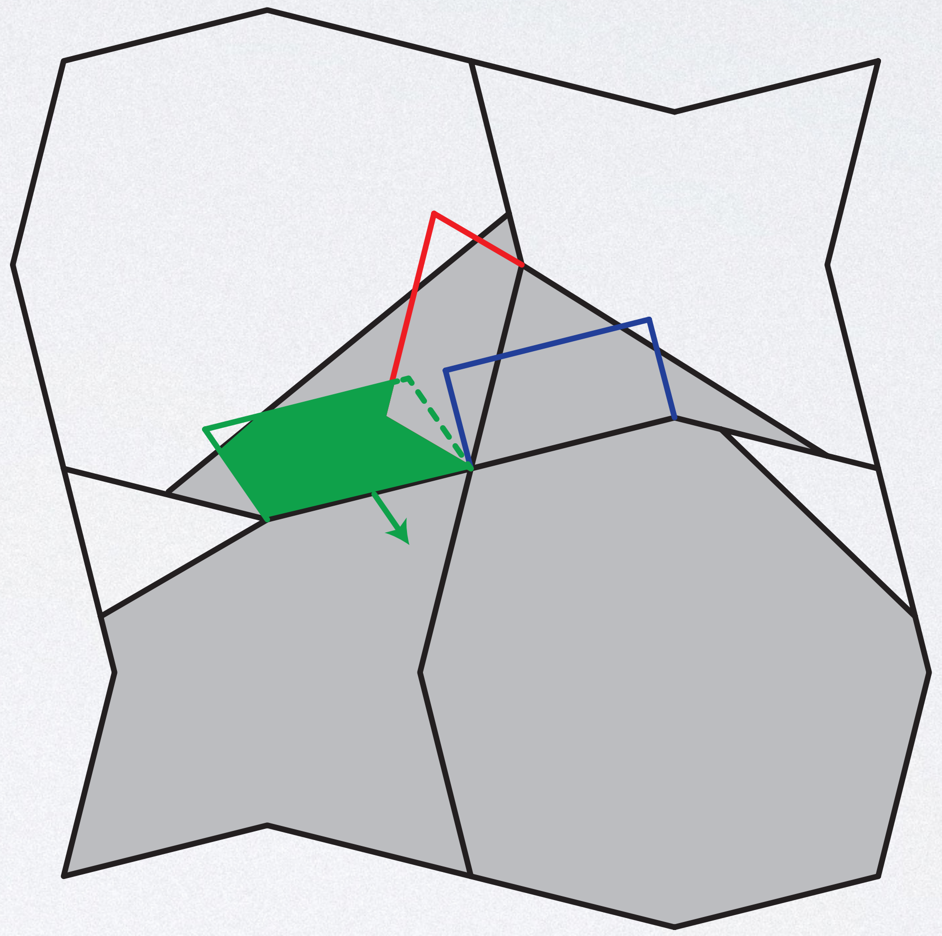
interpolation errors are amplified by differentiation

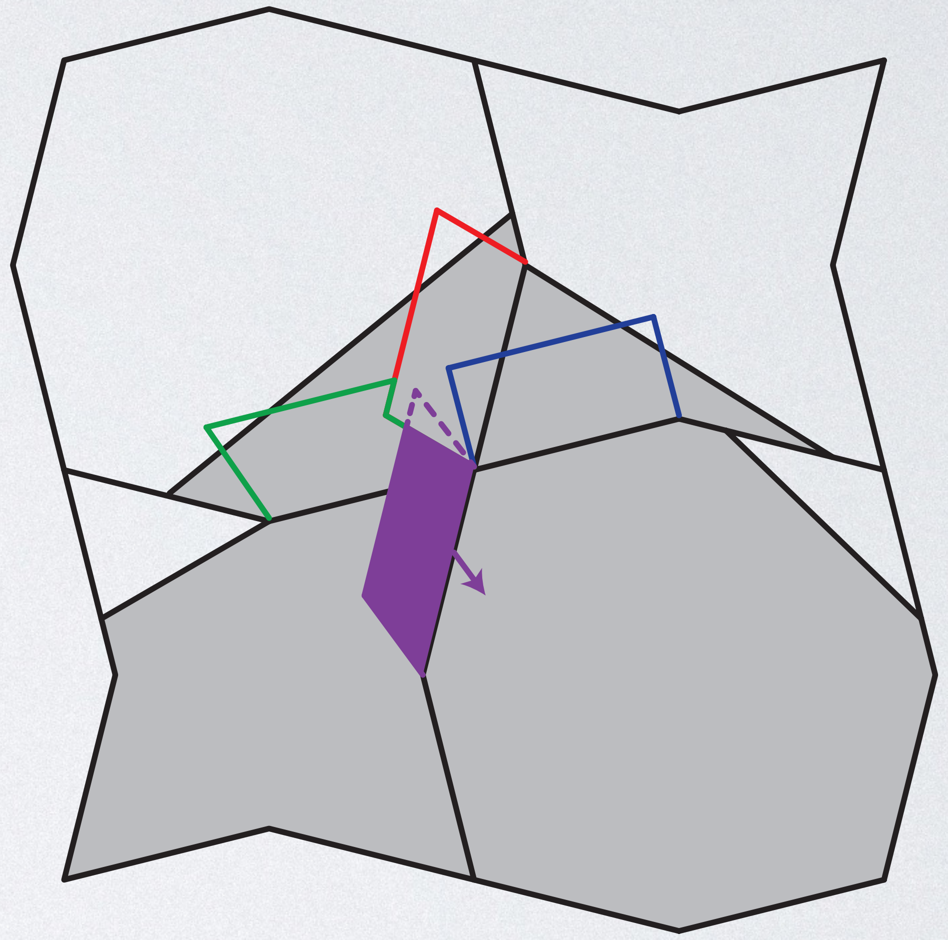
FLUX POLYHEDRA

- conservation and boundedness guaranteed by preventing overlap of flux polyhedra
- accuracy depends on flux polyhedron approximation of stream tube


form flux polyhedron extruding face opposite velocity direction to match flux volume


