Optimizing Medical Countermeasure Distribution Networks: Saving Lives with Operations Research

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Characteristics of Public Health Emergencies

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- Large-scale public health emergencies are rare, giving little opportunity to "practice".
- Realistic system exercises of the response network are very expensive, and hence are seldom performed.

Agenda

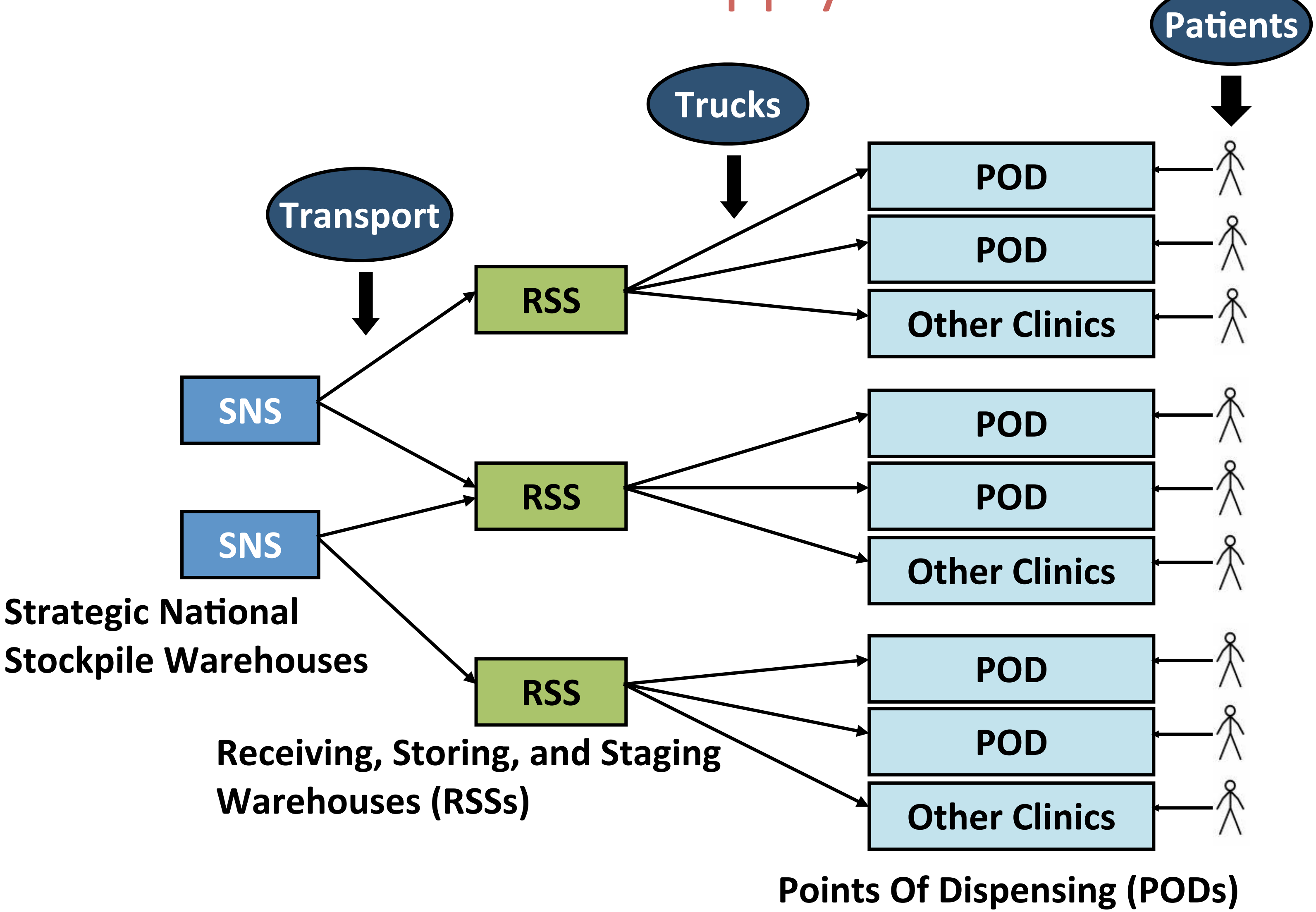
Responding to an Inhalational Anthrax Attack

- The United States' emergency response plans
- A multi-echelon inventory allocation model
- Public health policy implications

Inhalational Anthrax: A Significant Bioterrorist Threat

- Anthrax spores are very small and difficult to detect in the atmosphere.
- Inhalational anthrax is the disease contracted if a sufficient number of spores is inhaled.
- When symptoms begin to appear, it is often too late to save patients.
- Symptoms may appear within 48 hours of inhalation.
- If patients begin a course of antibiotics before becoming symptomatic, survival is highly likely.

The United States Strategic National Stockpile is Part of a Multi-Echelon Supply Chain



Planning Questions Asked By Policy-Makers

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- Lead times?

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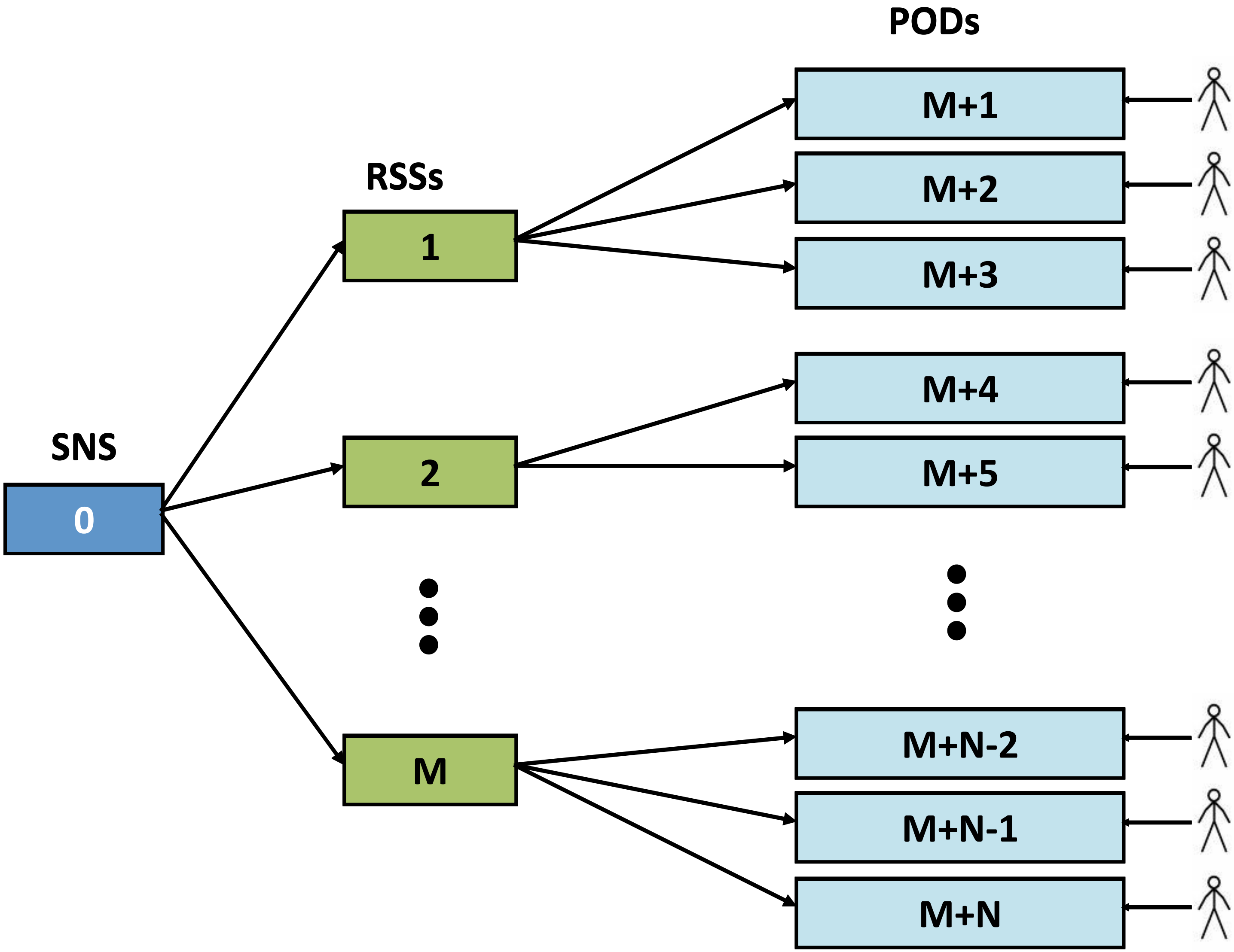
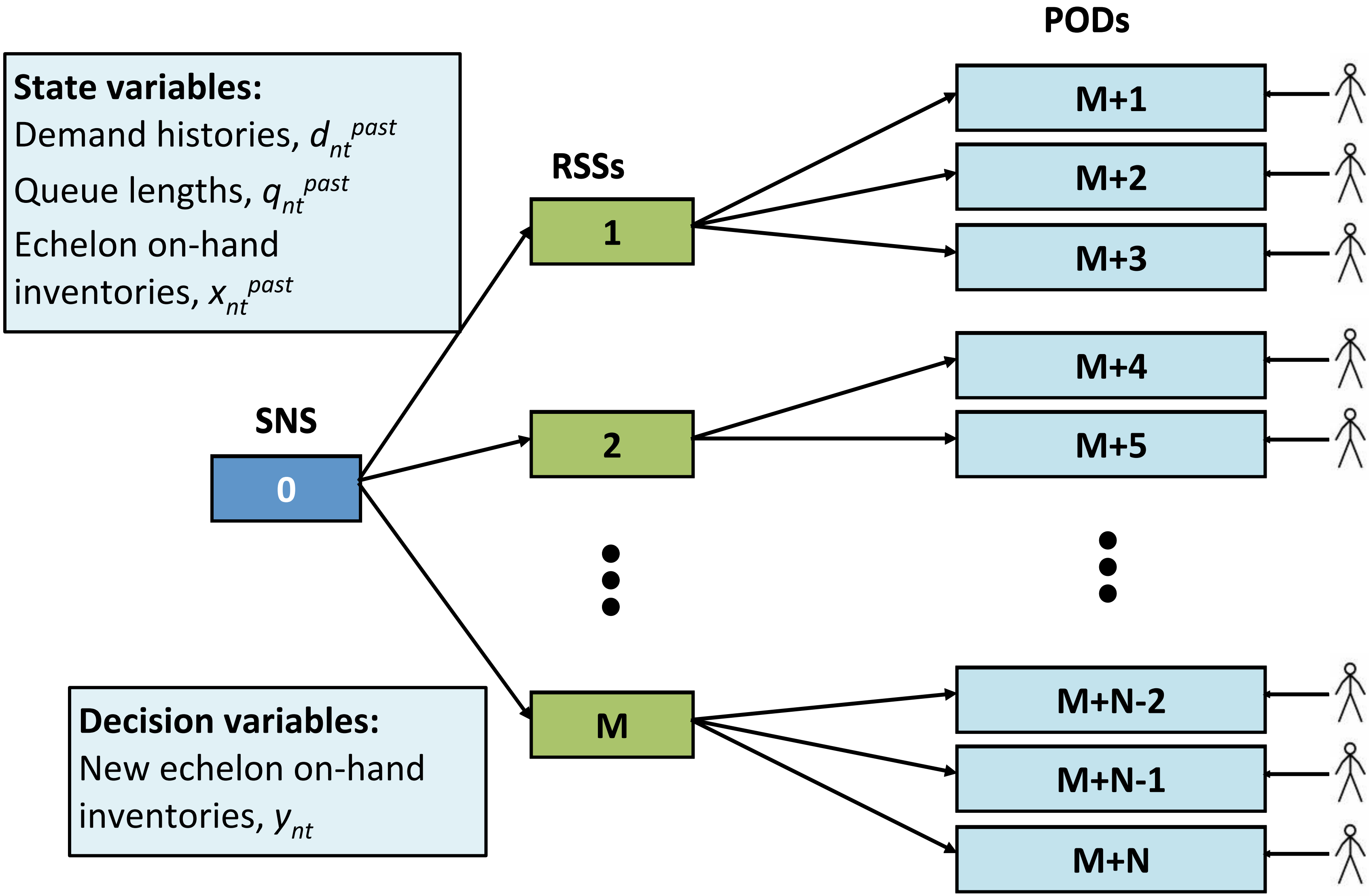
System Planning

