Electromagnetic Fluctuations in Microstructured Geometries

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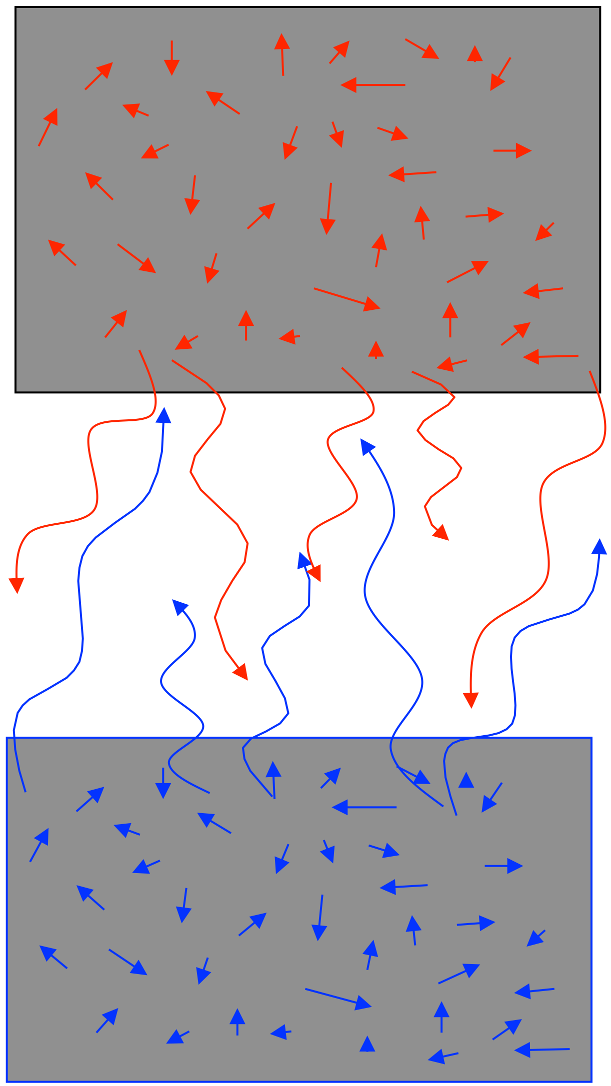
Electromagnetic Fluctuations



All matter is filled with constantly fluctuating current sources…

… radiating photons

= fluctuating fields everywhere

at all frequencies

[*even at zero temperature*

due to quantum effects]

causing nearby matter to interact

EM-fluctuation interactions are:

- Increasingly important in several contexts
  — as devices reach sub-μm scales

- Surprisingly hard to solve mathematically
  — even *two-cylinder* problems fully solved only recently

- Increasingly solvable by translation into classical wave problems.
  — many recent developments

- Effects of new geometries are exciting and ~ unexplored.

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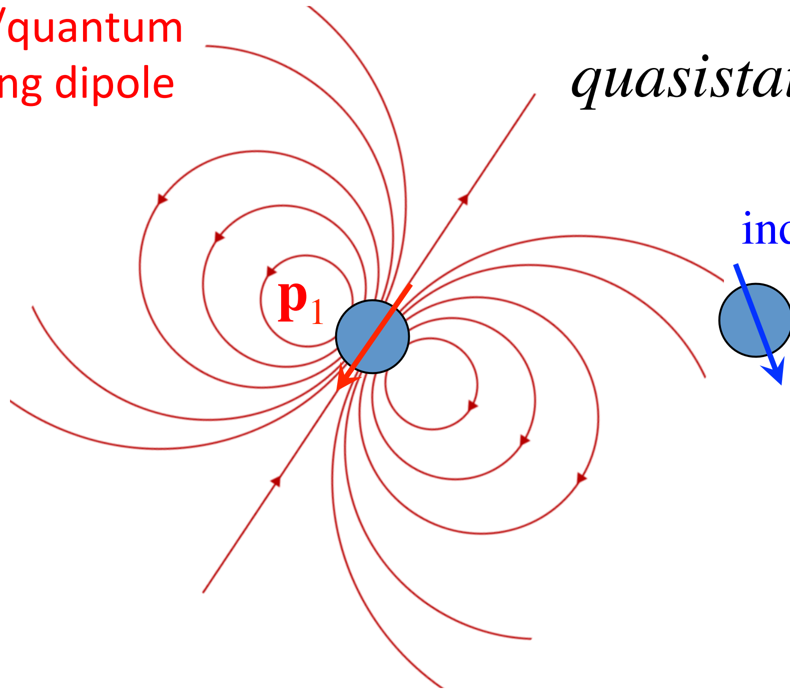
van der Waals Forces



$$\mathbf{p}_2 \sim \mathbf{E}_1 \sim \mathbf{p}_1 / d^3$$

$$U \propto -\langle \mathbf{p}_2 \cdot \mathbf{E}_1 \rangle \propto -\frac{\langle \mathbf{p}_2 \cdot \mathbf{p}_1 \rangle}{d^6}$$

dipole–dipole correlation (statistics of fluctuations)

$$\langle \mathbf{p}_2 \cdot \mathbf{p}_1 \rangle_\omega \sim \Theta_T(\omega) = \underbrace{\frac{1}{2} + \frac{\hbar\omega}{e^{\hbar\omega/kT} + 1}}_{} = \coth\left(\frac{\hbar\omega}{2kT}\right)$$

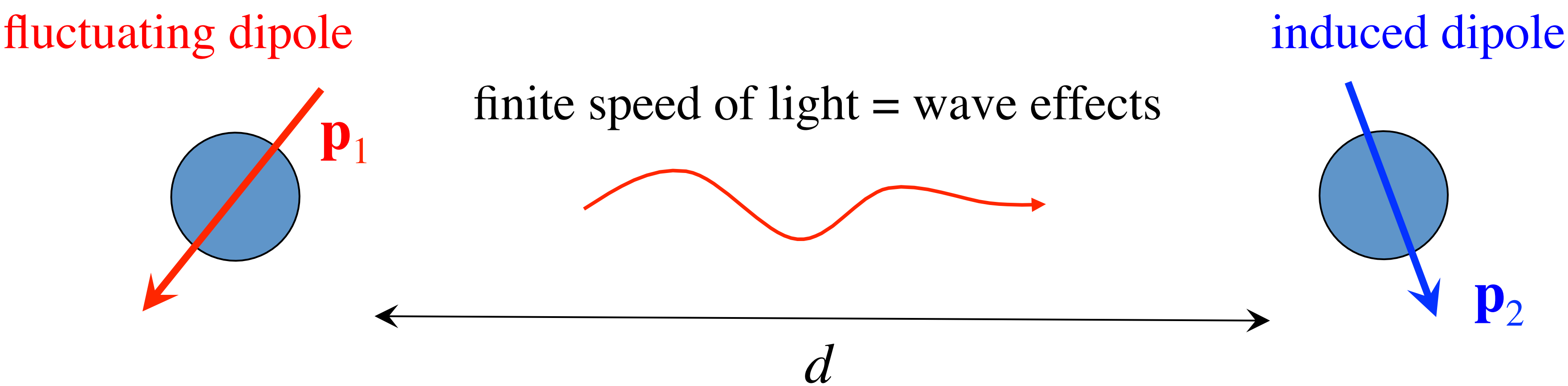
Planck spectrum (T=0 means only quantum fluctuations)

van der Waals Forces: Approximations

- Small separations only (e.g. < 10s of nm)
  = neglect wave effects

- Dilute / weakly-polarizable only
  = neglect multiple scattering

Casimir–Polder Forces

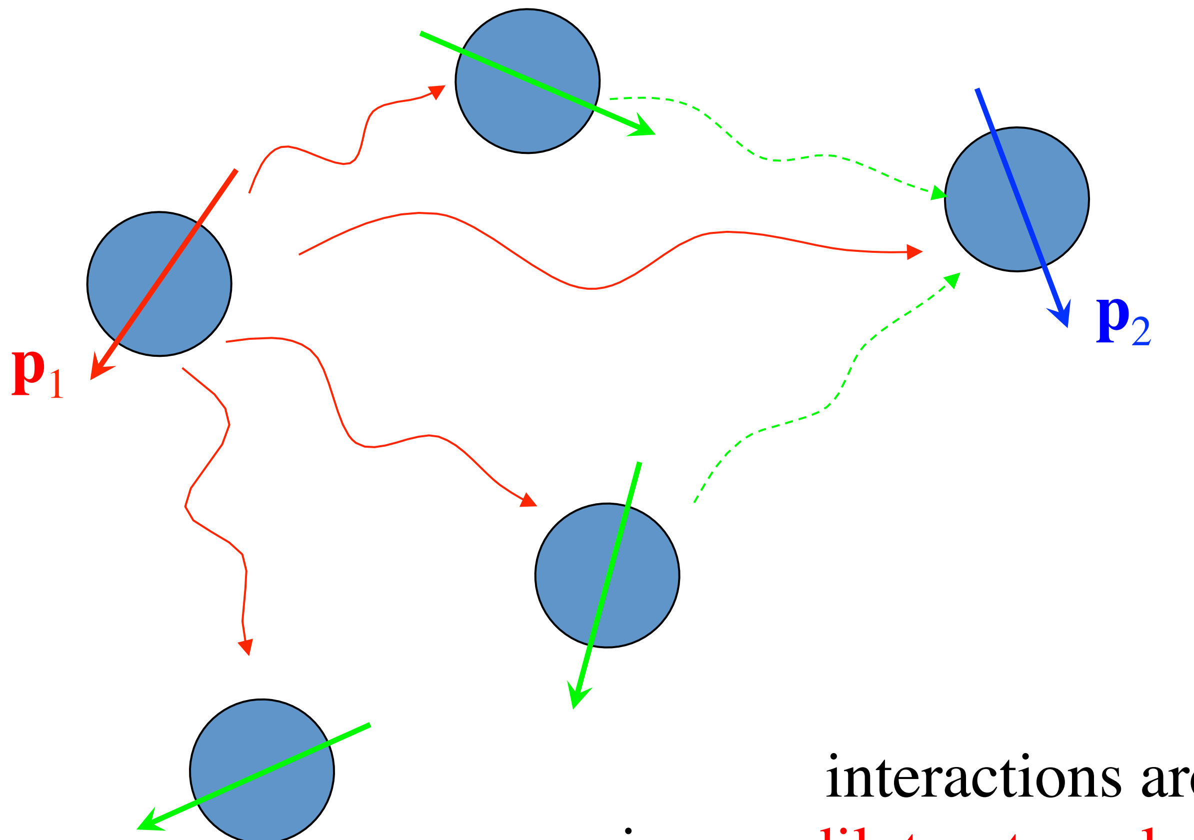


for separations >> resonant wavelength

$$U \sim -\frac{1}{d^7} \quad \Rightarrow \quad F \sim -\frac{1}{d^8}$$

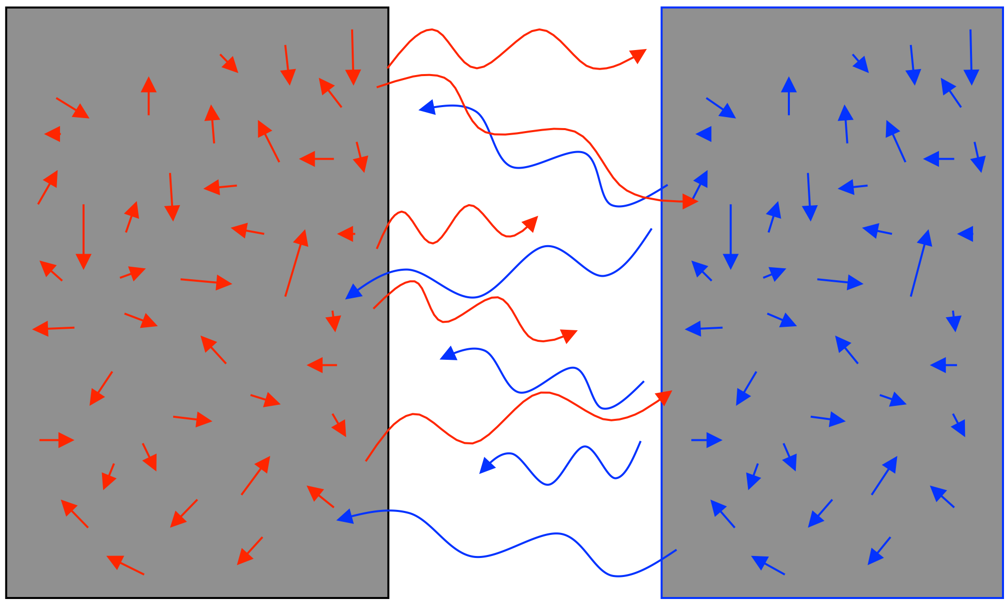
… in general, not a simple power law, because
polarizability is frequency-dependent (dispersion)

Multiple scattering



interactions are modified
in non-dilute, strongly polarizable media
= liquids/solids with large index contrasts

Casimir Forces (Macroscopic Bodies)

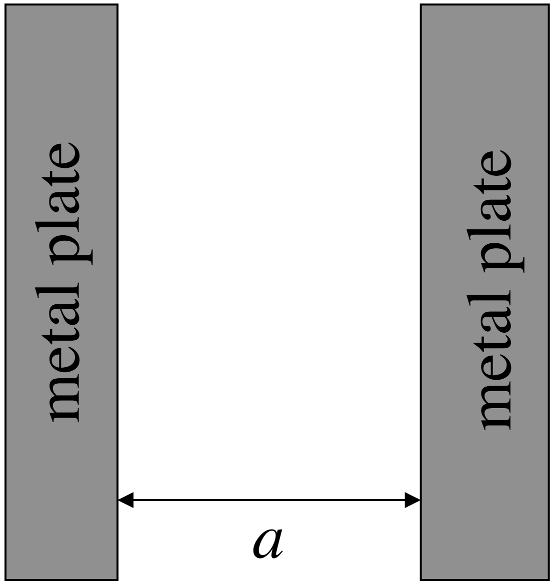


many interacting dipoles

EM field must satisfy boundary conditions at material interfaces …designable fluctuations!