flux polyhedron region is available to neighbor

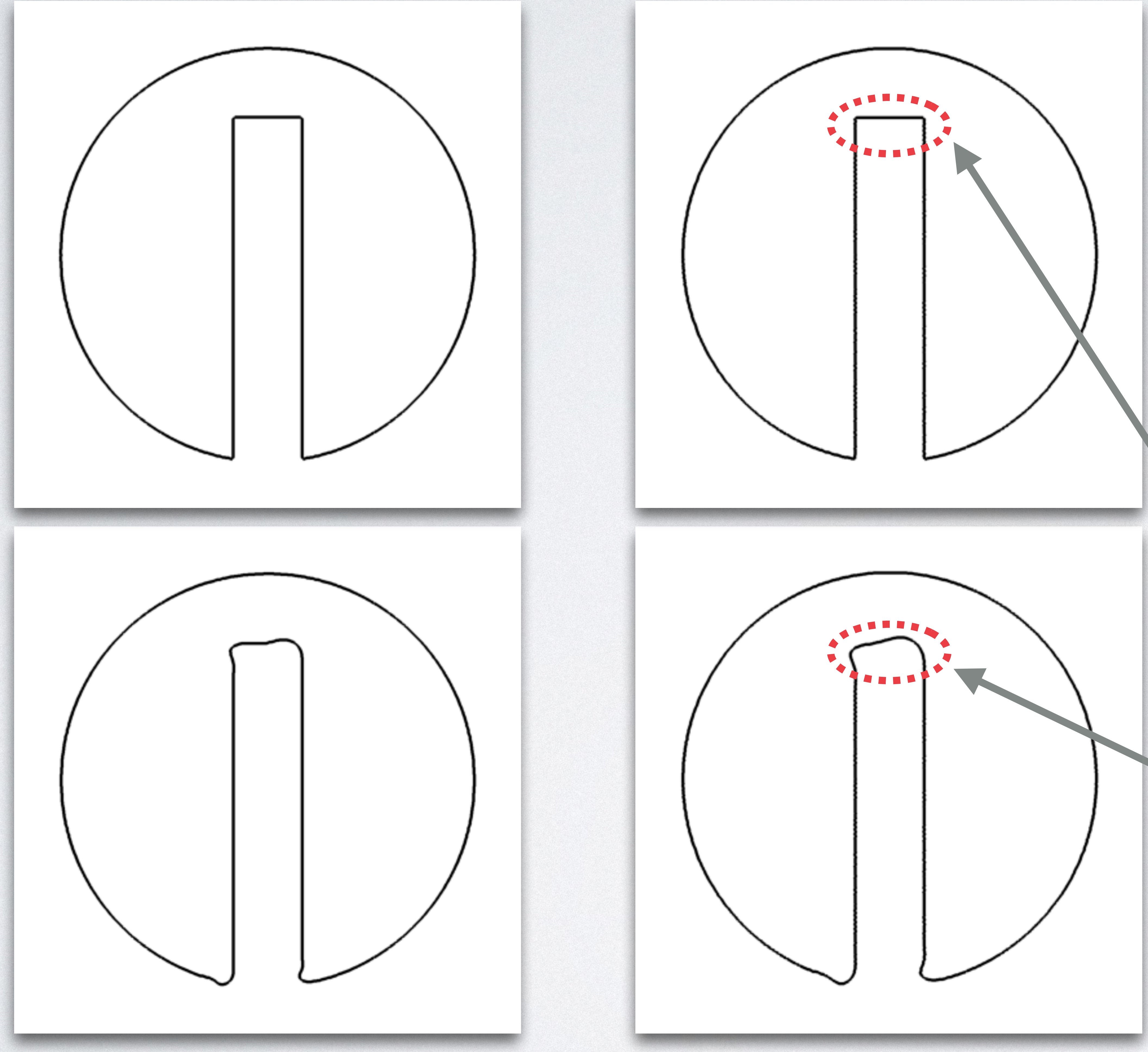

flux polyhedron region is unavailable to node


regions carry over into subsequent flux calculations

ROTATION OF A SLOTTED DISK

0

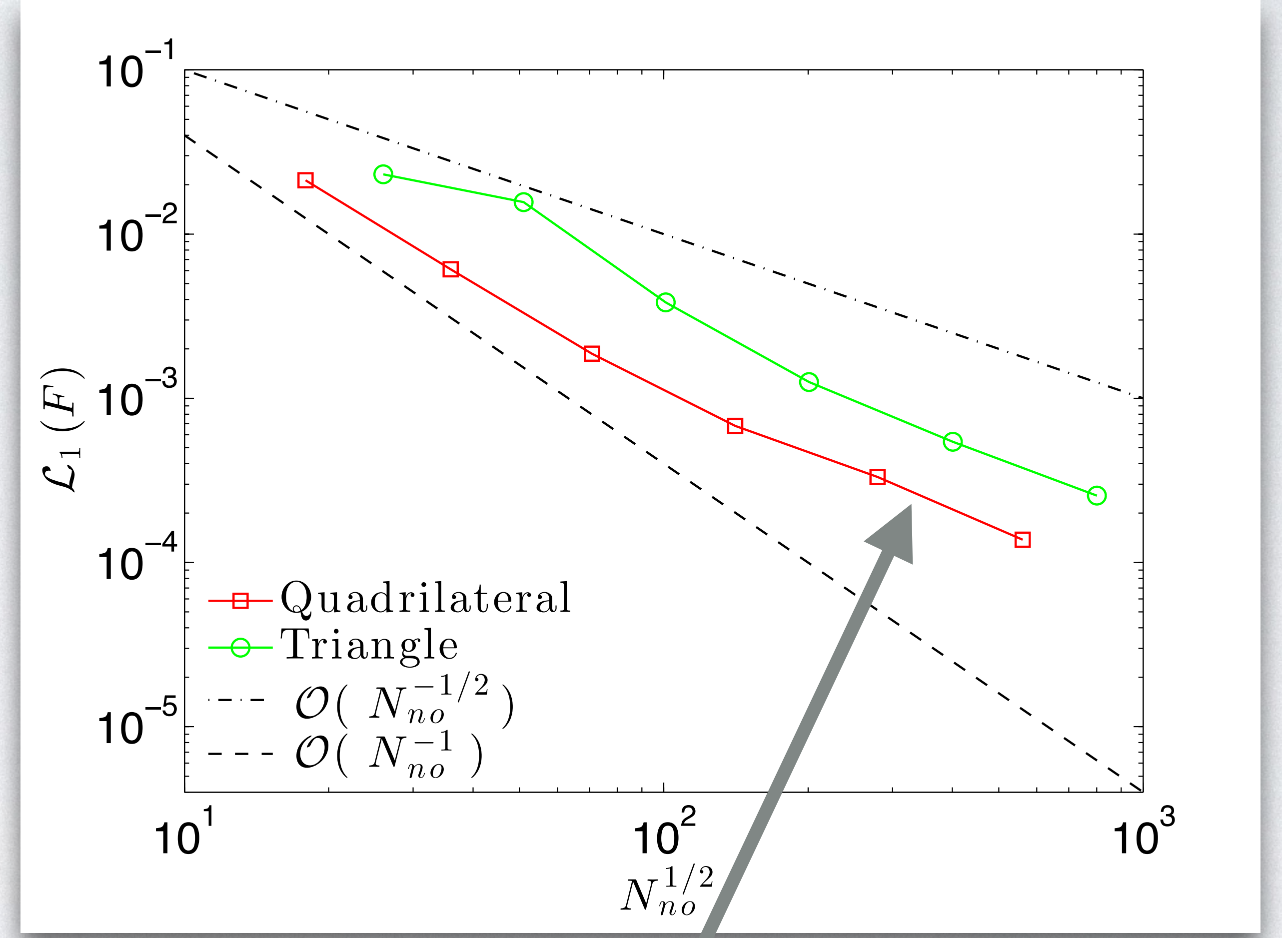
T



quadrilateral

triangle

shape error

