# Relics of Preheating after Inflation

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Now At: Argonne National Laboratory

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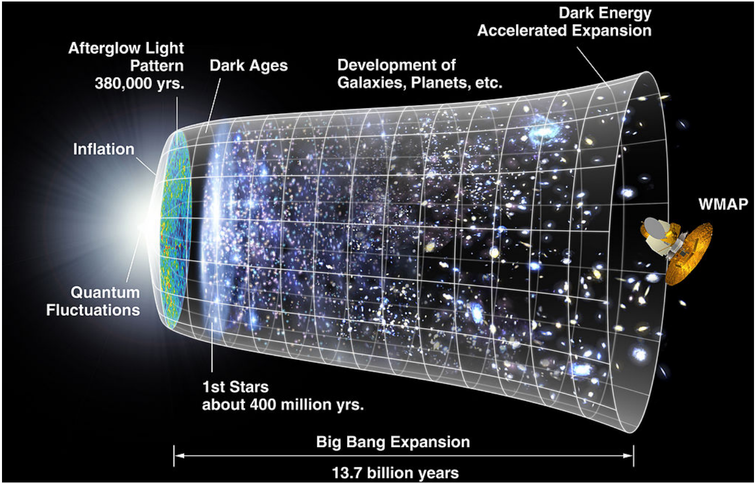
# Cosmology




(graphic by: NASA/WMAP Science Team)

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- The very-nearly-scale-free nature of the initial density-perturbation power spectrum
- And other more-subtle details (like certain statistical features of the CMB)

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- bubble nucleation and collisions.
- Primordial black-hole formation

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- Coupled modes can enter resonance bands which cause resonant amplification.

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- $\Box\phi^i = \frac{\partial V}{\partial \phi^i}$, $V$ is a nonlinear function of all of the $\{\phi^i\}$.

# FRW Background

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$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 \tag{1}$$

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The field equations of motion become:

$$\ddot{\phi}^i + 3H\dot{\phi}^i - \frac{\Delta}{a^2}\,\phi^i + \frac{\partial V}{\partial \phi^i} = 0 \tag{2}$$

# $V(\phi, \psi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\psi^2$

Using the potential:

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In terms of Fourier modes:

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left( \frac{k^2}{a^2} + m^2 + g^2 \psi^2 \right) \phi_k = 0 \tag{5}$$

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Easther, Finkel and Roth (2010) [arXiv:1005.1921, published JCAP 2010]

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- Requires lots of FFTs to evaluate the nonlinear terms (uses FFTW or Intel's MKL).
- Parallelized using OpenMP.
- Naturally integrates with Fourier-space $h_{\mu\nu}^{TT}$ evolution.

# $4^{th}$ Order vs $2^{nd}$ Order

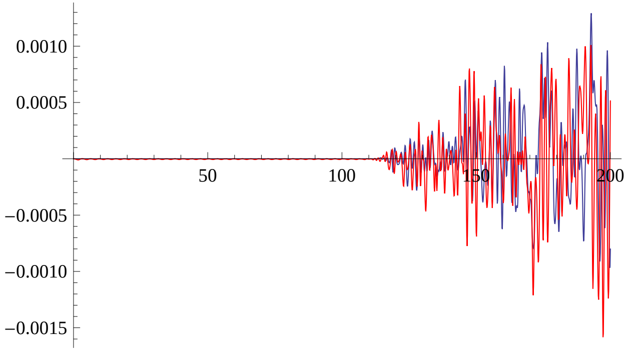


Figure: PSpectRe runs at $32^3$ ($L = 2$ and the time step is 0.005). The red line uses the Verlet integrator, blue shows the Runge-Kutta results.

# Padding (Mode-Aliasing Mitigation) Helps

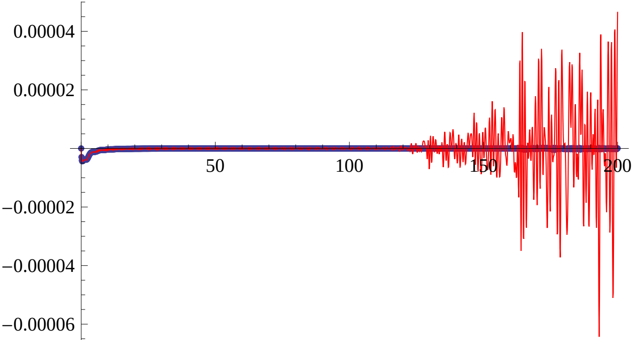


Figure: PSpectRe: Red is unpadded, blue is padded by a factor of 2.

