Spinning Detonation in a Circular Tube

Brian Taylor and D. S. Stewart
Dept. of Mechanical Science and Engineering
University of Illinois at Urbana-Champaign

Outline

1. What is detonation?
2. Spin detonation
3. Linear stability analysis
4. Numerical simulation

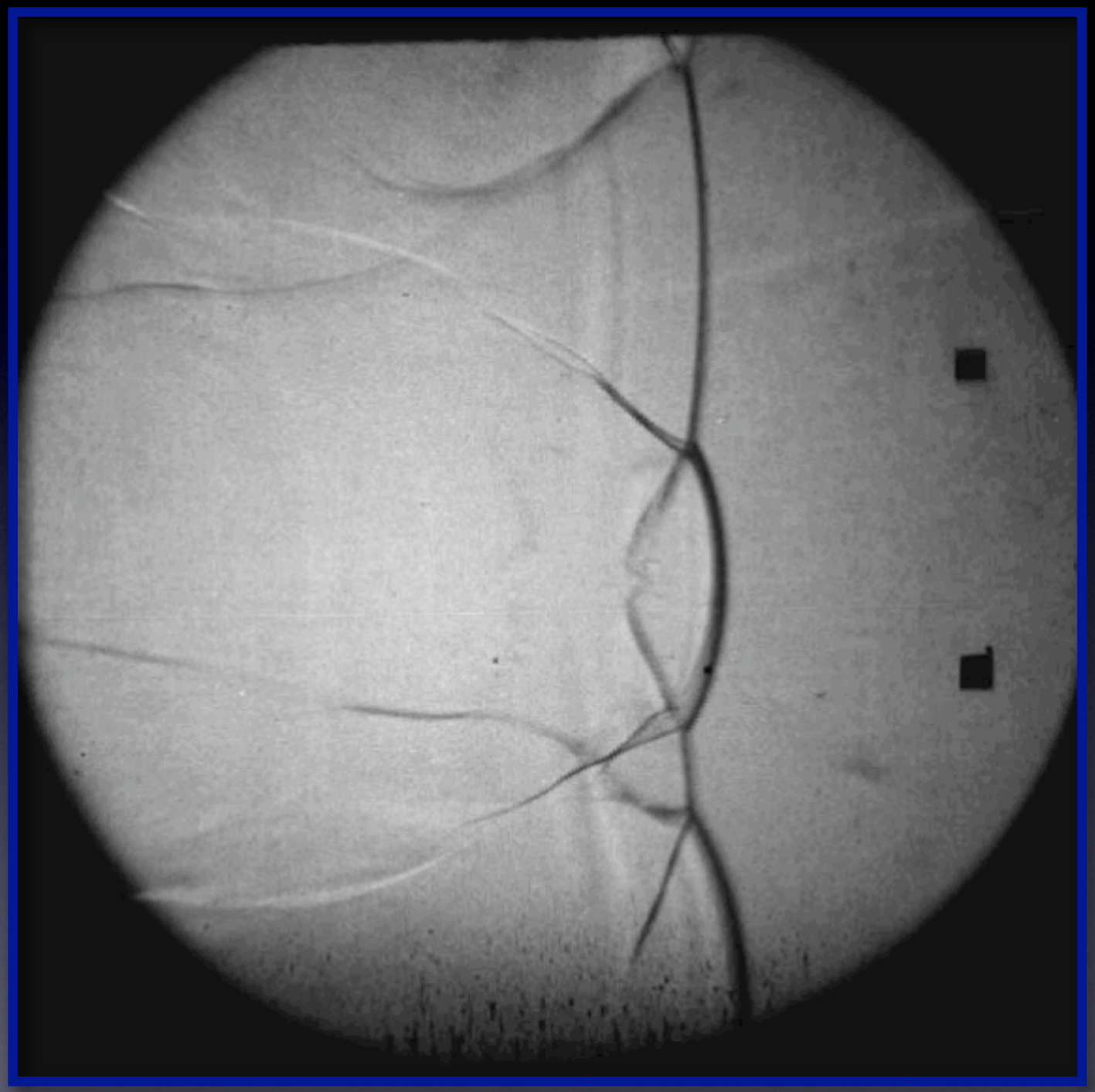
What is detonation?

- Shock-driven combustion in premixed fuel
- Unique features among physical processes
  - Velocity: 2 to 8 km/s (mm/us)
  - Pressure: 100 to 300,000 atm
  - Energy conversion rate: 10^{10} Watts/cm^2
- Multidimensional, unstable and unsteady