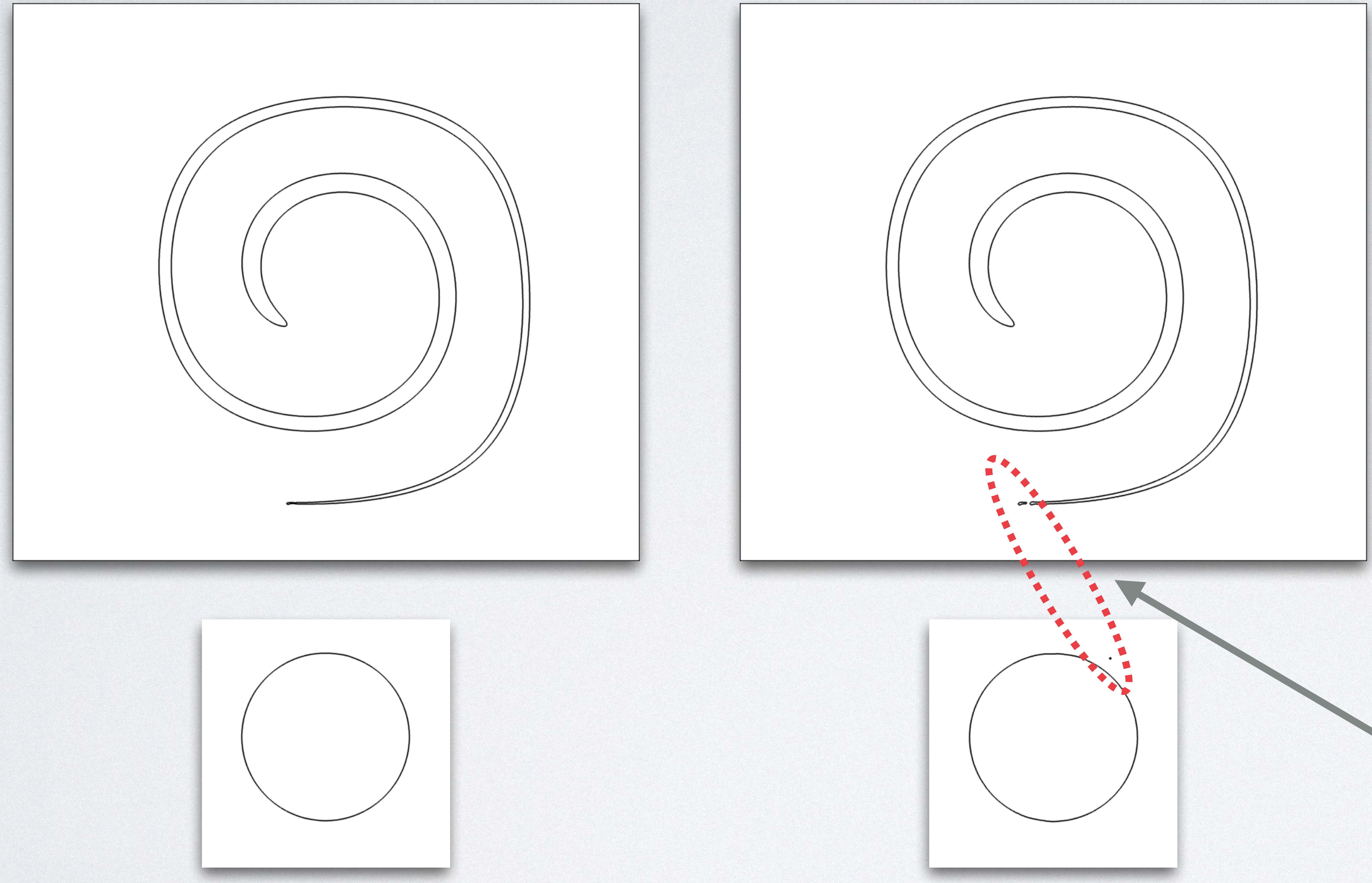


degradation in accuracy
comes from sharp corners

2D DEFORMATION OF A CIRCLE

T/2

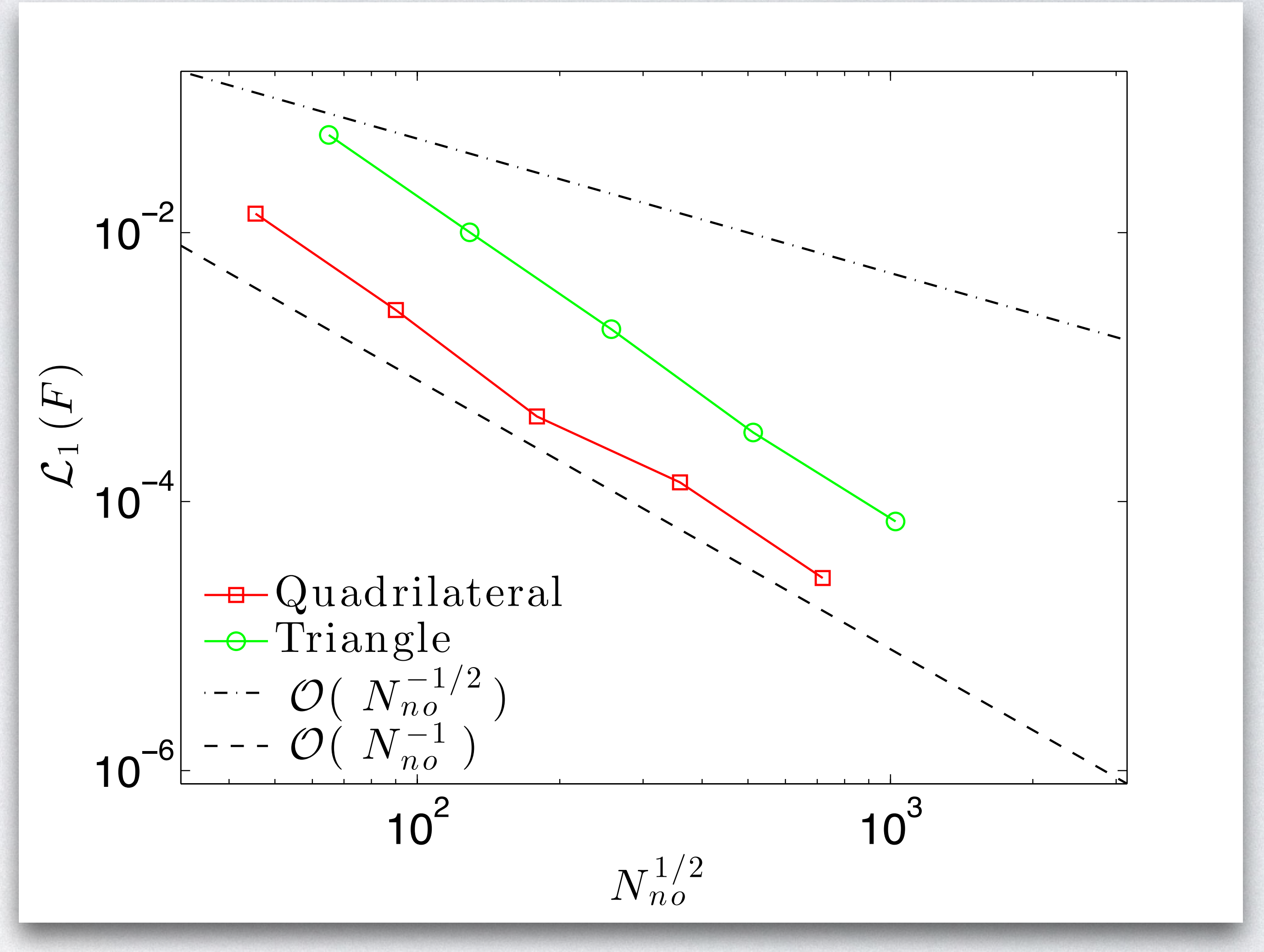
T



quadrilateral

triangle

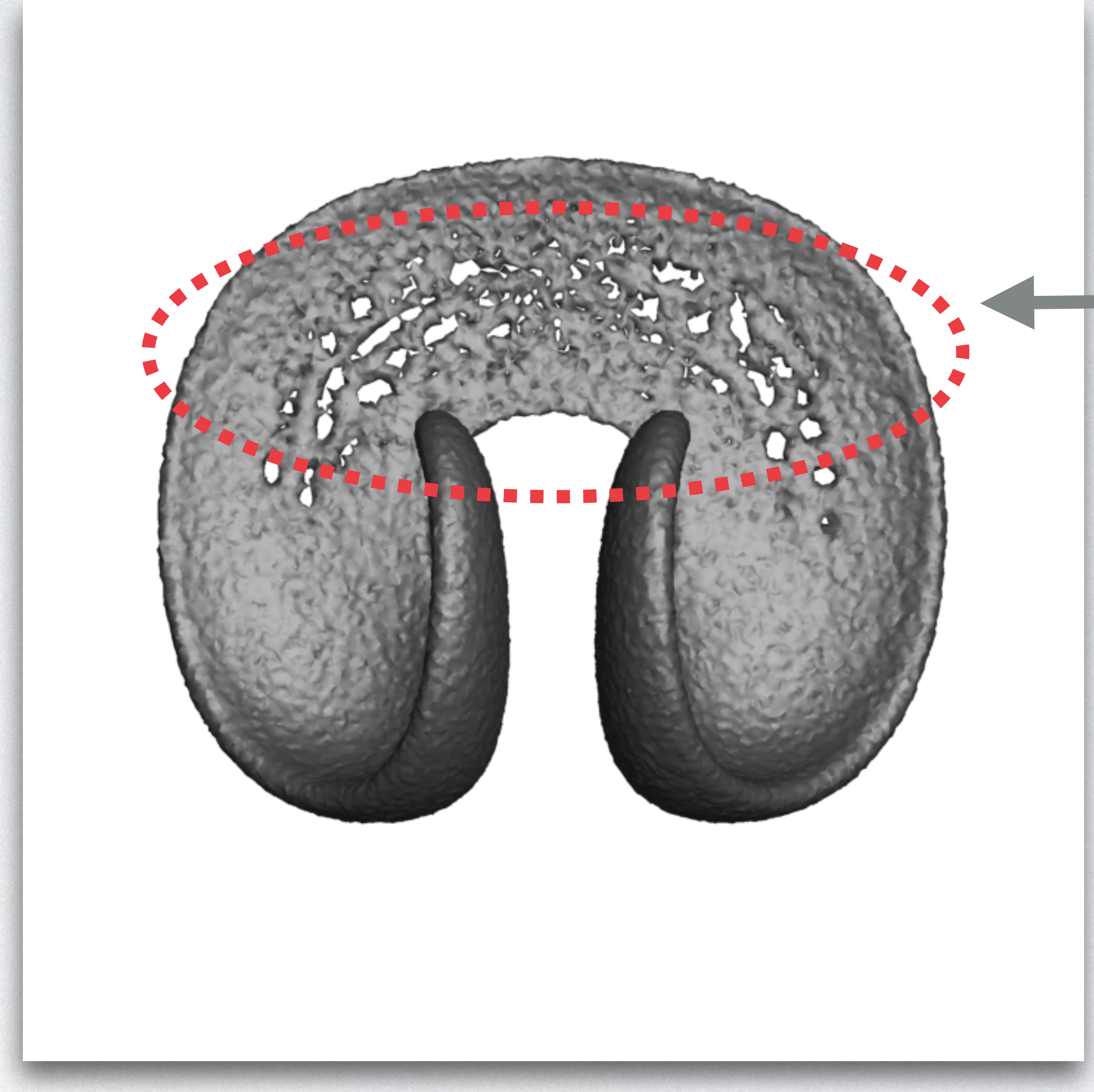
shape error



under-resolved features artificially breakup, leading to detached volume