- Inventory allocation over time?
- Information-sharing?

The Distribution Network has a Tree Structure

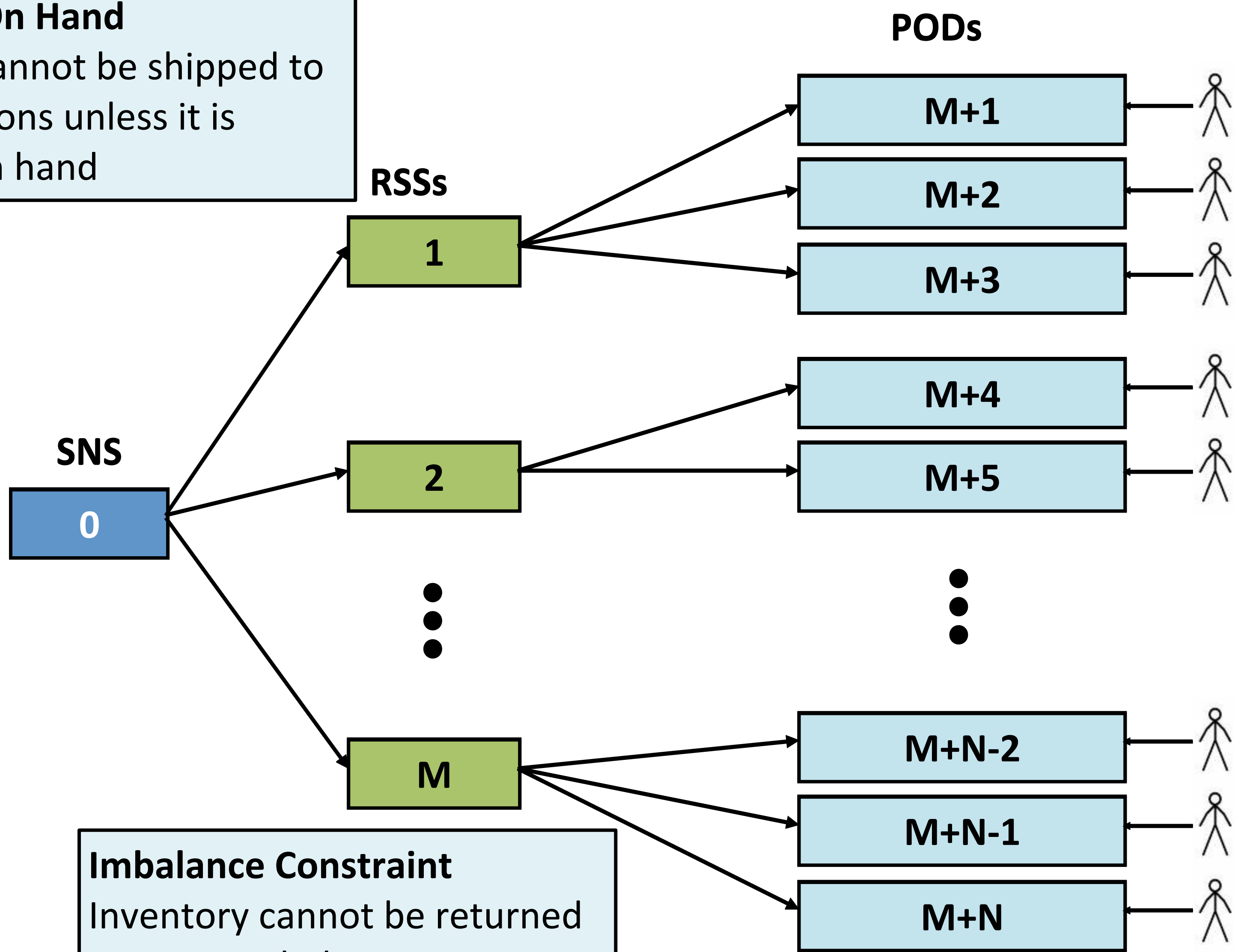


State Variables are Known at the Beginning of Each Period



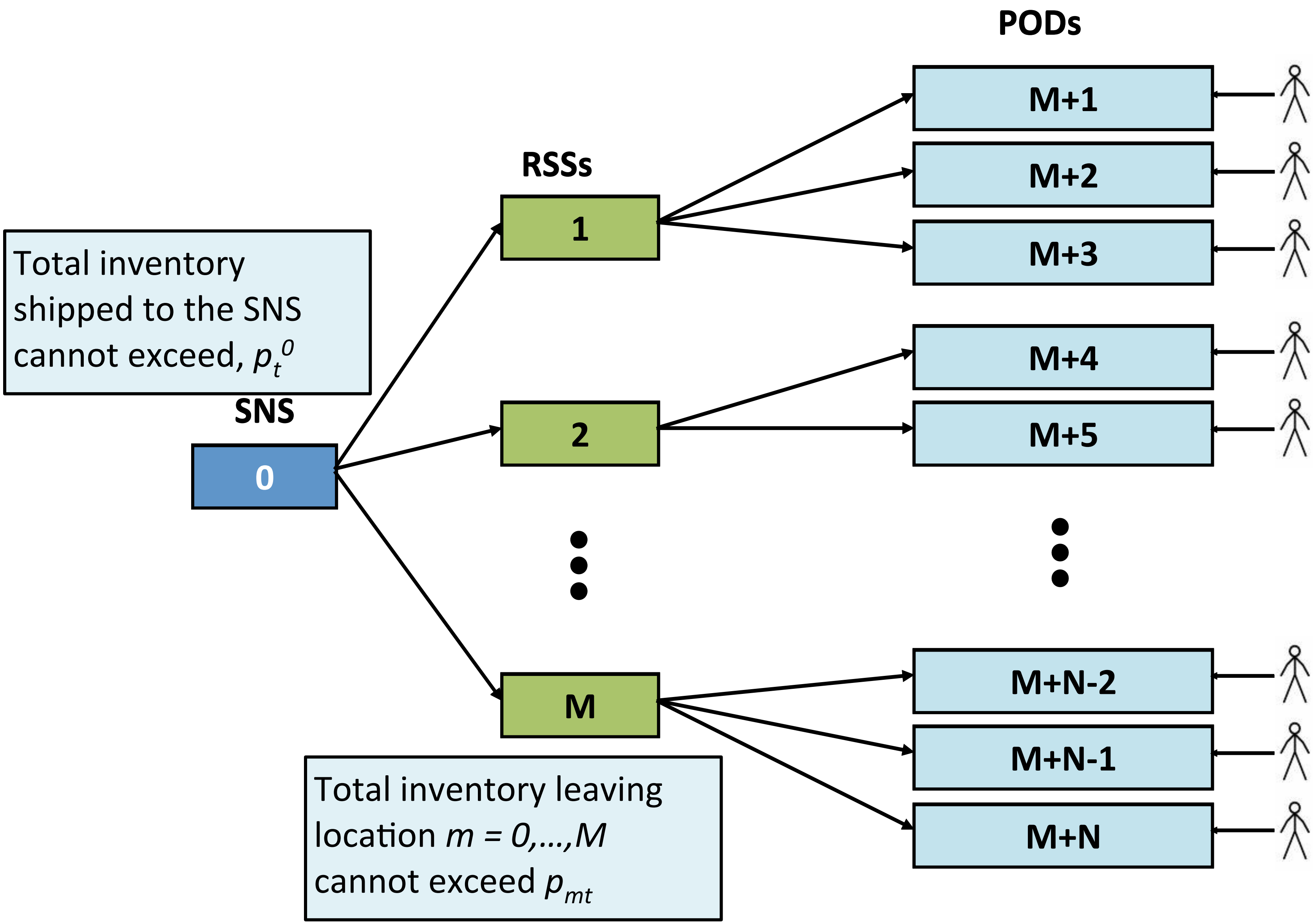
Allocation is Constrained By Physical Requirements