Source: Fickett & Davis. *Detonation*, 1979

Detonation in H_2 & O_2 with 85% Argon

138 mm



Source: Austin, *et al.* AIAA ASME 2005-1170, 2005

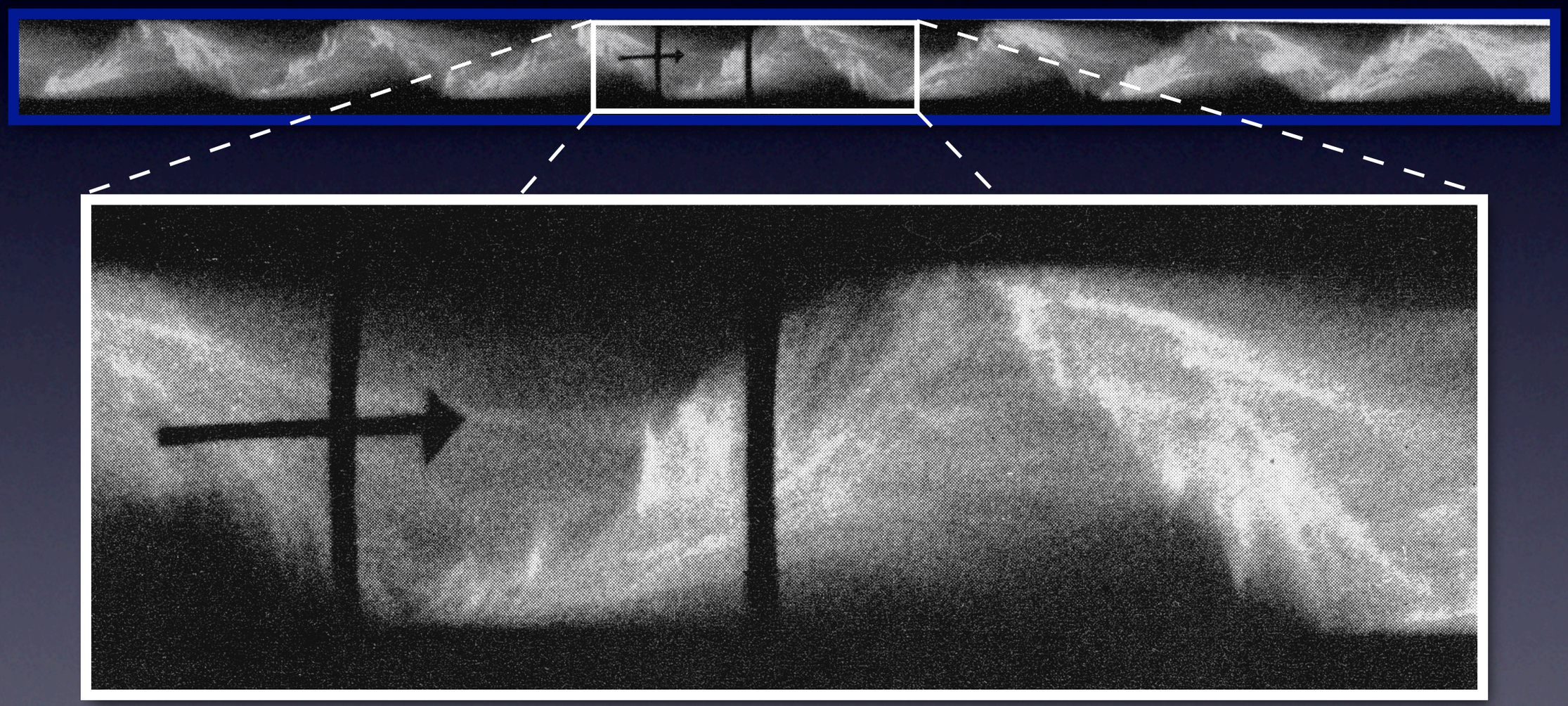
Spin Detonation

Spin Detonation

- Marginal form of gaseous detonation in tubes
  - Low initial pressure
  - Dilute and/or lean mixtures
- Axial velocity less than ordinary detonation
- Strong transverse wave(s)
- First form of multidimensional detonation observed experimentally

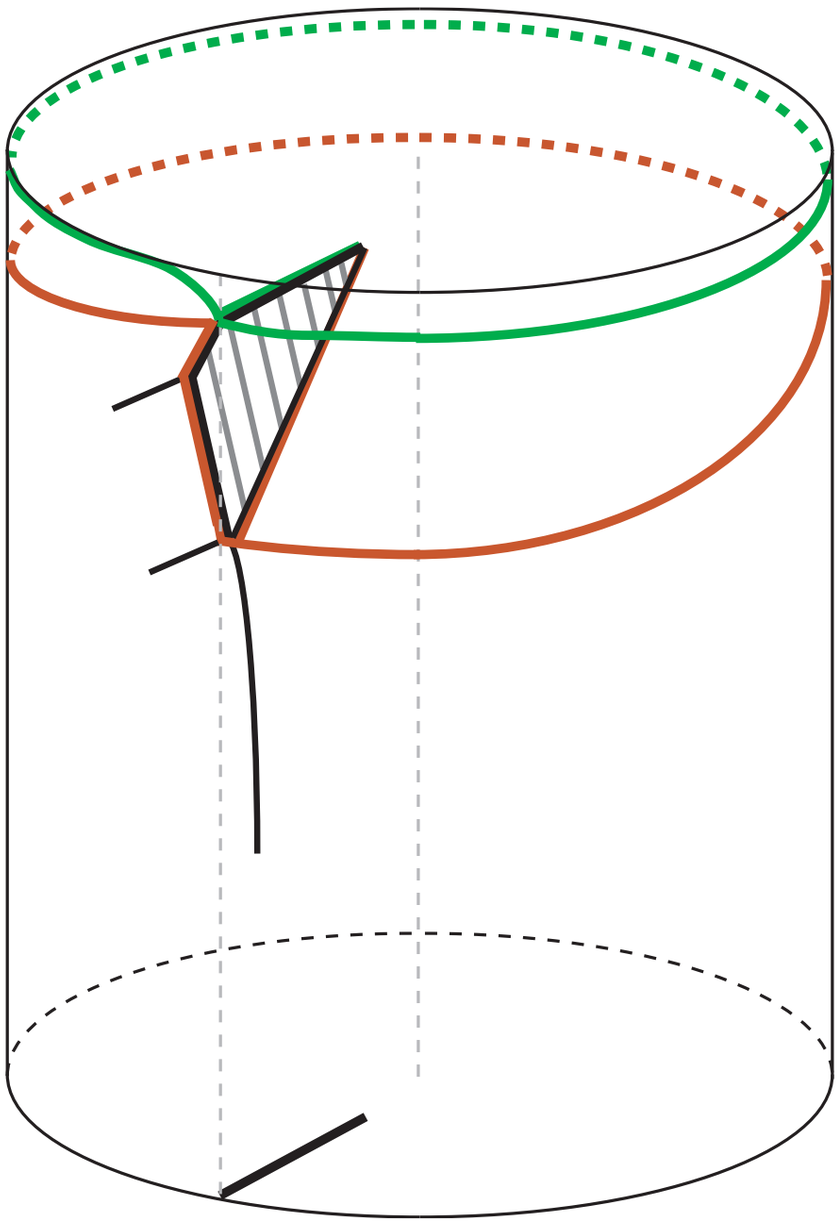
The major portion of the luminosity behind the wave-front is segregated into bands, each of which appears to have its origin in one of the undulations of the wave-front.

