Experimental validation of direct numerical simulation of the single-mode Rayleigh Taylor instability

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Outline

- Rayleigh-Taylor instability
  - Motivation
  - Boussinesq approximation
  - Single-mode simplification
- Comparison to experiment (Wilkinson and Jacobs 2007)
  - Validation of simulation
  - Interpolation: secondary flow
  - Extrapolation: late-time dynamics
- Conclusions

Rayleigh-Taylor instability (RTI)

The Rayleigh-Taylor instability occurs when pressure and density gradients oppose:

$$(\nabla P) \cdot (\nabla \rho) < 0$$

- Dense fluid above light fluid in gravity (reactor)
- Light fluid around dense fluid in explosion (supernova)
- Dense fluid around light fluid in implosion (fusion)

Topics limited by single-mode RTI understanding

- What is the safe operating envelope of a reactor
- How do we achieve inertial confinement fusion ignition
- How does mixing in core collapse supernovae occur
- What is the fine structure of oceanic salinity and thermal structures
- What is the self-similar RTI growth rate

State of the theory

We know:

- Linear (Duff, 1962) and weakly non-linear theories for early times
- Potential flow when the density difference is very large (Layzer 1955), (Goncharov 2002)

We don't know:

- Moderate and late-time theories (Ramaprabhu 2012) and (Wei 2012)

Only experiments and simulation can access late-time behavior

Governing equations

The Boussinesq approximation yields single fluid with active scalar:

$$\left[\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right]\mathbf{u} = \nu\nabla^2\mathbf{u} - \nabla P + A\mathbf{g}\phi$$

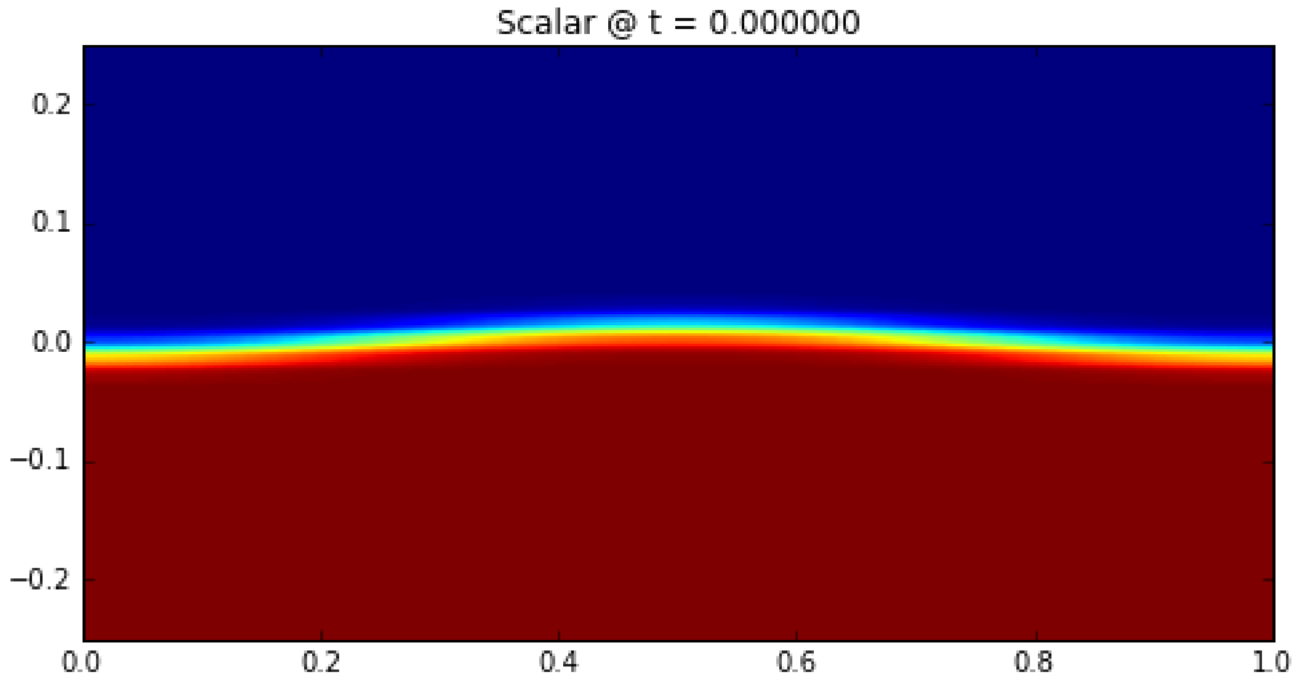
$$\left[\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right]\phi = D\nabla^2\phi$$