Inventory On Hand
Inventory cannot be shipped to lower echelons unless it is currently on hand



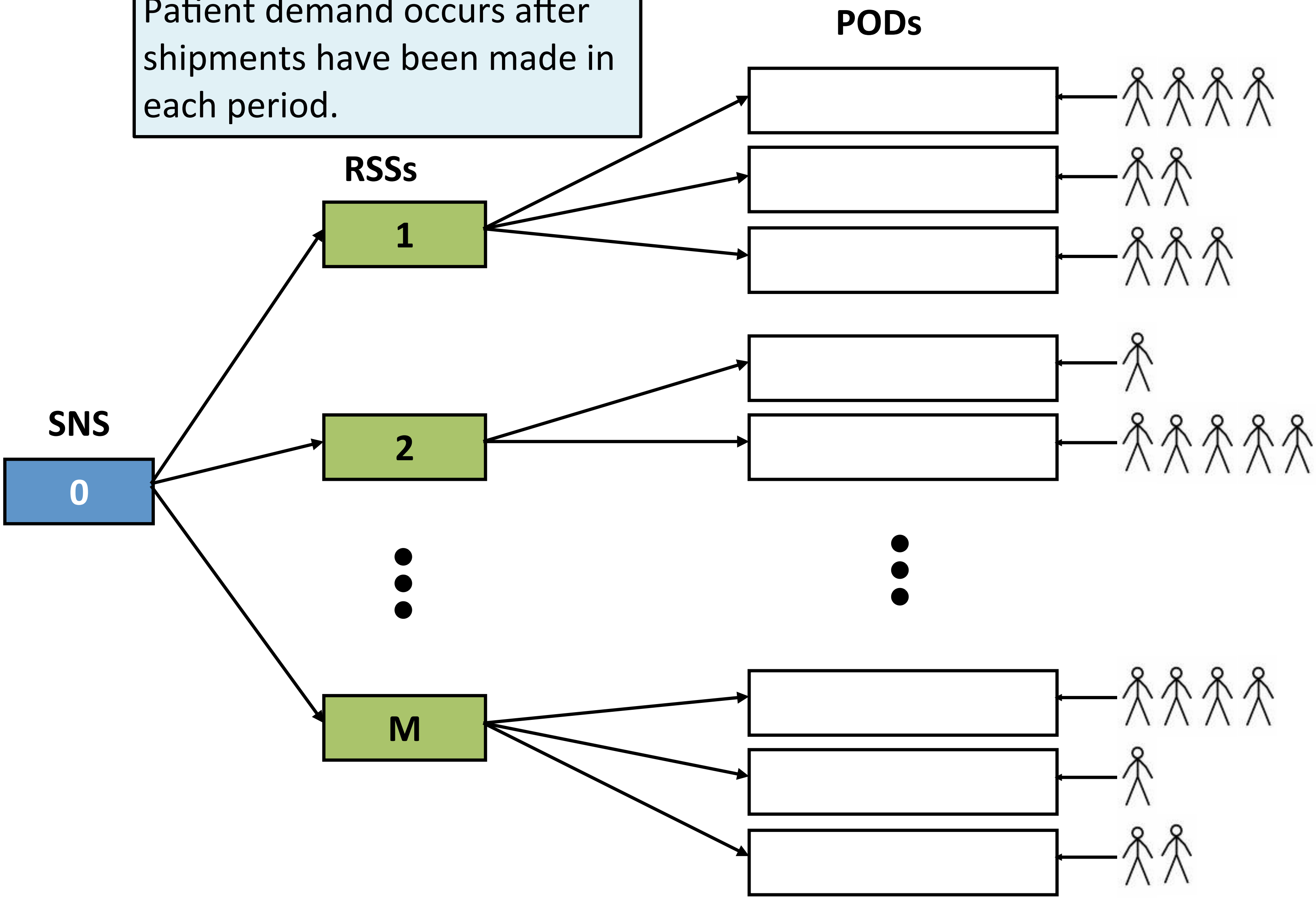
Imbalance Constraint
Inventory cannot be returned to upper echelons

Transportation Constraints Limit Shipment Sizes



Total Patient Demands in Each Period are Random

Patient demand occurs after shipments have been made in each period.



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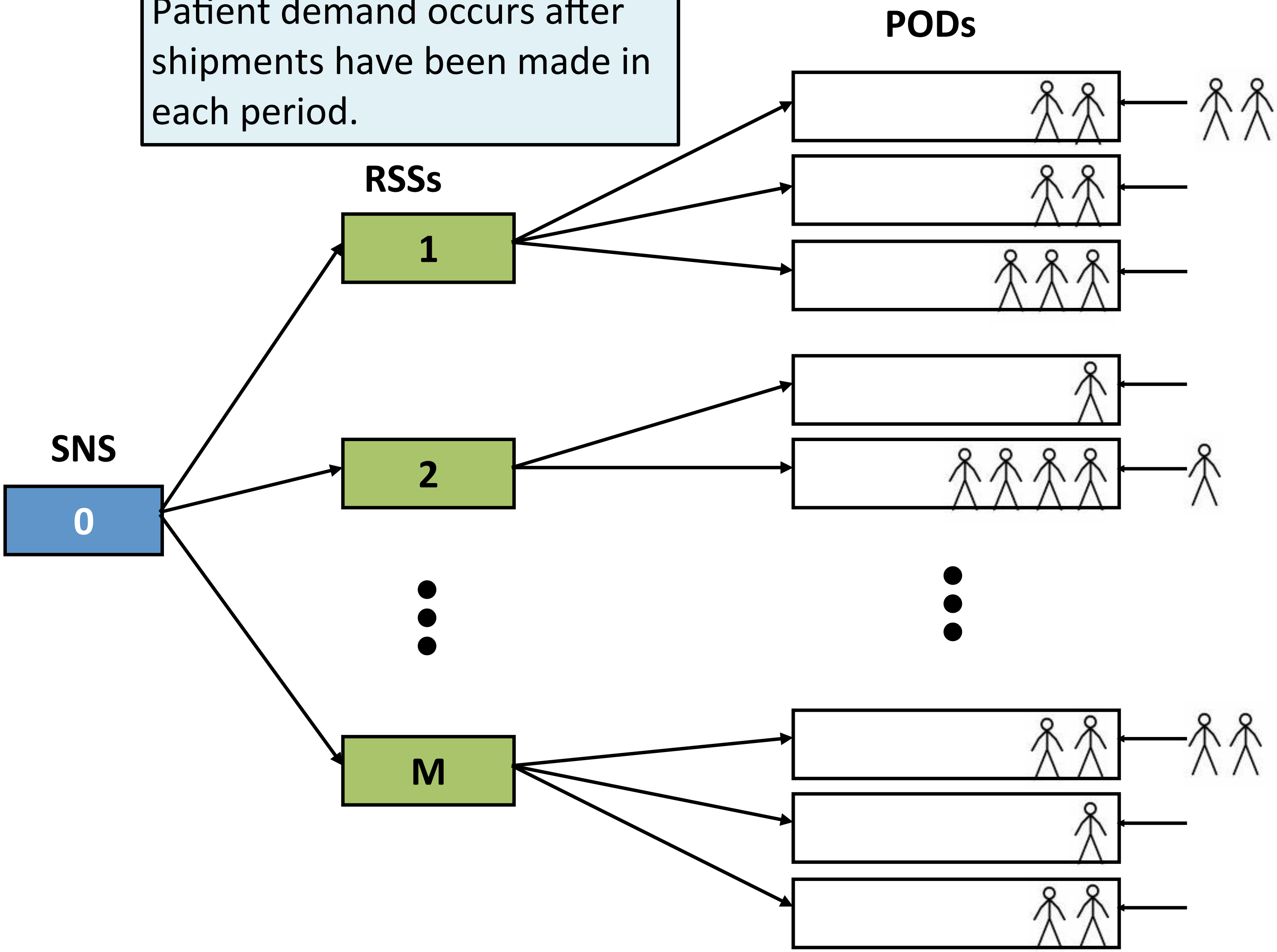
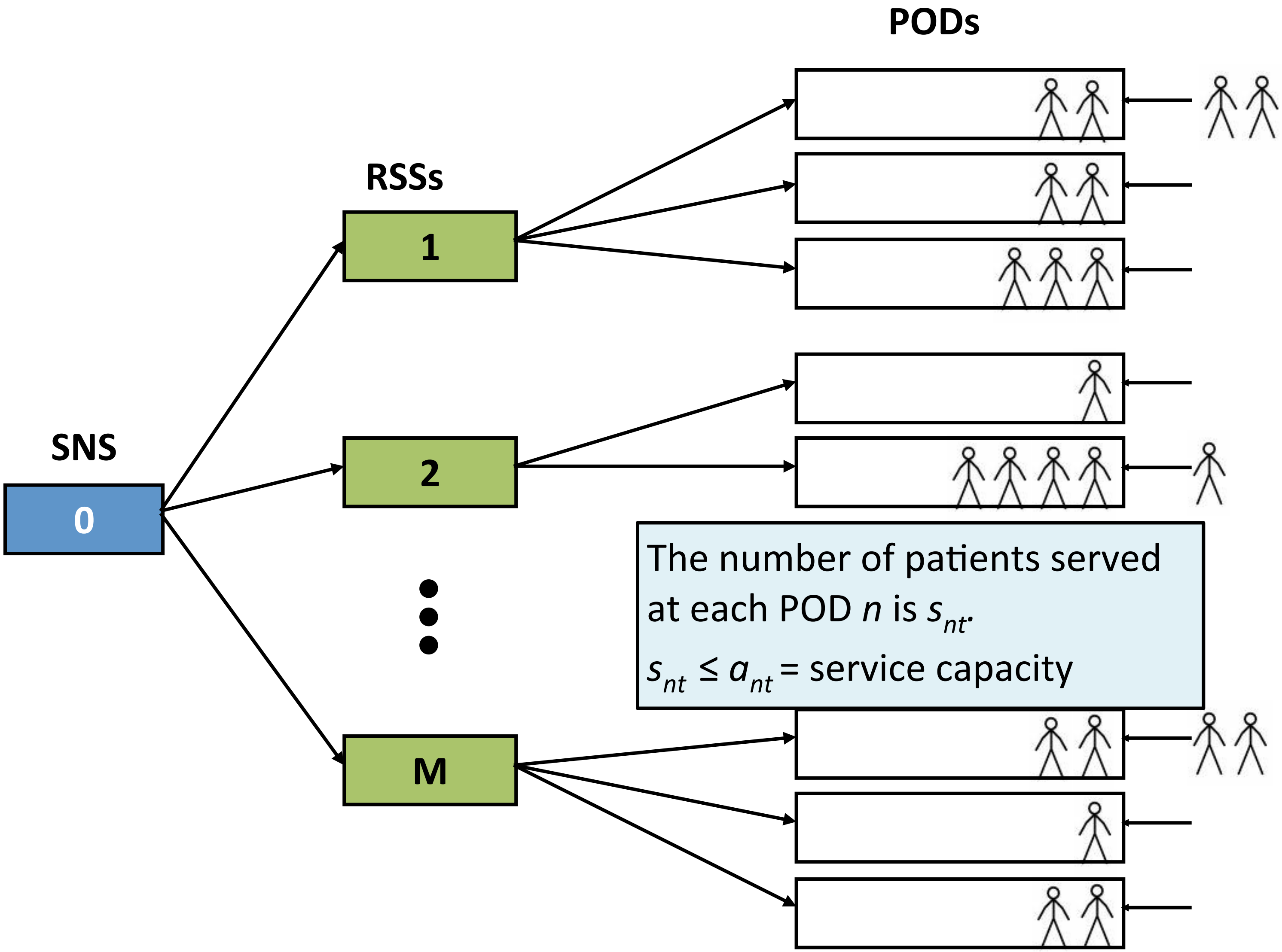