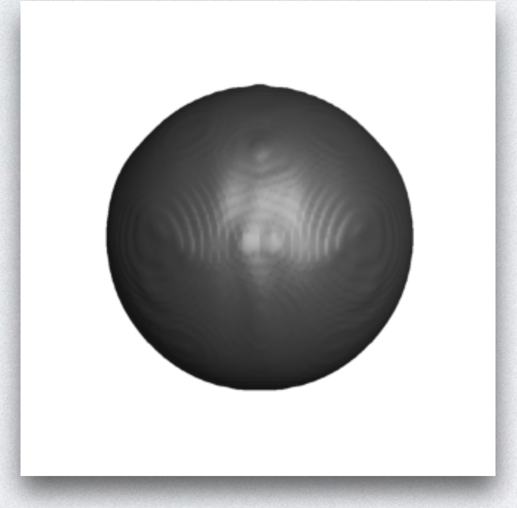
3D DEFORMATION OF A SPHERE

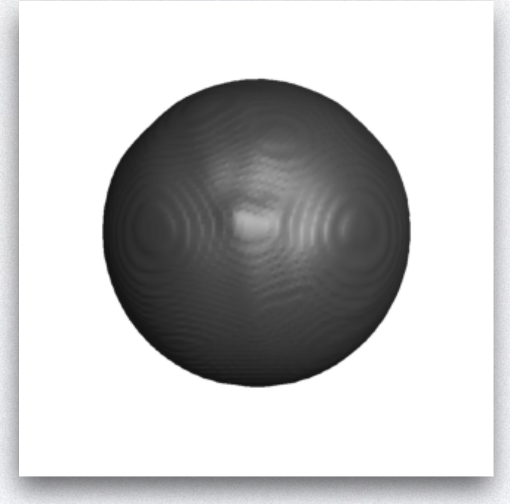
T/2

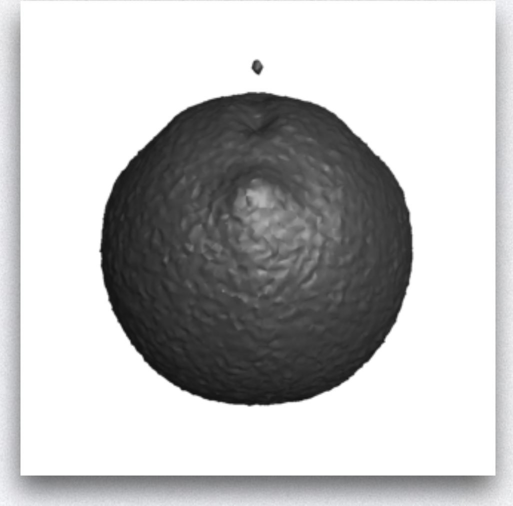


difficult to resolve thin features on tetrahedron primal mesh

T





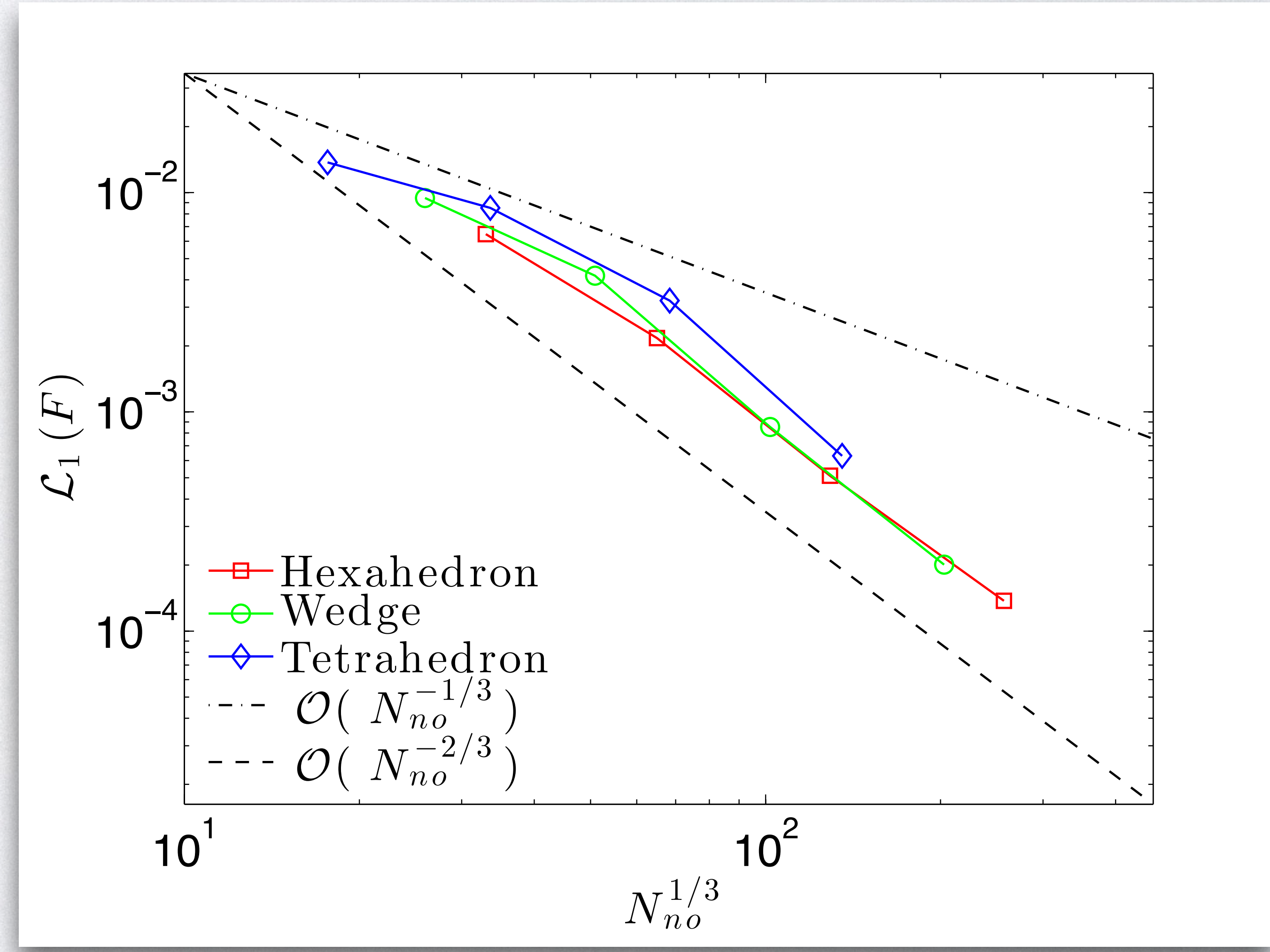


hexahedron

wedge

tetrahedron

shape error