-Campbell & Woodhead, 1926

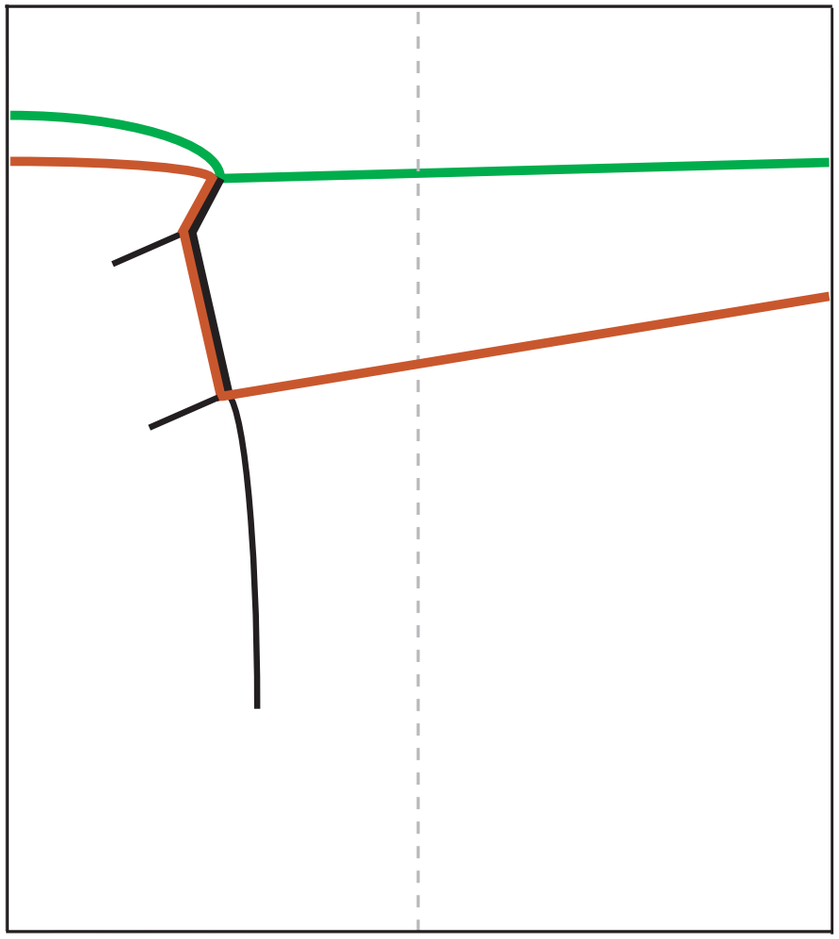


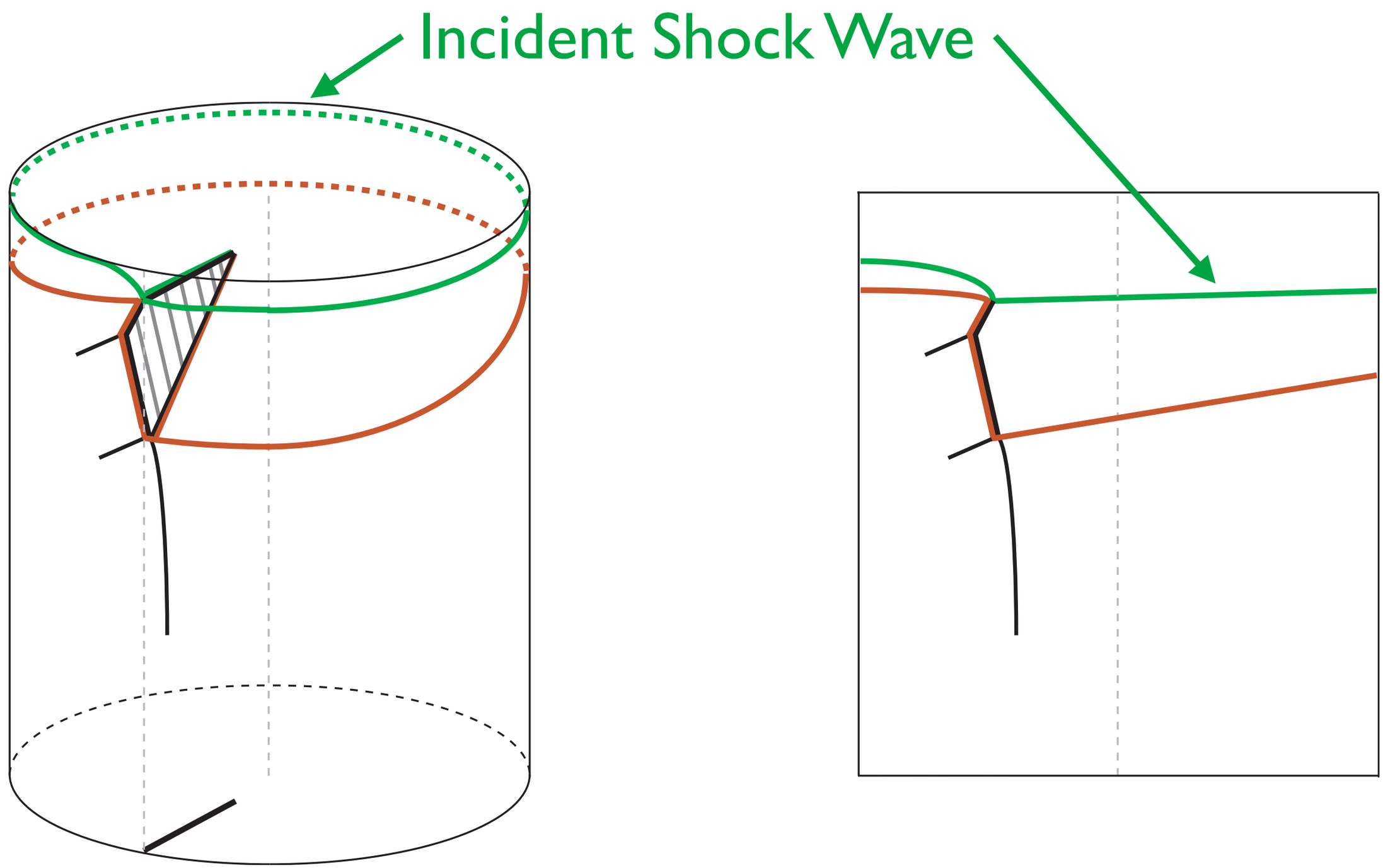
Source: G. Schott. *Physics of Fluids* 8:852, 1965