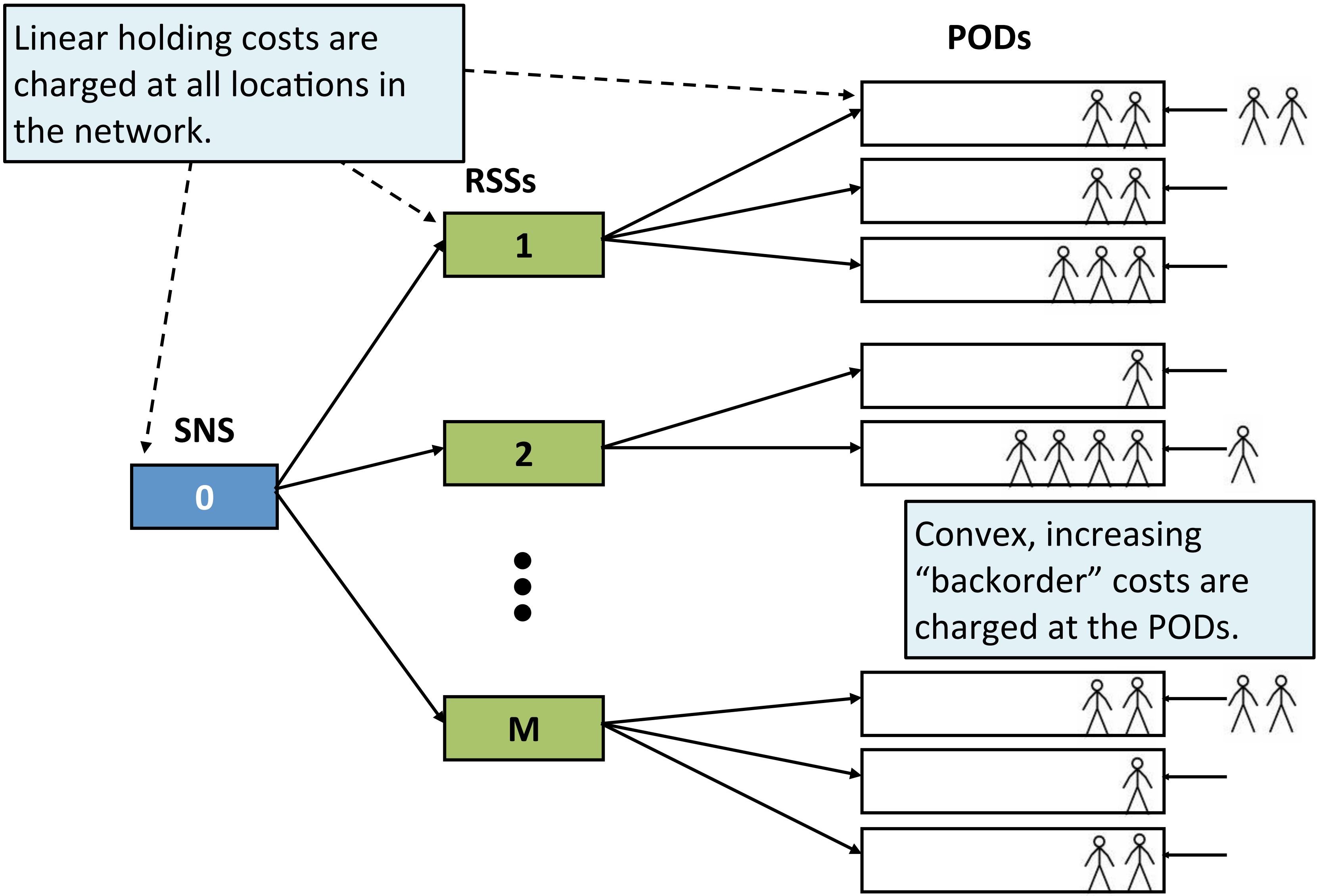
Demand, Inventory, and Capacity Limit the Patients Served



The number of patients served at each POD n is s_{nt}.

$s_{nt} \leq a_{nt}$ = service capacity

Holding and Backorder Costs are Incurred



Dynamic Programming Formulation

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- Let U_t be the feasible decision set in period t and $C_t(\boldsymbol{y}_t, \boldsymbol{q}_t)$ be the expected cost incurred for decisions made during period t.

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- Let U_t be the feasible decision set in period t and $C_t(\mathbf{y}_t, \mathbf{q}_t)$ be the expected cost incurred for decisions made during period t.

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- This problem is intractable due to its large state space. Instead, we construct two approximate solutions.

Solution Method 1: Truncated Cumulative Approximation (TCA)

- Characteristics of inventory allocation in this setting:
  - Limited service capacities → Inventory shipments will be small;
  - Decisions made in each period may not have long-term effects.

- Idea: In each time period, solve a myopic version of the problem to obtain a feasible solution.
  - Estimate the number of patients served using a cumulative approximation, and
  - Truncate the time horizon to “several” periods.

Solution Method 2: Lagrangian Relaxation and Decomposition (LR)

Idea: Use Lagrangian relaxation to simplify the structure of the problem, and then decompose by location.

Step 1: Relax the inventory imbalance and transportation constraints, and add Lagrange multipliers.

Step 2: Reduce the size of the state space by writing the problems in terms of inventory position.

Step 3: Relax one of the service constraints, and add Lagrange multipliers.

Step 4: Decompose the problem by location into M + N + 1 single variable dynamic programs.

Solution Method 2: Lagrangian Relaxation and Decomposition (LR)

- The relaxed problem is a lower bound on the original value function:

$$\tilde{V}_t(\mathbf{x_t} - \mathbf{q_t}) := \sum_{n=0}^{M+N} \psi_{nt}(x_{nt} - q_{nt}) \leq V_t(\mathbf{x_t^p}, \mathbf{q_t^p}, \mathbf{d_t^p})$$

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- We can use this in making inventory allocation decisions:

$$V_t(\mathbf{x_t^p}, \mathbf{q_t^p}, \mathbf{d_t^p}) \approx \min_{\mathbf{y_t} \in \mathcal{U}_t(\mathbf{x_t^p}, \mathbf{q_t^p}, \mathbf{d_t^p})} \Big(C_t(\mathbf{y_t}, \mathbf{q_t}) + E[\tilde{V}_{t+1}(\mathbf{x_{t+1}} - \mathbf{q_{t+1}}, \mathbf{d_t})] \Big)$$

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$V_{t+1}(\cdot)$ has been replaced by $\tilde{V}_{t+1}(\cdot)$

Evaluating the Inventory Allocation Strategies

Compare inventory allocation methods currently in use by public health authorities:

Fair Share Allocation: Inventory is "pushed" out from the SNS and RSSs in proportion to the total demand expected at each location.

