# Rotational Stabilization of Astrophysical Jets


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# Outline

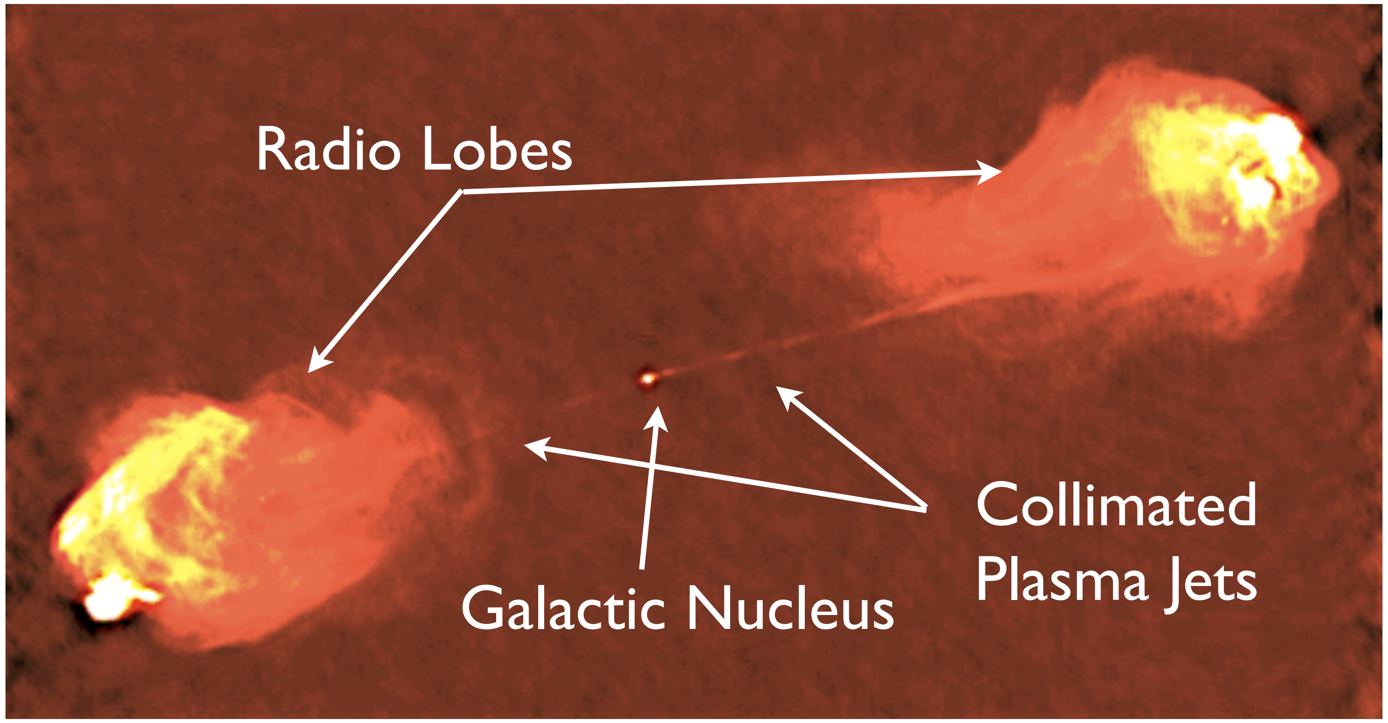
- Introduction to astrophysical jets
- Nonlinear astrophysical jet simulations
  - Magnetically collimated outflow forms
  - Outflow is unstable for slow-rotation accretion disk and stable for high-rotation disk
- Cylindrical linear eigenvalue calculations which investigate the effect of rotation on jet stability

# Introduction to Astrophysical Jets

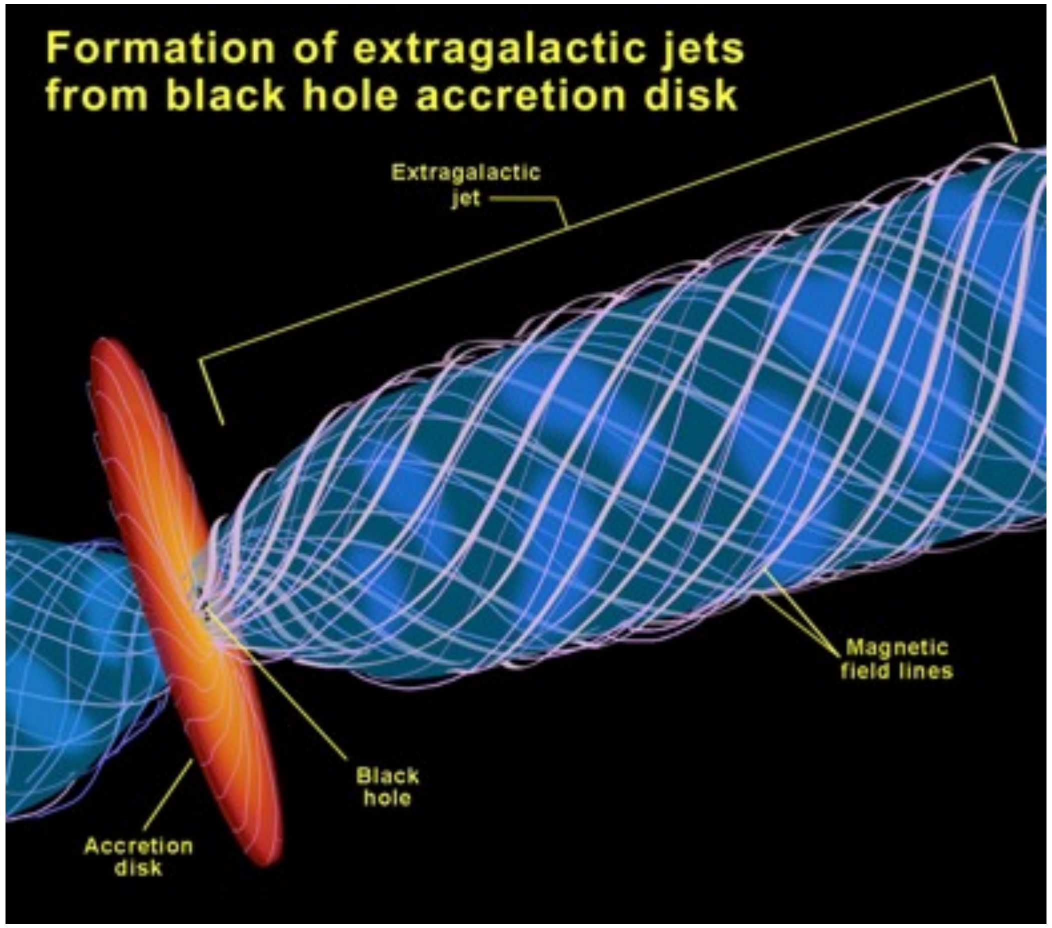
# Astrophysical jets are collimated by helical magnetic fields.

- In simplest terms, astrophysical jets are collimated plasma outflows from massive objects.
- Some active galaxies, young stars, and x-ray binaries form jets.
- The hoop stress of helical magnetic fields in the jets provide collimation of the outflow.
- Both theory and experiment show that pinched plasma configurations are susceptible to the current driven kink instability.