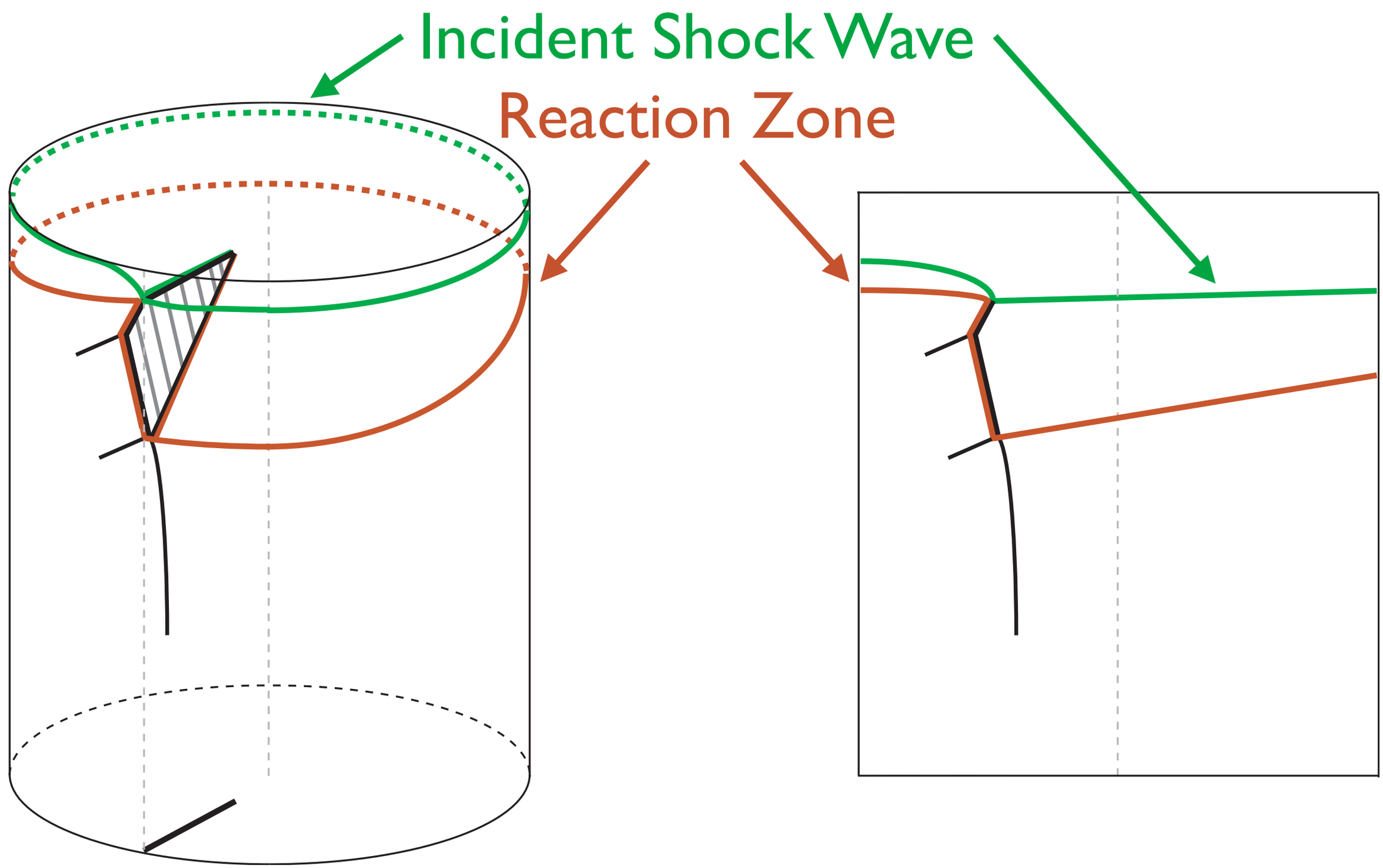
Perspective View

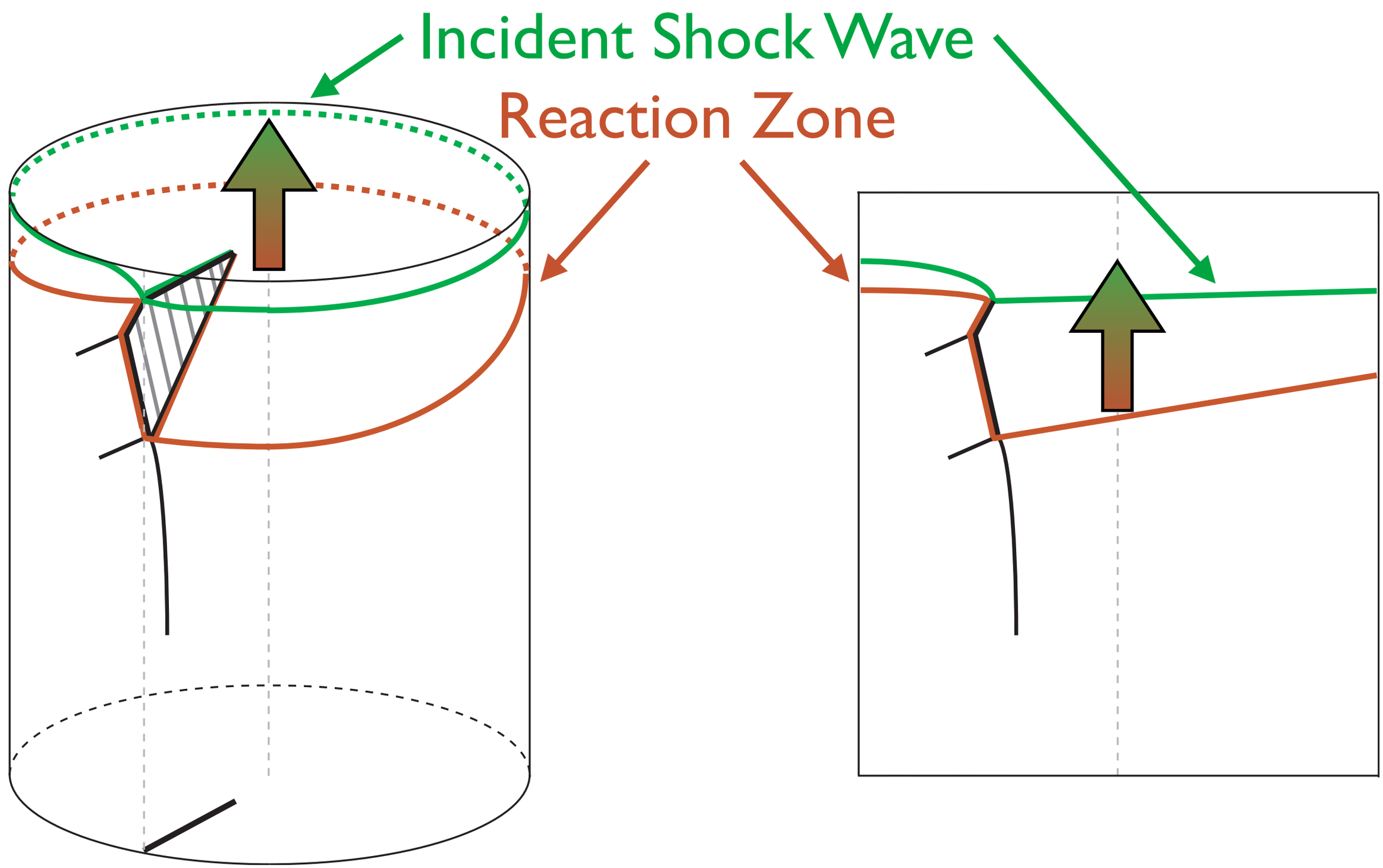


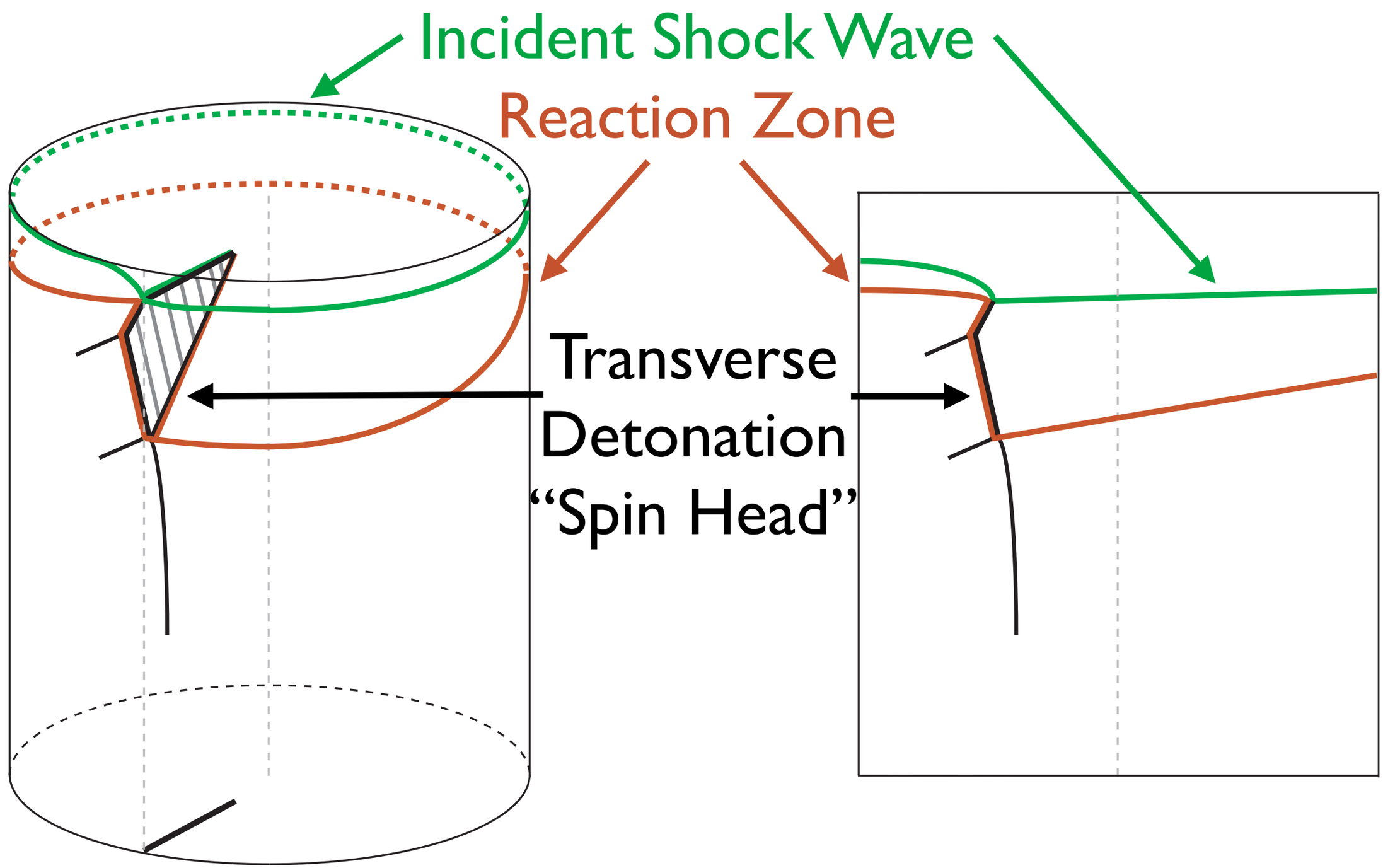
Tube Wall "Unwrapped"