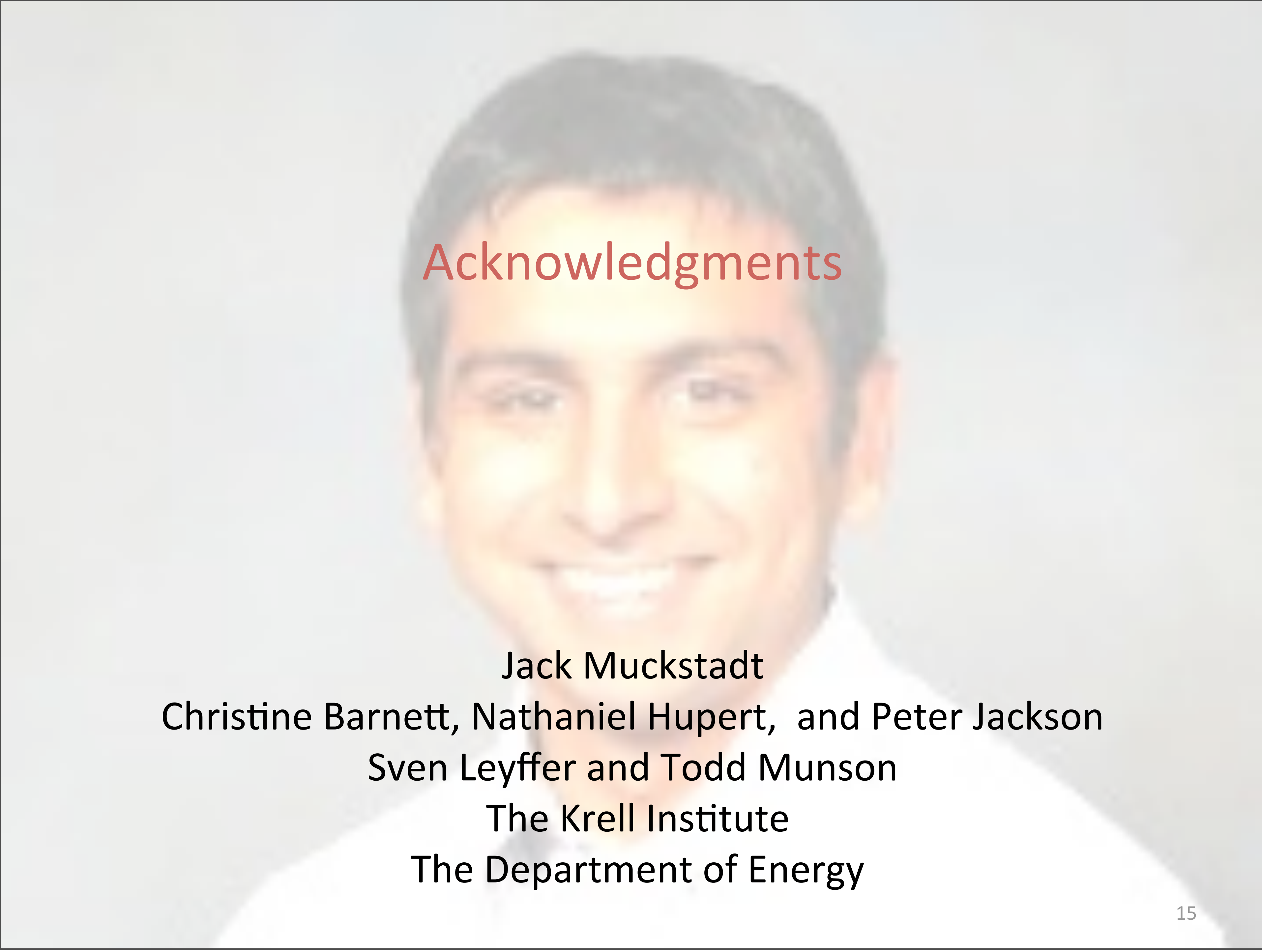
Independent Ordering Method (Order): Solve the myopic inventory problem for each location, and place "orders" accordingly.

Summary of Numerical Results

- Ran 23 simulation experiments
- Reported cost, average delay per patient, and average inventory use per patient
- Observed that:
  - TCA and LR methods perform best
  - Fair Share method is much worse (≥ 40% more inventory used and ≥ 25% longer patient delays)
  - Independent Ordering performs well under specific conditions

Implications of Numerical Results

- Centralized command and control is essential
- More PODs → Longer patient delays and more waste
- Dynamic staffing plans yield improved performance
- Independent Ordering is not robust
- Information must be collected and shared to enable responsive and effective decision-making



Acknowledgments

Jack Muckstadt

Christine Barnett, Nathaniel Hupert, and Peter Jackson

Sven Leyffer and Todd Munson

The Krell Institute

The Department of Energy

Questions?

Extra Slides

The Feasible Decision Set

$$\mathcal{U}_t(\mathbf{x}_\mathbf{t}^{\mathbf{past}}, \mathbf{q}_\mathbf{t}^{\mathbf{past}}, \mathbf{d}_\mathbf{t}^{\mathbf{past}}) = \Big\{ \mathbf{y}_\mathbf{t} : y_{nt} \geq x_{nt} \text{ for all } n = 0, \ldots, M + N;$$

$$\sum_{n=1}^{M+N} u_{mn}(y_{nt} - q_{nt}) \leq x_{m,t-\tau_m} - q_{m,t-\tau_m} - d_{m,t-\tau_m,t-1}$$

$$\text{for all } m = 0, \ldots, M;$$

$$\sum_{n=1}^{M+N} u_{mn}(y_{nt} - x_{nt}) \leq p_{mt} \quad \text{for all } m = 0, \ldots, M;$$

$$y_{0t} - x_{0t} \leq p_t^0 \Big\}$$

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On-Hand Inventory Constraints

The Feasible Decision Set

$$
\begin{aligned}
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& \sum_{n=1}^{M+N} u_{mn}(y_{nt} - q_{nt}) \leq x_{m,t-\tau_m} - q_{m,t-\tau_m} - d_{m,t-\tau_m,t-1} \\
& \qquad \text{for all } m = 0, \dots, M; \\
& \sum_{n=1}^{M+N} u_{mn}(y_{nt} - x_{nt}) \leq p_{mt} \quad \text{for all } m = 0, \dots, M; \\
& y_{0t} - x_{0t} \leq p_t^0 \Big\}
\end{aligned}
$$

Transportation Constraints

Lagrangian Relaxation and Decomposition (LR)

Step 1: Relax the inventory imbalance and transportation constraints, and add Lagrange multipliers.

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$$\text{s.t. } y_{mt} \geq x_{mt} \quad \text{for all } m = 0, \dots, M+N$$

$$\sum_{n=1}^{M+N} u_{mn}(y_{nt} - q_{nt}) \leq x_{m,t-\tau_m} - q_{m,t-\tau_m} - d_{m,t-\tau_m,t-1}$$

$$\text{for } m = 0, \dots, M$$

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$$- \sum_{n=1}^{M+N} \lambda_{nt}(y_{nt} - x_{nt}) - \sum_{m=0}^{M} \mu_{mt} \bigg(p_{mt} - \sum_{n=1}^{M+N} u_{mn}(y_{nt} - x_{nt}) \bigg) \Bigg\}$$

s.t. $y_{0t} \geq x_{0t}$

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$$\text{for } m = 0, \ldots, M$$

$$y_{0t} - x_{0t} \leq p^0_t$$

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Step 2: Reduce the size of the state space by setting $\overline{y}_{nt} = y_{nt} - q_{nt}$ and $\overline{x}_{nt} = x_{nt} - q_{nt}$.

$$V_t(\mathbf{x}_t^\mathbf{p}, \mathbf{q}_t^\mathbf{p}, \mathbf{d}_t^\mathbf{p}) = \min \left\{ \left(C_t(\mathbf{y}_t, \mathbf{q}_t) + E[V_{t+1}(\mathbf{x}_{t+1}^\mathbf{p}, \mathbf{q}_{t+1}^\mathbf{p}, \mathbf{d}_{t+1}^\mathbf{p})] \right) \right.$$

$$\left. - \sum_{n=1}^{M+N} \lambda_{nt}(y_{nt} - x_{nt}) - \sum_{m=0}^{M} \mu_{mt} \left(p_{mt} - \sum_{n=1}^{M+N} u_{mn}(y_{nt} - x_{nt}) \right) \right\}$$

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$$\text{s.t.} \quad \sum_{n=1}^{M+N} u_{mn}\bar{y}_{nt} \le \bar{x}_{m,t-\tau_m} - d_{m,t-\tau_m,t-1} \text{ for all } m = 0, \ldots, m$$

$$\bar{x}_{0t} \le \bar{y}_{0t} \le p^0_t + \bar{x}_{0t}$$

$$\bar{y}_{nt} \le a_{nt} + \bar{x}_{nt} \text{ for all } n = M+1, \ldots, M+N$$

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Step 2: Reduce the size of the state space by setting $\bar{y}_{nt} = y_{nt} - q_{nt}$ and $\bar{x}_{nt} = x_{nt} - q_{nt}$.

$$\bar{V}_t(\bar{\mathbf{x}}_{\mathbf{t}}^{\mathbf{p}}, \mathbf{d}_{\mathbf{t}}^{\mathbf{p}}) = \min \Bigg\{ C_t'(\bar{\mathbf{y}}_{\mathbf{t}}) + E[\bar{V}_{t+1}(\bar{\mathbf{x}}_{\mathbf{t+1}}^{\mathbf{p}}, \mathbf{d}_{\mathbf{t+1}}^{\mathbf{p}})]$$

$$- \sum_{n=1}^{M+N} \lambda_{nt}(\bar{y}_{nt} - \bar{x}_{nt}) - \sum_{m=0}^{M} \mu_{mt}\left(p_{mt} - \sum_{n=1}^{M+N} u_{mn}(\bar{y}_{nt} - \bar{x}_{nt})\right)\Bigg\}$$

$$\text{s.t.} \quad \sum_{n=1}^{M+N} u_{mn}\bar{y}_{nt} \leq \bar{x}_{m,t-\tau_m} - d_{m,t-\tau_m,t-1} \text{ for all } m = 0, \ldots, m$$

$$\bar{x}_{0t} \leq \bar{y}_{0t} \leq p_t^0 + \bar{x}_{0t}$$

$$\bar{y}_{nt} \leq a_{nt} + \bar{x}_{nt} \text{ for all } n = M+1, \ldots, M+N$$

$$\bar{y}_{nt} \leq a_{nt} \text{ for all } n = M+1, \ldots, M+N$$

Lagrangian Relaxation and Decomposition (LR)

Step 3: Relax one of the service constraints, and add Lagrange multipliers.