Single mode RTI

Consider a heavy fluid above a light fluid with a plane interface perturbed by a single wavenumber $\vec{k}$.

$$\phi(x, y, z, 0) = \text{erf}\left[\frac{z + a_0 \cos(k_x x)\cos(k_y y)}{\delta}\right]$$

In [7]:
```
plot_T(0, .0625)
```



A Typical Result

- $\text{Grashof} = Ag\lambda^3/(\nu^2) = 9.8 \times 10^4$
- $\text{Schmidt} = \nu/D = 10$

In [8]:

```
fig, axs = plt.subplots(1,3,sharey=True)
display_animation(animation.FuncAnimation(fig, partial(plot_TWV, zoom=.5
```

Out[8]:

Direct comparison to experiment

Experimental study of the single-mode three-dimensional Rayleigh-Taylor instability

J. P. Wilkinson and J. W. Jacobs
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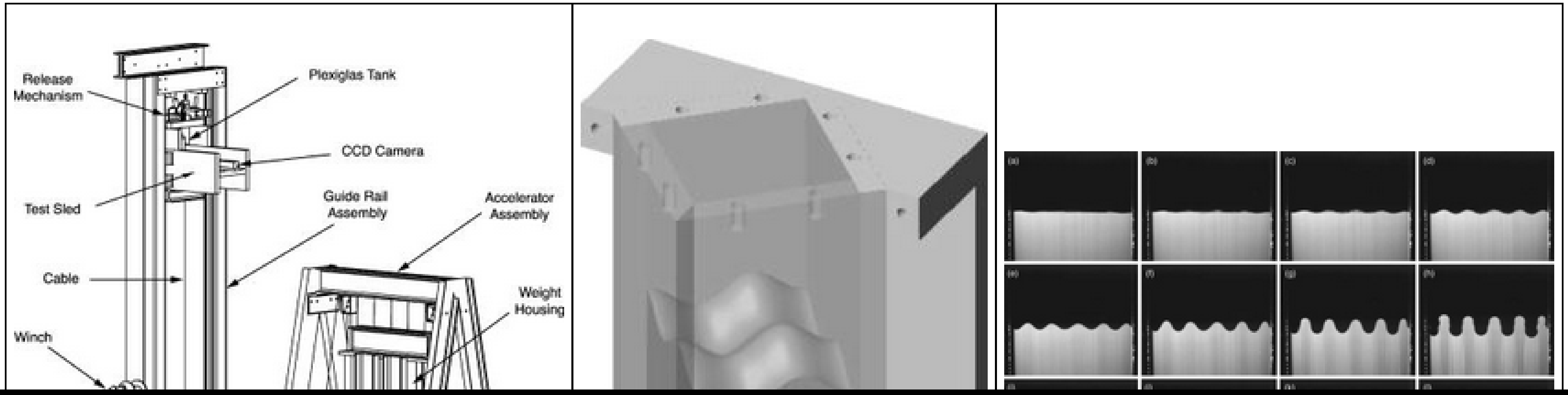
(Received 23 August 2007; accepted 24 October 2007; published online 10 December 2007)

The three-dimensional Rayleigh-Taylor instability is studied in a low Atwood number ($A \approx 0.15$) miscible fluid system. The two fluids are contained within a Plexiglas tank that is mounted on vertical rails and accelerated downward by a weight and pulley system. A net acceleration between 13 and 23 m/s^2 can be maintained, resulting in an effective body force equivalent to 0.33–1.35 times Earth's gravity. A single-mode, three-dimensional perturbation is produced by oscillating the tank, which has a square cross section, along its diagonal. Early time measured growth rates are shown to have good agreement with linear stability theory. At late time, the instability exhibits a nonconstant vertical interfacial velocity in agreement with the recent numerical computations of Ramaprabhu *et al.* [Phys. Rev. E **74**, 066308 (2006)]. Both the late-time bubble and spike velocities have values greater than those predicted by both the simple buoyancy-drag model developed by Oron *et al.* [Phys. Plasmas **8**, 2883 (2001)] and the potential flow model of Goncharov [Phys. Rev. Lett. **88**, 134502 (2002)]. The disagreement with the models can be attributed to the formation of vortices, in this case vortex rings, observed in the experiments but not accounted for by the models. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2813548]



Details

We replicated three runs from Wilkinson and Jacobs (2007):

$$\phi(x, y, z, 0) = \operatorname{erf}\left[\frac{z + (1.1)\cos\left((0.10)\frac{x+y}{2}\right)\cos\left((0.10)\frac{x-y}{2}\right)}{1.4}\right]$$

For

$$(x, y, z) \in [0, 76]^2 \times [-76, 76]$$

(all lengths in millimeters)

For CSGF-ers

- 3.4 B grid points, 67 M elements, 5 fields
- 1.5 s per step on 512K cores, 3 M core-hours each
- Grashof: 5 M, Rayleigh: 35 M

Validation of simulation

If the parameter spaces and observables of the experiment and computation overlap, then we can use the experimental results to validate the computation.