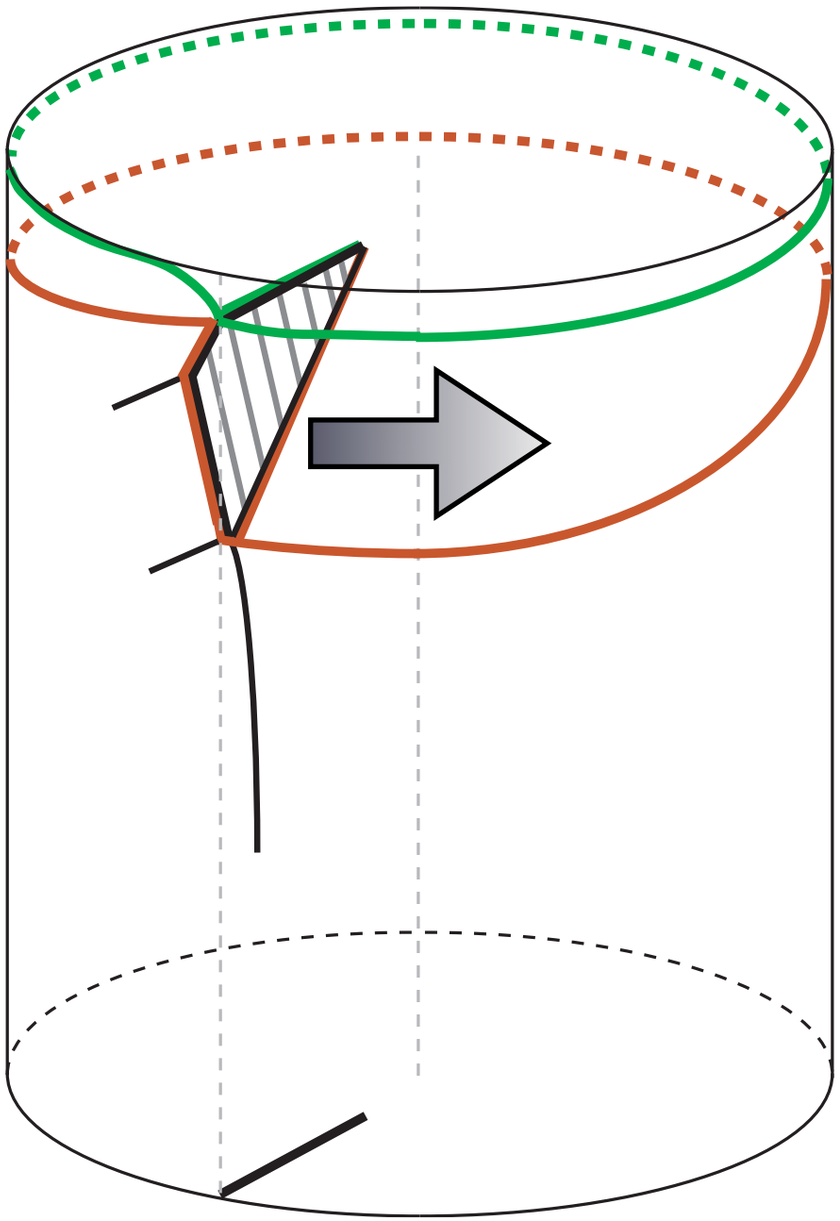


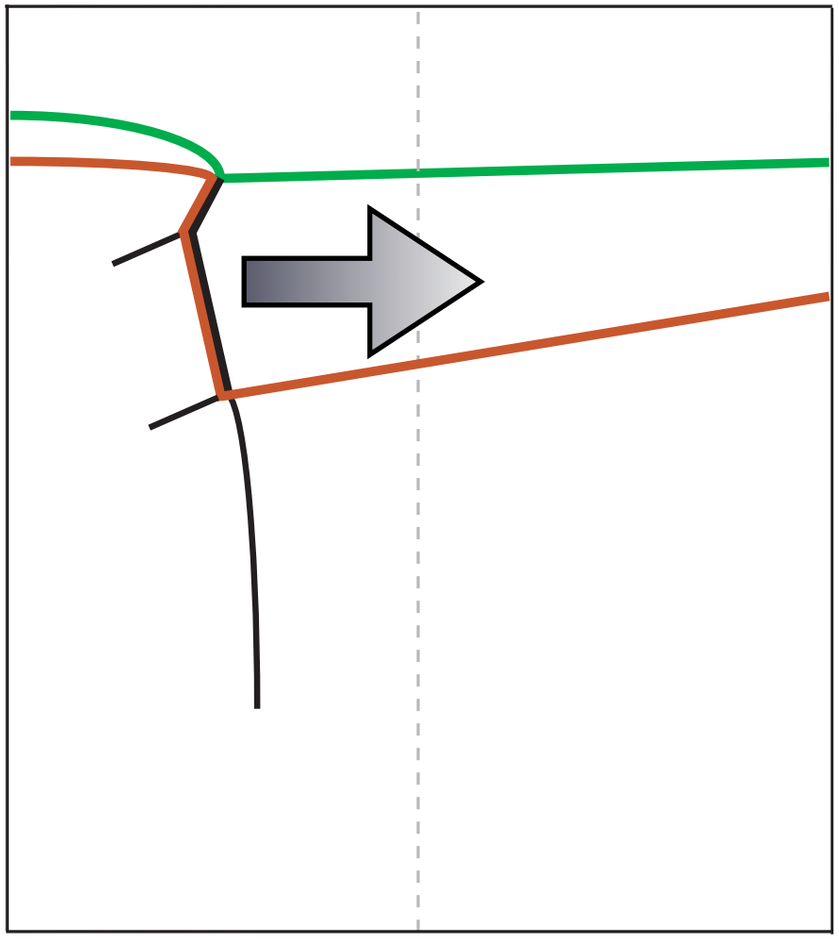

Incident Shock Wave


Incident Shock Wave
Reaction Zone

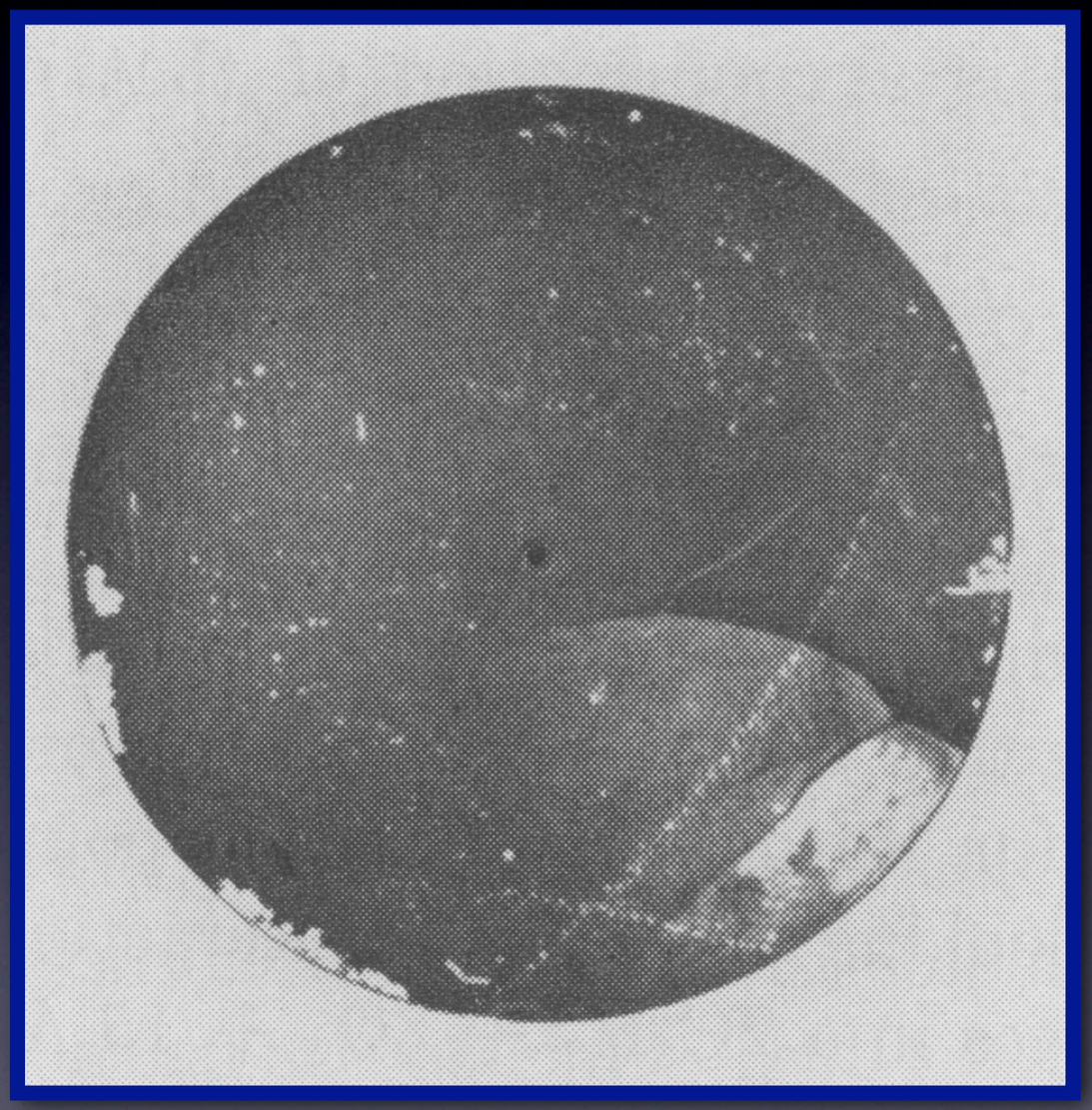