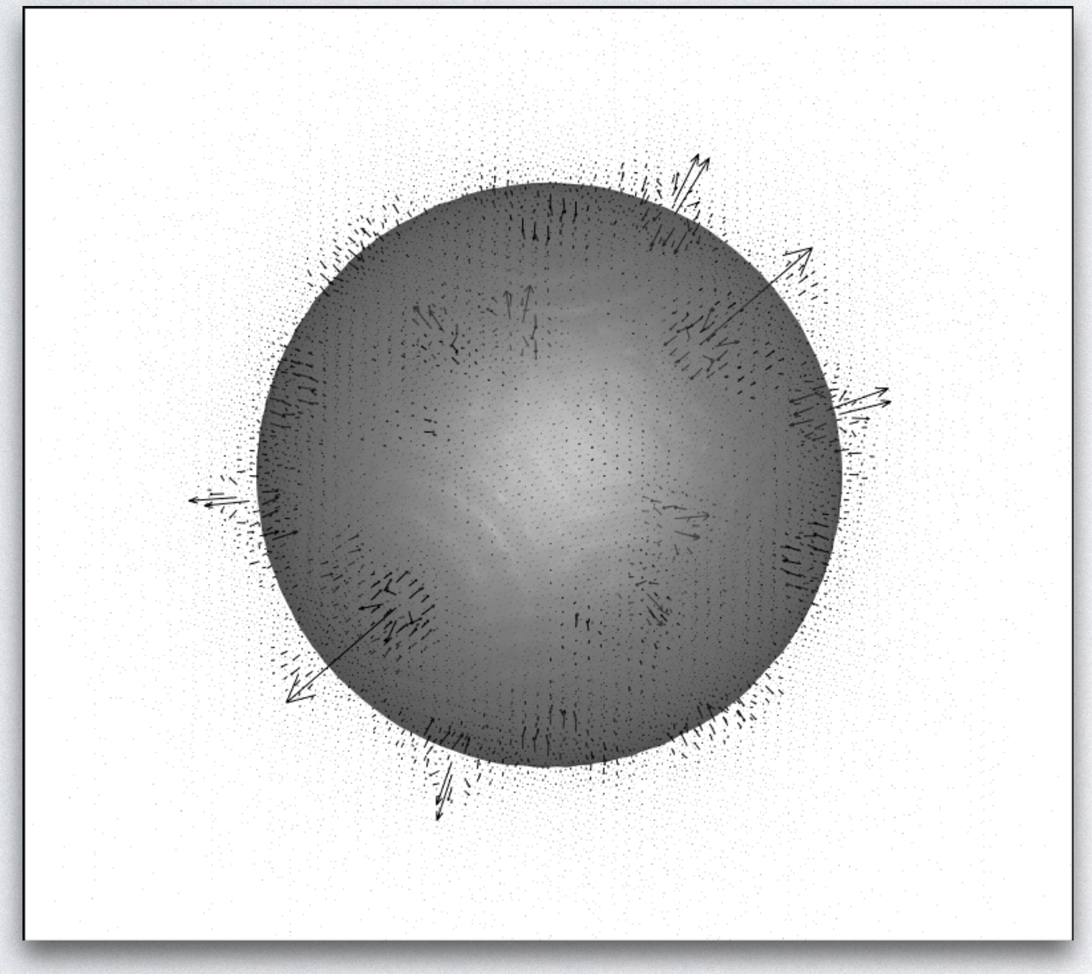


TWO-PHASE SOLVER OUTLINE

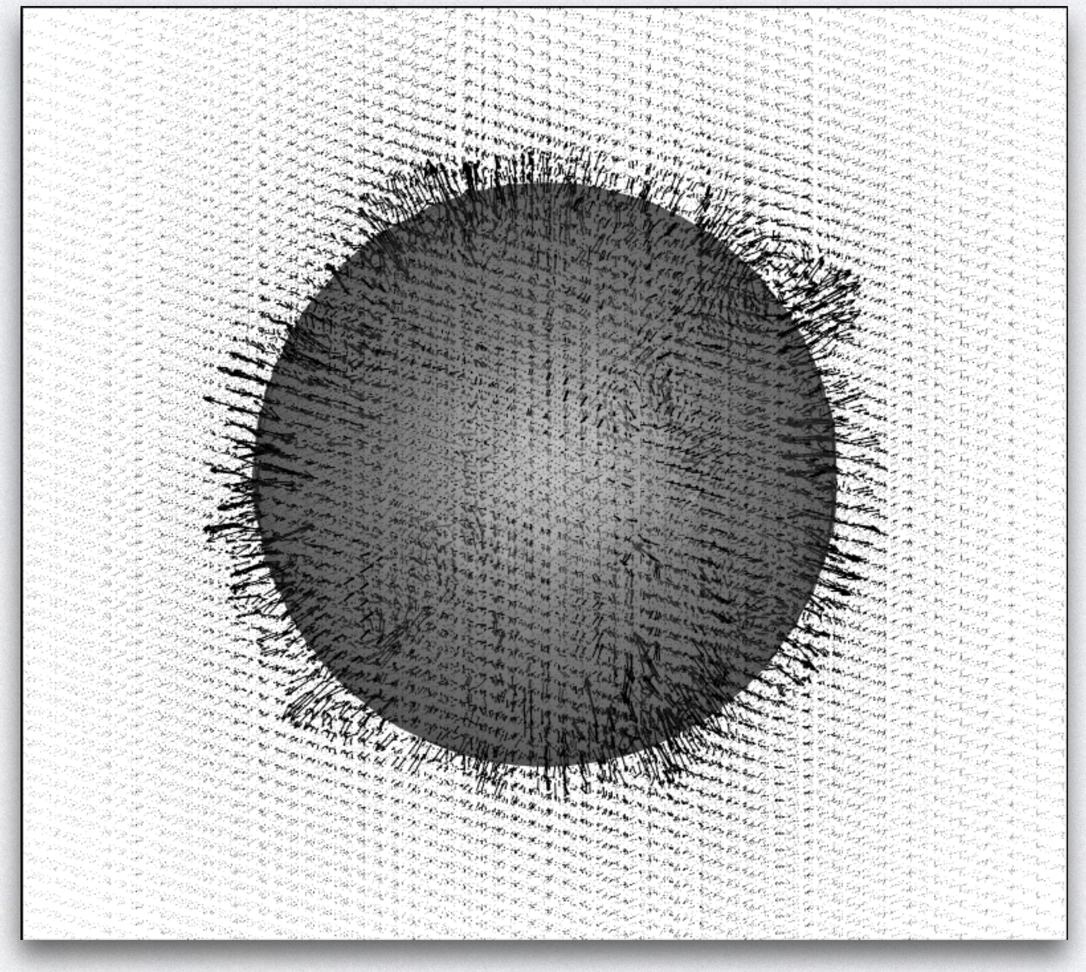
- Consistent mass (volume fraction) and momentum advection using extrapolated advection velocity
- Pressure gradient and surface tension discretely balance (so-called balanced-force method)
