# High stakes Where's Waldo

## Watching for nuclear weapon material

Josh Hykes

Department of Nuclear Engineering
North Carolina State University

CSGF Annual Conference
July 22, 2011


[hebster, commons.wikipedia.org]

# Credit for the worthwhile parts of this talk goes to:

Prof. Yousry Azmy
My advisor

Prof. Dan Cacuci
Sensitivities and data assimilation expert

Prof. John Mattingly
Materials detection expert

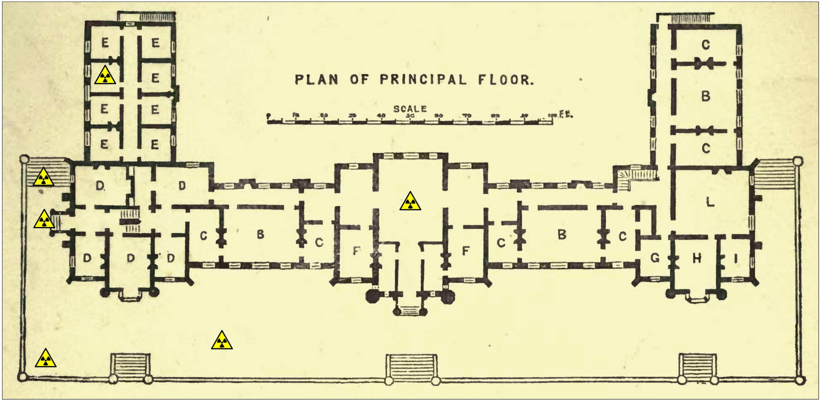
# The CSGF is a tremendous opportunity

DOE CSGF $\approx$



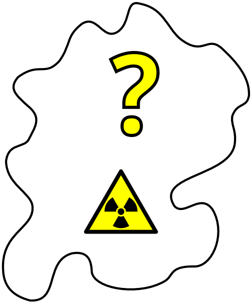
$\approx$ TA/RA

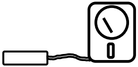
# My main thesis research is on computational methods for radiation source location




[based on an illustration by Henry Melville, 1852]

# This talk focuses on the problem of estimating material quantities with remote sensing

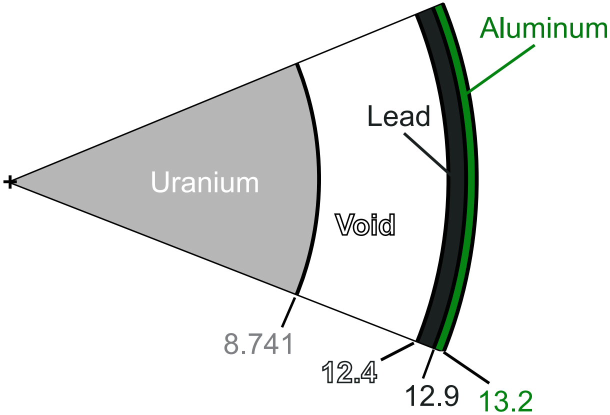




Radiation detector

Unknown materials

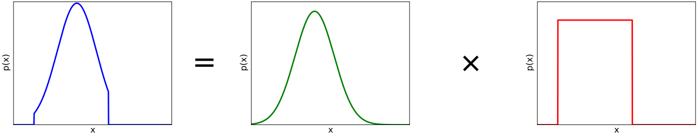
# In practice, the materials are assumed to be a set of spherical shells




[as suggested by Favorite in 2004]

# Bayesian inference is a general approach to solving inverse problems

$$\underbrace{p(\text{model}\,|\,\text{data})}_{\text{posterior}} \propto \underbrace{p(\text{data}\,|\,\text{model})}_{\text{likelihood}} \cdot \underbrace{p(\text{model})}_{\text{prior}}$$




See *Inverse Problem Theory* by Albert Tarantola for more details.