Jet in Cygnus A



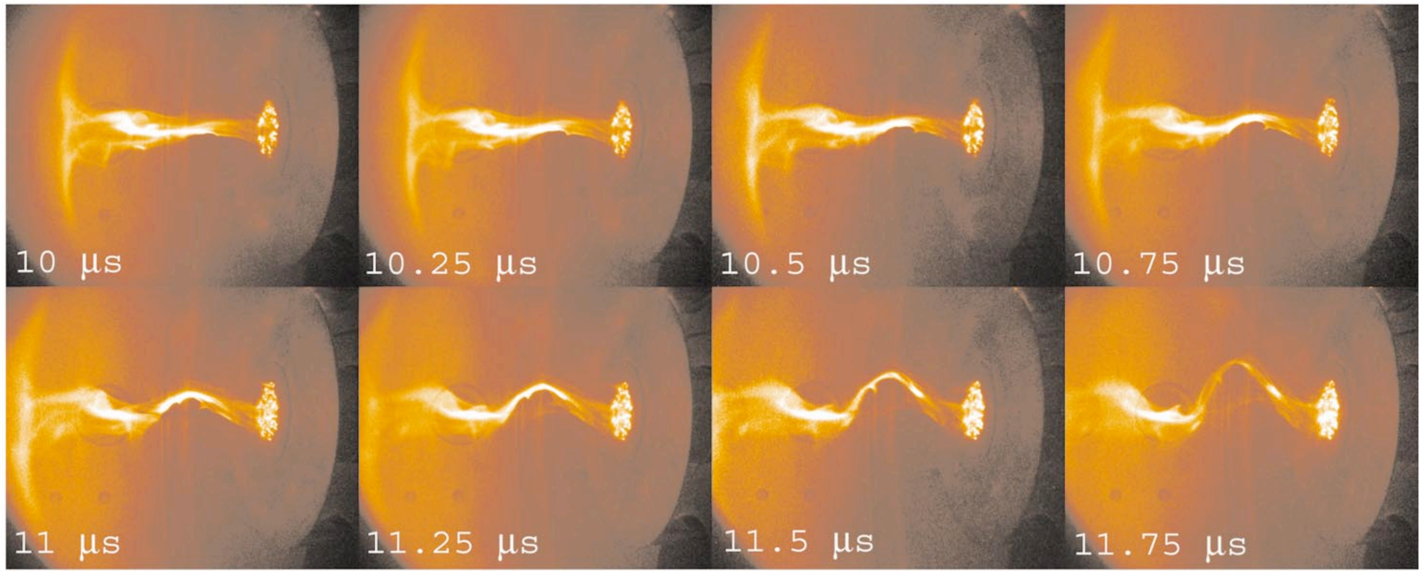

6cm image (courtesy of Chris Carilli)



# How does the kink instability effect jet evolution?

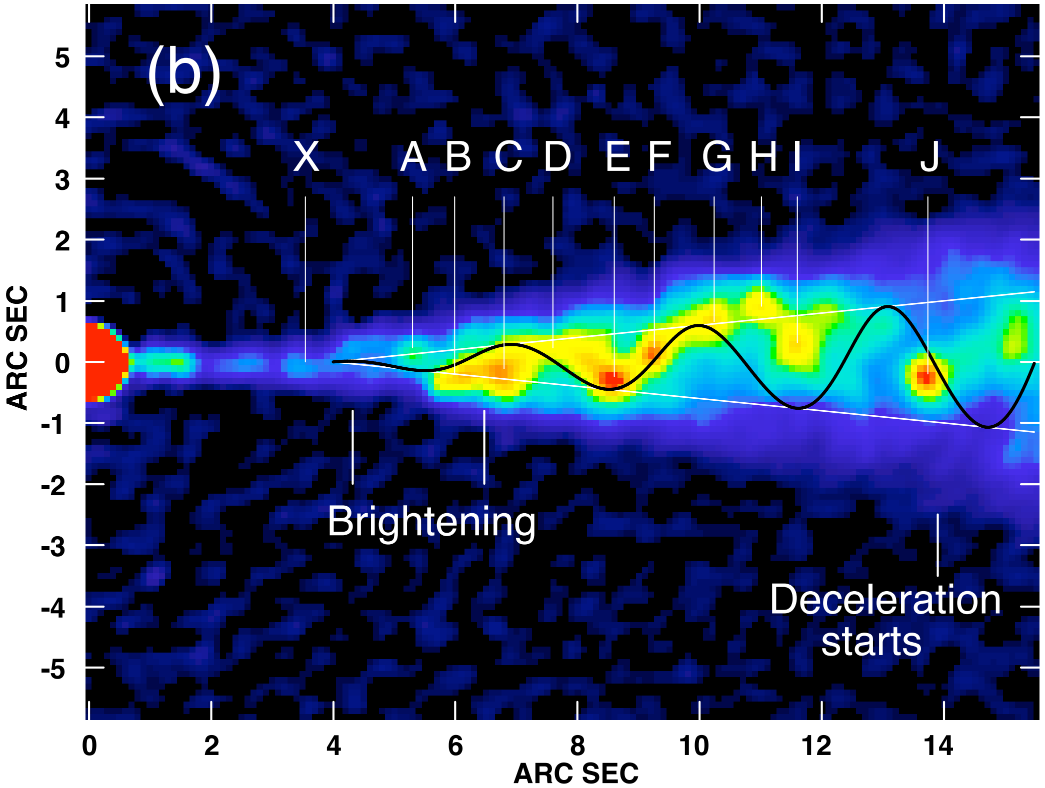
- The kink instability sources free energy in the magnetic field and relaxes the plasma column into a helical structure.
- Recent observations show that some jets may have a large scale helical structure.