# Compare with Defrost: Energy Conservation

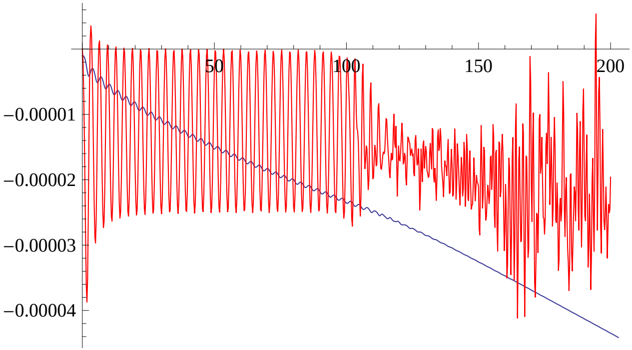


Figure: Runs with $256^3$ points and $L = 10$ (for Defrost's default model).

PSpectRe's convergence for equation-of-state observables is better than Defrost's too (see the paper).

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- Similar to a soliton, but not protected by a symmetry of the Lagrangian.
- Mustafa Amin (MIT), our collaborator, has done some of the best recent theory work.

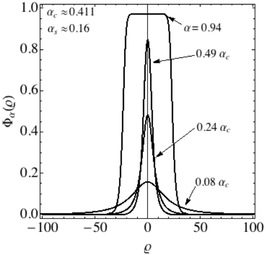
# Sextic Oscillon Potential

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \frac{g^2}{6m^2}\varphi^6 \quad (6)$$

with $\lambda > 0$ and $(\lambda/g)^2 \ll 1$. Assuming spherical symmetry and ignoring expansion gives:

$$\partial_t^2\varphi - \partial_r^2\varphi - \frac{2}{r}\partial_r\varphi + m^2\varphi - \lambda\varphi^3 + \frac{g^2}{m^2}\varphi^5 = 0 \quad (7)$$

Assuming a bounded, periodic solution gives an ODE which can be (approximately) solved to yield the radial profile of an oscillon. It is a one-parameter family of curves.



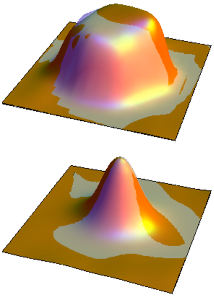



Figure: Oscillon profiles in the sextic potential

# Why Do We Care?

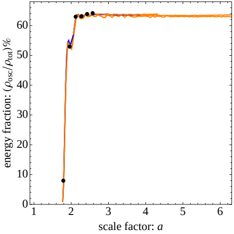
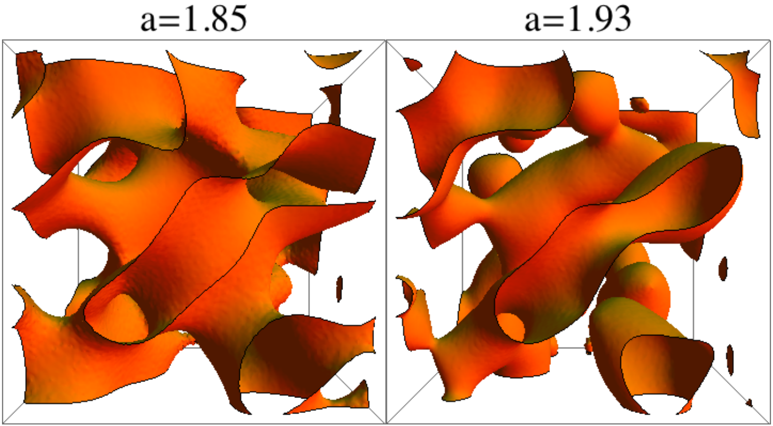


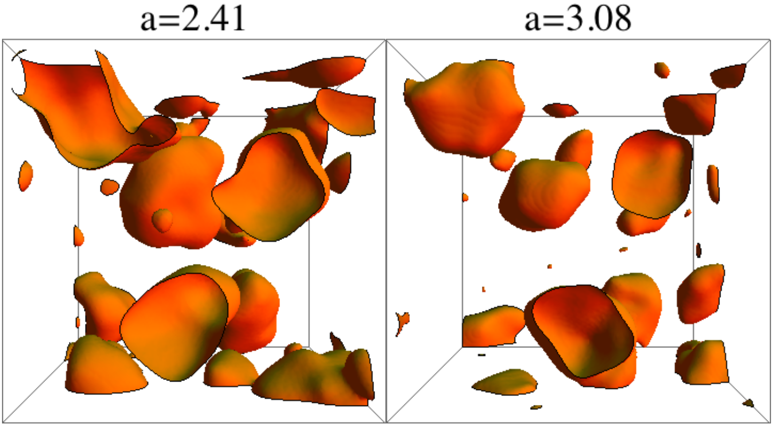

Figure: The fraction of the energy density of the universe after inflation which is in oscillons. The orange and blue curves are from PSpectRe runs (a lot of them) at $256^3$ and $384^3$ respectively. The black dots are $1024^3$ MPI Defrost runs.