# We compute the first and second moments around the maximum of the posterior

$$p(\vec{z} \mid C) = \frac{1}{\sqrt{|2\pi C|}} \exp\left[-\frac{1}{2} Q(\vec{z})\right]$$

where

$$\begin{aligned}
\vec{\alpha} &\equiv \text{model variables} \\
\vec{\alpha}^0 &\equiv \text{prior model values} \\
\mathrm{C}_\alpha &\equiv \vec{\alpha} \text{ covariances} \\
\vec{r} &\equiv \text{measurement variables} \\
\vec{r}_m &\equiv \text{measurement data} \\
\mathrm{C}_m &\equiv \vec{r}_m \text{ covariances} \\
\mathrm{C}_{\alpha r} &\equiv \vec{\alpha}\text{-to-}\vec{r}_m \text{ "covariances"}
\end{aligned}$$

$$\vec{z} \equiv \begin{bmatrix} \vec{\alpha} - \vec{\alpha}^0 \\ \vec{r} - \vec{r}_m \end{bmatrix}$$

$$C \equiv \begin{bmatrix} \mathrm{C}_\alpha & \mathrm{C}_{\alpha r} \\ \mathrm{C}_{\alpha r}^{\mathrm{T}} & \mathrm{C}_m \end{bmatrix}$$

$$Q(\vec{z}) \equiv \vec{z}^{\mathrm{T}} C^{-1} \vec{z}$$

[Cacuci 2010]

# Gauss-Newton optimization of $Q(\vec{z})$ searches for $\vec{\alpha}^*$ with the maximum posterior

$$\vec{\alpha}_{m+1} = \vec{\alpha}_m - \lambda_m \left(\nabla^2_\alpha Q(\vec{z}_m)\right)^{-1} \vec{\nabla}_\alpha Q(\vec{z}_m)$$

with the linearization

$$\vec{r} = \vec{R}(\vec{\alpha}_m) + \mathsf{S}(\vec{\alpha}_m)(\vec{\alpha} - \vec{\alpha}_m) \qquad \text{where} \qquad [\mathsf{S}(\vec{\alpha})]_{i,j} = \left.\frac{\partial \vec{R}_i}{\partial \alpha_j}\right|_{\vec{\alpha}}$$

[Kelley 1999]

# The thicknesses of each material are put in $\vec{\alpha}$,

$$\vec{\alpha} = \begin{bmatrix} \log_e \frac{\vec{\Delta\rho}}{\Delta\rho_0} \\ \\ \end{bmatrix}$$

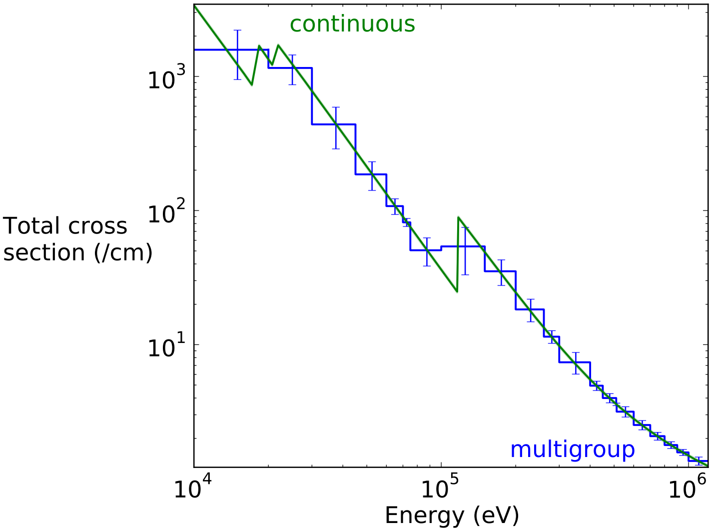
# The thicknesses of each material are put in $\vec{\alpha}$, as well as uncertain material properties

$$\vec{\alpha} = \begin{bmatrix} \log_e \frac{\vec{\Delta\rho}}{\Delta\rho_0} \\ \vec{\sigma}_t \end{bmatrix}$$