$$\bar{V}_t(\bar{\mathbf{x}}^{\mathbf{p}}_{\mathbf{t}}, \mathbf{d}^{\mathbf{p}}_{\mathbf{t}}) = \min \Bigg\{ C'_t(\bar{\mathbf{y}}_{\mathbf{t}}) + E[\bar{V}_{t+1}(\bar{\mathbf{x}}^{\mathbf{p}}_{\mathbf{t+1}}, \mathbf{d}^{\mathbf{p}}_{\mathbf{t+1}})] - \sum_{n=1}^{M+N} \lambda_{nt}(\bar{y}_{nt} - \bar{x}_{nt}) - \sum_{m=0}^{M} \mu_{mt} \left(p_{mt} - \sum_{n=1}^{M+N} u_{mn}(\bar{y}_{nt} - \bar{x}_{nt}) \right) \Bigg\}$$

$$\text{s.t.} \quad \sum_{n=1}^{M+N} u_{mn}\bar{y}_{nt} \leq \bar{x}_{m,t-\tau_m} - d_{m,t-\tau_m,t-1} \text{ for all } m = 0, \ldots, m$$

$$\bar{x}_{0t} \leq \bar{y}_{0t} \leq p_t^0 + \bar{x}_{0t}$$

$$\bar{y}_{nt} \leq a_{nt} + \bar{x}_{nt} \text{ for all } n = M+1, \ldots, M+N$$

$$\bar{y}_{nt} \leq a_{nt} \text{ for all } n = M+1, \ldots, M+N$$

Lagrangian Relaxation and Decomposition (LR)

Step 3: Relax one of the service constraints, and add Lagrange multipliers.

$$\bar{V}_t(\bar{\mathbf{x}}^{\mathbf{p}}_{\mathbf{t}}, \mathbf{d}^{\mathbf{p}}_{\mathbf{t}}) = \min \Big\{ C'_t(\bar{\mathbf{y}}_{\mathbf{t}}) + E[\bar{V}_{t+1}(\bar{\mathbf{x}}^{\mathbf{p}}_{\mathbf{t+1}}, \mathbf{d}^{\mathbf{p}}_{\mathbf{t+1}})]$$

$$- \sum_{n=1}^{M+N} \lambda_{nt}(\bar{y}_{nt} - \bar{x}_{nt}) - \sum_{m=0}^{M} \mu_{mt}\Big(p_{mt} - \sum_{n=1}^{M+N} u_{mn}(\bar{y}_{nt} - \bar{x}_{nt})\Big)\Big\}$$

$$\text{s.t.} \quad \sum_{n=1}^{M+N} u_{mn}\bar{y}_{nt} \leq \bar{x}_{m,t-\tau_m} - d_{m,t-\tau_m,t-1} \text{ for all } m = 0, \ldots, m$$

$$\bar{x}_{0t} \leq \bar{y}_{0t} \leq p^0_t + \bar{x}_{0t}$$

$$\bar{y}_{nt} \leq a_{nt} + \bar{x}_{nt} \text{ for all } n = M+1, \ldots, M+N$$

$$\bar{y}_{nt} \leq a_{nt} \text{ for all } n = M+1, \ldots, M+N$$

Lagrangian Relaxation and Decomposition (LR)

Step 3: Relax one of the service constraints, and add Lagrange multipliers.

$$\tilde{V}_t(\bar{\mathbf{x}}_\mathbf{t}^\mathbf{p}, \mathbf{d}_\mathbf{t}^\mathbf{p}) = \min \Big\{ C_t'(\bar{\mathbf{y}}_\mathbf{t}) + E[\tilde{V}_{t+1}(\bar{\mathbf{x}}_\mathbf{t+1}^\mathbf{p}, \mathbf{d}_\mathbf{t+1}^\mathbf{p})]$$

$$- \sum_{n=1}^{M+N} \lambda_{nt}(\bar{y}_{nt} - \bar{x}_{nt}) - \sum_{m=0}^{M} \mu_{mt}\Big(p_{mt} - \sum_{n=1}^{M+N} u_{mn}(\bar{y}_{nt} - \bar{x}_{nt})\Big)$$

$$- \sum_{n=M+1}^{M+N} \gamma_{nt}(a_{nt} + \bar{x}_{nt} - \bar{y}_{nt})\Big\}$$

$$\text{s.t.} \quad \sum_{n=1}^{M+N} u_{mn}\bar{y}_{nt} \le \bar{x}_{m,t-\tau_m} - d_{m,t-\tau_m,t-1} \text{ for all } m = 0, \ldots, m$$

$$\bar{x}_{0t} \le \bar{y}_{0t} \le p_t^0 + \bar{x}_{0t}$$

$$\bar{y}_{nt} \le a_{nt} \text{ for all } n = M+1, \ldots, M+N$$

Lagrangian Relaxation and Decomposition (LR)

Step 4: Decompose the problem by location into M + N + 1 single variable dynamic programs.

Lagrangian Relaxation and Decomposition (LR)

- For the PODs:

$$\psi_{nt}(\bar{x}_{nt}) = \min_{\{\bar{y}_{nt} \le a_{nt}\}} \Big\{ C'_{nt}(\bar{y}_{nt}) + (\mu^U_{nt} + \gamma_{nt} - \lambda_{nt})(\bar{y}_{nt} - \bar{x}_{nt}) - \gamma_{nt} a_{nt} + E[\psi_{n,t+1}(\bar{y}_{nt} - D_{nt})] \Big\}$$

- For the RSSs:

$$\psi_{mt}(\bar{x}_{mt}) = \min \Big\{ C'_{mt}(\bar{y}_{mt}) + E[\Delta_{m,t+\tau_m}(\bar{x}_{mt} - D_{mt,t+\tau_m-1})] - \lambda_{mt}(\bar{y}_{mt} - \bar{x}_{mt}) - \mu_{mt} p_{mt} + E[\psi_{m,t+1}(\bar{y}_{mt} - D_{mt})] \Big\}$$

Lagrangian Relaxation and Decomposition (LR)

- For the PODs:

$$\begin{aligned}\psi_{nt}(\bar{x}_{nt}) \quad = \quad & \min_{\{\bar{y}_{nt} \le a_{nt}\}} \Big\{ C'_{nt}(\bar{y}_{nt}) + (\mu^U_{nt} + \gamma_{nt} - \lambda_{nt})(\bar{y}_{nt} - \bar{x}_{nt}) - \gamma_{nt} a_{nt} \\ & + E[\psi_{n,t+1}(\bar{y}_{nt} - D_{nt})] \Big\}\end{aligned}$$

- For the RSSs:

$$\begin{aligned}\psi_{mt}(\bar{x}_{mt}) \quad = \quad & \min \Big\{ C'_{mt}(\bar{y}_{mt}) + E[\Delta_{m,t+\tau_m}(\bar{x}_{mt} - D_{mt,t+\tau_m-1})] \\ & -\lambda_{mt}(\bar{y}_{mt} - \bar{x}_{mt}) - \mu_{mt} p_{mt} + E[\psi_{m,t+1}(\bar{y}_{mt} - D_{mt})] \Big\}\end{aligned}$$

- For the SNS:

$$\begin{aligned}\psi_{0t}(\bar{x}_{0t}) \quad = \quad & \min_{\{\bar{x}_{0t} \le \bar{y}_{0t} \le \bar{x}_{0t} + p^0_t\}} \Big\{ C'_{0t}(\bar{y}_{0t}) + E[\Delta_{0,t+\tau_0}(\bar{x}_{0t} - D_{0t,t+\tau_0-1})] \\ & -\mu_{0t} p_{0t} + E[\psi_{0,t+1}(\bar{y}_{0t} - D_{0t})] \Big\}\end{aligned}$$