The Casimir Force

[H. Casimir, 1948]



Parallel, neutral, perfect-metal plates, separation a

$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$$

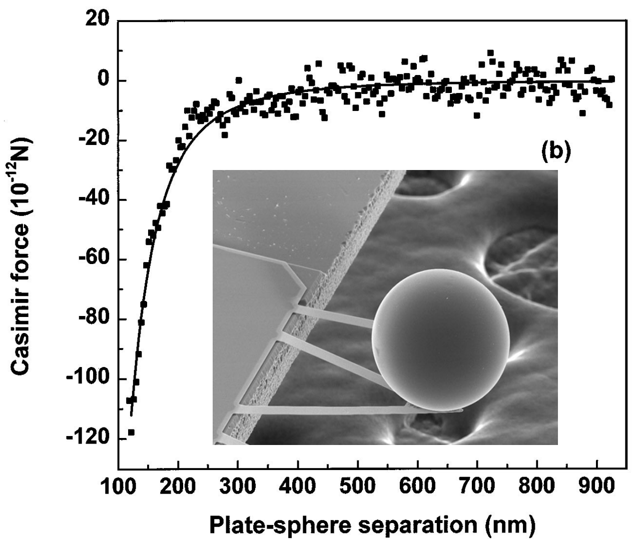
attractive force, monotonic decreasing

- 10^{-7} N for a=1μm, A=1cm^2
- 1atm for $a \sim 100$nm, A=1μm^2

Experimental Progress

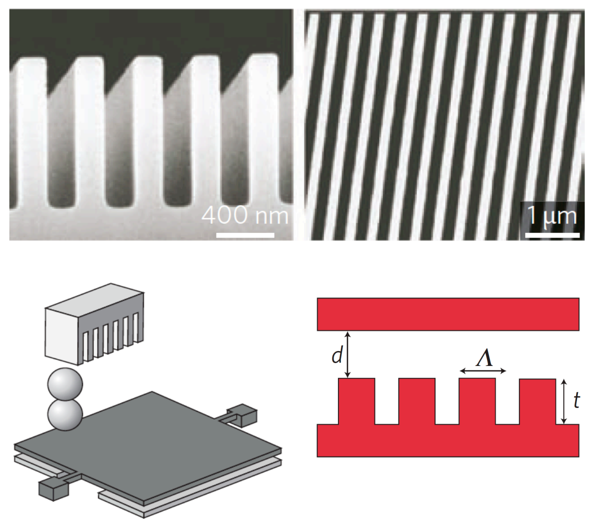
- Recently verified via high precision experiments (~ 5% accuracy)
- Experiments beginning to explore more complex geometries

sphere–plate



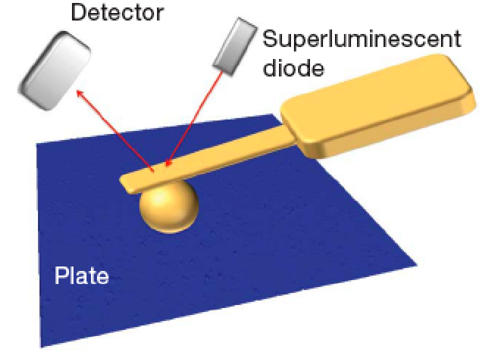
[U. Mohideen *et. al.* **PRL, 81** (1998)]

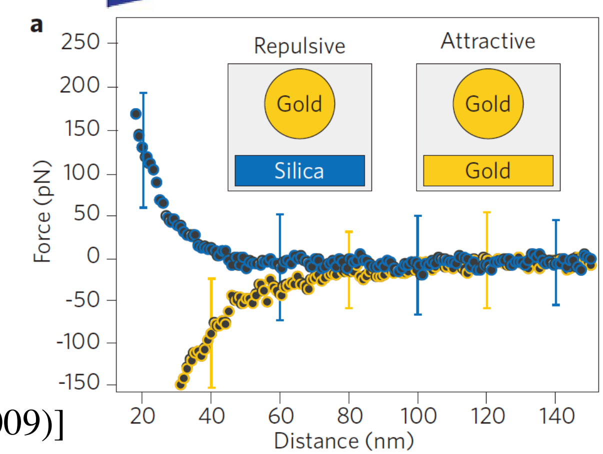
trenches



[H. B. Chan *et. al.*
PRL 101, 030401,(2008)]

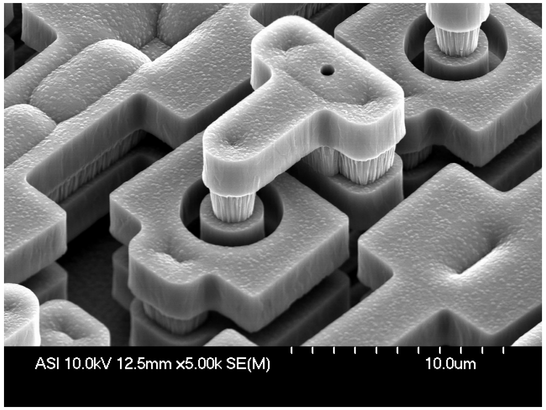
fluid experiments

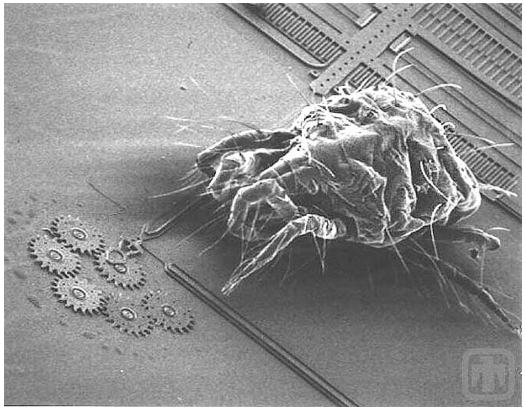




[Munday *et. al.*
Nature, 157 (2009)]

Micromechanical Devices



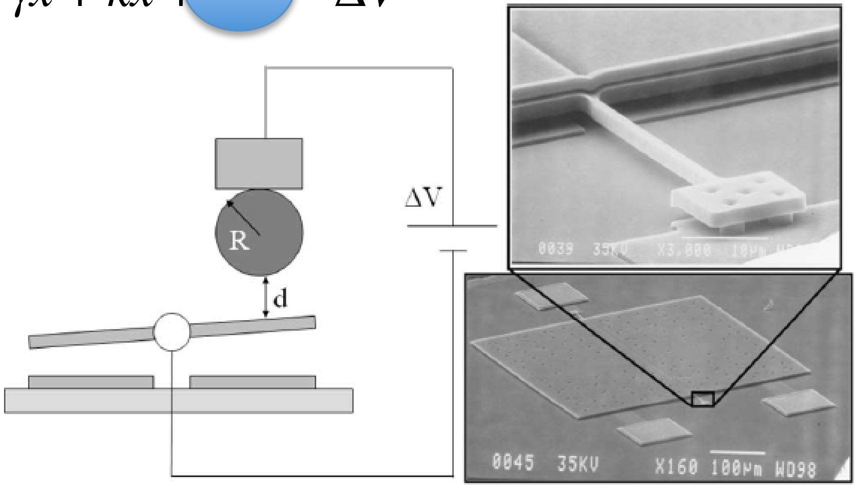


attractive Casimir force causes stiction

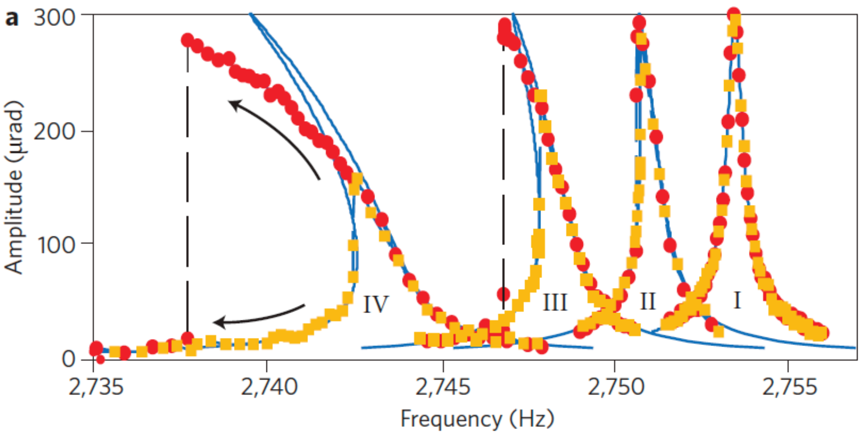
reduce stiction
design the force?

nonlinear oscillator

$$m\ddot{x} + \gamma\dot{x} + kx + \quad = \Delta V$$



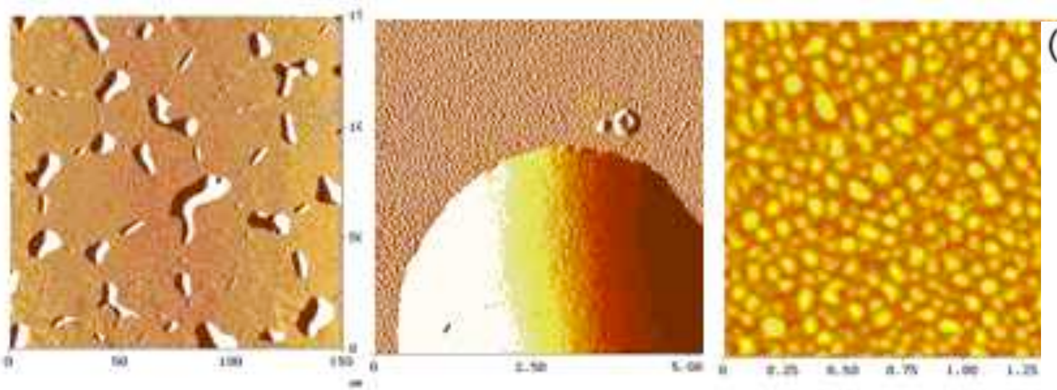
frequency response at different *d*



[H. Chan *et*. *al*., **Science 91**, (2001)]

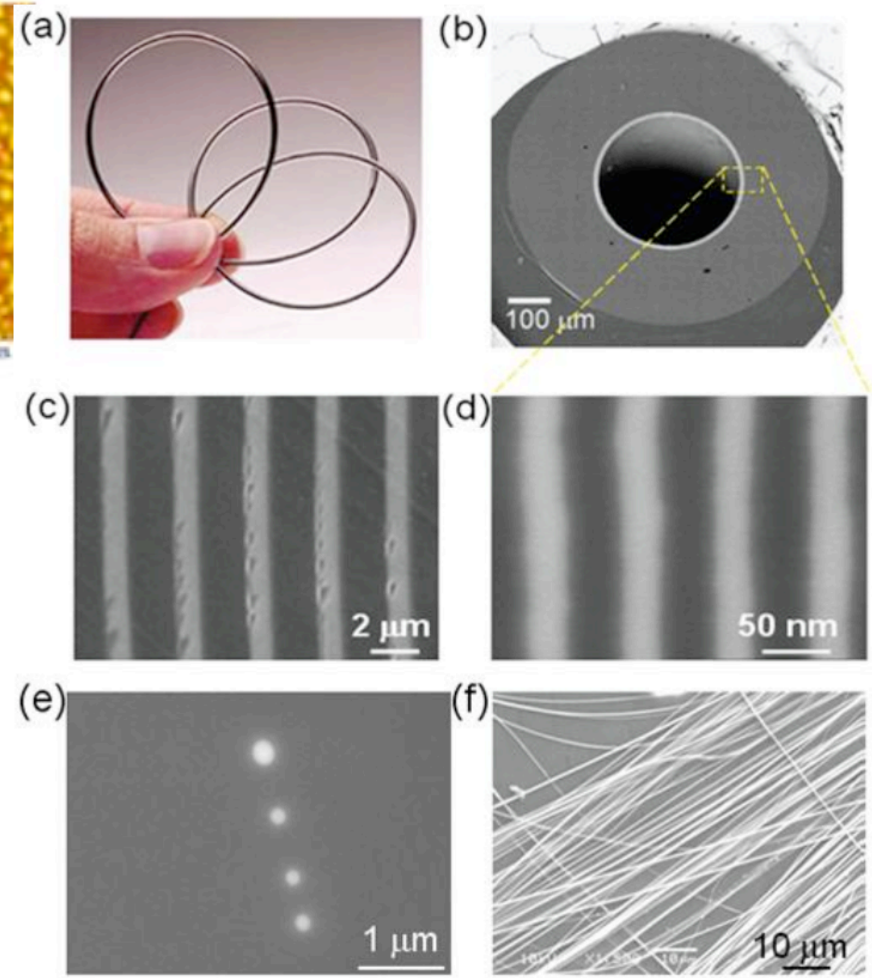
De-wetting of Thin Fluid Films

thin polymer films on Si



[P. Müller, Tech. Univ. Munich]

glass/polymer drawn fibers



[D. S. Deng, Y. Fink, MIT]

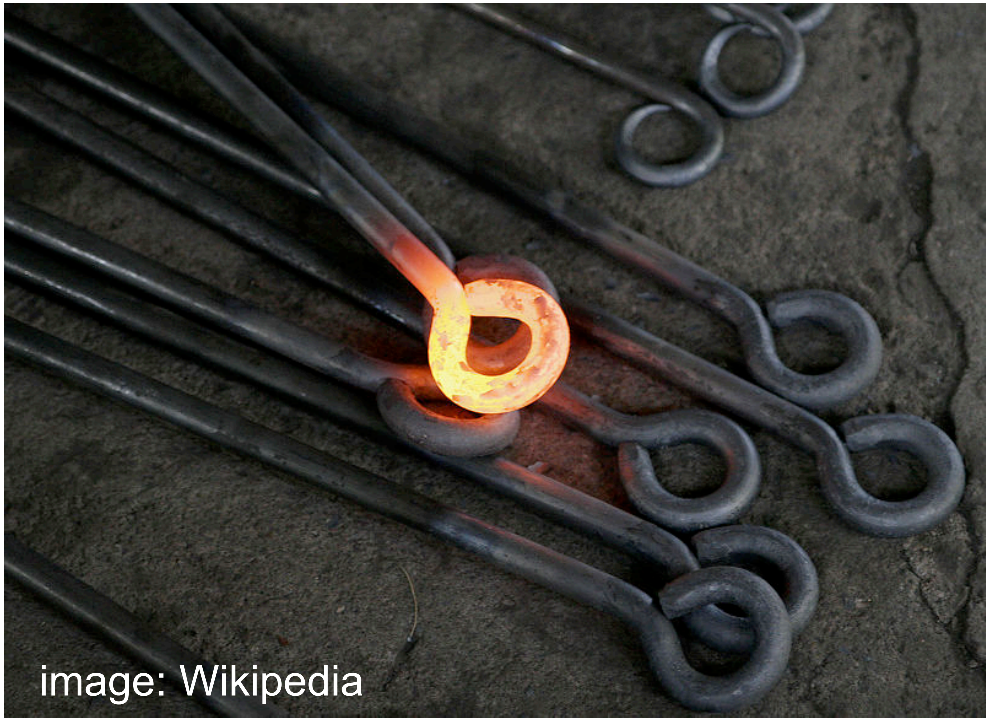
Many researchers investigating "van der Waals" forces (e.g. as competition for surface tension)

…

but additive "vdW" power laws are often fundamentally invalid (for non-planar interfaces)

Radiative Transport

Far-field: Black/grey body radiation



Easy to compute by Kirchhoff's law:

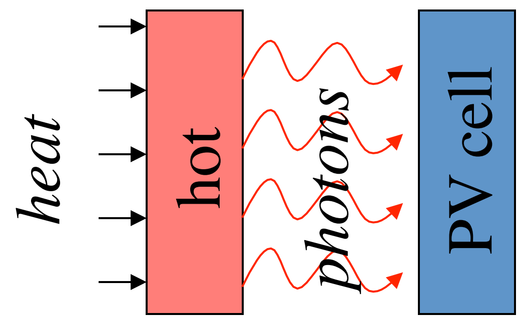
emissivity = absorptivity

< 1 … limited by black body

… greatly modified by λ-scale structure

Near-field (evanescent) thermal transport can exceed black-body limit

e.g. for future thermo-photovoltaic systems?



difficulty:

current near-field theory only for planes/spheres

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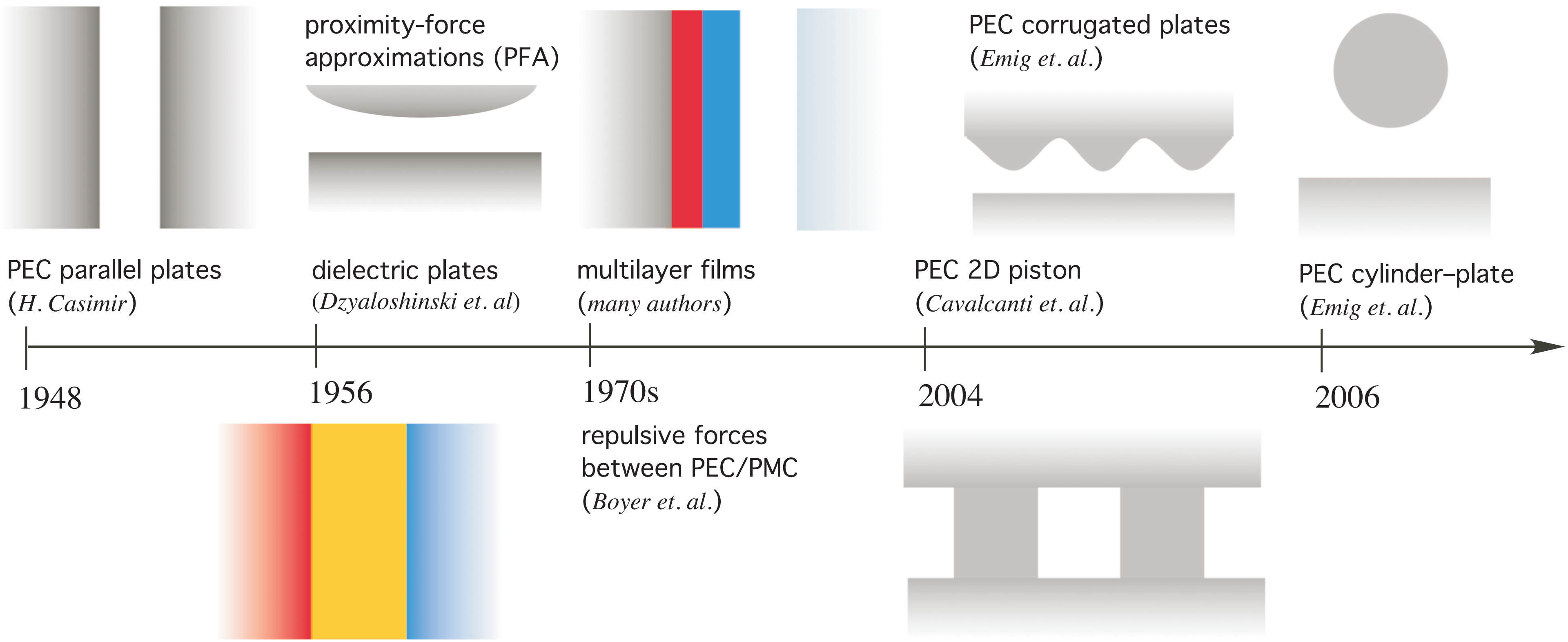
  — many recent developments
- Effects of new geometries are exciting and ~ unexplored.