Incident Shock Wave
Reaction Zone

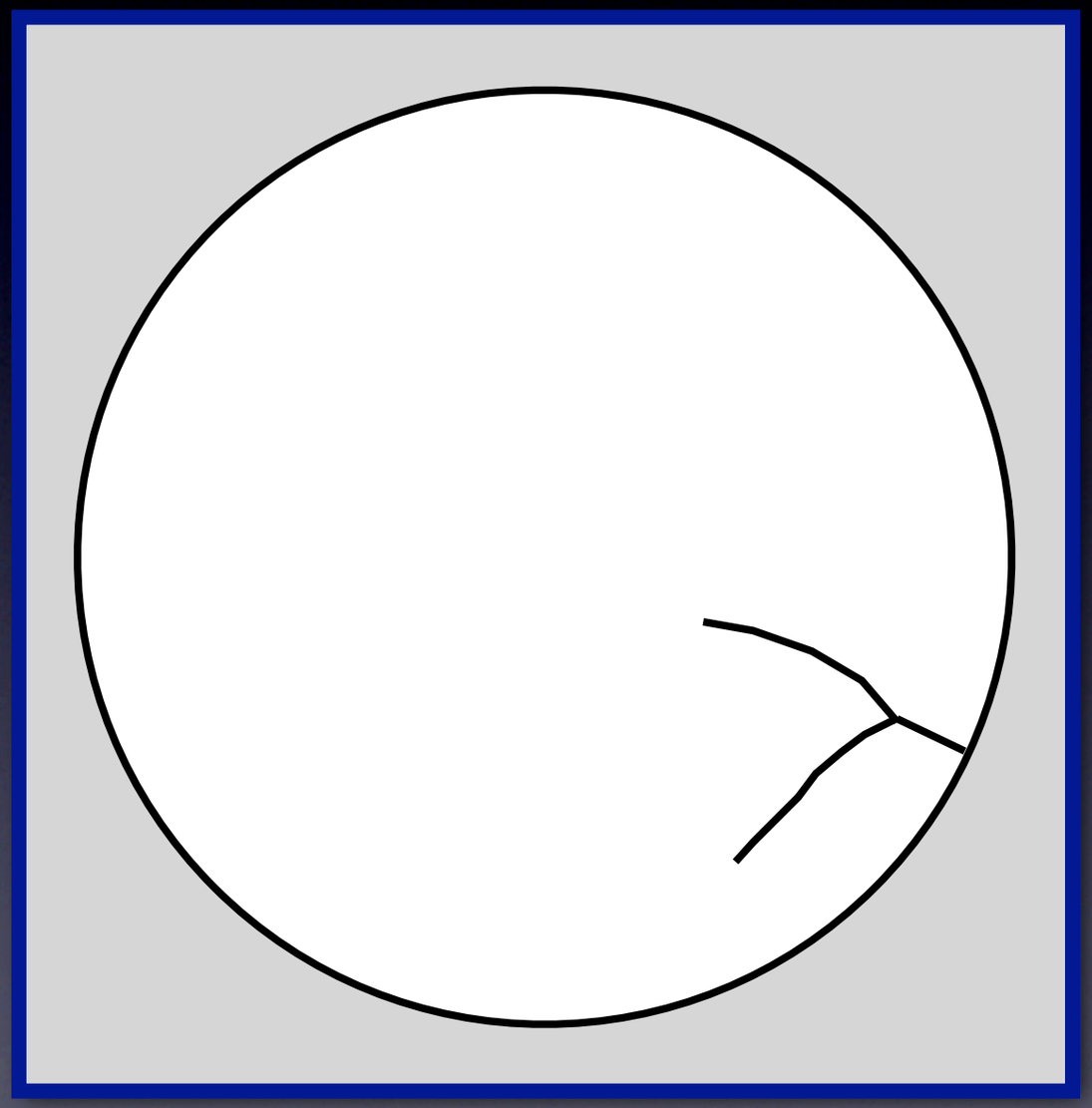

Incident Shock Wave
Reaction Zone
Transverse
Detonation
"Spin Head"