# A Universe of Oscillons

Simulation using PSpectRe at $L = 200$ and $N = 256$.



# A Universe of Oscillons (cont.)



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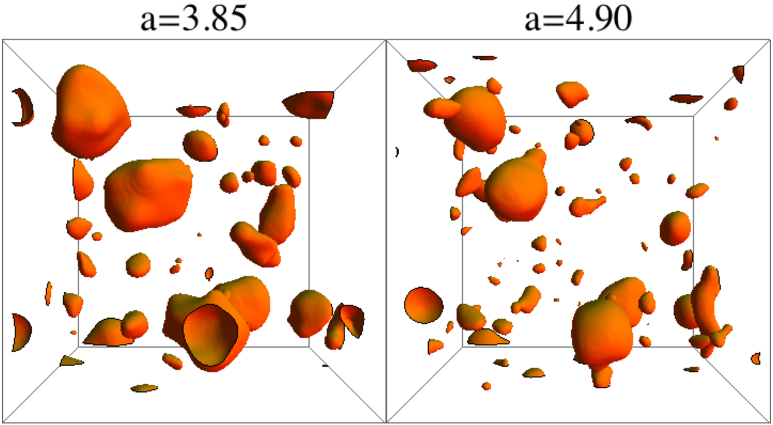

# Conclusion

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- PSpectRe is good for localized objects (like oscillons).
- Oscillons appear in many theories of inflation, and affect the evolution history of the universe.

# Acknowledgments

I would like to thank:

- My adviser, Richard Easther.
- and the rest of my thesis committee.
- Mustafa Amin
- Tom Giblin and Eugene Lim
- DOE CSGF, and the Krell staff.
- Yale GSA&S, and the physics department.

# The End

"Begin at the beginning and go on till you come to the end: then stop." - Lewis Carroll, Alice's Adventures in Wonderland.

# H = 0 and the Mathieu Equation

When $H = 0$, substitute:

$$q = \frac{g^2\psi^2}{4m^2},\ A = \frac{k^2}{m^2} + 2q,\ z = mt \tag{8}$$

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where primes denote differentiation with respect to $z$. All solutions:

$$\phi_k \propto f(z)e^{\pm i\mu z} \tag{10}$$

# Solution Stability

$m \geq 0$ implies that $A \geq 2q$. $\phi_k$ grows exponentially if $\mu$ has an imaginary part:

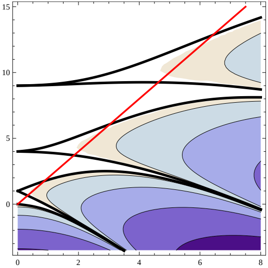


Figure: The imaginary part of the Mathieu critical exponent. Outside the heavy black lines the exponent is real-valued. The diagonal line is $A = 2q$.

# Other Codes

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- LatticeEASY: Felder and Tkachev (2000)
- DEFROST: Frolov (2008)
- PSpectRe: Easther, Finkel and Roth (2010)
- HLattice: Huang (2011)

# FFT

FFT = Fast Fourier Transform (transforms from (discrete) position space to "frequency space")

$$\Phi(\vec{k}) = \sum_{\vec{r}} \phi(\vec{r}) e^{-i\vec{k}\cdot\vec{r}}, \tag{11}$$

$$\phi(\vec{r}) = \frac{1}{N^3} \sum_{\vec{k}} \Phi(\vec{k}) e^{i\vec{k}\cdot\vec{r}}. \tag{12}$$

FFT evaluates these using a recursive decomposition: $O(n \log n)$.
$\phi$ is real: $\phi(\vec{r}) = \phi(\vec{r})^\star$, so $\Phi(\vec{k}) = \Phi(-\vec{k})$ and the number of free parameters matches in both representations.