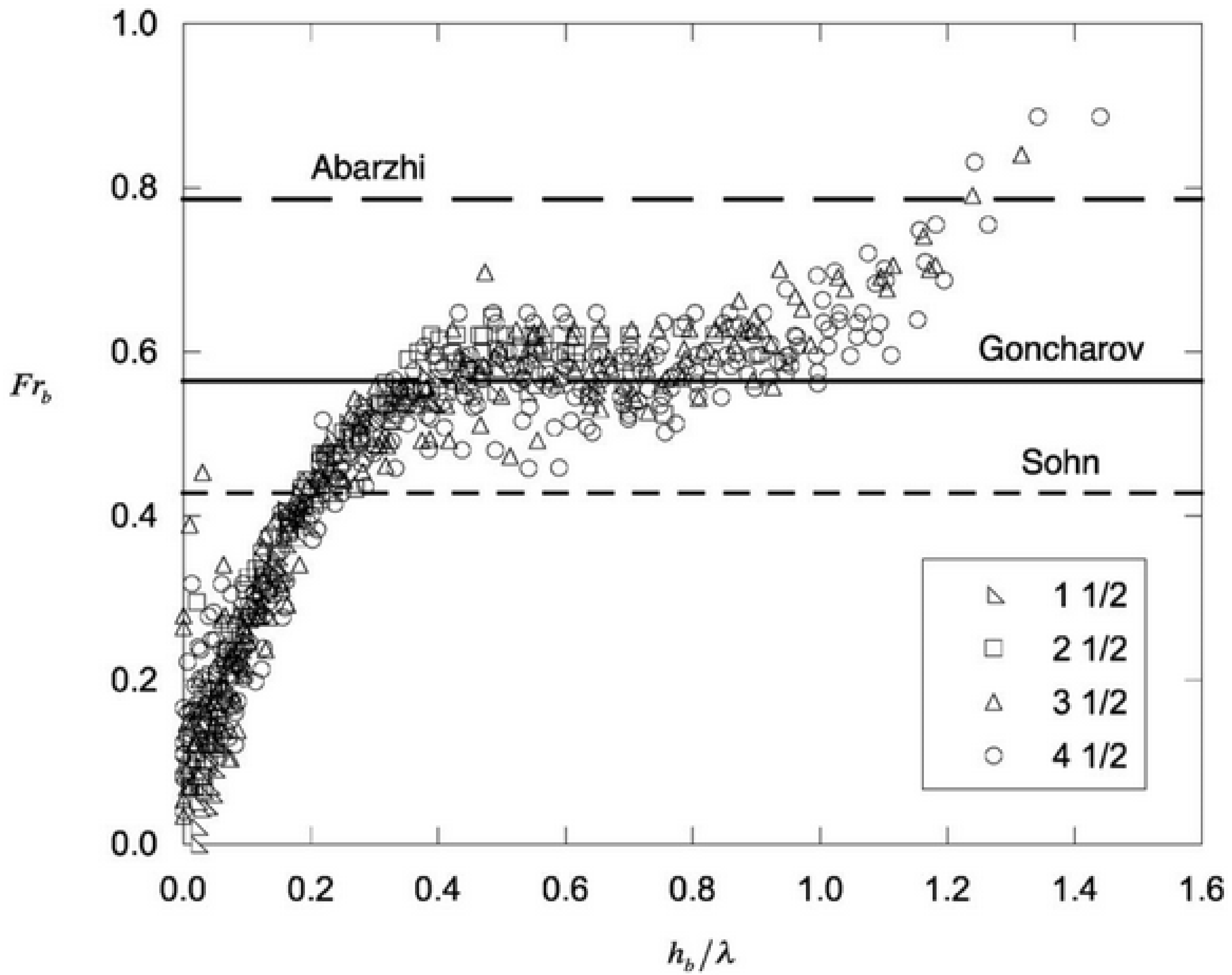
Matched:

- Reynolds number
- Boundary, initial conditions

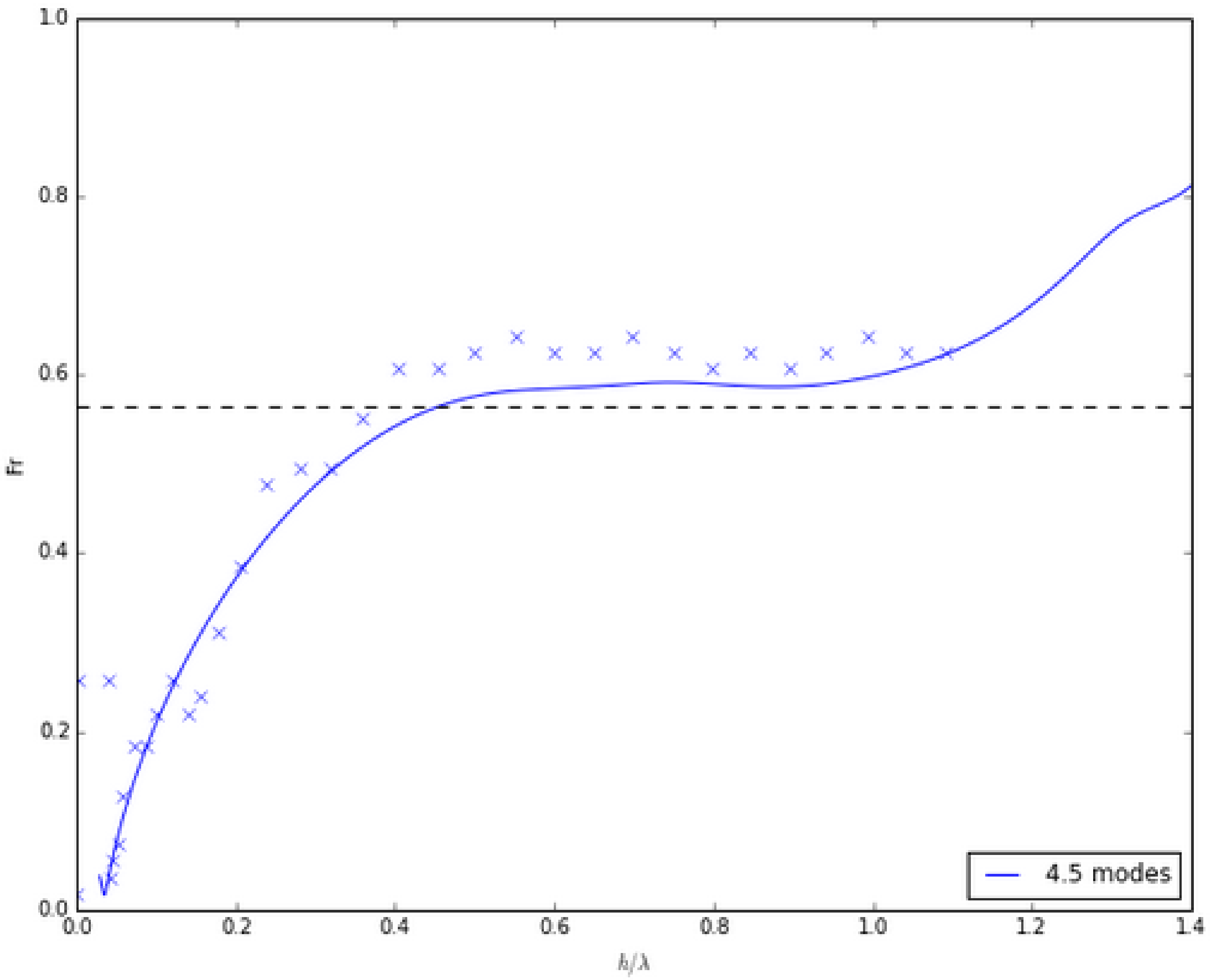
Unmatched:

- Schmidt number
- Density, viscosity asymmetry

The summary plot is the non-dimensional velocity (Fr) vs the non-dimensional height (h/λ).



We've picked out the experimental points that correspond to our initial conditions.



https://github.com/maxhutch/thesis-notebooks/WilkinsonAndJacobs.ipynb

Interpolation: secondary flow

One advantage of simulations is their exposure of the full physical fields, not the sparse set of observables found in experiments.

Without much marginal effort, we can explore the data to find new phenomena.

Here are slices of the fields through the mid-plane:

In [10]:
```
fig, axs = plt.subplots(2,2,figsize=(12,9))
anim = animation.FuncAnimation(fig, partial(plot_wilk_span), frames=64, :
display_animation(anim)
```

Out[10]:

0:00

This is secondary flow of the first kind, which enhances mixing **at low Reynolds number**.