Laboratory Jet Simulation Being Reconfigured by a Kink Instability




Hsu, SC, Bellan PM, Physics of Plasmas, Feb. 2005, 12, 032103

5-GHz Emission of the Main Jet of The Active Galaxy NGC315




D.M. Worrall, M. Birkinshaw, et. al., 2007

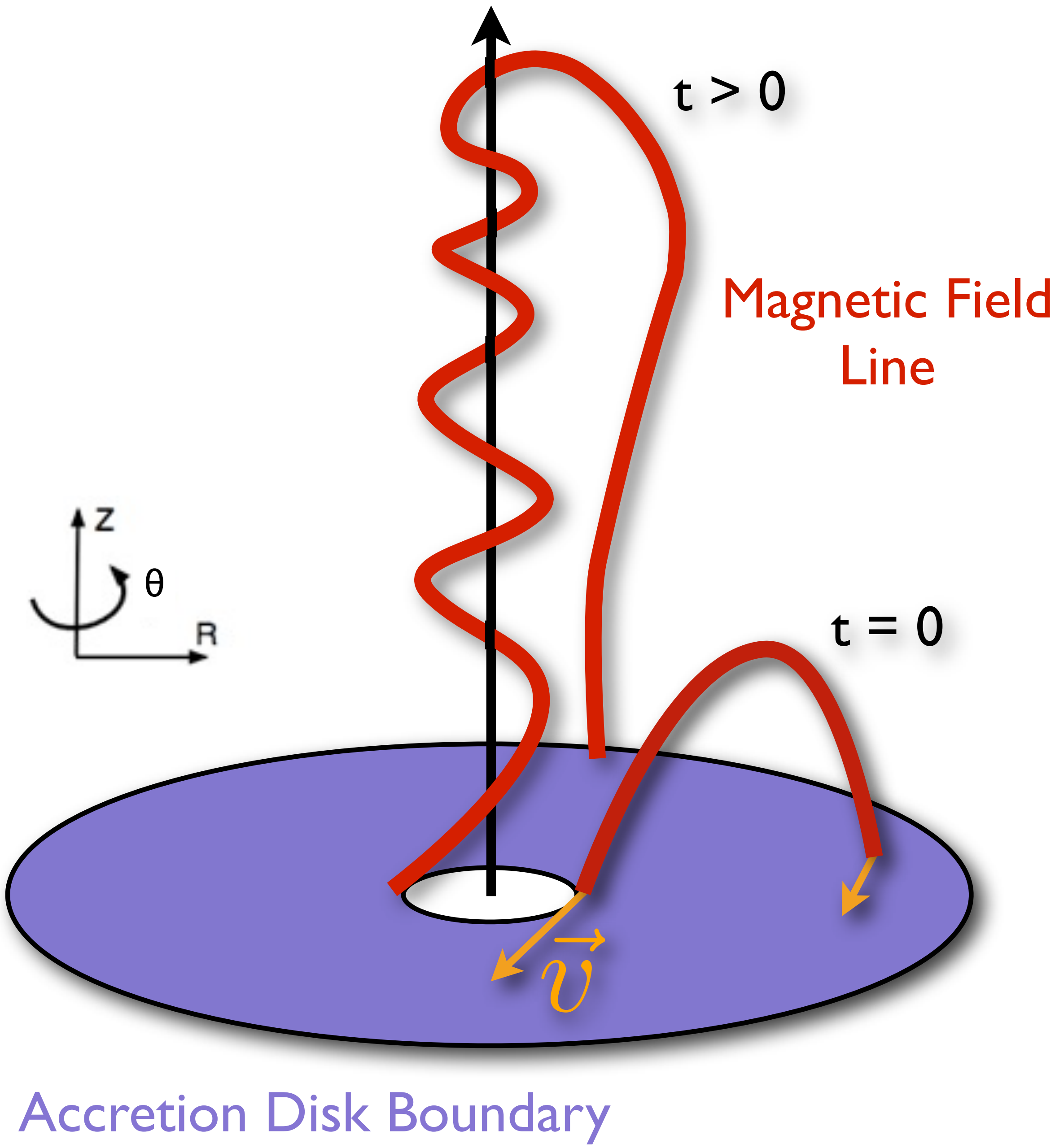
# Nonlinear MHD Jet Simulations

# Our jet model treats the accretion disk as a boundary condition for the numerical simulation.

- Lower boundary of computational domain is treated as an accretion disk by applying a Keplerian profile to $v_\theta$. (Ustyugove, Lovelace, Ap. J., 541, 200)

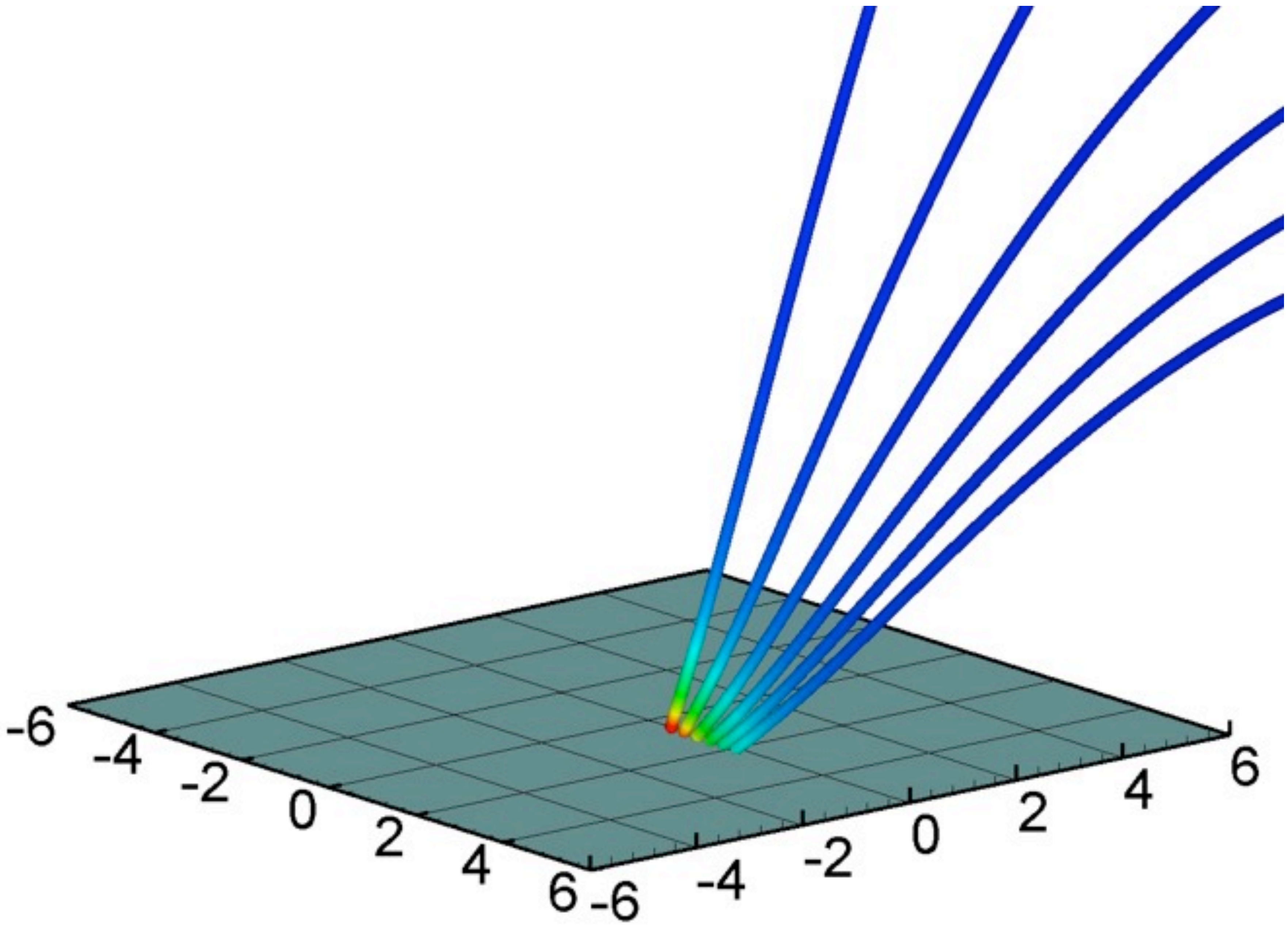
$$v_\theta(r, \theta, z = 0) = \frac{\sqrt{GM}\, r}{(r^2 + r_i^2)^{3/4}}$$

- The initial magnetic field is chosen such that there is no net magnetic flux through the disk boundary.
- The NIMROD code is used to simulate this model by evolving the visco-resistive Magnetohydrodynamic (MHD) equations.
- The MHD model treats the plasma as a conducting fluid.
- Time Units: $T_o$ - Period of the inner disk



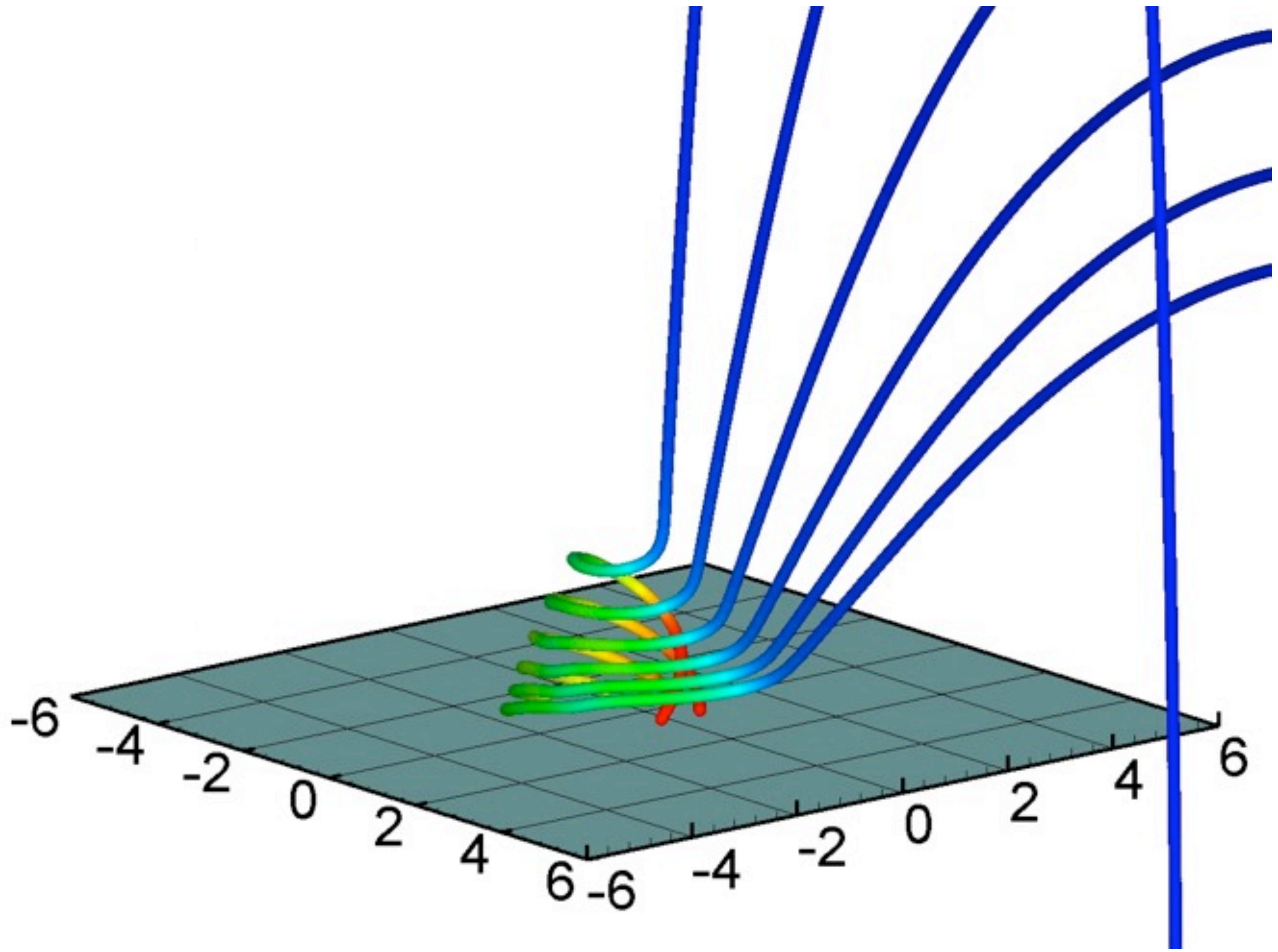
# The disk rotation twists up the magnetic field generating a collimated jet.