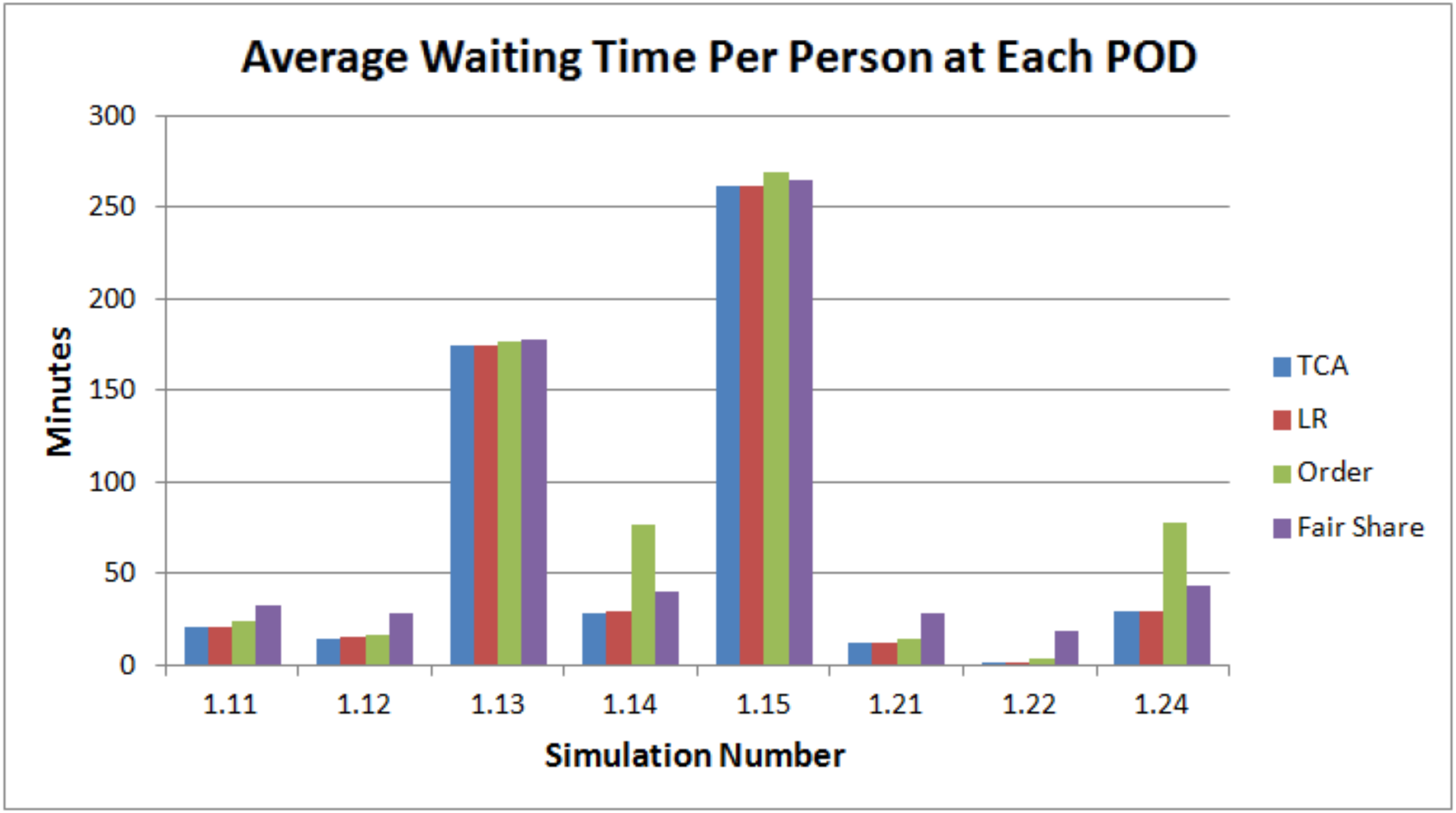
The RSS Penalty Function

$$
\begin{aligned}
\Delta_{mt}(\bar{x}'_{mt}|\lambda,\mu,\gamma) \quad = \quad & \min \sum_{n\in\mathcal{P}} u_{mn}\Big[[(\mu^U_{nt}+\gamma_{nt}-\lambda_{nt}) \\
& -(\mu^U_{n,t+1}+\gamma_{n,t+1}-\lambda_{n,t+1})](\bar{y}_{nt}-\bar{y}^*_{nt}) \\
& +C'_{nt}(\bar{y}_{nt})-C'_{nt}(\bar{y}^*_{nt})\Big] \\
\text{such that} \quad & \sum_{n\in\mathcal{P}} u_{mn}\bar{y}_{nt} \le \bar{x}'_{mt} \\
& \bar{y}_{nt} \le a_{nt} \text{ for } n \in \mathcal{P}
\end{aligned}
$$

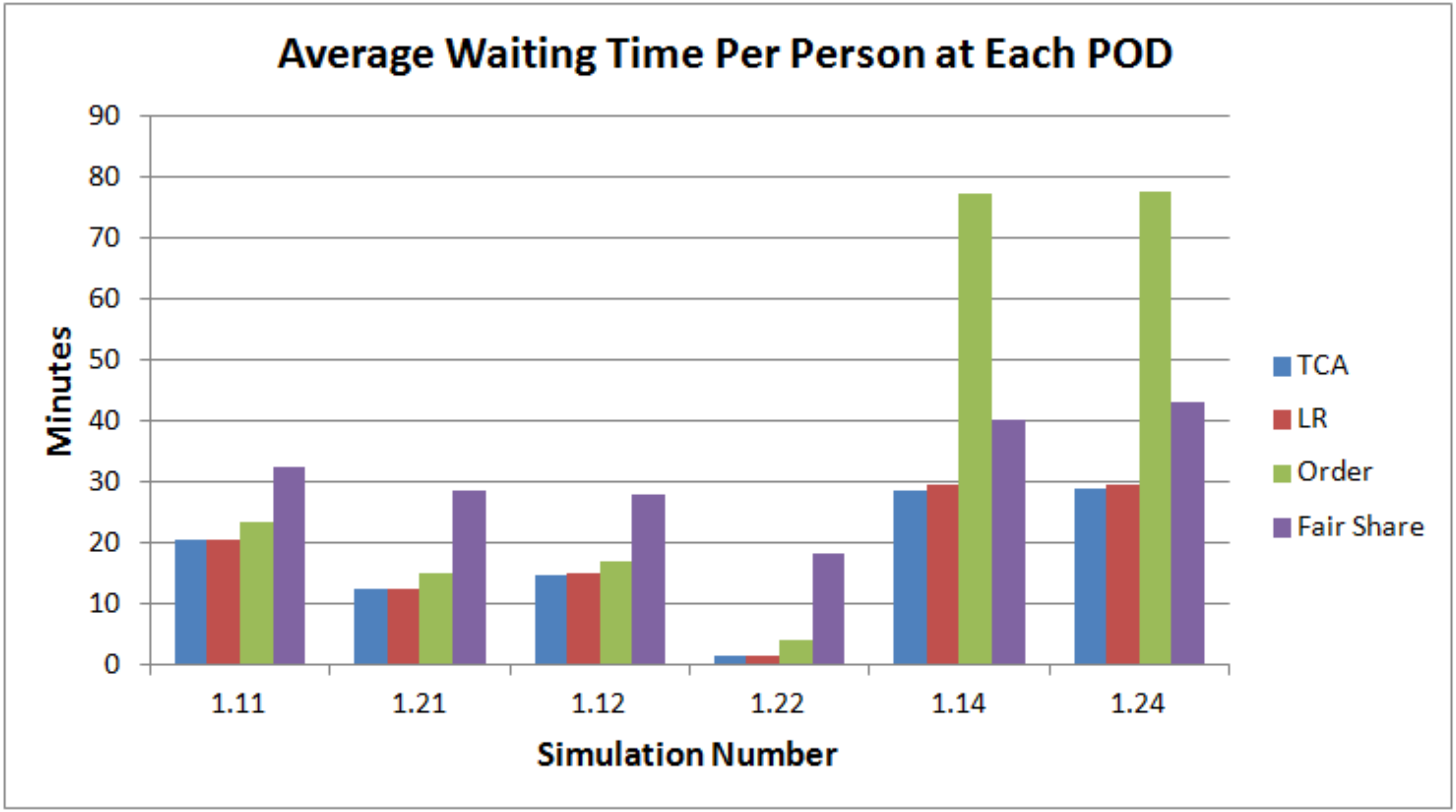
The SNS Penalty Function

$$\begin{aligned}
\Delta_{0t}(\bar{x}'_{0t}|\lambda,\mu,\gamma) &= \min \sum_{m\in\mathcal{R}} \Big(C'_{mt}(\bar{y}_{mt}) - C'_{mt}(\bar{y}^*_{mt}) \\
&\quad -\lambda_{mt}(\bar{y}_{mt} - \bar{y}^*_{mt}) \\
&\quad +E[\psi_{m,t+1}(\bar{y}_{mt} - D_{mt}|\lambda,\mu,\gamma) \\
&\qquad -\psi_{m,t+1}(\bar{y}^*_{mt} - D_{mt}|\lambda,\mu,\gamma)]\Big) \\
\text{such that} &\quad \sum_{m\in\mathcal{R}} \bar{y}_{mt} \le \bar{x}'_{0t}.
\end{aligned}$$