Extrapolation: late time dynamics

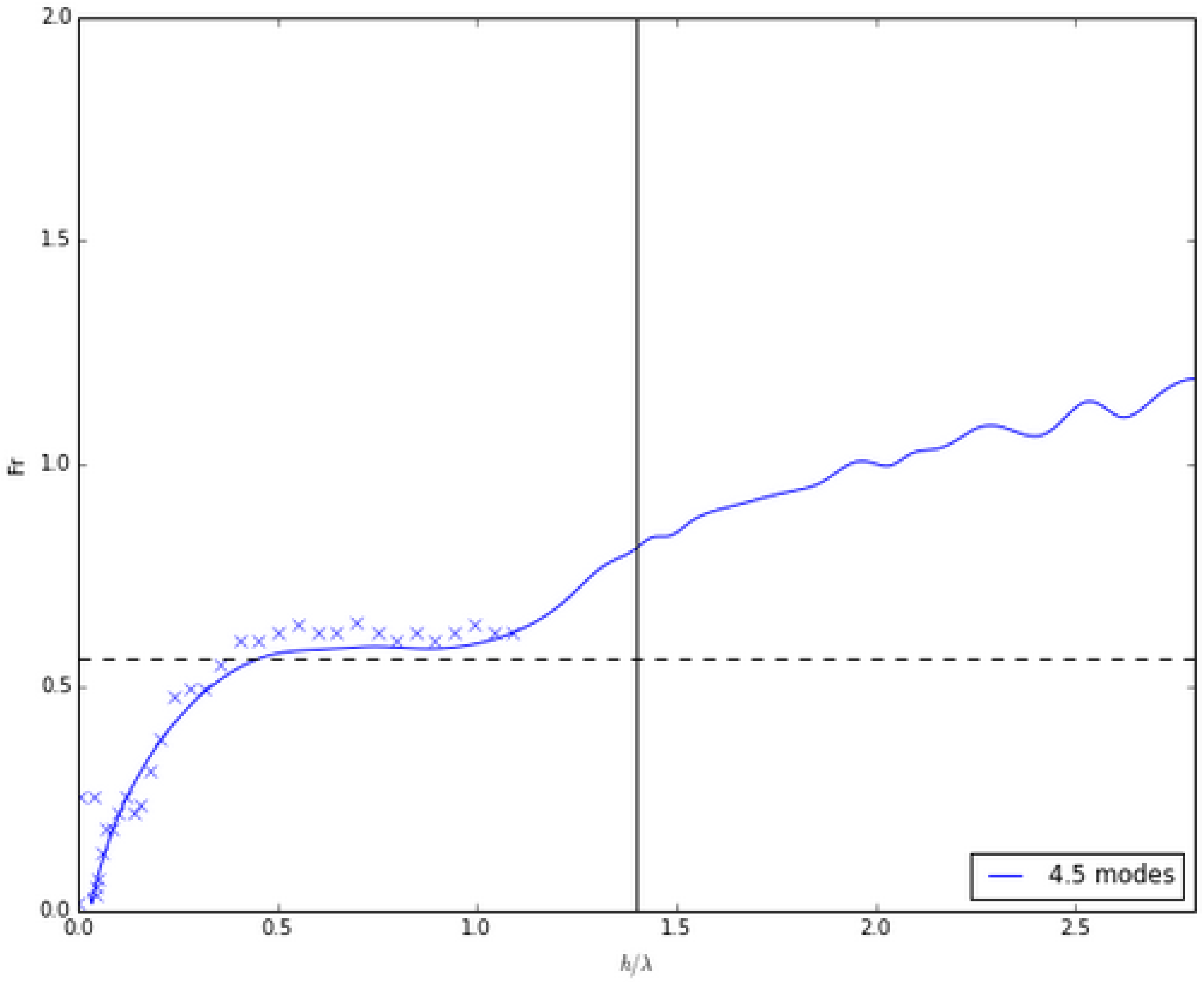
Another advantage of simulations is their flexibility.

We can try things in simulations that would be too costly/difficult/time-consuming to try experimentally. The marginal cost is mostly computer, not scientist, time.

What if the rig was 4x as tall?

- The calculation is 4x more expensive, but uses all the same code

We extend the velocity vs height plot from before.



Conclusions

- Direct numerical simulations agree with single-mode lab experiments
  - Boussinesq can be corrected for by the theory
- Access to full-field data reveals new span-wise flows
  - Secondary flows of the first kind
  - Could enhance mixing at low Reynolds number
- Higher aspect ratio simulations are easy
  - But they run into aforementioned wall-related issues
  - High aspect simulations should stick to periodic boundary conditions

Thank you
Questions

https://github.com/maxhutch/rti-talks/blob/master/FinalTalkCSGF.ipynb