Spinning Detonation in a Circular Tube

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Outline

1. What is detonation?
2. Spin detonation
3. Linear stability analysis
4. Numerical simulation
What is detonation?

- Shock-driven combustion in premixed fuel
- Unique features among physical processes
  - Velocity: 2 to 8 km/s (mm/us)
  - Pressure: 100 to 300,000 atm
  - Energy conversion rate: $10^{10}$ Watts/cm²
- Multidimensional, unstable and unsteady

Source: Fickett & Davis. *Detonation*, 1979
Detonation in \( \text{H}_2 \) & \( \text{O}_2 \) with 85% Argon

Spin Detonation
Spin Detonation

- Marginal form of gaseous detonation in tubes
- Low initial pressure
- Dilute and/or lean mixtures
- Axial velocity less than ordinary detonation
- Strong transverse wave(s)
- First form of multidimensional detonation observed experimentally
The major portion of the luminosity behind the wave-front is segregated into bands, each of which appears to have its origin in one of the undulations of the wave-front.

-Campbell & Woodhead, 1926

Source: G. Schott. Physics of Fluids 8:852, 1965
Perspective View

Tube Wall
“Unwrapped”
Incident Shock Wave
Incident Shock Wave

Reaction Zone
Incident Shock Wave

Reaction Zone
Incident Shock Wave

Reaction Zone

Transverse Detonation

“Spin Head”
Spin Structure

Spin Structure

Spin Structure

Helical Path - Single Spin

Source: G. Schott. Physics of Fluids 8:855, 1965
Complex Path - 2x3 Spin

Linear Stability Analysis
Steady, Planar Detonation

- Density
- Velocity
- Pressure
- Reaction Progress

Graphs showing the relationships between these parameters.
Steady, Planar Detonation
• Linear stability analysis can tell us:

• Is the steady solution stable for a given set of parameters?

• What types of instabilities exist? Axial oscillations, radial, spin, etc

• How does a parameter affect stability?

• Stability analysis can guide simulation: Where are single spin modes predicted?
Governing Equations

Reactive Euler Equations
\[
\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0
\]
\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla P = 0
\]
\[
\frac{\partial P}{\partial t} + \mathbf{u} \cdot \nabla P + \rho c^2 \nabla \cdot \mathbf{u} = \rho c^2 \sigma \omega
\]
\[
\frac{\partial \lambda}{\partial t} + \mathbf{u} \cdot \nabla \lambda = \omega
\]

Rankine-Hugoniot Relations
\[
- \rho_0 \mathbf{D} \cdot \hat{\mathbf{n}} = \rho (\mathbf{u}_S - \mathbf{D}) \cdot \hat{\mathbf{n}} = \mathcal{M}
\]
\[
\rho S - \rho 0 = \mathcal{M}^2 (\nu_0 - \nu S)
\]
\[
h_0 + \frac{D^2}{2} = h_S + \frac{(\mathbf{u}_S - \mathbf{D})^2}{2}
\]

Reaction Rate Law
\[
\omega = k (1 - \lambda)^{\nu} \exp \left( - \frac{E \rho}{P} \right)
\]

Equation of State
\[
e (\rho, P; \lambda) = \frac{P}{\rho (\gamma - 1) - \lambda Q}
\]
Expansion

- Expand governing equations in normal modes

\[ q = q^*(z) + \varepsilon q'(z, r) \exp(\alpha t + i\theta) \]

- Resulting equations are separable in \( r \) and \( z \)

\[ q'(z, r) = \begin{bmatrix}
\rho'(z) J_n(ikr) \\
u'_r(z) \frac{\partial J_n(ikr)}{\partial r} \\
u'_\theta(z) \frac{J_n(ikr)}{r} \\
u'_z(z) J_n(ikr) \\
P'(z) J_n(ikr) \\
\lambda'(z) J_n(ikr)
\end{bmatrix} \]

Result of Derivation

- System of 5 complex-valued ODE’s
  \[ A^* \frac{dq'}{dz} + C^* q' + \alpha q' + b^* = 0 \]
- Coefficients are functions of steady solution
- Initial conditions from linearization of the Rankine-Hugoniot conditions
- Closure condition specifies forward characteristic in far-field has zero strength
- Solve for eigenvalues using shooting method
Results of Stability Analysis

Q-E Neutral Stability ($f=1, \gamma=1.2$)

Root Migration from $f=1$ to $f=1.2$
($\gamma=1.2$, $Q=50$, $E=25$)
Numerical Simulation
• Numerically solve the reactive Euler equations

• Goals:
  • Verify results of linear stability analysis
  • Observe single and multi-headed spin
Overlapping Grid
Simulation Results

Reactive Euler (one step) rho

$t=50.000, \ dt=1.09e-02$
Conclusion

- Extending linear stability analysis of detonation to cylindrical geometry
- Verifying stability analysis through simulation
- Using stability analysis and simulation to understand spin detonation
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