Selected *Pre*-2007 Theoretical Work



two-cylinders problems fully solved only five years ago

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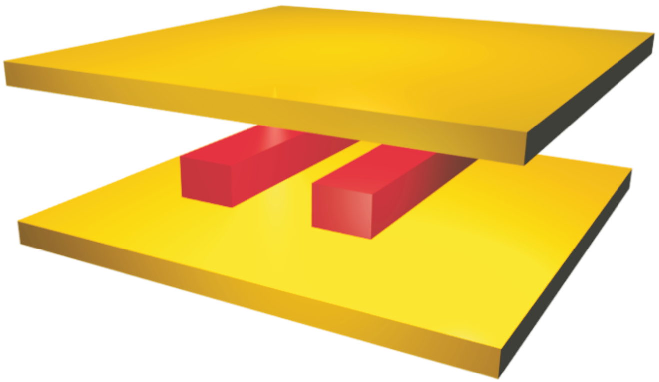
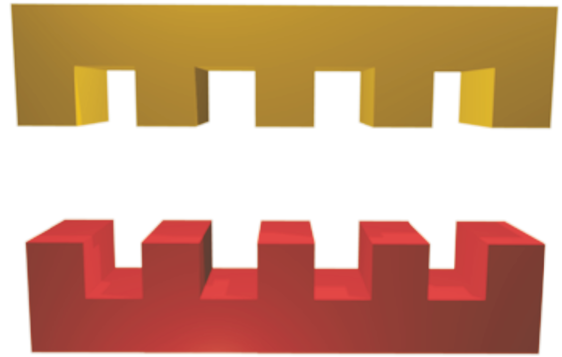
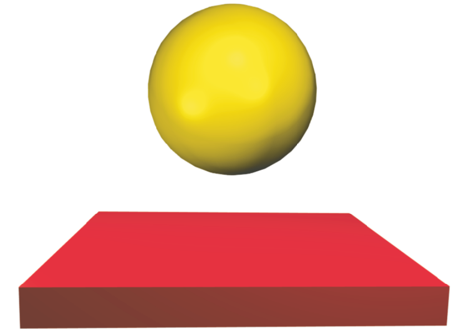
Selected Recent Theoretical Structures

multi-body geometries
[Rodriguez *et. al*. 2007]

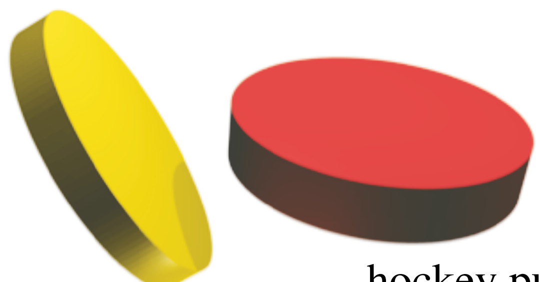




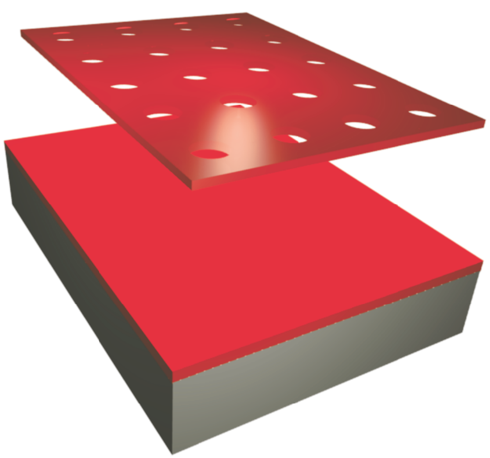
trench [Lambrecht *et. al*. 2009]



sphere–plate
[Maia Neto *et. al*. 2008]

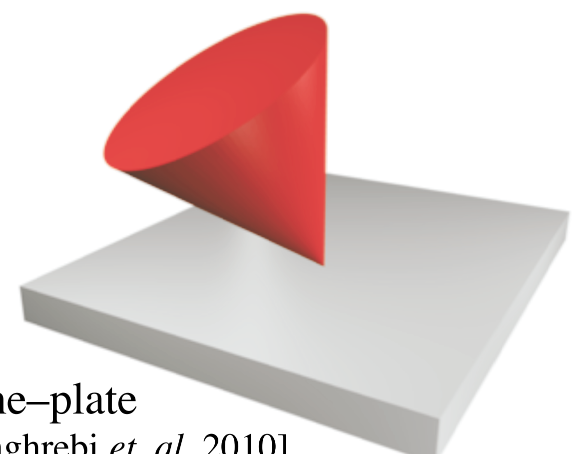


hockey pucks
[Reid *et. al*. 2010]



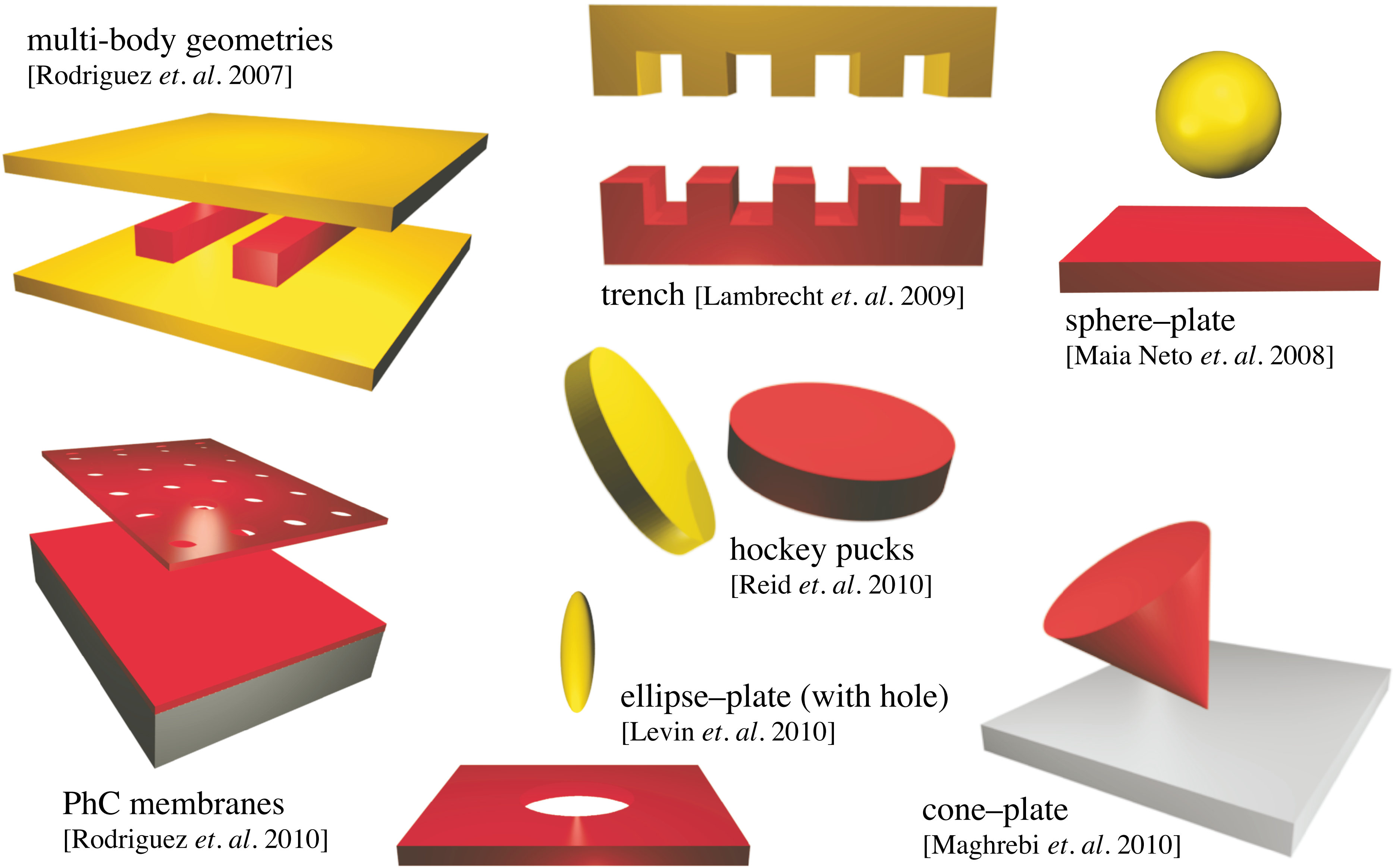
PhC membranes
[Rodriguez *et. al*. 2010]

ellipse–plate (with hole)
[Levin, Rodriguez *et. al*. 2010]



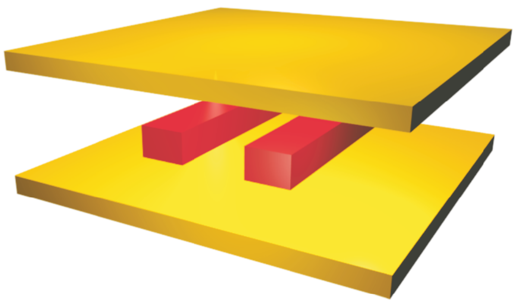
cone–plate
[Maghrebi *et. al*. 2010]

Selected Recent Theoretical Structures



Theoretical Progress

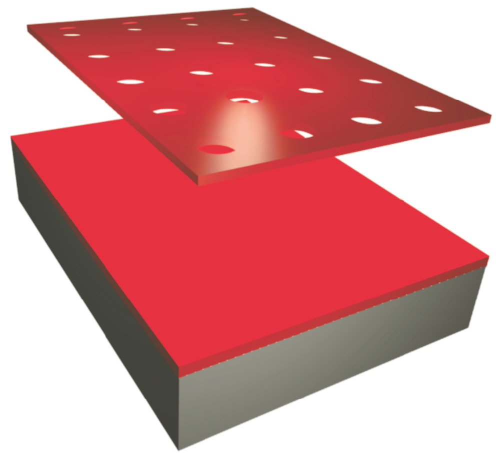
non-monotonic
multi-body effects



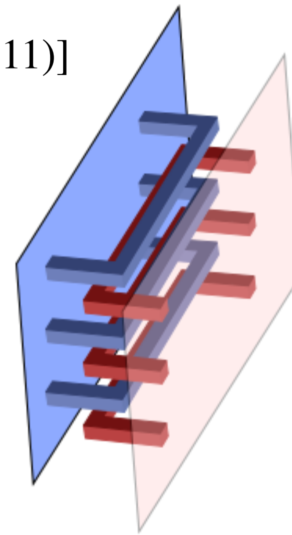
[Rodriguez *et al*. ***PRL*** **104**, 7303 (2007);
PRA **76**, 032106 (2007)]

interplay of Casimir
and optomechanical forces

[Rodriguez *et*. *al*. ***APL,*** **98** 194105 (2011)]



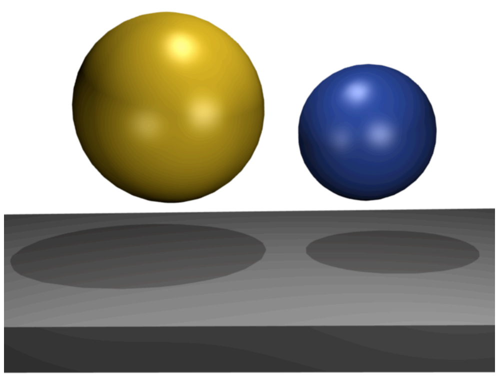
repulsive forces

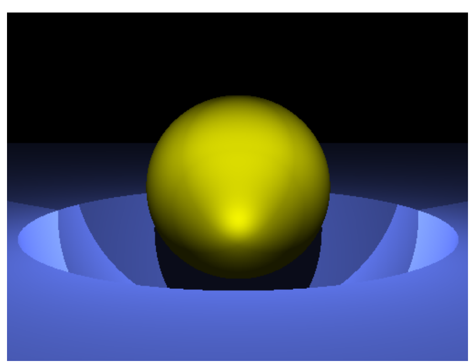


glide
symmetric
geometries

[Rodriguez *et al*.,
PRA **97**
160401 (2008)]

stable suspension of objects

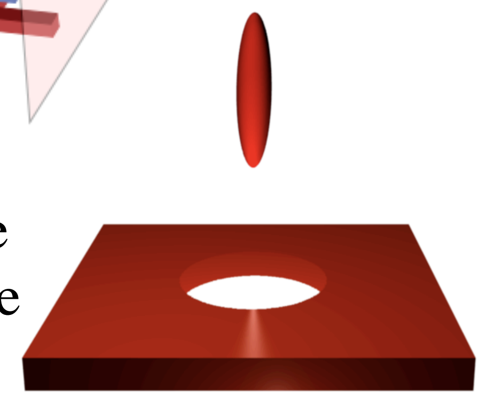




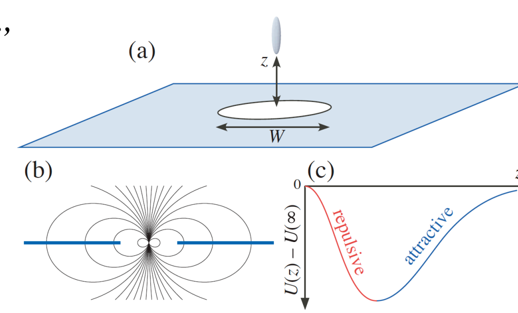
[Rodriguez, *et*. *al*. ***PRL*** **101** 190404 (2008);
PRL **104** 160402; ***PRL*** **105** 060401 (2010)]

[McCauley, Rodriguez *et*. *al*.
PRA **97** 160401 (2008)]

repulsive
plate–hole



[Levin,
Rodriguez *et*. *al*.,
PRL 105,
090403 (2010)]



Is this problem really that hard?

non-interacting bosons — linear Maxwell-like PDEs,
continuum material models
polynomial complexity

- Surprisingly hard to solve numerically

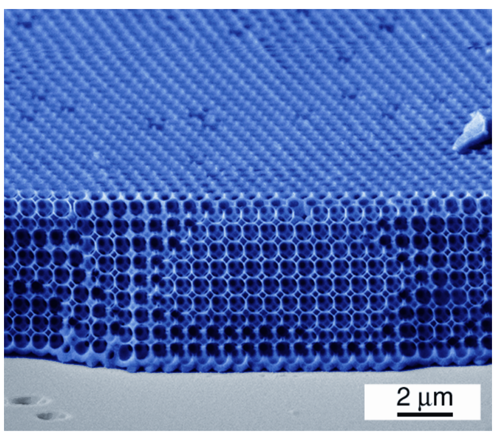
  — solution can easily involve solving PDE's 10,000's of times
  — increasingly solvable by translation into classical wave problems (many recent developments)

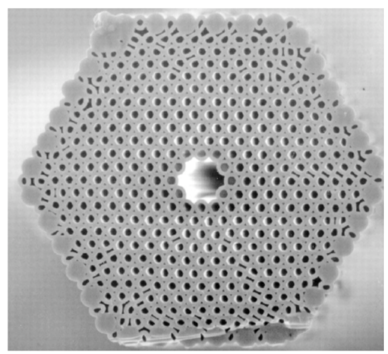
- Which PDE you solve makes a huge difference

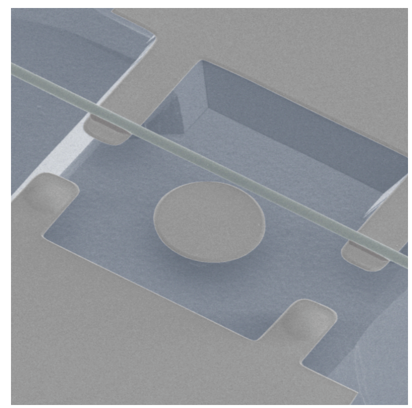
[Rodriguez *et al* ***PRA*** **76**, 032106 (2007)]

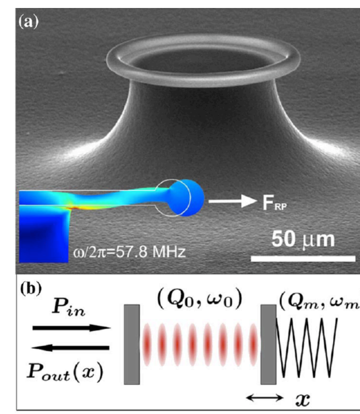
  — many equivalent formulations
  … which are well suited for numerics?

goal: exploit mature, scalable methods from classical EM for arbitrary geometries/materials









…need to relate quantum fluctuations
to classical nanophotonics?