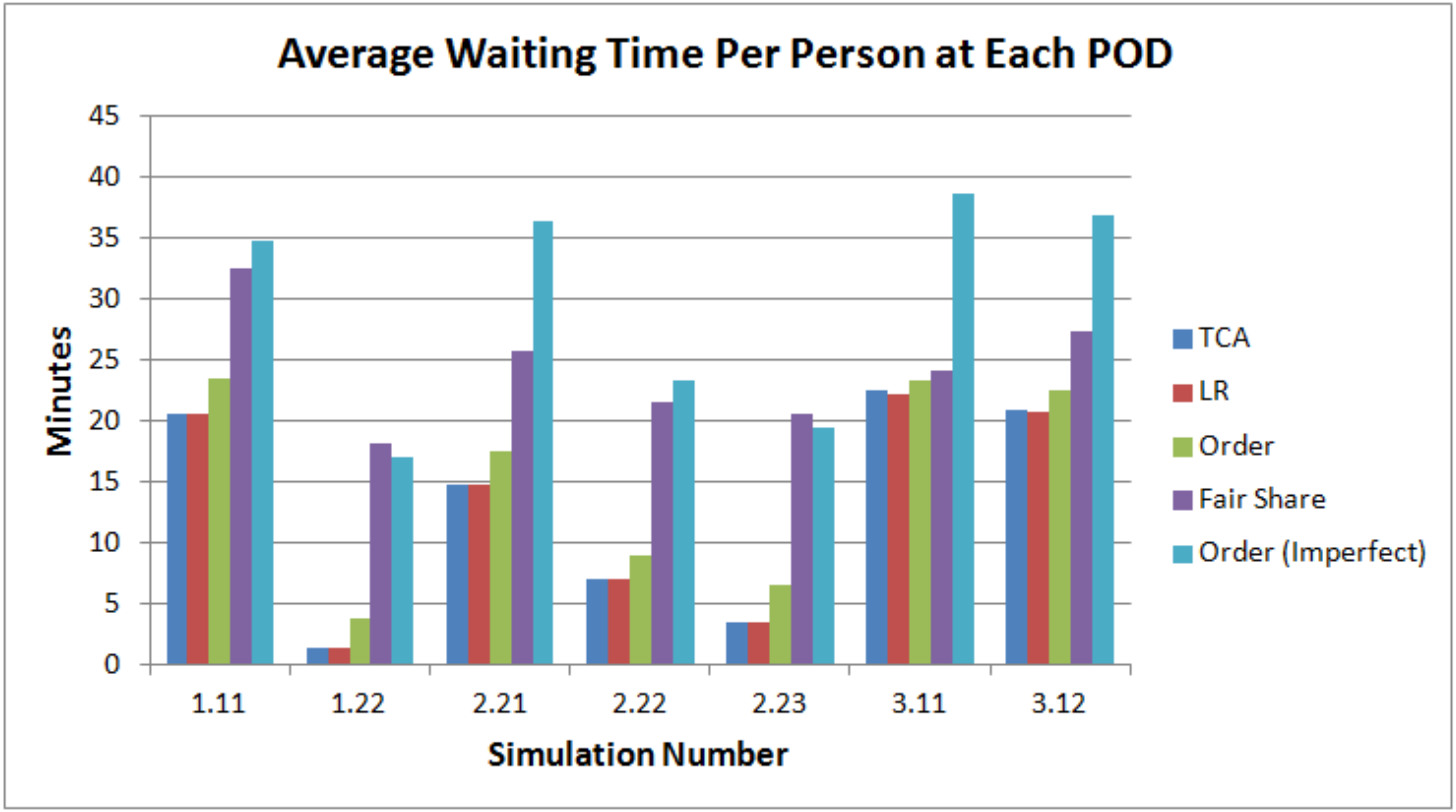
Sample Computational Results



Sample Computational Results



Sample Computational Results



Inventory Allocation Numerical Results

	TCA		**LR**		**Order**		**Fair Share**	
Simulation	WS LB	LR LB	WS LB	LR LB	WS LB	LR LB	WS LB	LR LB
1.11	1.431	1.634	**1.423**	1.625	1.575	1.798	2.465	2.814
1.12	**1.094**	8.931	1.135	9.263	1.155	9.433	2.047	16.715
1.13	**1.074**	2.208	1.089	2.240	1.089	2.240	1.122	2.307
1.14	**1.130**	2.609	1.166	2.693	3.865	8.928	1.839	4.248
1.15	**1.035**	1.992	1.040	2.001	1.050	2.020	1.052	2.025
1.21	1.344	1.045	1.343	**1.044**	1.408	1.095	3.069	2.386
1.22	3.393	1.374	3.602	1.459	3.385	**1.370**	19.519	7.903
1.24	**1.137**	1.973	1.171	2.033	3.787	6.572	2.067	3.587
2.11	**1.140**	4.413	1.160	4.491	1.261	4.880	1.499	5.802
2.12	**1.557**	2.194	1.553	2.188	1.699	2.395	2.386	3.362
2.13	1.550	**1.406**	1.558	1.413	1.680	1.524	2.784	2.526
2.21	**1.536**	3.379	1.578	3.472	1.723	3.791	2.936	6.459
2.22	2.072	**1.527**	2.136	1.575	2.291	1.689	7.032	5.185
2.23	3.793	**1.202**	4.051	1.284	4.033	1.278	16.598	5.259
2.31	**1.176**	2.263	1.181	2.273	1.265	2.436	1.353	2.604
2.32	1.195	2.902	**1.195**	2.900	1.247	3.028	1.332	3.232
2.33	1.243	2.003	**1.240**	1.998	1.280	2.061	1.410	2.271
3.11	1.145	1.619	**1.138**	1.609	1.149	1.625	1.249	1.766
3.12	1.324	1.361	**1.312**	1.348	1.336	1.372	1.876	1.928
3.13	1.590	2.021	**1.581**	2.010	1.770	2.250	2.599	3.304
3.14	1.978	2.303	**1.935**	2.253	2.248	2.617	3.673	4.276
3.23	3.964	**1.135**	4.059	1.162	4.227	1.210	26.509	7.588
3.24	5.151	1.374	5.039	**1.344**	5.493	1.465	31.156	8.311

Simulation Models

Goals: Help public health planners to

1. Explore how capacities interact with and affect one another;

2. Understand the impact of uncertainty on their emergency response plans.

Models:

1. Dynamic POD Simulator (D-PODS)

2. Emergency Supply Chain Operations Evaluator (ESCOE)

D-PODS: The Dynamic POD Simulator

Cornell University

Step 1: Construct the Model

Main Menu

Construct Model

Generate Arrivals

Set Service Parameters

Establish Staffing Levels

Run Simulation

Step A	Input duration of the campaign in days	2
	Input hours of operation per day	22
Step B	Number of Stations (maximum is 15)	4
	Input station names and exits	Station Names
	Input station transition probabilities	Transition Rates
Step C	Input PoD capacity	50000
Step D	Return to Main Page	Return

Transition Rates Table

	Greeting	Triage	Medical Eval	Drug Dispensing	Exit
Greeting		0.95	0.05	0.00	0.00
Triage			0.05	0.95	0.00
Medical Eval				0.95	0.05
Drug Dispens					1.00

The first station is the immediate station after entering the PoD. All subsequent stations should be listed in a downward flowing manner. Meaning, if station j can be reached from station i, then i should be listed before station j. Likewise, one must exit from the last station.

Station Names Table

Station	1	2	3	4	
Name	Greeting	Triage	Medical Eval	Drug Dispensing	Exit

The transition rate matrix below indicates the probability of flowing from state i to state j within the PoD. The Exit column in the transition rate matrix below the probability of flowing out of the PoD from any of the stations.

D-PODS: The Dynamic POD Simulator

Cornell University

Step 1: Construct the Model

Main Menu

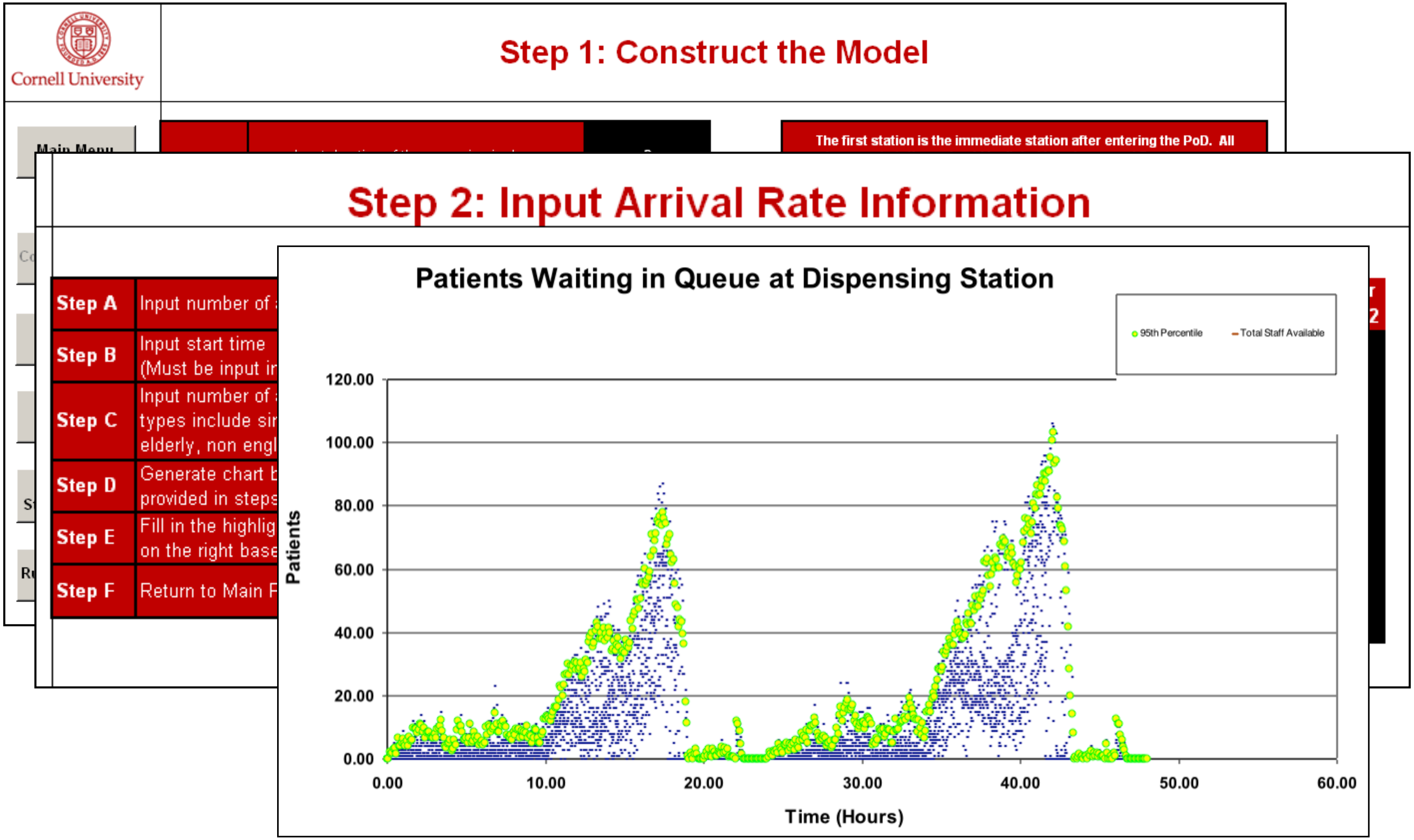
The first station is the immediate station after entering the PoD. All

Step 2: Input Arrival Rate Information

Step A	Input number of arrival intervals	12
Step B	Input start time (Must be input in time format)	7:00 AM
Step C	Input number of arrival types. Arrival types include single person, families, elderly, non english speakers, etc.	2
Step D	Generate chart based on information provided in steps A and B	Chart
Step E	Fill in the highlighted cells of the chart on the right based on the headers	
Step F	Return to Main Page	Return

Interval Number	Hours per Interval	Interval Start	Interval End Time	Arrivals Per Hour: Type 1	Arrivals Per Hour: Type 2
1	2.00	7:00 AM	9:00 AM	300	150
2	2.00	9:00 AM	11:00 AM	200	100
3	2.00	11:00 AM	1:00 PM	500	250
4	2.00	1:00 PM	3:00 PM	500	250
5	2.00	3:00 PM	5:00 PM	700	350
6	2.00	5:00 PM	7:00 PM	400	200
7	2.00	7:00 PM	9:00 PM	200	100
8	2.00	9:00 PM	11:00 PM	100	50
9	2.00	11:00 PM	1:00 AM	100	50
10	2.00	1:00 AM	3:00 AM	50	25
11	2.00	3:00 AM	5:00 AM	0	0
12	2.00	5:00 AM	7:00 AM	200	100

D-PODS: The Dynamic POD Simulator



ESCOE: The Emergency Supply Chain Operations Evaluator

ESCOE Menu

In order to run the program, follow the steps as shown below. The program may not work if executed in the incorrect order.

Step 1		Simulation Start Time:	8:00 AM
		Period Length (Hours):	2
		Number of Periods:	24
	Simulation End Time:	Day:	3
		Time:	8:00 AM
Step 2	Construct the Network		Network
Step 3	Describe the Lead Times		Lead Times
Step 4	Describe the Inventory		Inventory
Step 5	Describe the SNS		SNS
Step 6	Describe the FDSs		FDSs
Step 7	Describe the RSSs		RSSs
Step 8	Describe the POD Types		POD Types

ESCOE: The Emergency Supply Chain Operations Evaluator