# Derivatives in Fourier Space

Each derivative operator brings down a factor of $-ik$, so:

$$\nabla^2 \rightarrow \vec{k} \cdot \vec{k}$$

And so (for example):

$$\int_{\text{box}} |\nabla \phi|^2 = \frac{1}{N^3} \sum_{\vec{k}\_\text{space}} |\vec{k}|^2 |\Phi(\vec{k})|^2 \tag{13}$$

# A Complication: Mode Aliasing

In the discrete case, there is a complication:

- A discrete (upper-half) mode $k$ corresponds not only to the continuum mode $k$, but also to the continuum mode $k - \frac{2\pi N}{L}$.

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- This works if the modes $\frac{\pi(N+2)}{L}$ through $\frac{2\pi(N-1)}{L}$ are negligible, compared to the modes $-\frac{\pi(N-2)}{L}$ through $-\frac{2\pi}{L}$.

# Nonlinear Terms

Terms such as $\chi^2\phi$ are implemented as:

- (Optionally) Pad the Fourier-space grid.
- Perform an inverse FFT (transform to position space).
- Compute the nonlinear operation.
- Perform an FFT (transform to Fourier space).
- (Optionally) Unpad the Fourier-space grid.

Padding in Fourier space is equivalent to performing a polynomial fit using all of the available data points and then filling in using interpolation. It is a bit tricky to implement when using a conjugate-symmetry-reduced storage layout; the details are in the paper.

# Energy Conservation?

GR's dynamic metric does not generally allow for a conserved energy. In this case, the FRW background is changing, but homogeneous, and so we have (the averaged Friedmann equation):

$$\frac{\langle\rho\rangle}{3H^2} - 1 \tag{14}$$

And it should be as good as the homogeneity assumption (parts in $10^7$).

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- Potentials for which $V(\phi) \sim \phi^{2\alpha}$ with $\alpha < 1$ at large $\phi$ arise in a wide variety of string and supergravity scenarios!
- Quartic inflation ($\alpha = 2$) is ruled out, and even quadratic inflation ($\alpha = 1$) is somewhat disfavored, relative to models with $\alpha < 1$.

Oscillons can form in potentials of the form:

$$V(\phi) = \frac{m^2\phi^2}{2} + U(\phi) \tag{16}$$

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- For our monodromy model this requirement is satisfied if $\alpha < 1$.
- If $M$ is significantly sub-Planckian, $U(\phi)$ is both negative and non-vanishing as the field oscillates about $\phi = 0$. This yields resonance and oscillon production!

# Monodromy Oscillons!

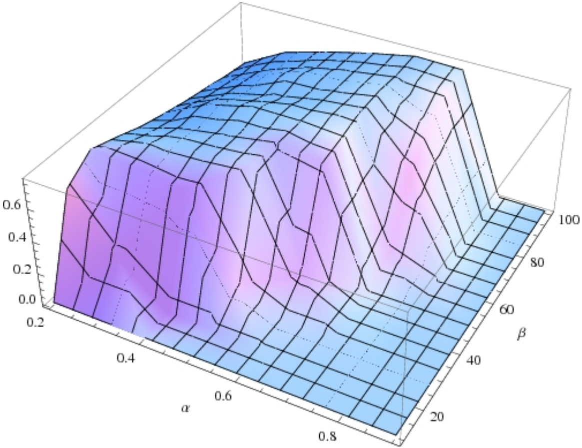


Figure: The fractional energy density in oscillons after a monodromy-inflation preheating phase as a function of $\alpha$ and $\beta$.

# Metric-Perturbation Evolution

Write the metric as:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu} \tag{17}$$

Then metric perturbation obeys:

$$\Box h_{\alpha\beta} - \hat{g}_{\alpha\beta}\Box h + h_{;\alpha\beta} + 2\hat{R}^{\mu\nu}_{\alpha\beta} h_{\mu\nu} - h^{;\mu}_{\alpha\mu;\beta} - h^{;\mu}_{\beta\mu;\alpha} + \hat{g}_{\alpha\beta} h^{;\mu\nu}_{\mu\nu} = -16\pi G \delta T_{\alpha\beta} \tag{18}$$

which simplifies after a gauge is chosen.

# Gravitational-Wave $T_{\mu\nu}$

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$$\rho_{gw} = \frac{1}{32\pi G} \left\langle h_{ij,0} h^{ij}_{,0} \right\rangle = \sum_{i,j} \frac{1}{32\pi G} \left\langle h^2_{ij,0} \right\rangle \quad (20)$$

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The fractional contribution to the overall density per logarithmic interval in wave-number:

$$\frac{d\Omega_{gw}}{d \ln k} = \frac{1}{\rho_{crit}} \frac{d\rho}{d \ln k} = \frac{\pi k^3}{3H^2 L^2} \sum_{i,j} \left| h_{ij,0}(k) \right|^2 \tag{21}$$

# Code by Easther, Giblin and Lim

Easther, Giblin and Lim (2006, 2007, 2008), published work on preheating models. Bubble-collision and Oscillon calculations in progress.