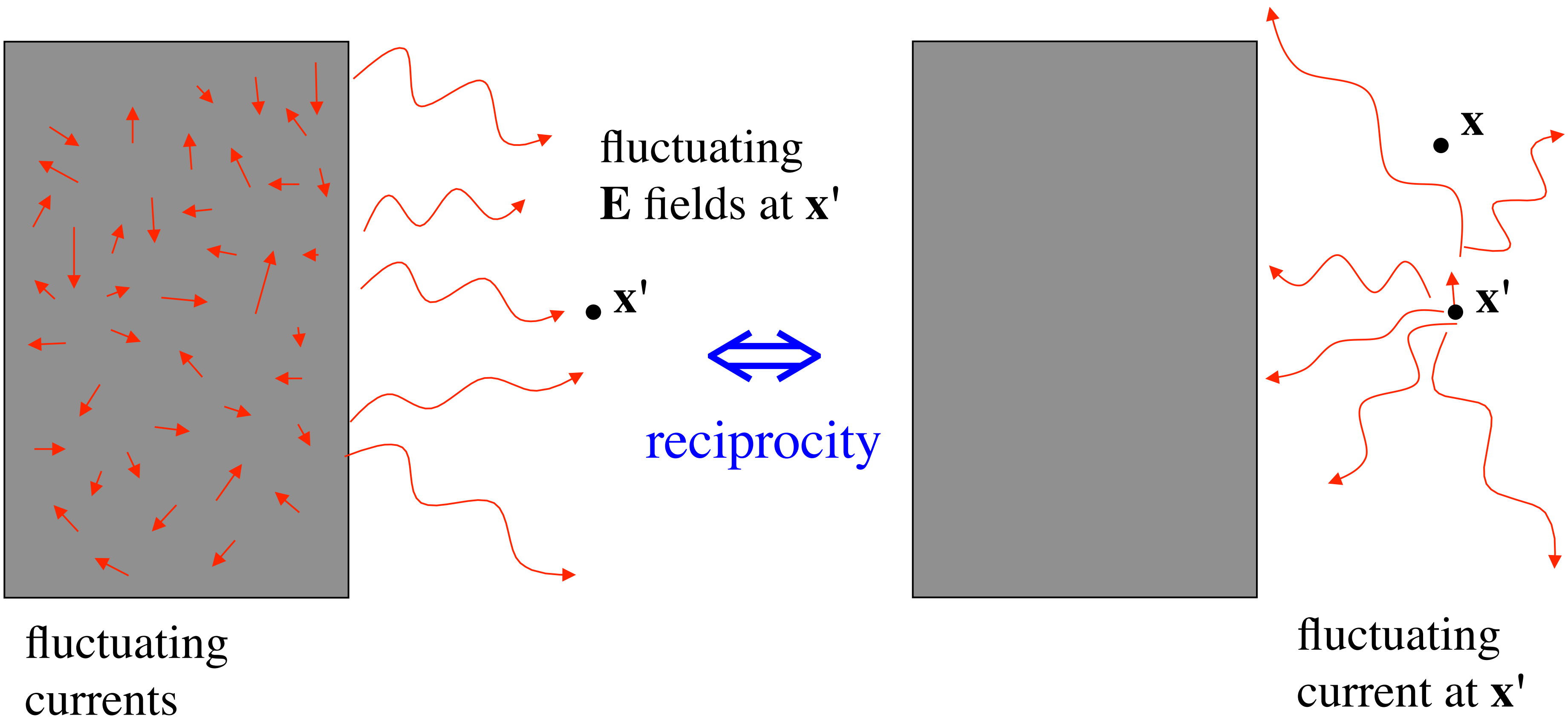
One ~ accessible formulation (~1960):

Connecting fluctuations to dissipation & Green's functions

to get e.g. mean $\boldsymbol{E}^2$ *and* $\boldsymbol{H}^2$

…compute *energy densities*, *stresses*, …

Why do Green’s functions appear?

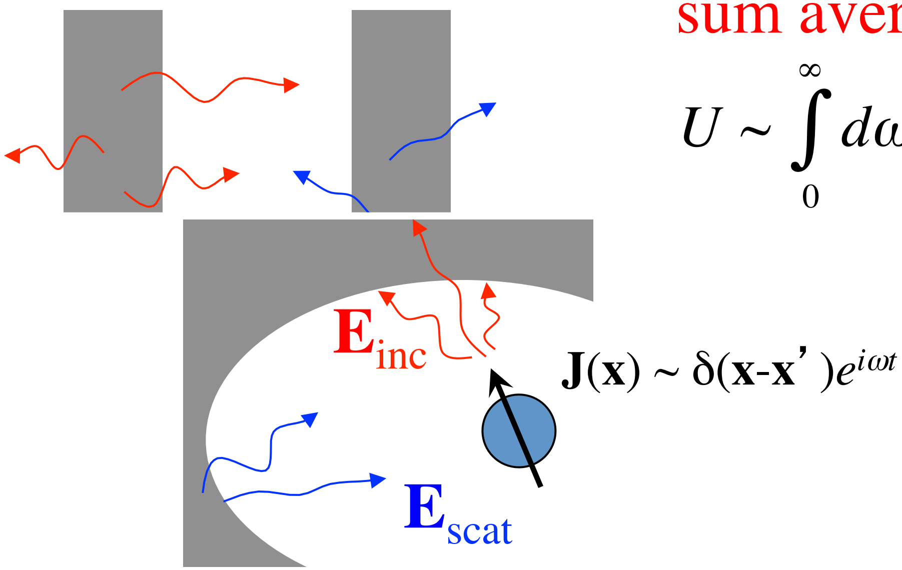


equilibrium ⇒ standard thermal statistics

…mean value of **E**(**x**)**E**(**x**') ~ field at **x** from current at **x**'

~ Green’s function

Scattering and Green's Functions



sum average energy density in all space

$$U \sim \int_0^\infty d\omega \int_V d^3\mathbf{x} \left\langle \mathbf{E}(\mathbf{x})^2 \right\rangle_\omega + \mathbf{H}^2 \text{ terms}$$

electric response due to dipole current

$$\mathbf{E}_{\mathrm{tot}}(\mathbf{x}) = \mathbf{E}_{\mathrm{inc}}(\mathbf{x}) + \mathrm{E}_{\mathrm{scat}}(\mathbf{x})$$
$$\sim i\omega G(\omega; \mathbf{x}, \mathbf{x}')$$

classical "photon" Green's function

$$\left[\nabla \times \nabla \times -\omega^2 \varepsilon(\mathbf{x},\omega)\right] G_{ij}(\omega,\mathbf{x},\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}')\hat{e}_j$$

Fluctuation–Dissipation Theorem

[e.g. Lifshitz, Pitaevskii & Dzyaloshinskii, ~1960]

current–current correlation
~ Planck distribution
$\Theta(\omega,T) = \coth(h\omega/kT)$

$$\left\langle E_i(\mathbf{x})E_j(\mathbf{x}')\right\rangle_\omega = \frac{1}{2}\mathrm{Re}\left\langle E_i(\mathbf{x})J_j(\mathbf{x}')\right\rangle_\omega$$
$$= \mathrm{Re}[-i\omega G_{ij}(\omega;\mathbf{x},\mathbf{x}')] \cdot \left\langle J_i(\mathbf{x})J_j(\mathbf{x}')\right\rangle$$
$$= \hbar\omega^2 \,\mathrm{Im}\, G_{ij}(\omega;\mathbf{x},\mathbf{x}') \text{ as } T \to 0^+$$

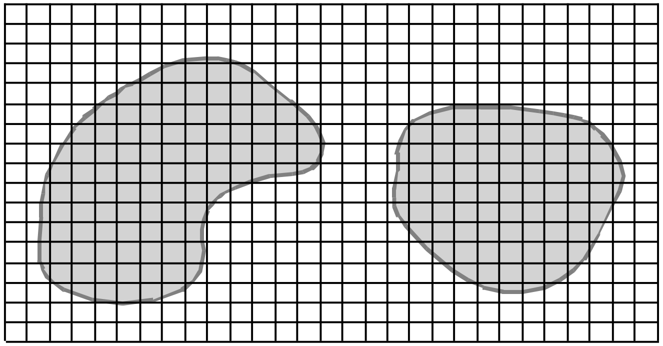
Computing Green's Functions

solve Maxwell's equations in a localized basis:
…a standard problem in classical electromagnetism

$$\frac{\partial \mu \mathbf{H}}{\partial t} = -\nabla \times \mathbf{E}$$
$$\frac{\partial \varepsilon \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}$$

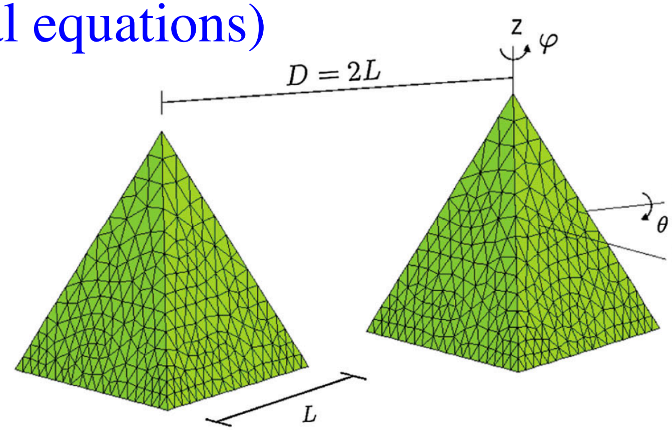
- choice of basis functions depends on problem
- ultimately, solving linear equation A$\mathbf{x}$ = b

finite differences



[A. Rodriguez, S. G. Johnson (2007)]

boundary element methods
(integral equations)



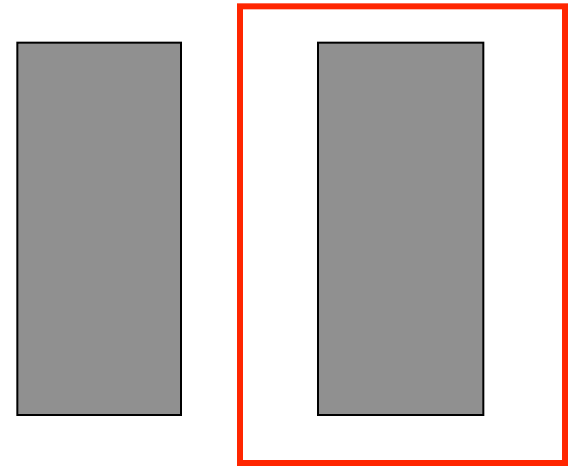
[H. Reid, Jacob White, S. G. Johnson (2009)]

Stress Tensors from Green's Functions

[Pitaevskii & Dzyaloshinskii, ~1960]

$$F = \int_0^\infty d\omega \oint\!\!\!\oint_S \left\langle \overleftrightarrow{\mathrm{T}} \right\rangle \cdot d\vec{\mathrm{A}}$$

stress tensor $\sim \left\langle \mathbf{E}^2 \right\rangle + \left\langle \mathbf{H}^2 \right\rangle$ terms



surface S

fluctuation-dissipation theorem

$$\left\langle E_i(\mathbf{x}) E_j(\mathbf{x}') \right\rangle_\omega = \hbar\omega^2 \operatorname{Im} G_{ij}(\omega, \mathbf{x}, \mathbf{x}') \times \coth(h\omega/2kT)$$

for $T > 0$

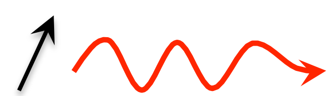
(similar for H correlations)

classical "photon" Green's function

$$\left[\nabla \times \nabla \times -\omega^2 \varepsilon(\mathbf{x}, \omega)\right] G_{ij}(\omega, \mathbf{x}, \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}')\hat{e}_j$$

electric response
to current source

$\mathbf{J} = \delta(\mathbf{x}\text{-}\mathbf{x}')$



$\mathbf{E}(\mathbf{x})$

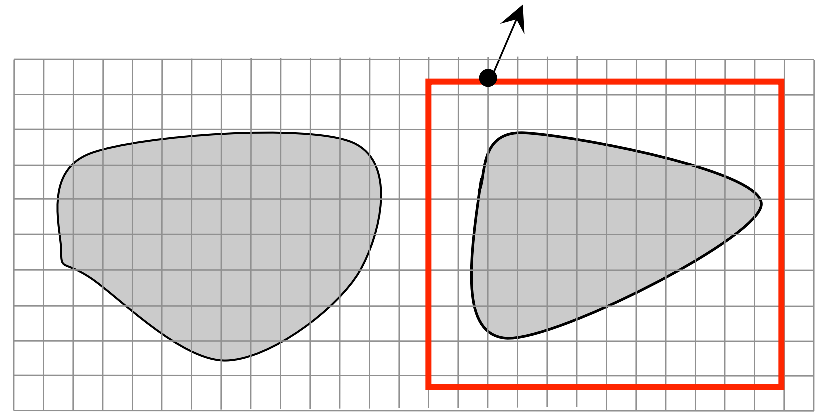
The Finite Difference Method

[Rodriguez *et al* ***PRA*** **76**, 032106 (2007)]

$$F = \int_0^\infty d\omega \oiint_S \langle \overleftrightarrow{\mathrm{T}} \rangle \cdot d\vec{\mathrm{A}}$$

stress tensor $\sim \langle \mathbf{E}^2 \rangle + \langle \mathbf{H}^2 \rangle$ terms

$\mathbf{J}(\mathbf{x},\mathrm{t}) \sim \delta(\mathbf{x}-\mathbf{x}')e^{i\omega t}$



surface surrounding body *S*

simplest approach

– for every frequency ω:

– for every point **x** on the surface S:

– compute classical Green's function

$$\left[\nabla \times \nabla \times -\omega^2 \varepsilon(\mathbf{x},\omega)\right] G_{ij}(\omega, \mathbf{x} - \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}')\hat{e}_j$$