- Fractionally-stepped with mid-pointed pressure and surface tension
- Finite-element discretization of viscous terms with Crank-Nicholson splitting

INVISCID STATIC DROP

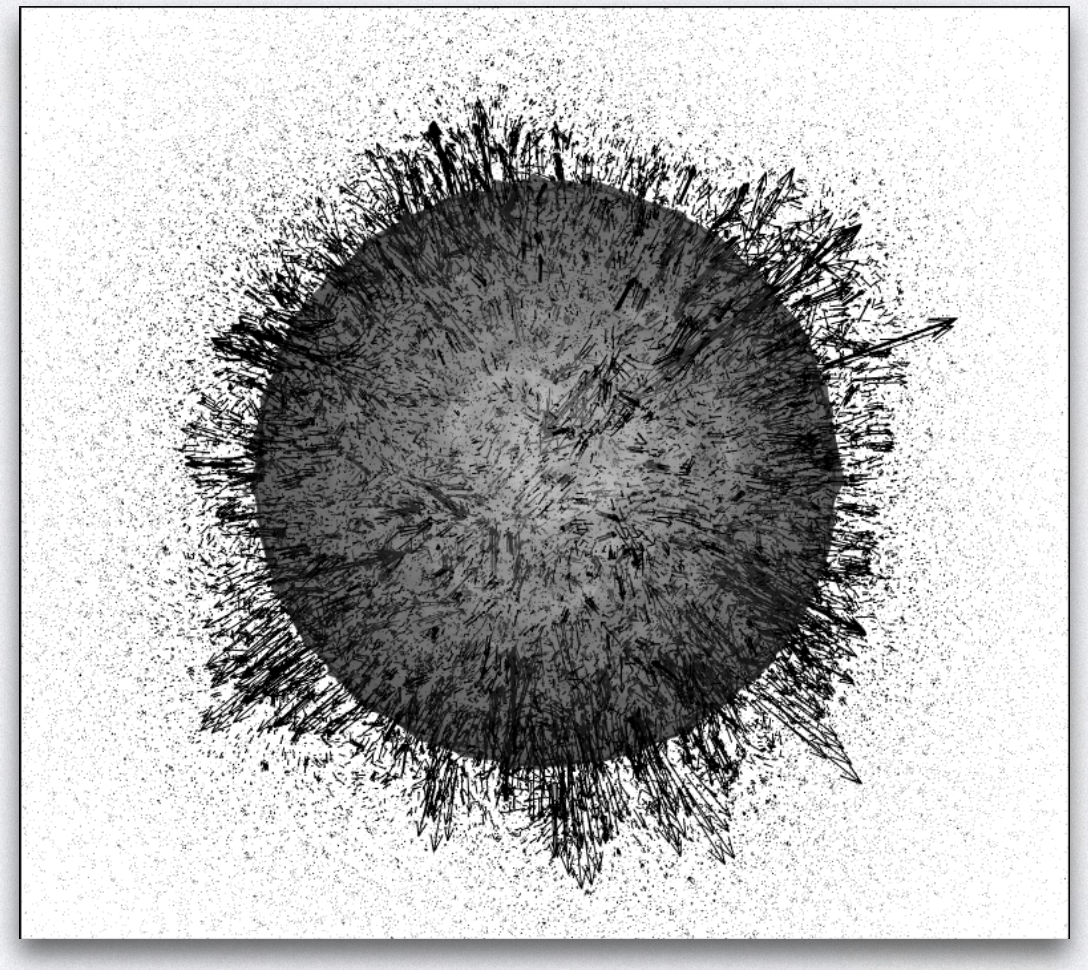
- Place a dense sphere in a static gas with a large surface tension coefficient
- Spurious currents generated only by errors in curvature