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- Evolves $h_{\mu\nu}^{TT}$ in Fourier space given real-space inputs.
- Has stability issues with high-frequency noise.
- No scalar or vector pieces, no back-reaction.

# What Others Do

So what does everyone else do? Generally, they estimate based on calculating $T_{\mu\nu}^{TT}$. There are different techniques:

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- Estimate by integrating a Green's function for $h^{TT}_{\mu\nu}$ assuming a particular expansion history (matter or radiation dominated, etc.). (Price and Siemens, 2008).
- Assuming Gaussian initial conditions and that (mean) vorticity vanishes, evolve uncoupled $h_{ij}$ (back-reaction included, approx. effect unknown). (García-Bellido, et, al., 2008).

# Results for Preheating are Similar

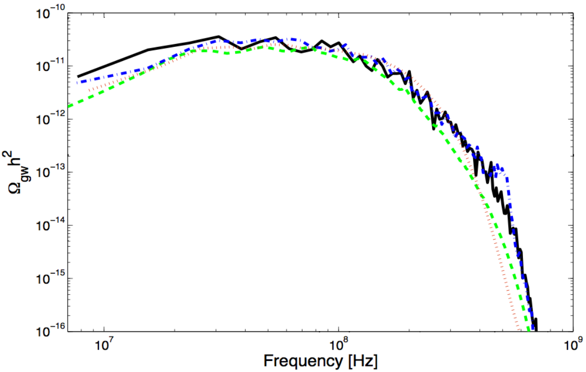


Figure: Plot by Price and Siemens showing their results along with results by: Easther, Giblin and Lim; Dufaux, et al.; and García-Bellido, et, al.

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- The higher the inflationary scale the more post-inflation growth takes place and the smaller the wavelength of the resonant modes.
- Maximal production: $\frac{d\Omega_{gw}}{d\ \ln(k)} \approx 10^{-5}$, $10^{-10}$ today.

# PSpectRe Does Quite Well!

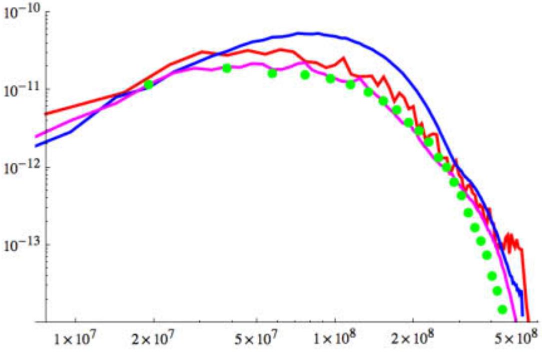


Figure: PSPectRE+$h_{ij}$ at $128^3$ (dots) vs. LatticeEasy+$h_{ij}$ at $128^3$, $256^3$, $512^3$. The PSpectRe+$h_{ij}$ run beats the largest LatticeEasy+$h_{ij}$ run (which had been used for publication).

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- Small regions may tunnel to another minimum $\phi_2$, $V(\phi_2) < V(\phi_1)$, forming a "bubble."

# Example Potential for Bubble Collision Scenario

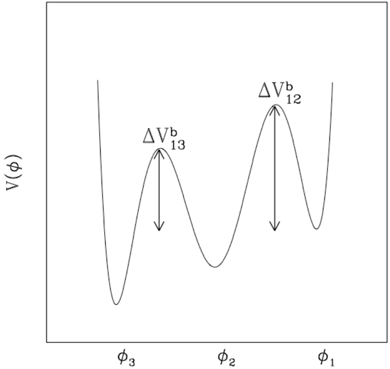


Figure: Diagram of $V(\phi)$ supporting bubble collisions scenarios. Figure from Easther, et al., 2009

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- Produce Hawking radiation as they decay, including gravitational radiation. Could cause a matter-dominated phase.
- For average masses larger than $\approx 1$ gram, constrained by nucleosynthesis, x-ray background, dark-matter abundance.

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- Preprint posted on Feb. 1, so I'll have more to say after I've tried it...

# MPI Defrost

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- Parallelized using MPI, used PETSc's parallel SNES (nonlinear solver).