Fluctuation theorem in colloidal systems with quenched disorder

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Second Law Violations

2nd Law of Thermodynamics:
“The entropy of the universe tends to a maximum.” (Clausius)


Miniaturized devices do not behave like their larger counterparts!
Fluctuation Theorem


- Quantifies the probability of short-term second law violations in a finite system
- Valid far from equilibrium

Compute average injected power over an interval $\tau$

$$J_\tau(t) = \frac{1}{\tau} \int_t^{t+T} \mathbf{F}(t') \cdot \mathbf{v}(t') \, dt'$$

FT: The probability density function of $J_\tau$ follows

$$\frac{P(J_\tau = \epsilon)}{P(J_\tau = -\epsilon)} = e^{\beta \epsilon \tau} \quad \text{as } \tau \to \infty$$

$\beta^{-1}$ can be considered an “effective temperature”
Simulation parameters

- Screened Coulomb (Yukawa) interparticle interaction
- Quenched disorder simulated by randomly placed parabolic traps
- Overdamped dynamics; velocity Verlet integration

\[ V(r_{ij}) = \frac{Q^2}{|\vec{r}_i - \vec{r}_j|} \exp^{-\kappa |\vec{r}_i - \vec{r}_j|} \]

\[ \frac{d\vec{r}_i}{dt} = \sum \vec{F}_{ij} + \vec{F}_p + \vec{F}_d \]

- Interparticle repulsion
- Pinning force
- Drive
Probability Density Function
Effective Temperature

\[ \beta^{-1} \]

- elastic flow
- plastic flow

\[ \rho_c \]

- density

\[ | \vec{F}_d | \]

- drive
Crick, 1970: Nature has difficulty evolving elaborate biochemical mechanisms. Diffusion is a likely candidate for establishing gradients.

S.F. Gilbert, *Developmental Biology*, Sinauer

Some Observed Gradients

- D. melanogaster (fruit fly)

FlyEx database, Reinitz lab, SUNY-Stony Brook

Caudal – Even-skipped -- Bicoid

Bicoid in vivo
Problems with the Crick model

Stokes’ radius:

\[ D = \frac{k_B T}{6\pi \eta r} \]

Stirring?

Observed in vivo gradients

T. Gregor et al., PNAS 102, 18403 (2005)
Analog with effective temperature

T. Gregor et al., PNAS 102, 18403 (2005)

Stokes' radius

additive non-thermal offset
Summary

- We verify that colloidal particles dragged over a disordered substrate with constant applied drive have probability density functions of injected power corresponding to predictions of the fluctuation theorem.

- An “effective temperature” can be defined in both the plastic and elastic flow regimes despite the fact that the colloid is a dissipative, non-equilibrium system.

- Quenched disorder should be considered as a model in explaining transport behavior of biological molecules.
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