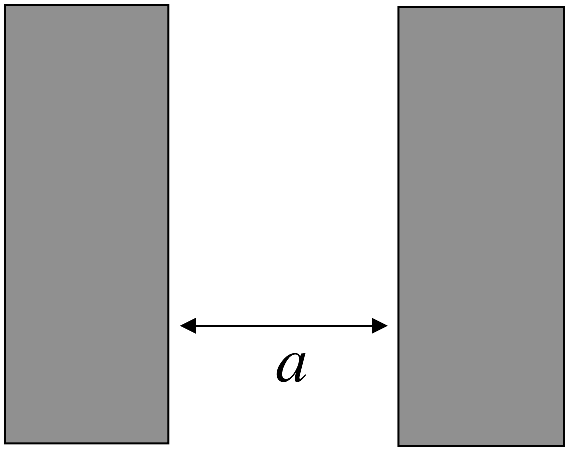
= Green's function
or **E** at **x** from current at **x**

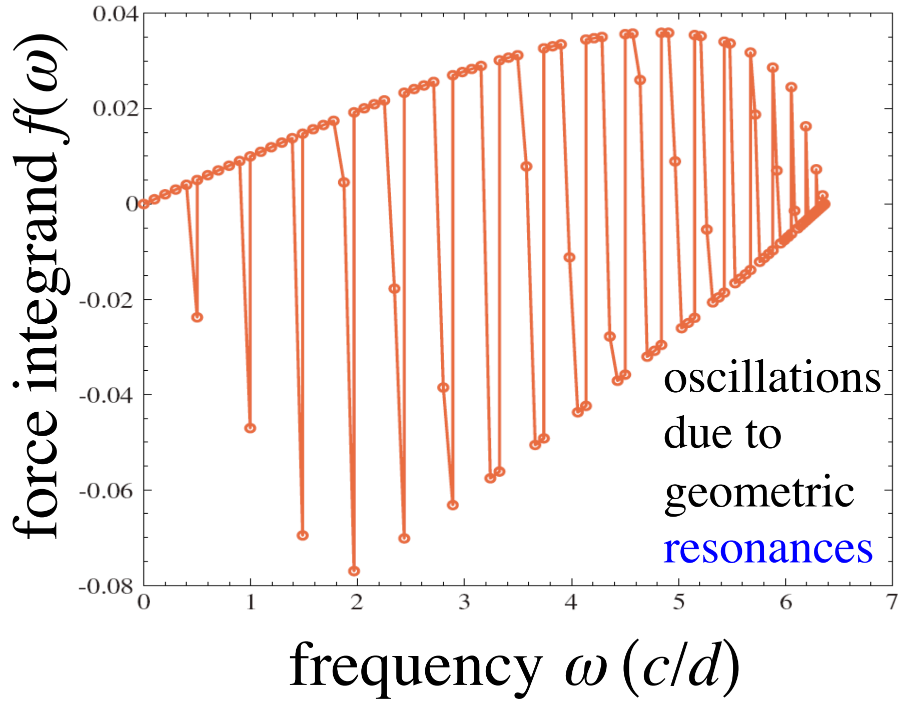
= solving linear system

$$A\mathbf{y} = b$$

$$A^{-1} = \frac{1}{\nabla \times \nabla \times -\omega^2 \varepsilon(\mathbf{x},\omega)}$$

Problems with real frequencies



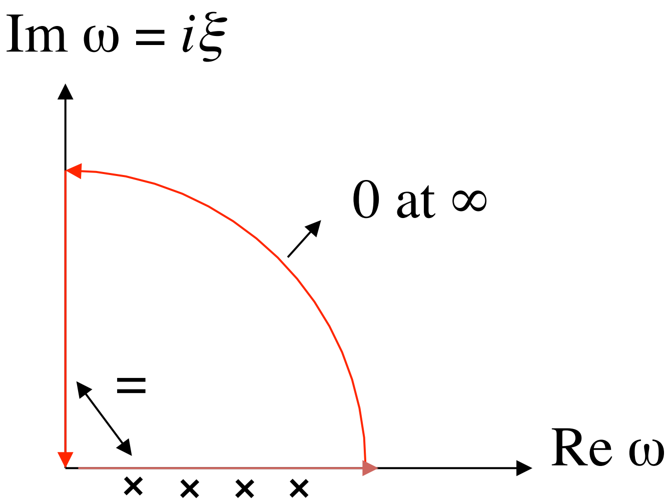


$$F = \int_0^\infty d\omega\, f(\omega)$$

integrand $f(\omega)$ is ill-behaved

- wildly oscillating
- broad bandwidth contributions up to Nyquist frequency

Complex frequency: Wick rotation



vacuum Green's function:

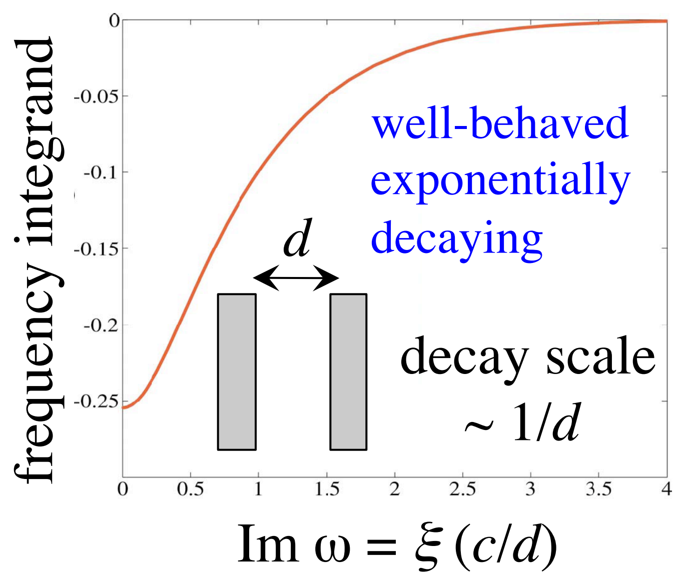
$$G_\omega \sim \frac{e^{i\omega r/c}}{r} \rightarrow G_{i\xi} \sim \frac{e^{-\xi r/c}}{r}$$

exponentially decaying
non-oscillatory
no resonance/interference

causality ⇒ poles only in lower-half plane

Wick rotation (contour integration):
real ω to imaginary ω → $i\xi$
— move contour away from poles
(same integration result)

[standard analytical technique,
critical for numerics]



Wick Rotations

standard and especially nice numerical problem:

—inverting real-symmetric positive-definite (conjugate-gradient)

[Rodriguez *et. al*. ***PRA*** **76**, 032106 (2007)]

$\varepsilon(\mathbf{x},i\xi)$ is purely *real*

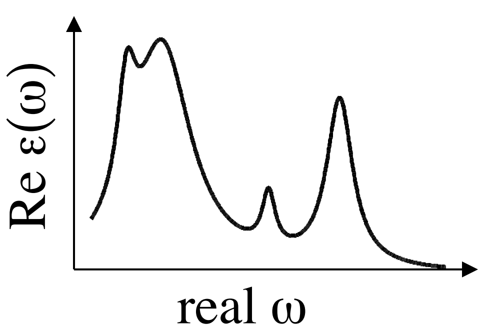
$$\frac{1}{\nabla\times\nabla\times-\omega^2\varepsilon(\mathbf{x},\omega)} \quad \omega\to i\xi \quad \frac{1}{\nabla\times\nabla\times+\xi^2\varepsilon(\mathbf{x},i\xi)}$$

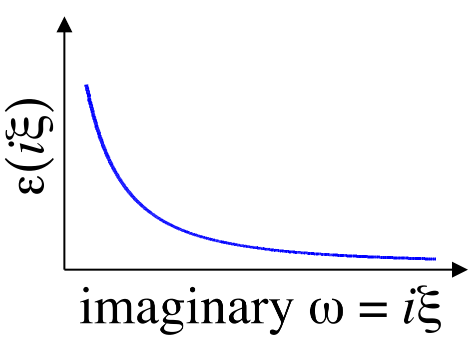
unusual in classical nanophotonics:

—cannot rely on geometric/material resonances

[Rodriguez *et. al*. ***PNAS*** **106** 6883 (2010)]

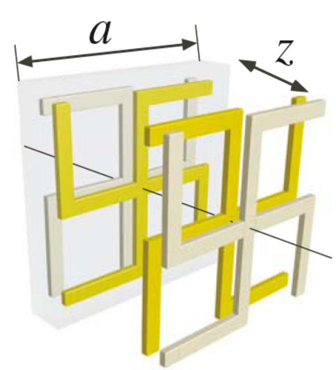
material resonances





geometric resonances

(*e.g*. chiral metamaterials)



—achieving negative ε, μ over narrow bandwidth

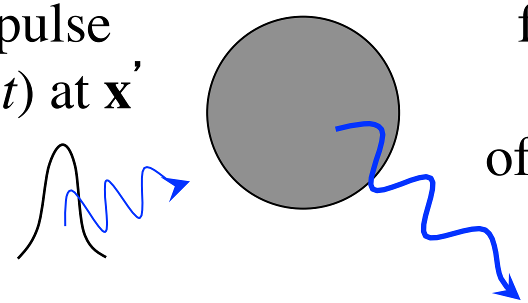
⇒ little change in force

[McCauley *et. al*. ***PRB*** **82**, 165108 (2010)]

Computing Forces in the Time Domain

want response integrated over broad range of frequencies

pulse $\delta(t)$ at $\mathbf{x}'$



frequency response = Fourier transform of scattered field $\mathbf{E}(\mathbf{x},\omega)$

FDTD solvers widespread (off the shelf), efficient and versatile

e.g. anisotropic dielectrics, many types of boundary conditions, parallelizable

need to perform Wick rotation…in time!

Wick-rotated Green's function $\mathbf{G}(i\xi)$:
field $\mathbf{E}(\mathbf{x})$ in response to *exponentially growing* current in time $\mathbf{J}(\mathbf{x},i\xi) \sim \delta(\mathbf{x}-\mathbf{x}')e^{\xi t}$

Complex Frequencies in Real Time?

Green's function inverts: $\nabla \times \nabla \times - \omega^2 \varepsilon(\omega, x)$

ω and ε appear together!

complex contour deformation

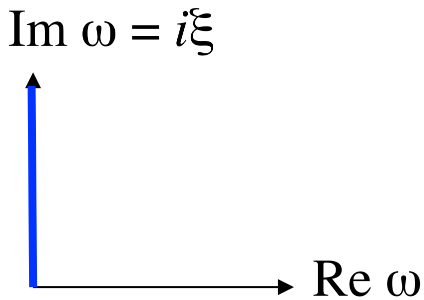
⇒ mapping from $\omega \to \xi f(\xi)$ is *equivalent* to
leaving ω unchanged and instead changing material:

$$\omega \to \xi$$
$$\varepsilon(\omega, \mathbf{x}) \to f(\xi)^2 \, \varepsilon(\xi f(\xi), \mathbf{x})$$

can obtain all the advantages of complex-frequency but
for real frequency/time with transformed materials

note: ξ promoted from contour parameter to real frequency

Wick Rotations in the Time Domain

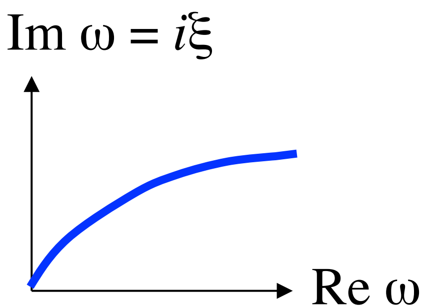


Wick rotations $\quad$ $f(\xi) = i$ $\quad$ gain media

$$\omega \to i\xi \quad \Longleftrightarrow \quad \varepsilon(\omega) \to -\varepsilon(\xi)$$

exponentially growing solutions
if negative at **all** *frequencies*

different contour?



$$\omega \to \xi\sqrt{1+\frac{i\sigma}{\xi}} \quad \Longleftrightarrow \quad \varepsilon \to \left(1+\frac{i\sigma}{\xi}\right)\varepsilon$$

time domain: real-frequency response in **conductive medium**

$$\left.\begin{aligned} \frac{\partial \mu \mathbf{H}}{\partial t} &= -\nabla \times \mathbf{E} \\ \frac{\partial \varepsilon \mathbf{E}}{\partial t} &= \nabla \times \mathbf{H} - \sigma\varepsilon\mathbf{E} - \mathbf{J} \end{aligned}\right\}$$

most off-the-shelf FDTD software already supports conductive media:

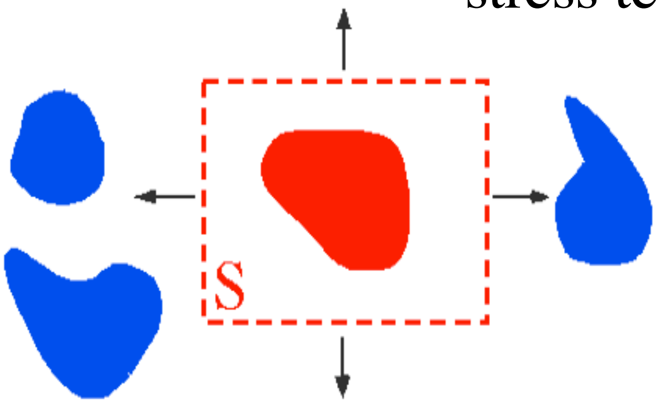
– frequency domain ~ 10^5 CPU hours
– time domain ~ 100–10^3 CPU hours

[Rodriguez *et al*. ***PRA*** **80** 012115 (2009)]

Stress tensor in Time-domain

$$\text{stress tensor}(x) \sim \int_0^\infty g(\omega)G(\omega,x)\,d\omega = \int_0^\infty \hat{g}(-t)\hat{G}(t,x)\,dt$$

(Fourier transforms)

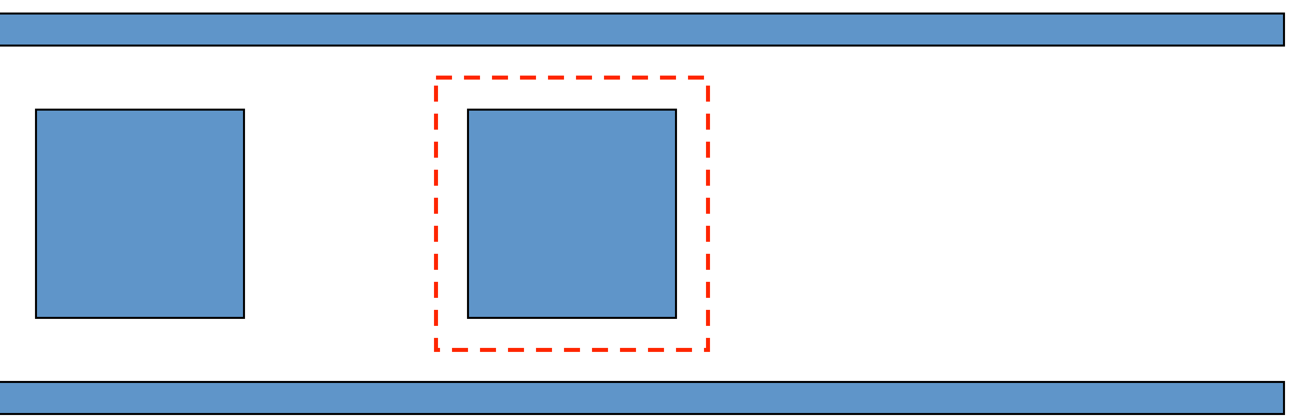


geometry-independent kernel

geometry-dependent Green's function: field at x from source at x

Finite-difference *time*-domain simulation for each point on S
to get the *entire spectrum's* contribution to stress tensor at that point

example structure:



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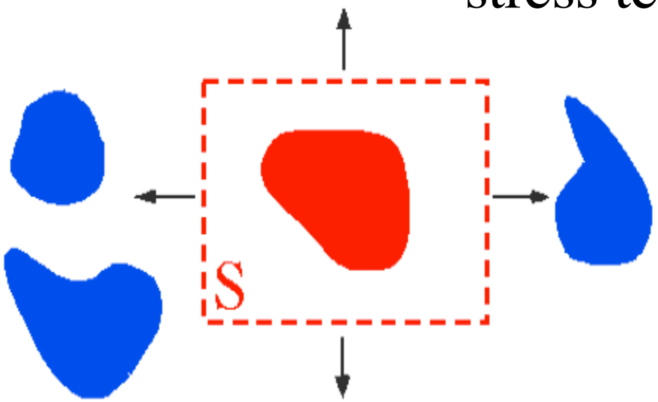