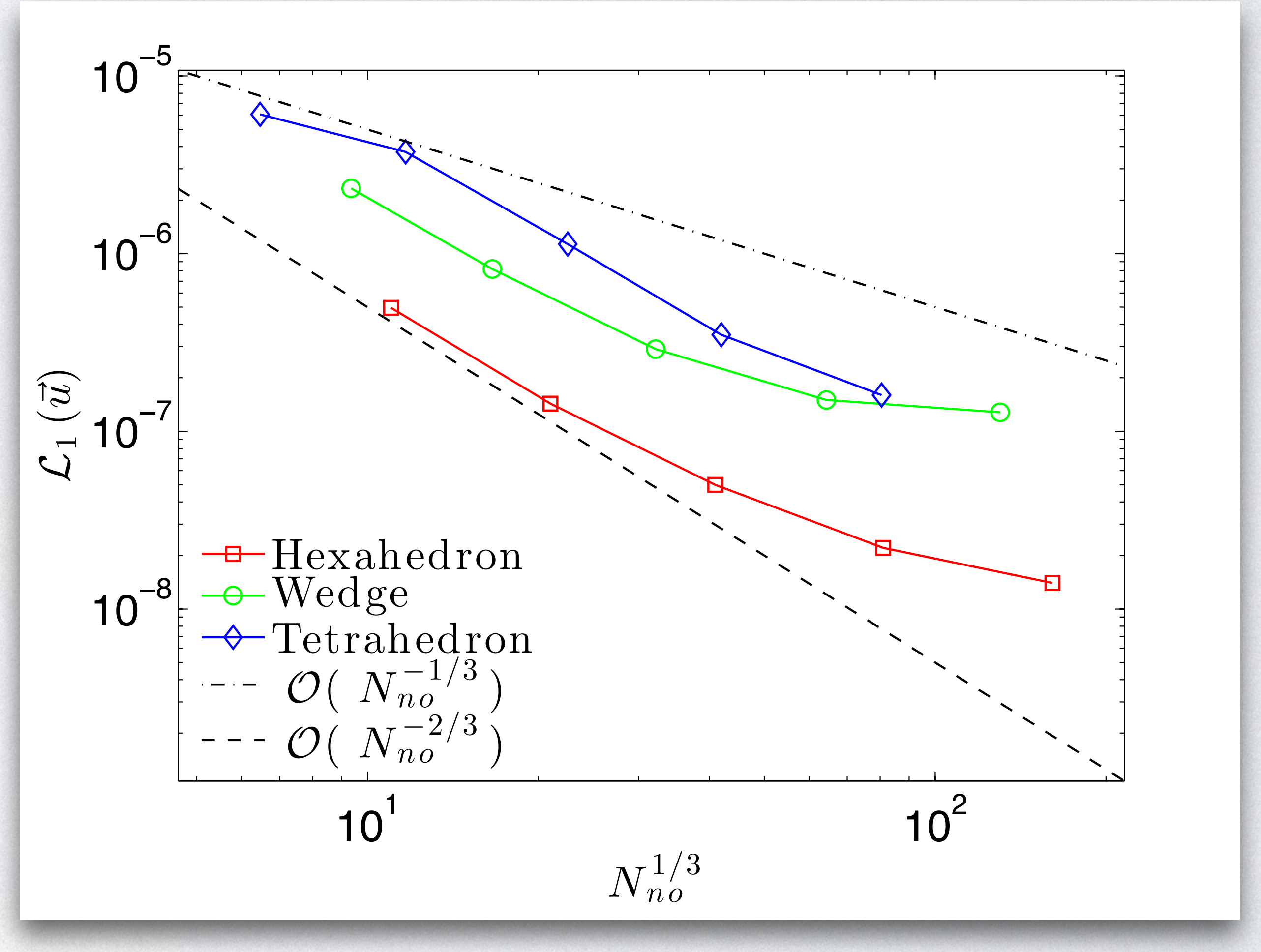
hexahedron



wedge

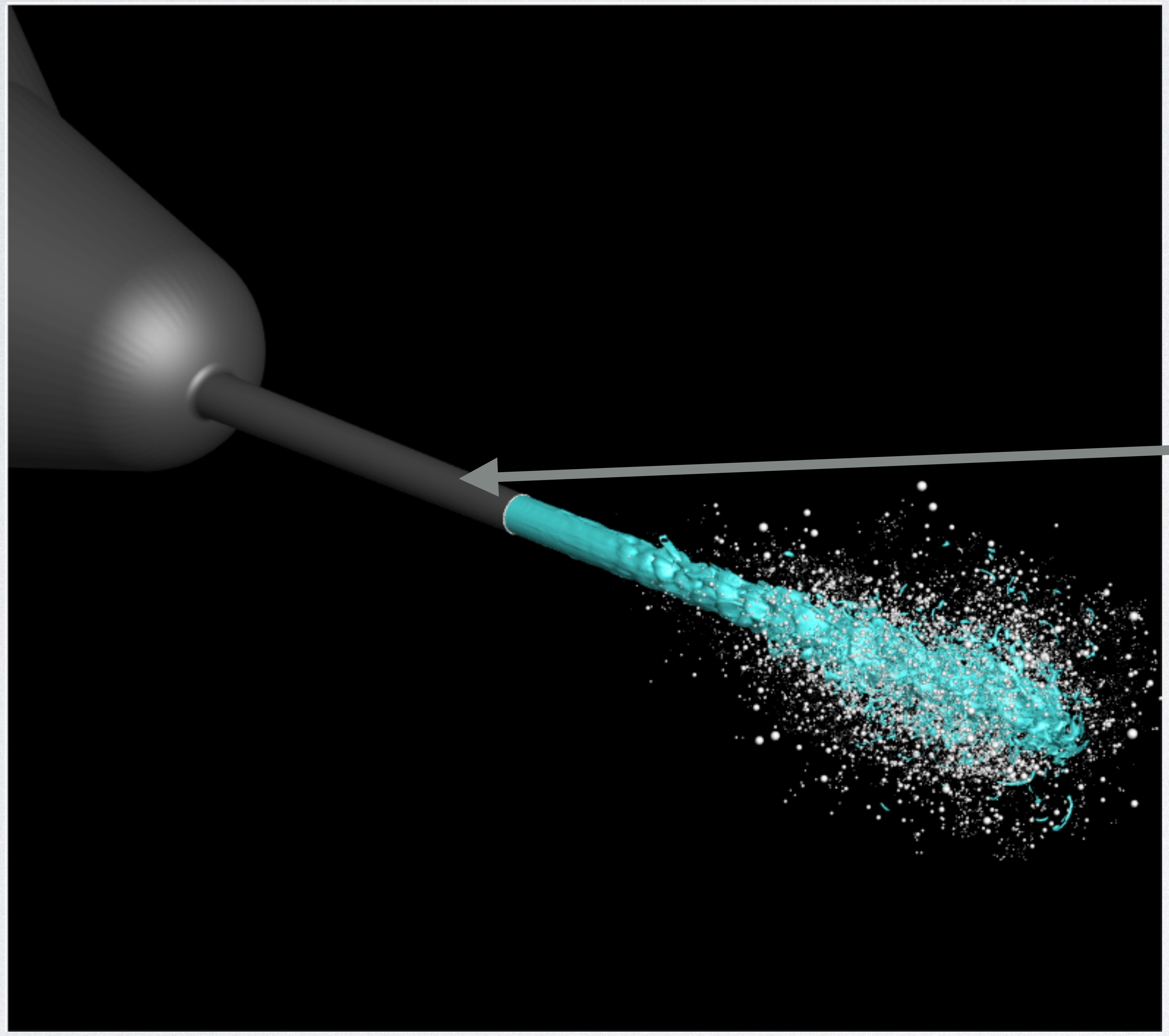


tetrahedron

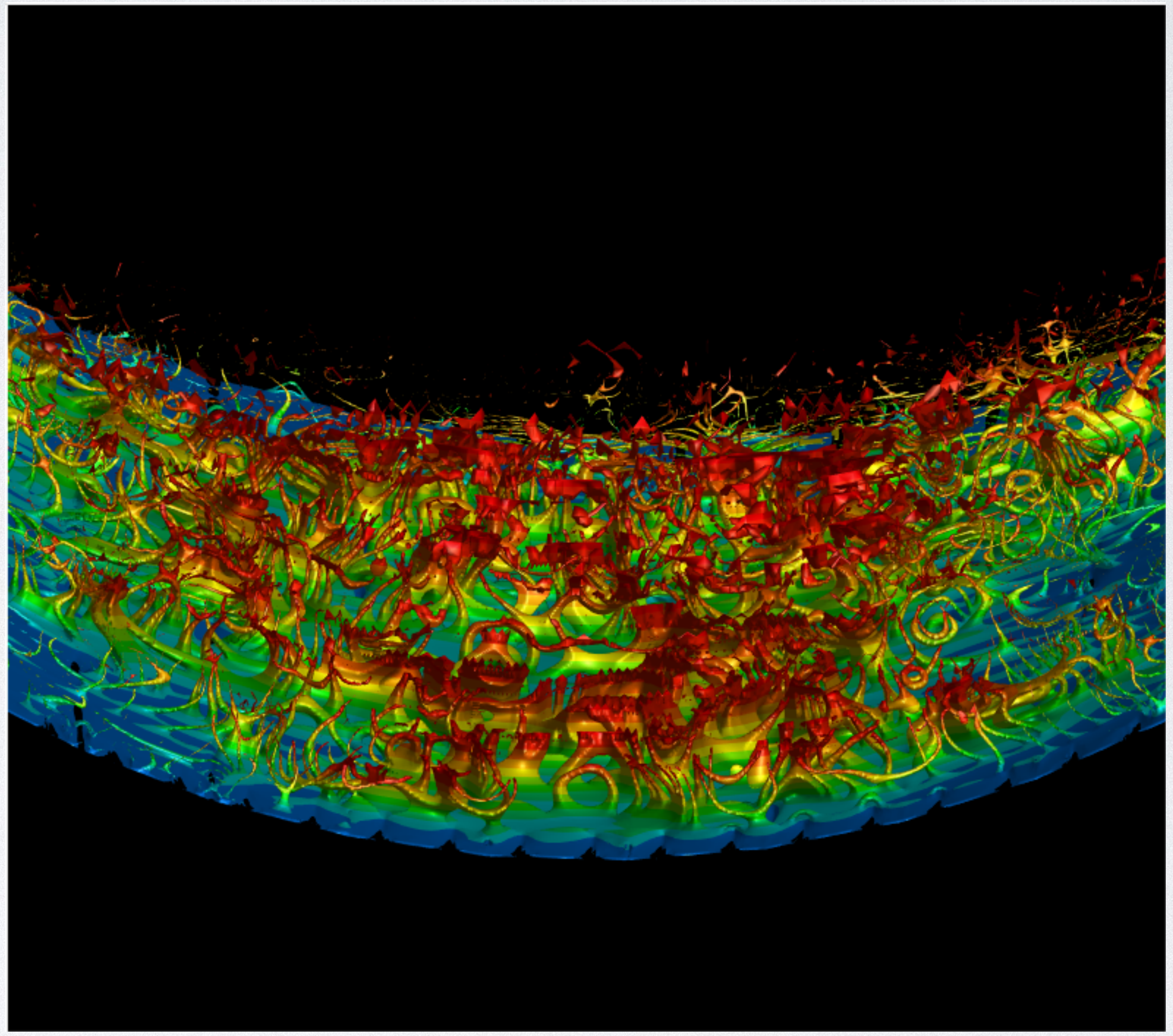


velocity error

PRIMARY ATOMIZATION OF DIESEL SPRAY FROM COMMON RAIL FUEL INJECTOR



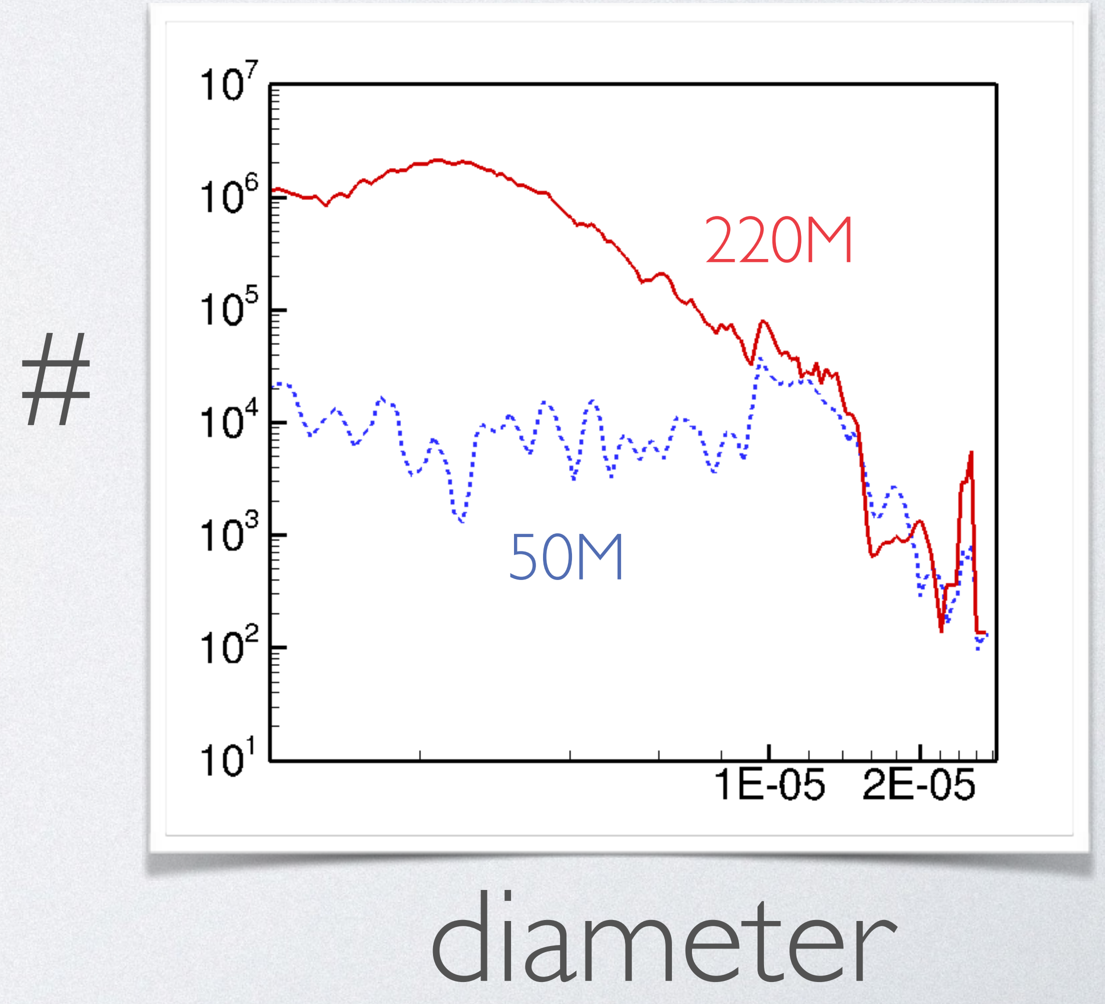
nozzle geometry generated using x-ray tomography



zoom on bottom wall of nozzle showing turbulent structures

Surface perturbations are NOT simple Fourier modes

$$
\begin{aligned}
\theta_{simulation} &= 5.1125^\circ \\
\theta_{theory} &= 5.8325^\circ \\
\theta_{experiment} &\approx 4^\circ
\end{aligned}
$$



CONCLUDING REMARKS

- Solving two-phase flows with large density ratios is difficult
- Solving them on unstructured node-centered meshes makes matters worse
- Leveraged a user-developed non-convex polyhedral library to develop:
  - a discretely conservative and bounded advection operator
  - a means to extract accurate and convergent normals and curvatures

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