$t = 0.0\,T_o$



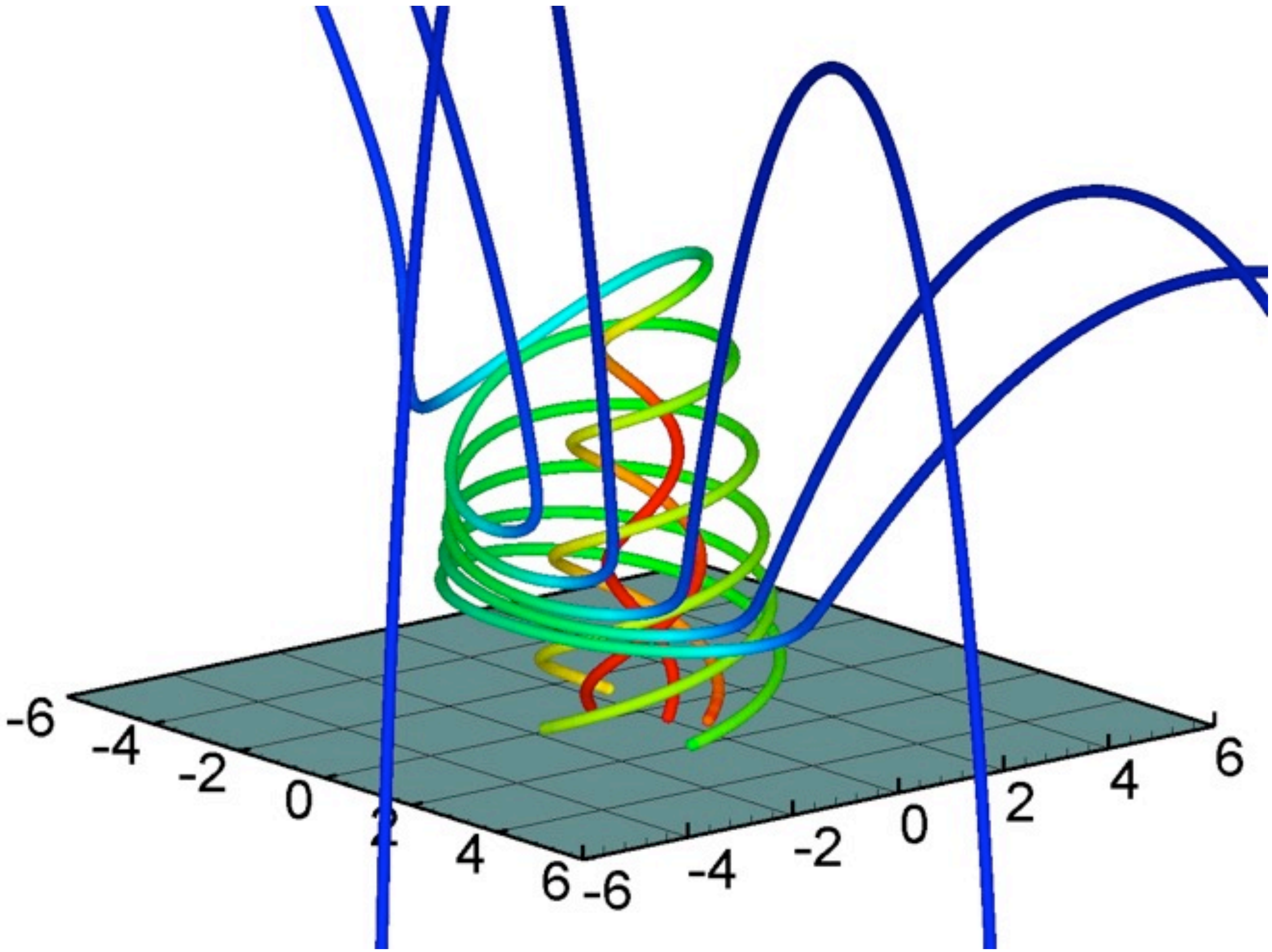
# The disk rotation twists up the magnetic field generating a collimated jet.

$t = 0.7\, T_o$



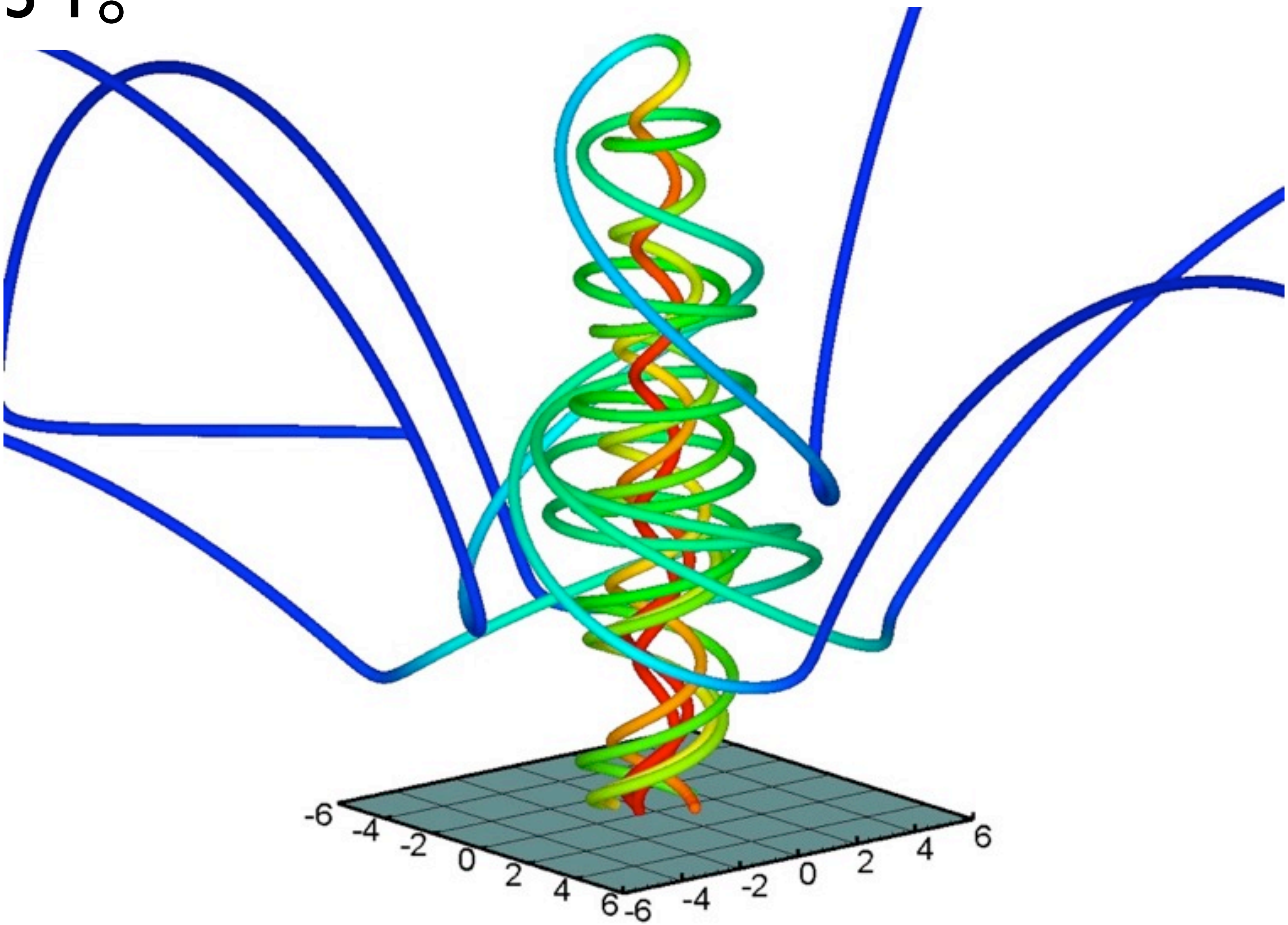
# The disk rotation twists up the magnetic field generating a collimated jet.