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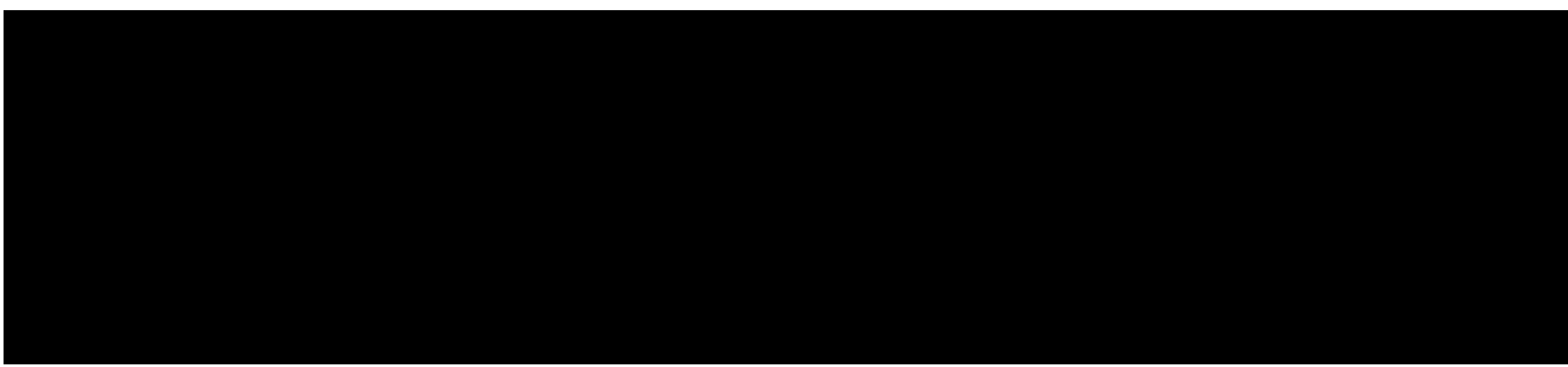
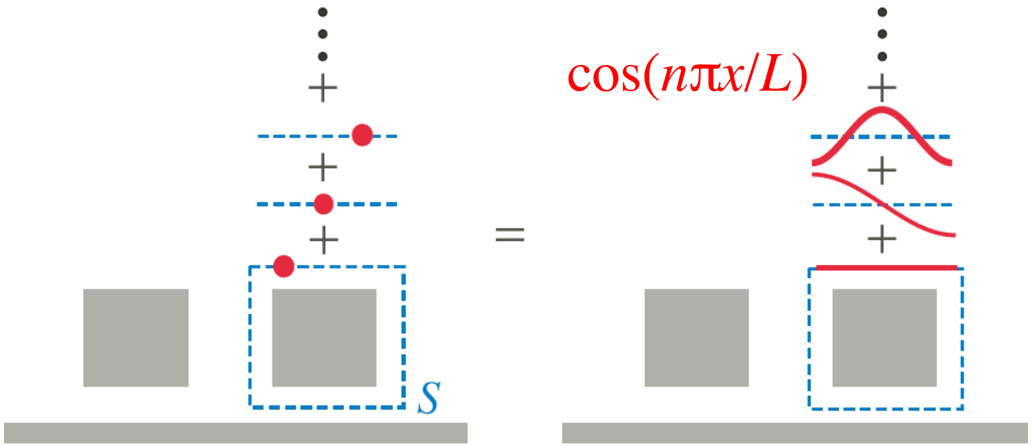
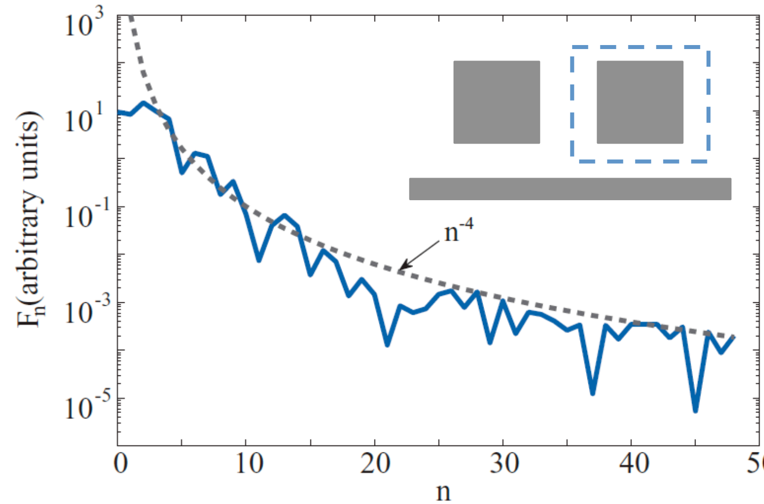
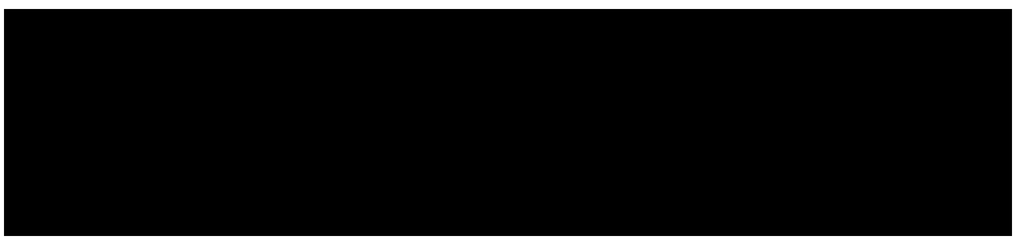


One more trick…replace point sources with Fourier surfaces

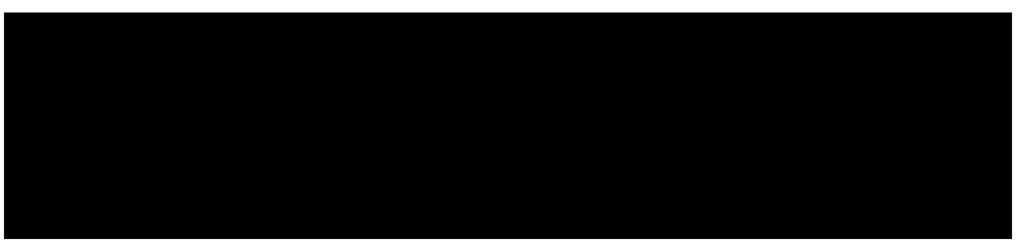




n=0



n=4



- rapid convergence ~ $1/n^4$
- need ~ 10 sources

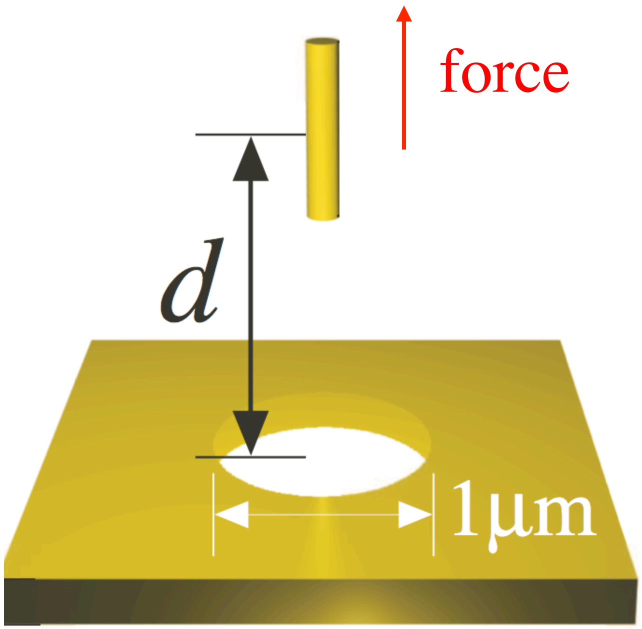
EM-fluctuation interactions are:

- Increasingly important in several contexts
  — as devices reach sub-μm scales
- Surprisingly hard to solve mathematically
  — even *two-cylinder* problems fully solved only recently
- Increasingly solvable by translation into classical wave problems.
  — many recent developments
- Effects of new geometries are exciting and ~ unexplored.

… almost any geometry you can imagine is unstudied …

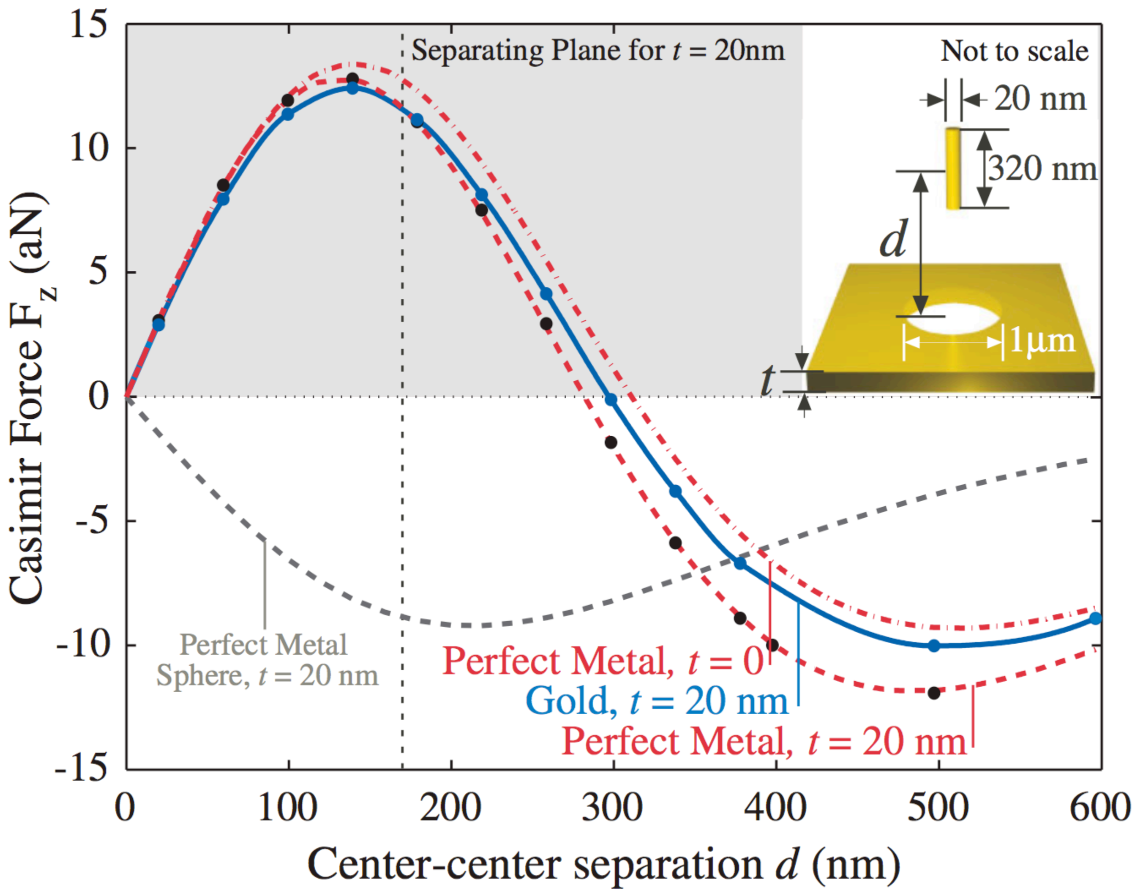
How much can we alter the effect?

A repulsive geometry in vacuum



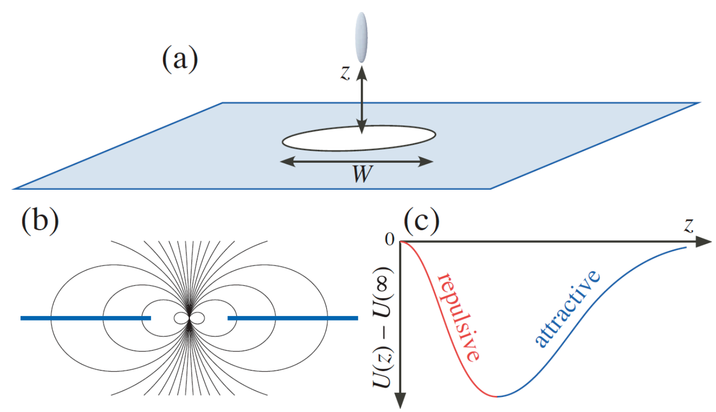
simple: an elongated “needle”
above a metal plate with a hole

A repulsive geometry in vacuum



needle is repelled,
sphere is attracted

… why?



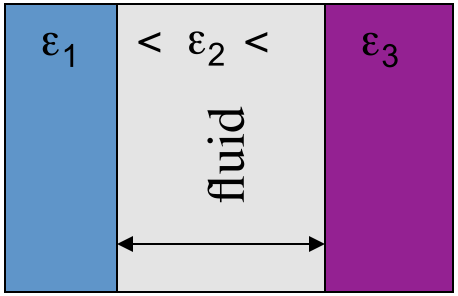
Repulsion and stability via materials?

Theorem 1: [Kenneth, ***PRL*** 2006]

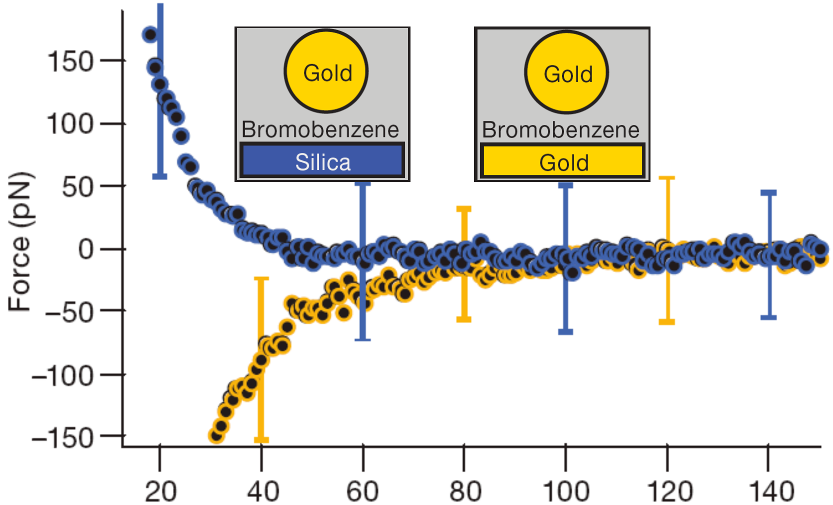
Repulsion impossible
for mirror-symmetric
geometries

Theorem 2: [Rahi, ***PRL*** 2010

stable equilibrium impossib
for metals/dielectrics in vac

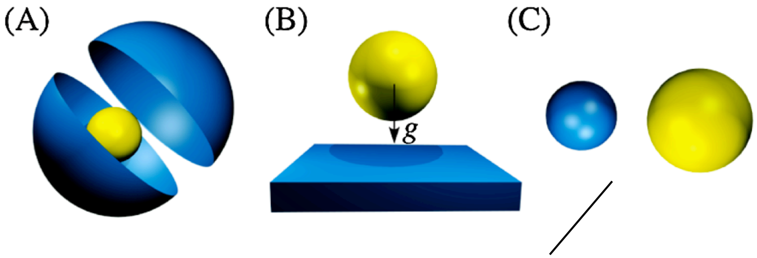


[theory: Dzyaloshinskii, 1961]