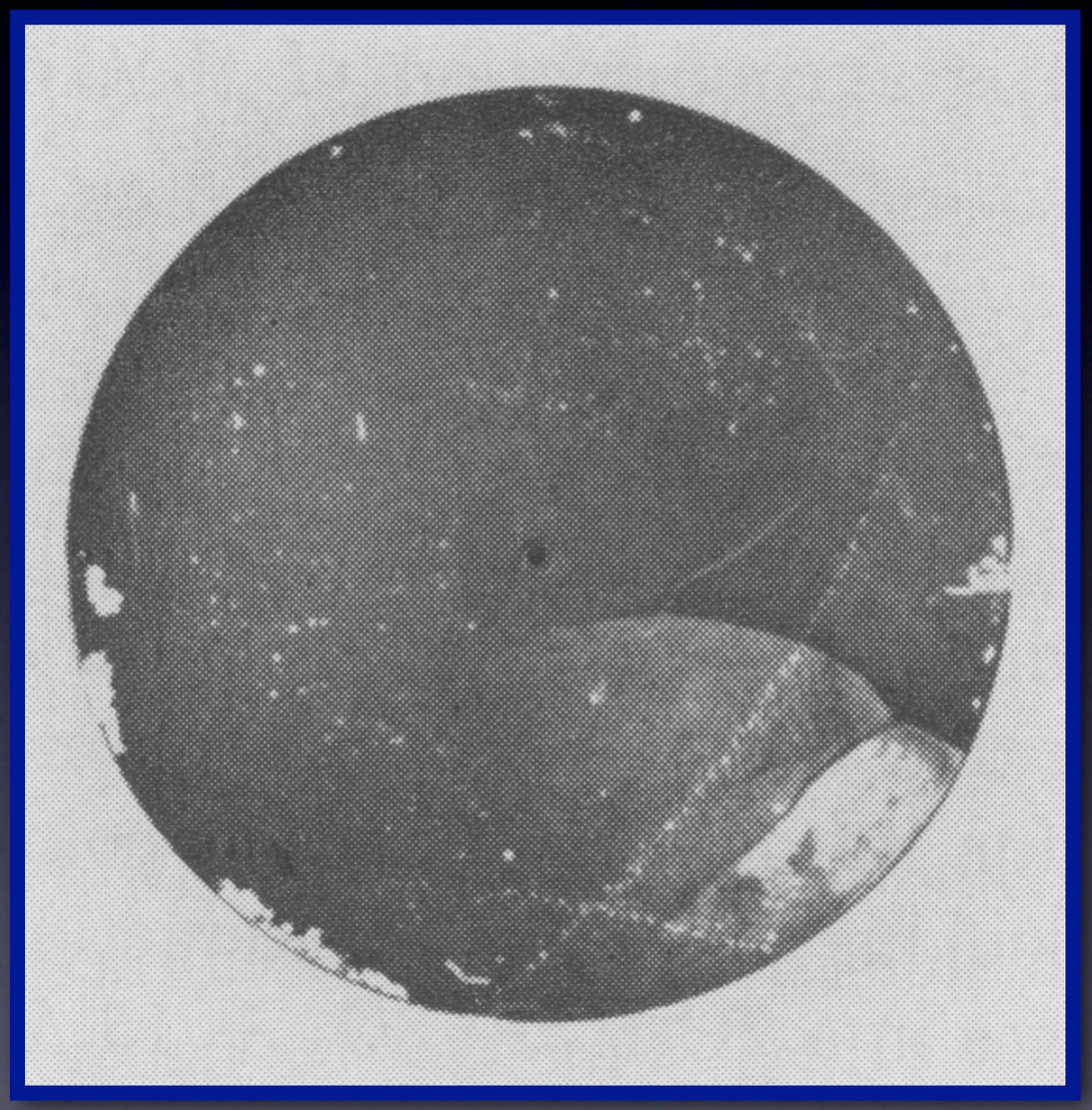
Spin Structure

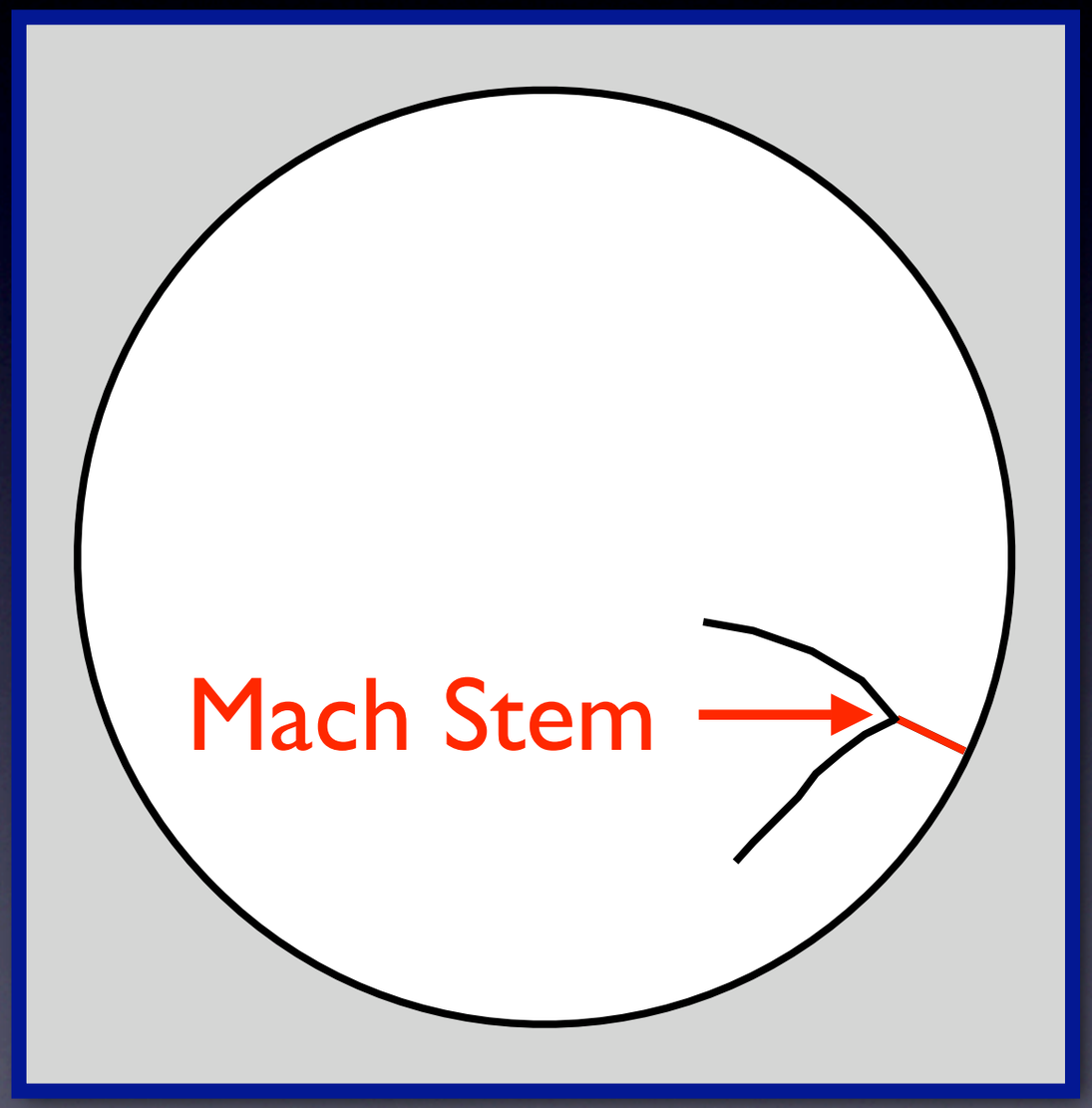