$t = 2.1\ T_o$

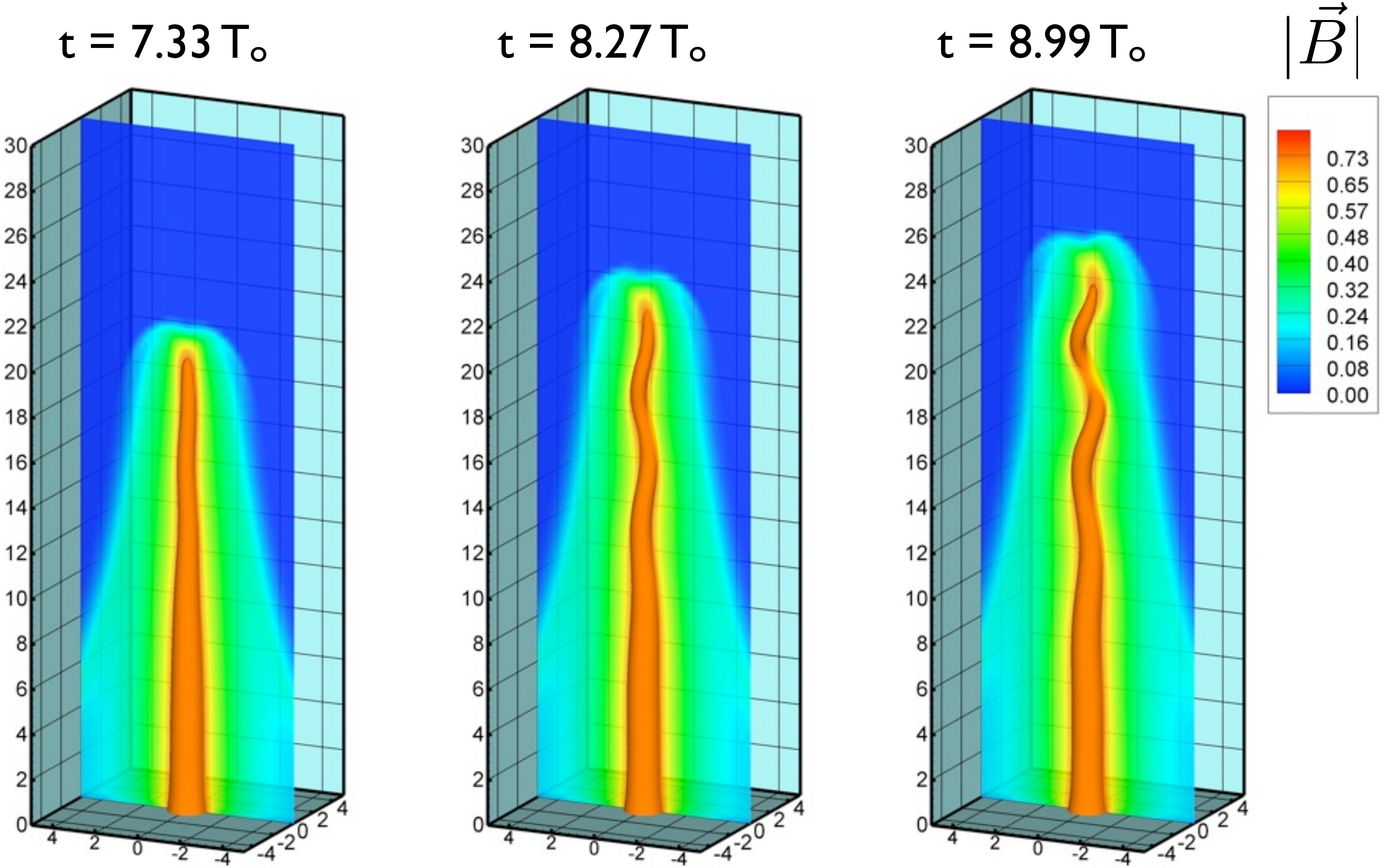


# The disk rotation twists up the magnetic field generating a collimated jet.

$t = 7.3\ T_o$

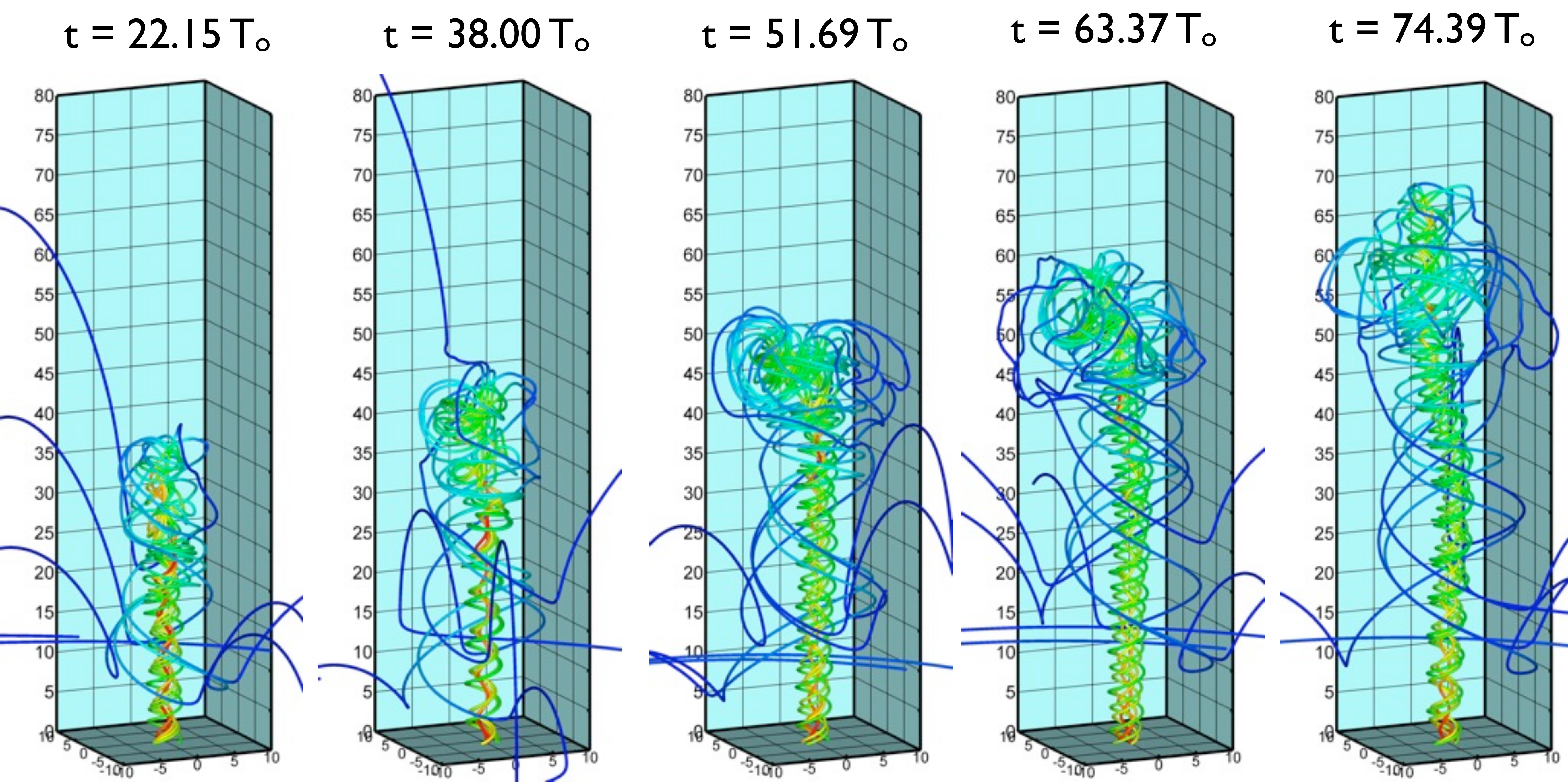


# The kink instability initially generates a helical structure in the jet.



# The turbulence tangles magnetic field lines at the end of the jet.