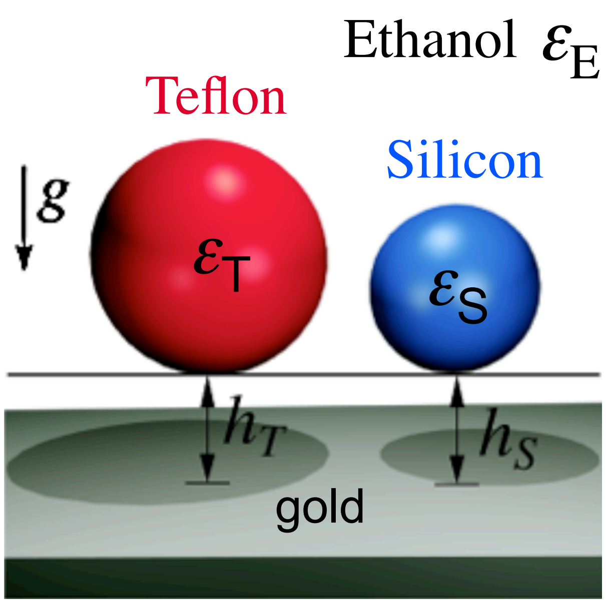


[experiment: Munday 2009]

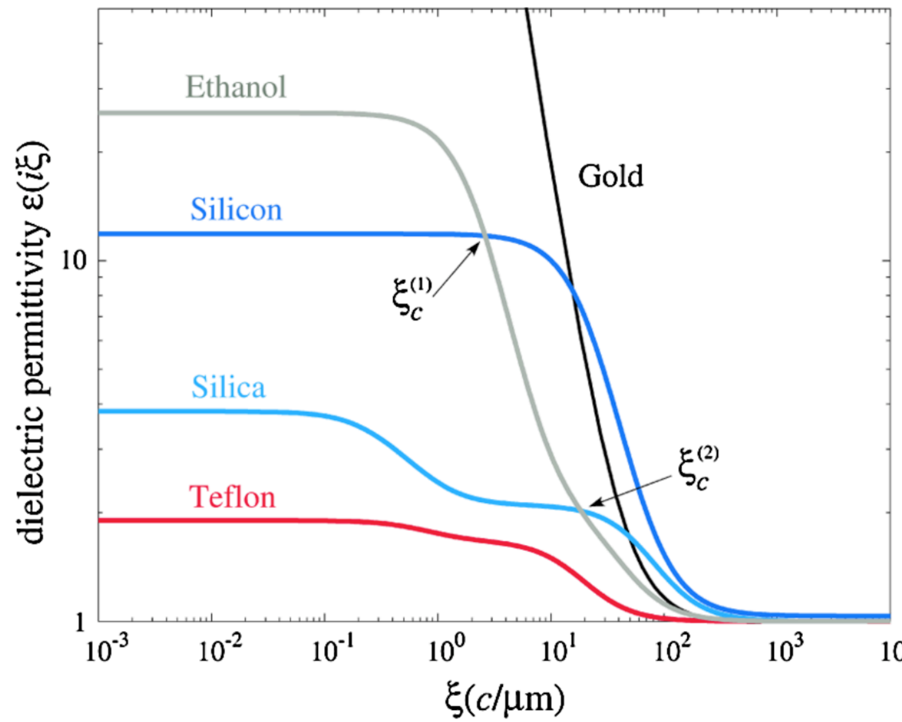
Stability via Material Dispersion



stable non-touching pair



[Rodriguez *et*. *al*. ***PRL*** **104**, 160402 (2010)]



large separation

$\varepsilon_E > \varepsilon_S > \varepsilon_T$

attraction

small separation

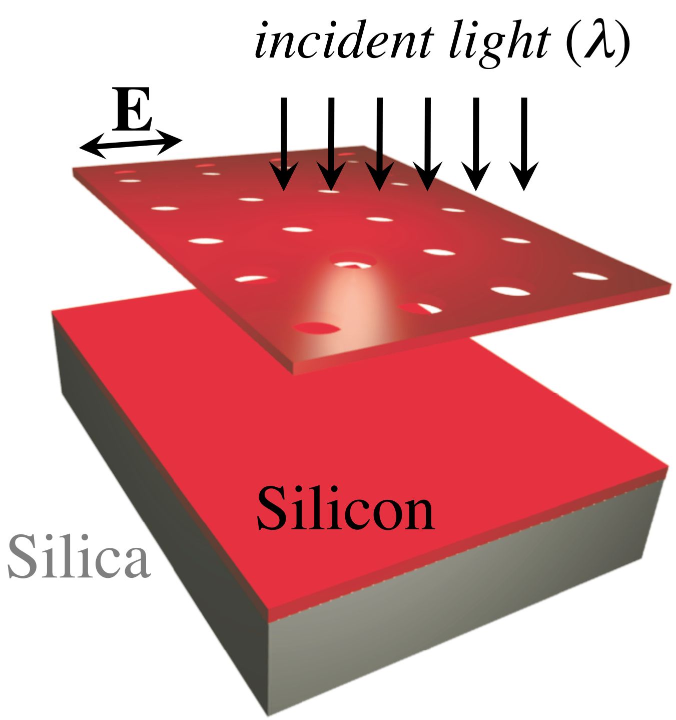
$\varepsilon_S > \varepsilon_E > \varepsilon_T$

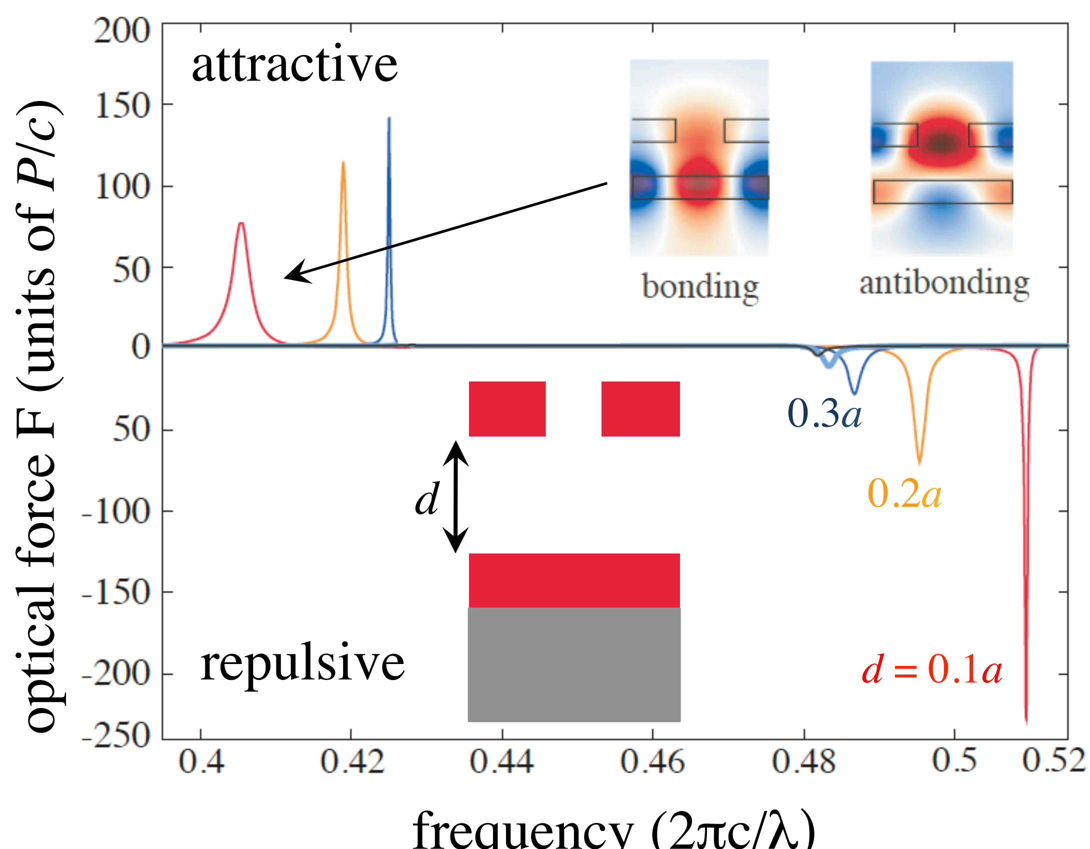
repulsion

Casimir Optomechanics

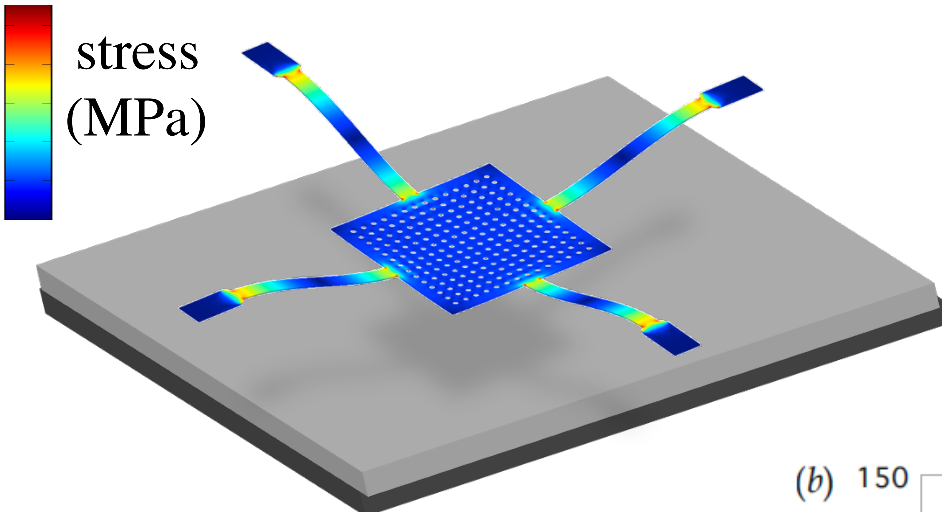
[Rodriguez *et. al*. ***OE*** 19, 2215 (2010)]

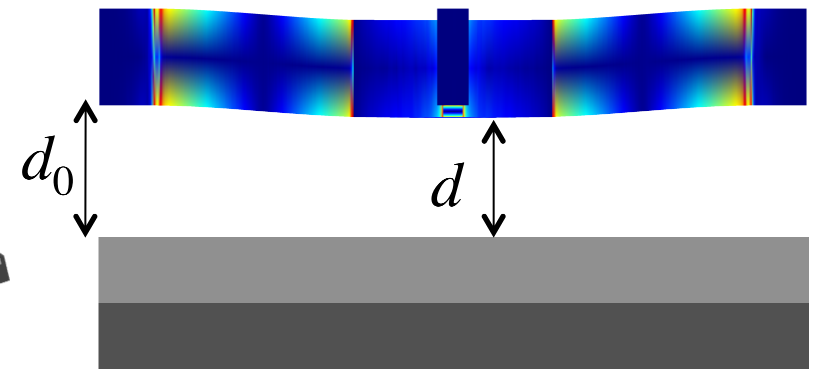
goal: control and exploit Casimir force in MEMS devices using evanescent optical forces



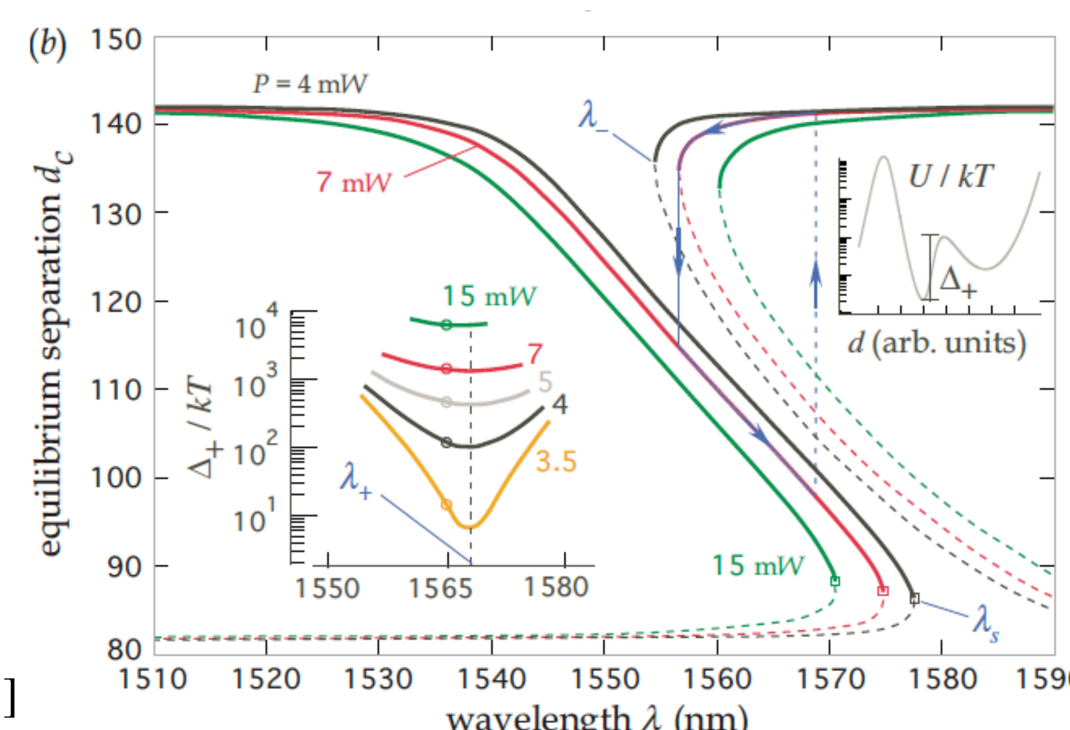


Probing the Casimir force



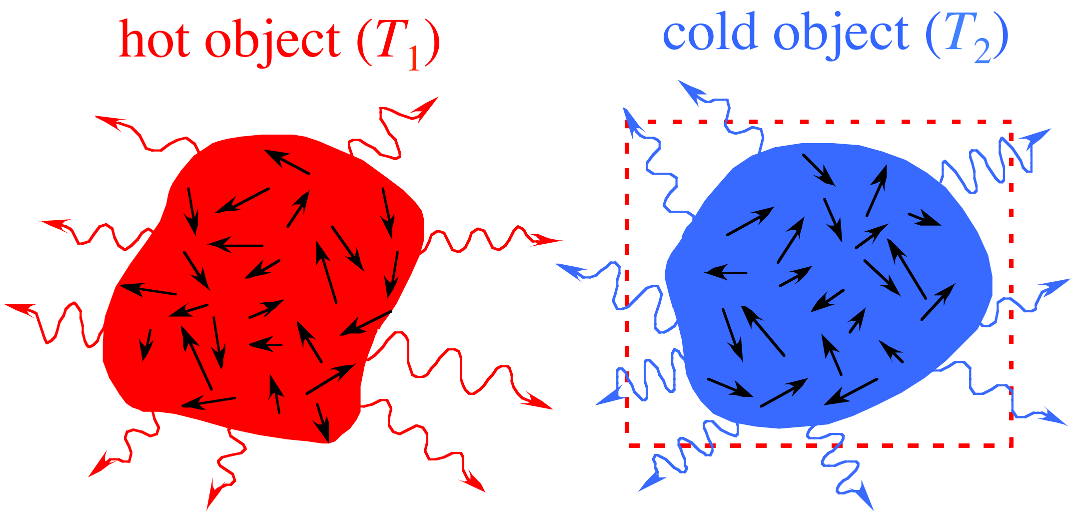


- tuning equilibrium separation via λ
- multistability behavior
- Casimir force induces unstable equilibrium



[Rodriguez *et. al.* ***APL*** 19, 2215 (2011)]