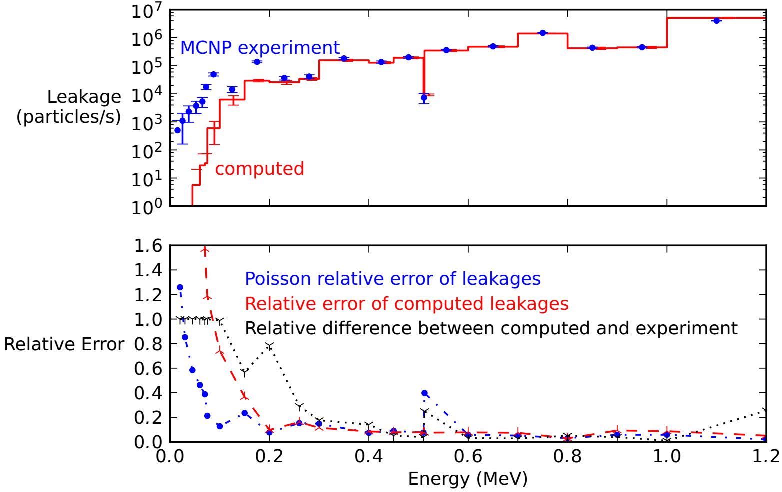
# The thicknesses of each material are put in $\vec{\alpha}$, as well as uncertain material properties

$$\vec{\alpha} = \begin{bmatrix} \log_e \frac{\vec{\Delta\rho}}{\Delta\rho_0} \\ \vec{\sigma}_t \end{bmatrix} \begin{array}{l} \text{4 components} \\ \sim 10^3 \text{ components} \end{array}$$

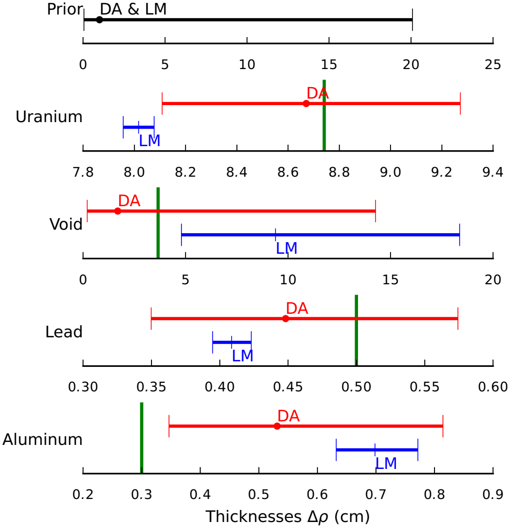
# We can't know the input data $\vec{\sigma}_t$ perfectly



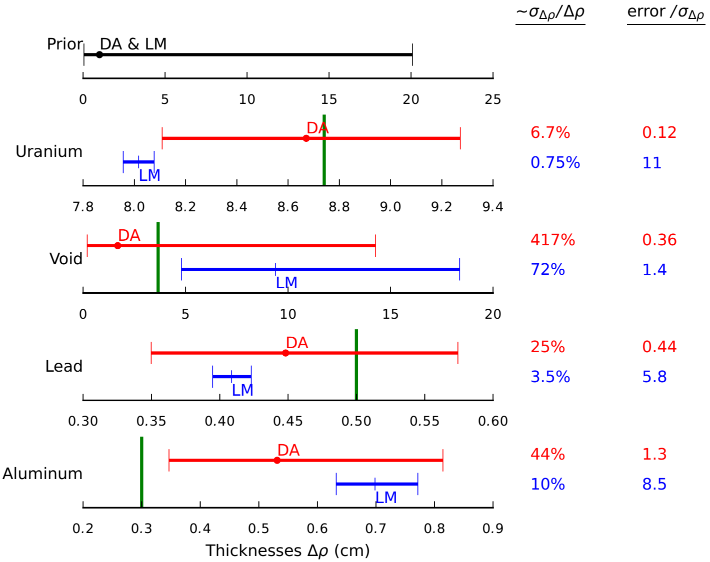
# Uncertainty in the input helps to explain modeling discrepancies



# Including $\vec{\sigma}_t$ uncertainties, the posterior PDF gives a reasonable estimate of the thicknesses



# Including $\vec{\sigma}_t$ uncertainties, the posterior PDF gives a reasonable estimate of the thicknesses



# References


- Albert Tarantola, *Inverse problem theory and methods for model parameter estimation*, SIAM 2005.
- Jeffrey Favorite, “Using the Schwinger variational functional for the solution of inverse transport problems” *Nuclear Science and Engineering* **146** 2004.
- Dan Cacuci and Mihaela Ionescu-Bujor, “Best-estimate model calibration and prediction through experimental data assimilation–I: mathematical framework” *Nuclear Science and Engineering* **165** 2010.
- CT Kelley, *Iterative Methods for Optimization*, SIAM 1999.