## Magnetic Field Lines Colored by Alfvén Velocity

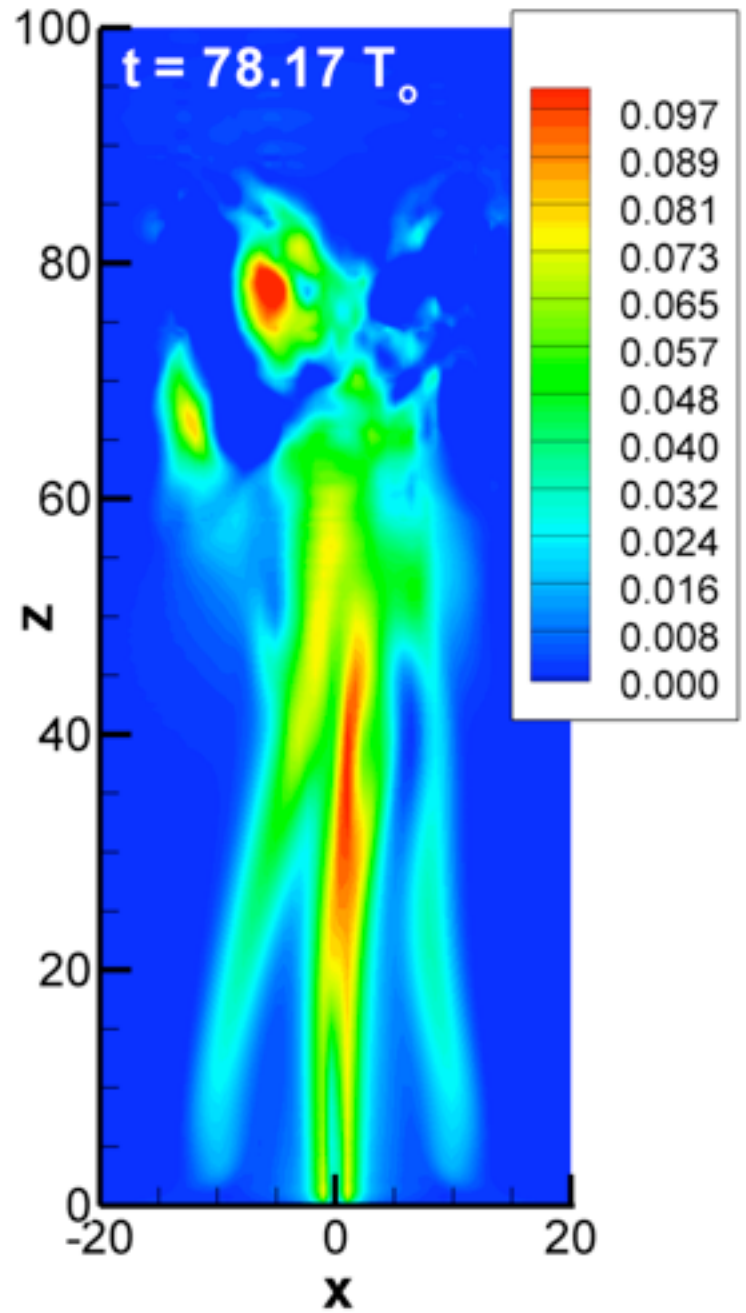


# The jet is stable for fast rotation of the accretion disk.

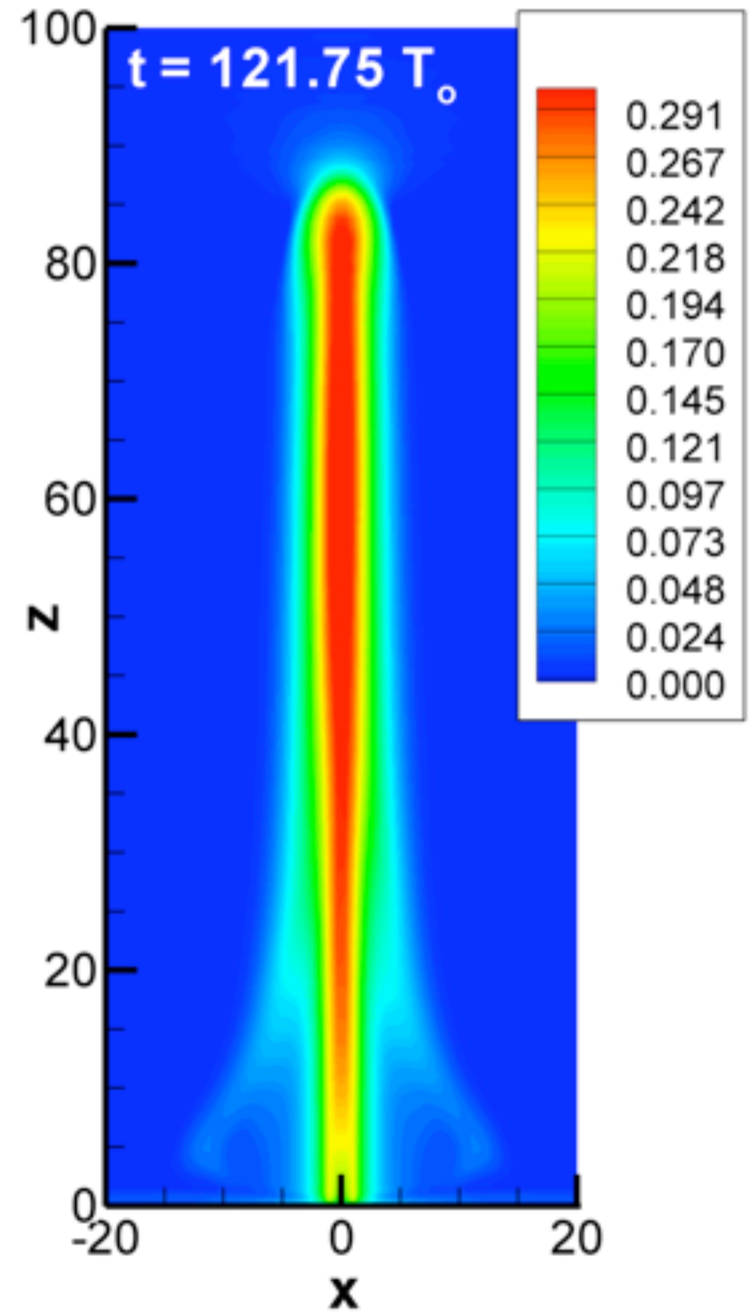
## Axial Component of the Fluid Velocity

Jet is unstable for a slow-rotation accretion disk.