Non-equilibrium Heat Transfer



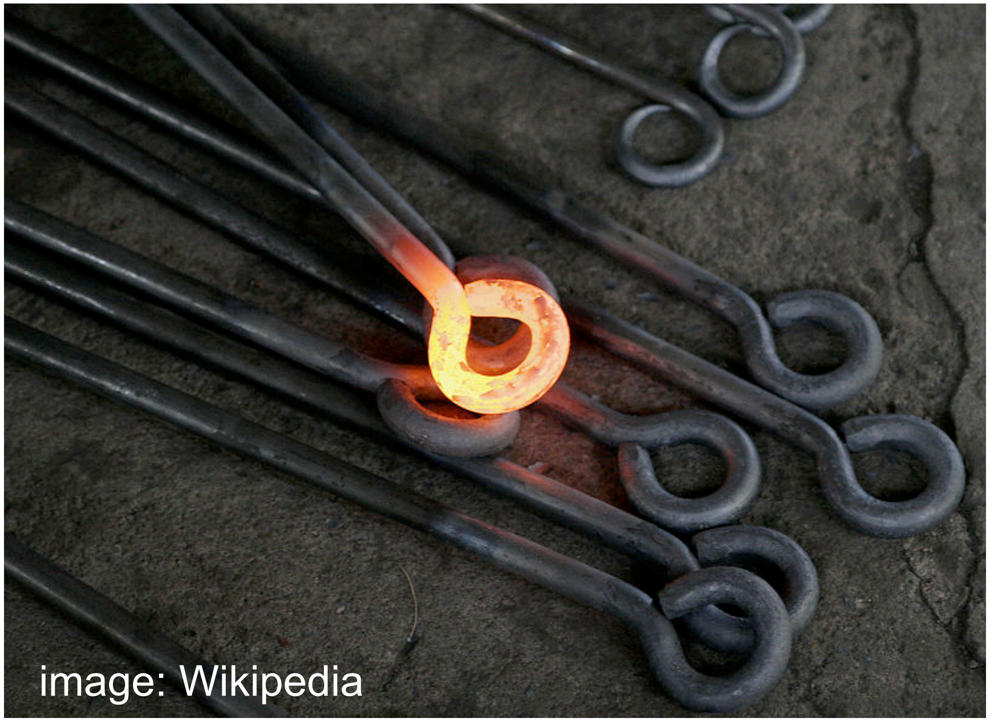
radiative electromagnetic waves (heat) flows from hot → cold object

currently, little known about heat transfer beyond sphere–plate and parallel-plate geometries

as bodies are brought closer, evanescent modes couple…
⇒ can exceed blackbody (far-field) heat transfer

Non-equilibrium Heat Transfer

Far-field: Black/grey body radiation



Easy to compute by Kirchhoff's law:

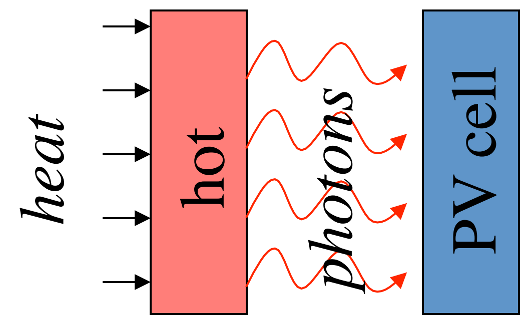
emissivity = absorptivity

< 1 … limited by black body

… greatly modified by λ-scale structure

Near-field (evanescent) thermal transport can exceed black-body limit

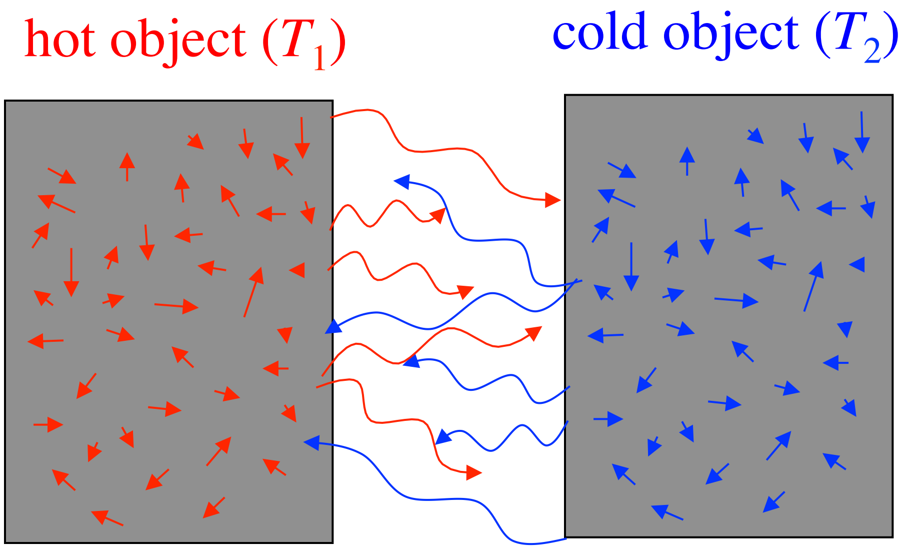
e.g. for future thermo-photovoltaic systems?



difficulty:

current near-field theory only for planes/spheres

Stochastic Time Domain Method



Fluctuation-Dissipation Theorem

current–current correlation
at local temperature T

$$\left\langle J_i(\mathbf{x}) J_j(\mathbf{x}') \right\rangle_\omega = \Theta(\omega, T)\delta(\mathbf{x} - \mathbf{x}')\delta_{ij}$$

Langevin method

Maxwells equations with auxiliary equation for the material response

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

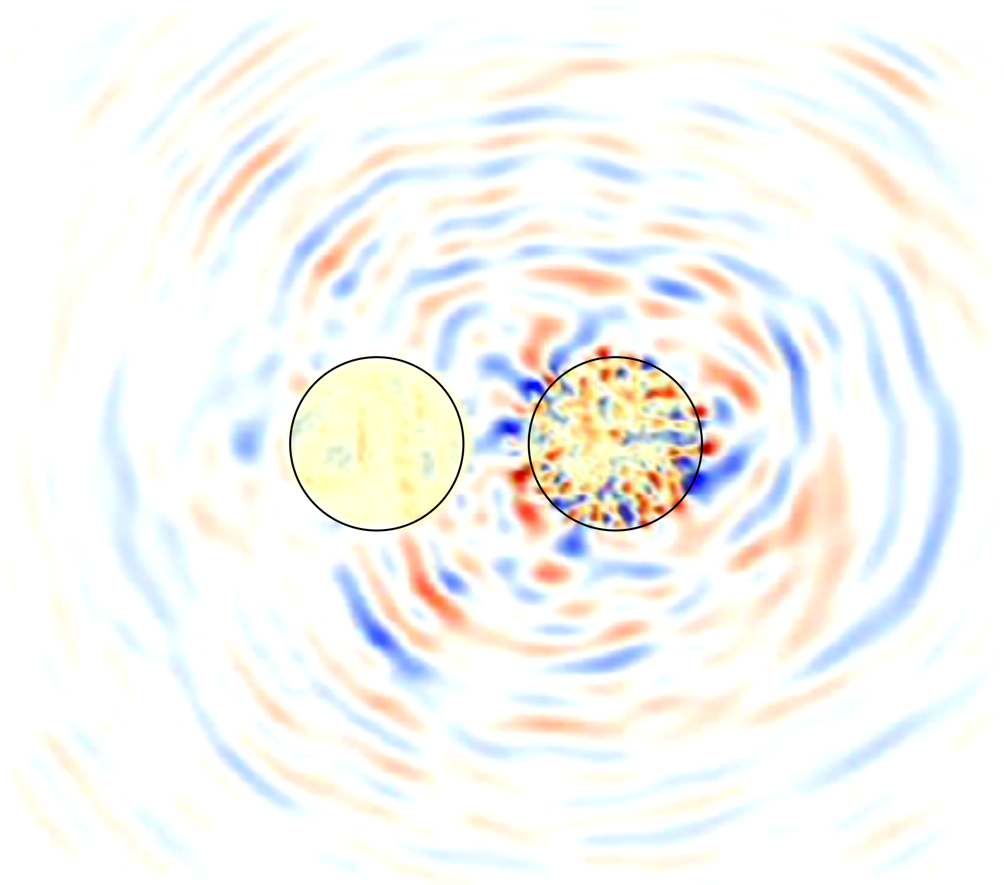
$$\mathbf{D} = \mathbf{E} + \mathbf{P}$$ (polarization/material response)

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} + \mathbf{J}$$

$$\frac{d^2\mathbf{P}}{dt^2} + \gamma \frac{d\mathbf{P}}{dt} + \omega_0^2 \mathbf{P} = \sigma \mathbf{E} + \mathbf{J}$$

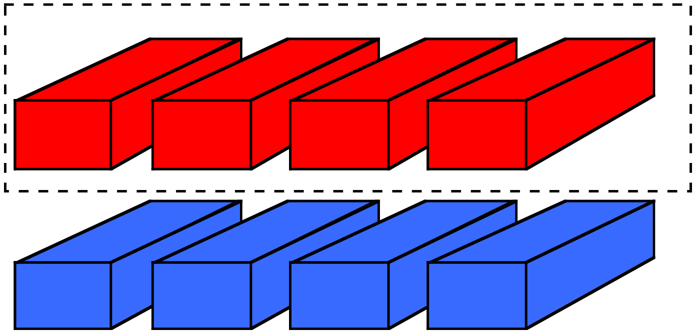
correlated random #s

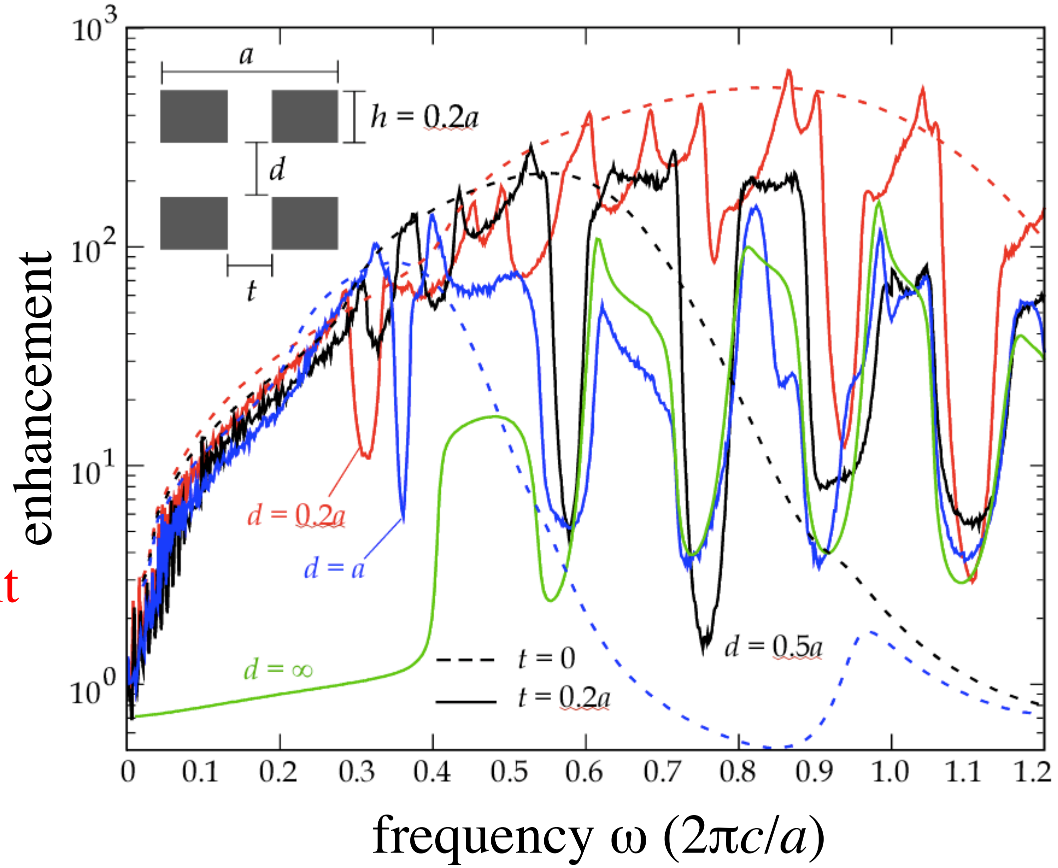
FDTD example: two cylinders



Frequency-Selective Enhancement

patterned PhC slabs





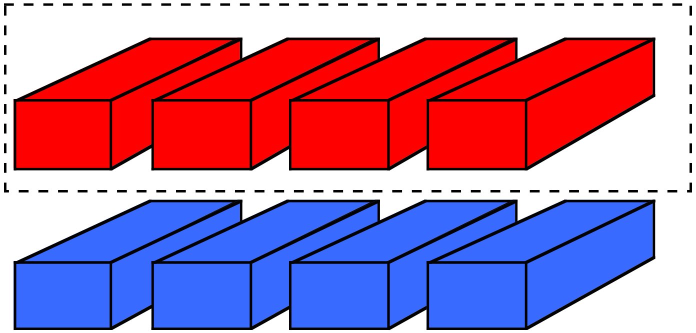
- dramatic near-field enhancement over un-patterned slabs at selective frequencies

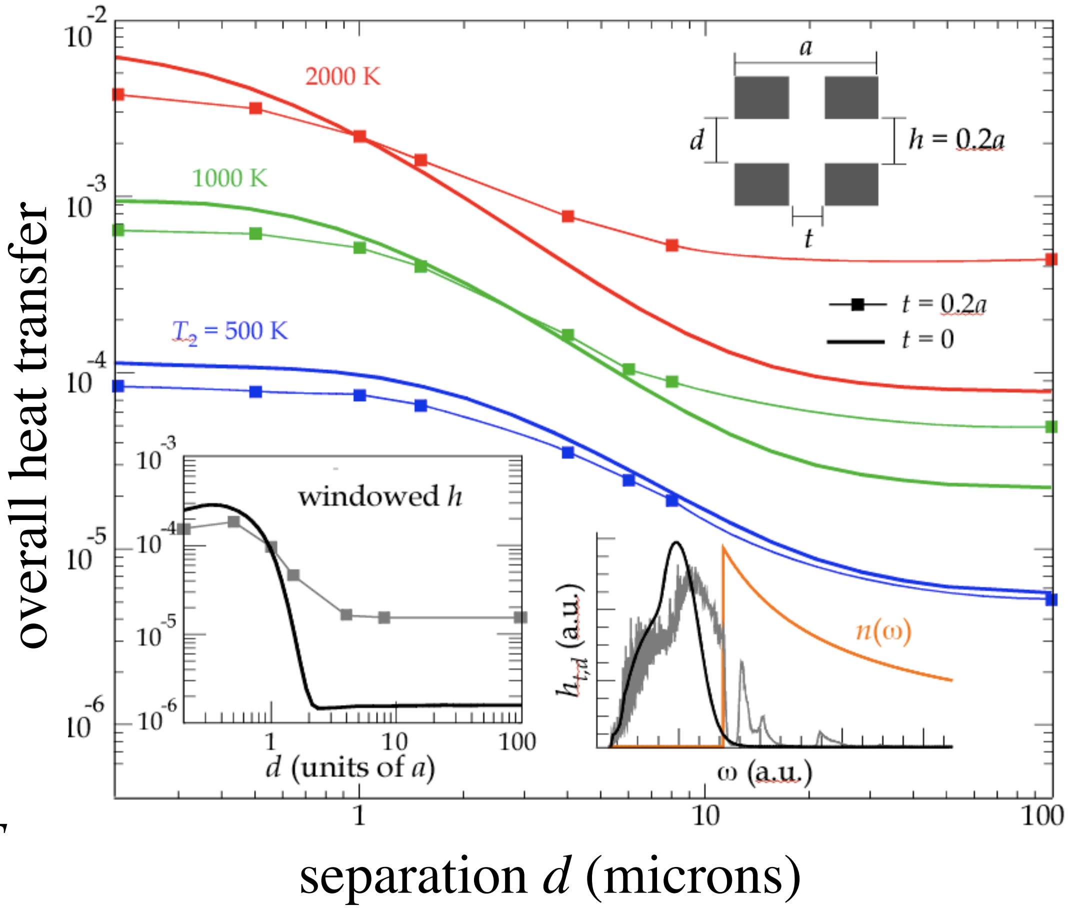
- selectivity important for applications involving thermophotovoltaic devices

[Rodriguez *et. al.* ***PRL***, in press (2011)]

Overall heat transfer

patterned PhC slabs





- overall heat transfer better than un-patterned slabs over wide range of separations
- optimal separation depends on T

[Rodriguez *et*. *al*. ***PRL***, In press, (2011)]

Fin

review article: Rodriguez *et. al*., "Casimir effect in microstructured geometries", **Nature Photonics 5**, 211–221 (2011)

- The Casimir field is virgin territory for mathematical modeling & computation

- Similar opportunities in non-equilibrium transport, thin-film dewetting, critical-Casimir effects, …

Thanks!

DOE CSGF

DARPA