$V_i = 0.5$



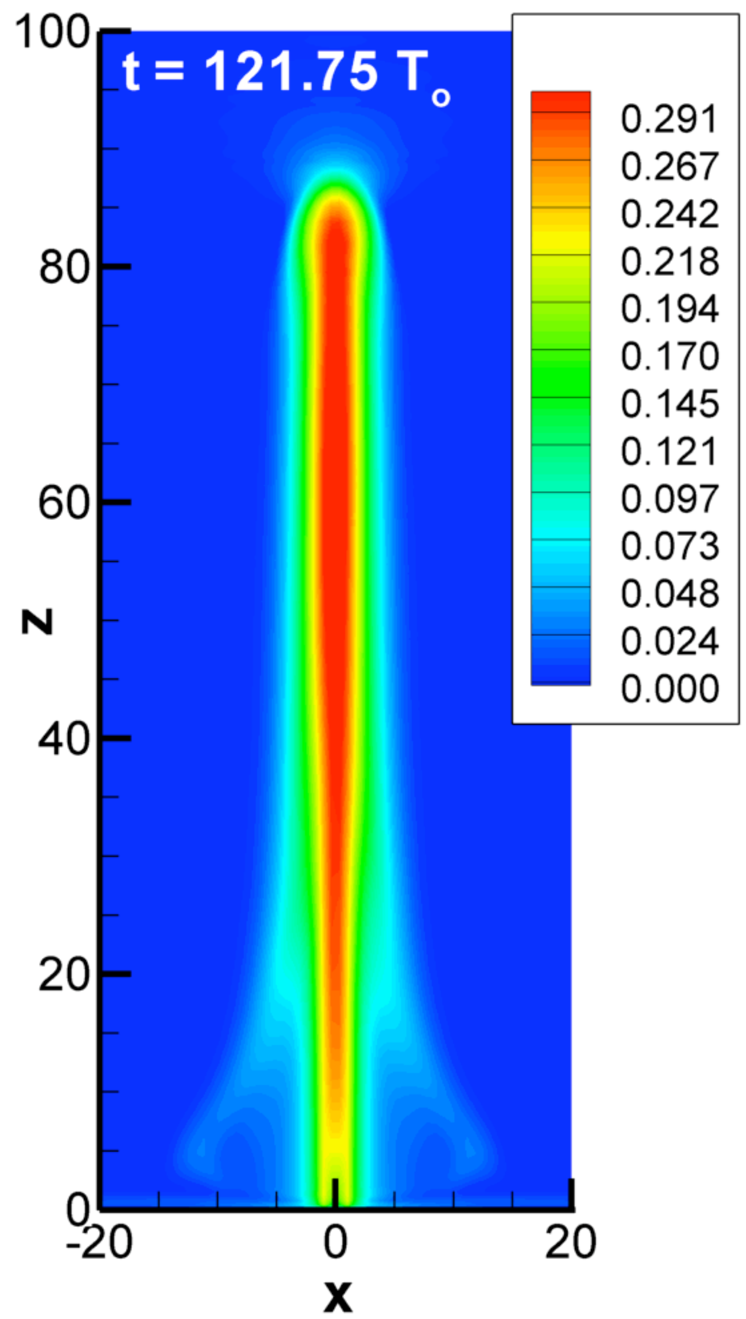

$V_i = 4.0$




Jet is stable for a fast-rotation accretion disk

# Rotation stabilizes the jet for the fast disk case.

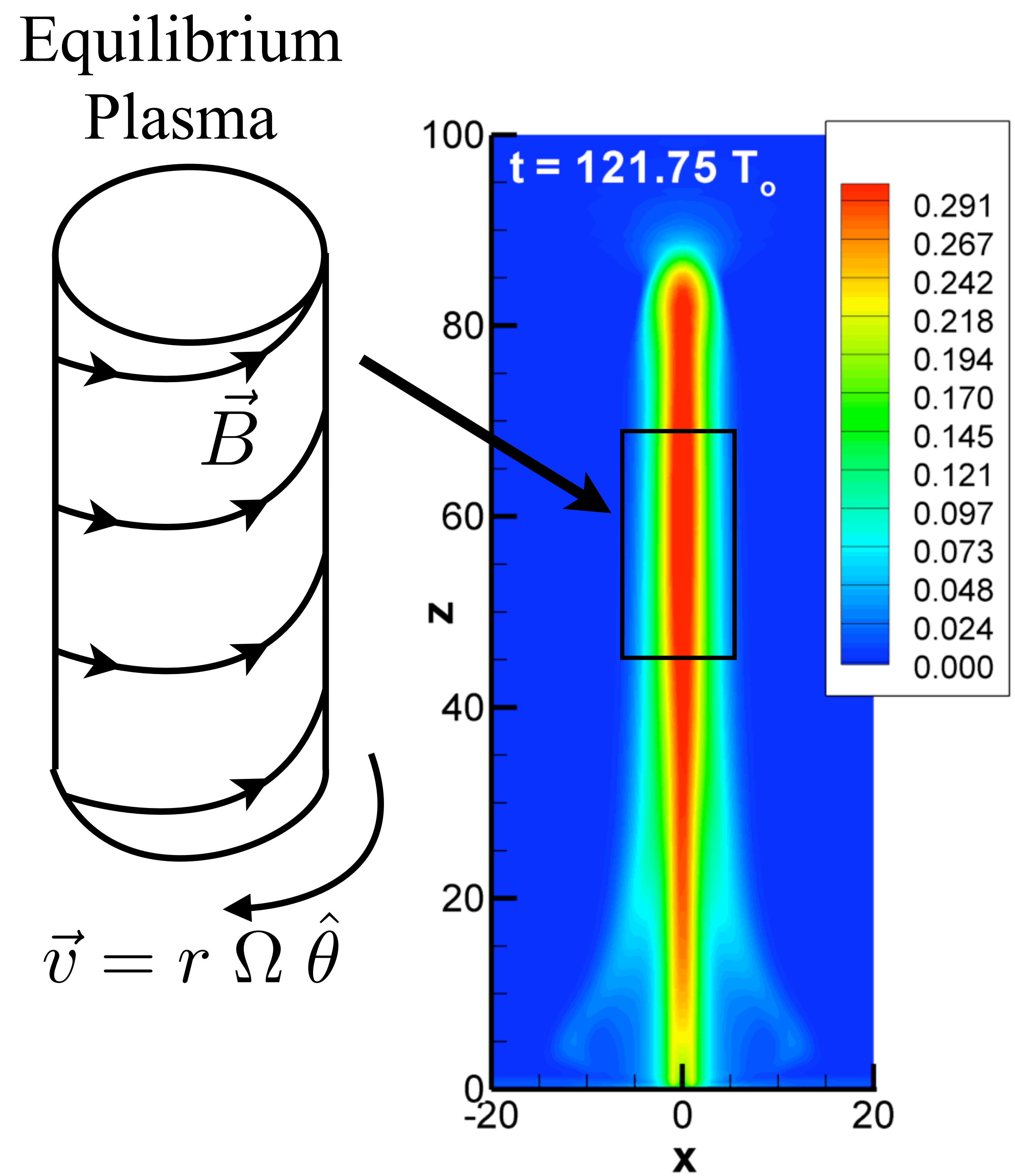
- Accretion disk rotation injects angular momentum into the jet.
- The jet transports this angular momentum as it expands
- A faster rotating disk produces a faster rotating jet.



# Analytic Linear Eigenvalue Calculations

# Motivation for a Linear Theory

- In the nonlinear simulations, it is difficult to determine what effect is stabilizing the kink mode.
  - Complicated geometry
  - Nonlinear effects
  - Diffusive effects
- We consider a cylindrical equilibrium plasma which models the jet far from its base and end.
- We linearize the ideal MHD equations.
- We systematically scan rotation to examine its effect on stability.



# We linearize the nonlinear MHD equations.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\,\vec{v}) = 0$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho\,(\vec{v} \cdot \nabla \vec{v}) = \frac{1}{\mu_o}(\nabla \times \vec{B}) \times \vec{B} - \nabla p$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$$

$$\frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p = -\gamma\, p \nabla \cdot \vec{v}$$

# We linearize the nonlinear MHD equations.

- All fields quantities in the MHD equations are written as an equilibrium plus a perturbation to the equilibrium

$$\vec{B} = \vec{B}_o + \vec{B}_1$$

- During initial instability growth the perturbed fields are much smaller than the equilibrium.
- Any terms which are products of the perturbed fields are dropped from the equations.

$$\begin{aligned}
\vec{B} \cdot \vec{v} &= (\vec{B}_o + \vec{B}_1) \cdot (\vec{v}_o + \vec{v}_1) \\
&= \vec{B}_o \cdot \vec{v}_o + \vec{B}_1 \cdot \vec{v}_o + \vec{B}_o \cdot \vec{v}_1 + \vec{B}_1 \cdot \vec{v}_1 \\
&\approx \vec{B}_o \cdot \vec{v}_o + \vec{B}_1 \cdot \vec{v}_o + \vec{B}_o \cdot \vec{v}_1
\end{aligned}$$

# We linearize the nonlinear MHD equations.

- Lagrangian displacement vector, $\vec{\xi}$

$$\vec{v}_1 = \frac{\partial \vec{\xi}}{\partial t} + \nabla \times (\vec{\xi} \times \vec{v}_o)$$

- Momentum equation

$$-\omega_D^2 \vec{\xi} - 2i\Omega\omega_D(\hat{z} \times \vec{\xi}) + r\Omega^2(\nabla \cdot \vec{\xi}) \left[ \left(3 + \frac{m\Omega}{\omega_D}\right) \hat{r} + i\frac{\omega_D}{\Omega}\hat{\theta} \right] = \frac{1}{\rho_o}\tilde{F}(\vec{\xi})$$

$$\tilde{F}(\vec{\xi}) = \frac{1}{\mu_o}\left(\vec{B}_o \cdot \nabla \vec{B}_1 + \vec{B}_1 \cdot \nabla \vec{B}_o\right) - \nabla \left(p_1 + \frac{1}{\mu_o}\vec{B}_1 \cdot \vec{B}_o\right)$$

- Induction equation

$$\vec{B}_1 = \nabla \times (\vec{\xi} \times \vec{B}_o) + \frac{\Omega B_{oz}}{\omega_D}(\nabla \cdot \vec{\xi})(rk\hat{\theta} - m\hat{z})$$

- Pressure equation

$$p_1 = -|\vec{\xi}|\frac{dp_o}{dr} - \gamma p_o \left(1 + \frac{m\Omega}{\omega_D}\right)(\nabla \cdot \vec{\xi})$$

S Appl, M Amenzind, *Astronomy and Astrophysics* **256**, 354 (1992)

# We linearize the nonlinear MHD equations.

- The perturbed fields are assumed to depend on space and time as

$$\xi \propto e^{-i\omega t + im\theta + ikz}.$$

- The equations are reduced to a set of coupled first order ODE's

$$\underline{\underline{A}}(r,\omega)\,\frac{d}{dr}\begin{pmatrix} r\xi_r \\ \tilde{P} \end{pmatrix} = \underline{\underline{B}}(r,\omega)\,\begin{pmatrix} r\xi_r \\ \tilde{P} \end{pmatrix},$$

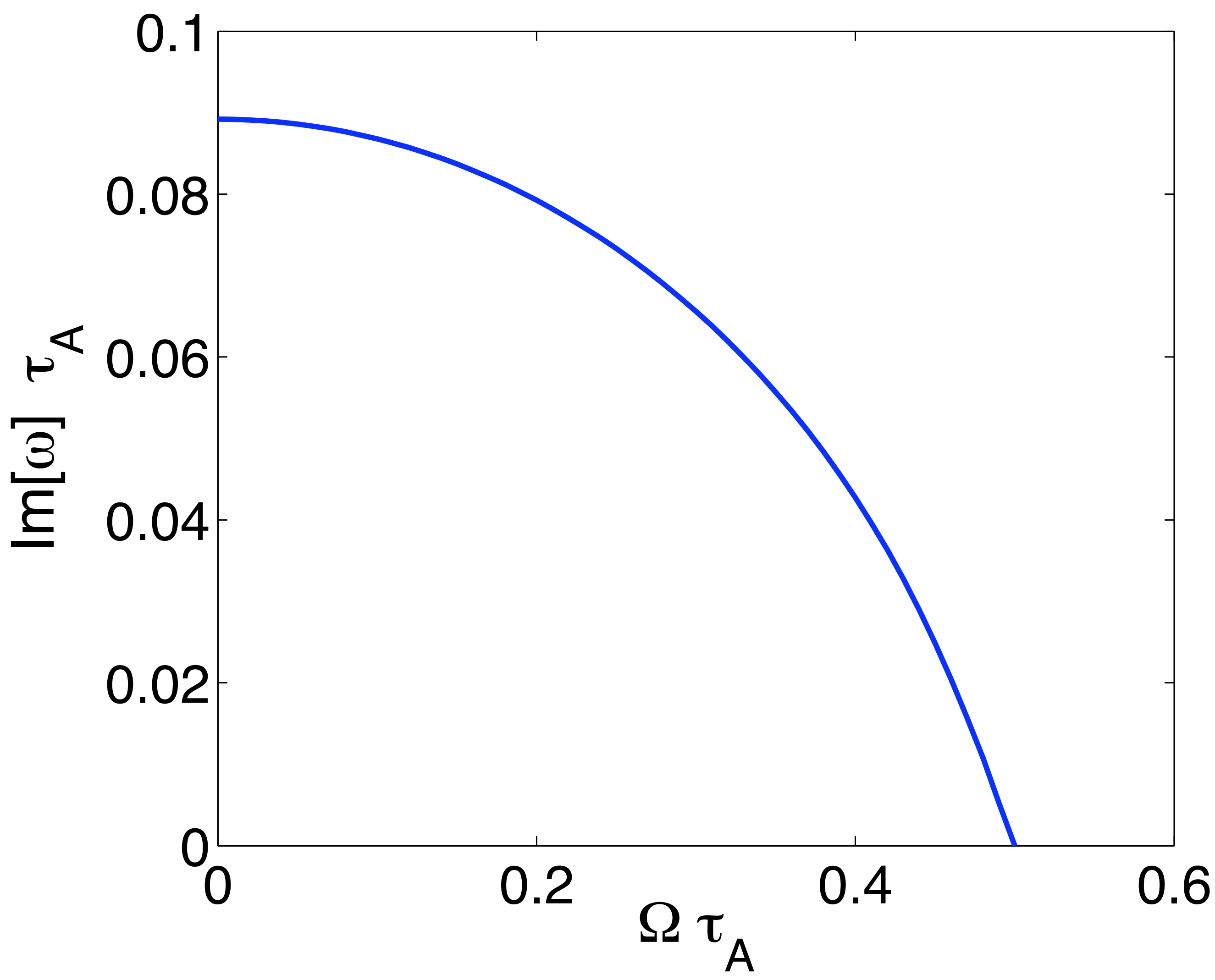
$$\tilde{P} = p_1 + B_1 B_o/\mu_o.$$

- The complex eigenvalue problem is solved numerically using a shooting method.

# Rigid rotation stabilizes the kink mode.

- The growth rate of the kink mode is calculated as a function of the equilibrium rotation frequency.
- Growth rates calculated in the Lagrangian frame and Eulerian frame agree.
- Further analysis shows that the Coriolis force is responsible for the stabilization.

Kink Growth Rate as a Function of Rotation Frequency



# Conclusions

- Nonlinear jet simulations which wind up a coronal magnetic field via accretion disk rotation form magnetically collimated jets.
- The jet is unstable for a slow-rotation disk and stable for a fast-rotation disk
- Linear eigenvalue calculations show that the Coriolis force stabilizes the kink instability in a rotating cylindrical plasma.
- Jet rotation may stabilize magnetically collimated astrophysical jets allowing them to extend to such dramatic length scales without distortion.
- This work is an example of discovery through simulation.

Started out looking to study the kink instability in jets → Rotational stabilization observed in simulation → Analytic theory for Coriolis stabilization of the kink instability

I would like to thank the Department of Energy and the Krell Institute for both the financial support and for building this community of computational scientists.