Source: R. E. Duff. *Physics of Fluids* 4:1432, 1961

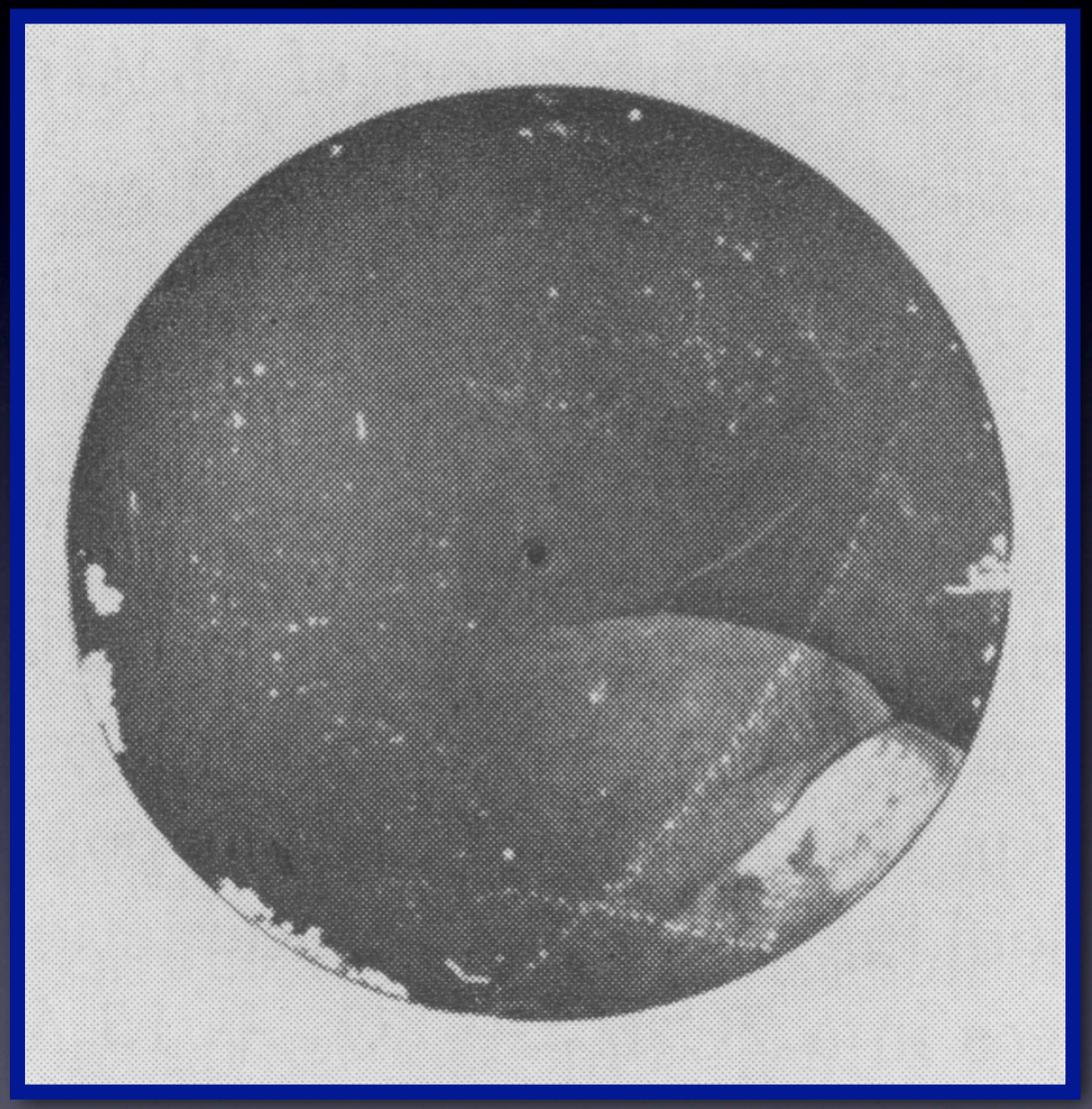
Spin Structure

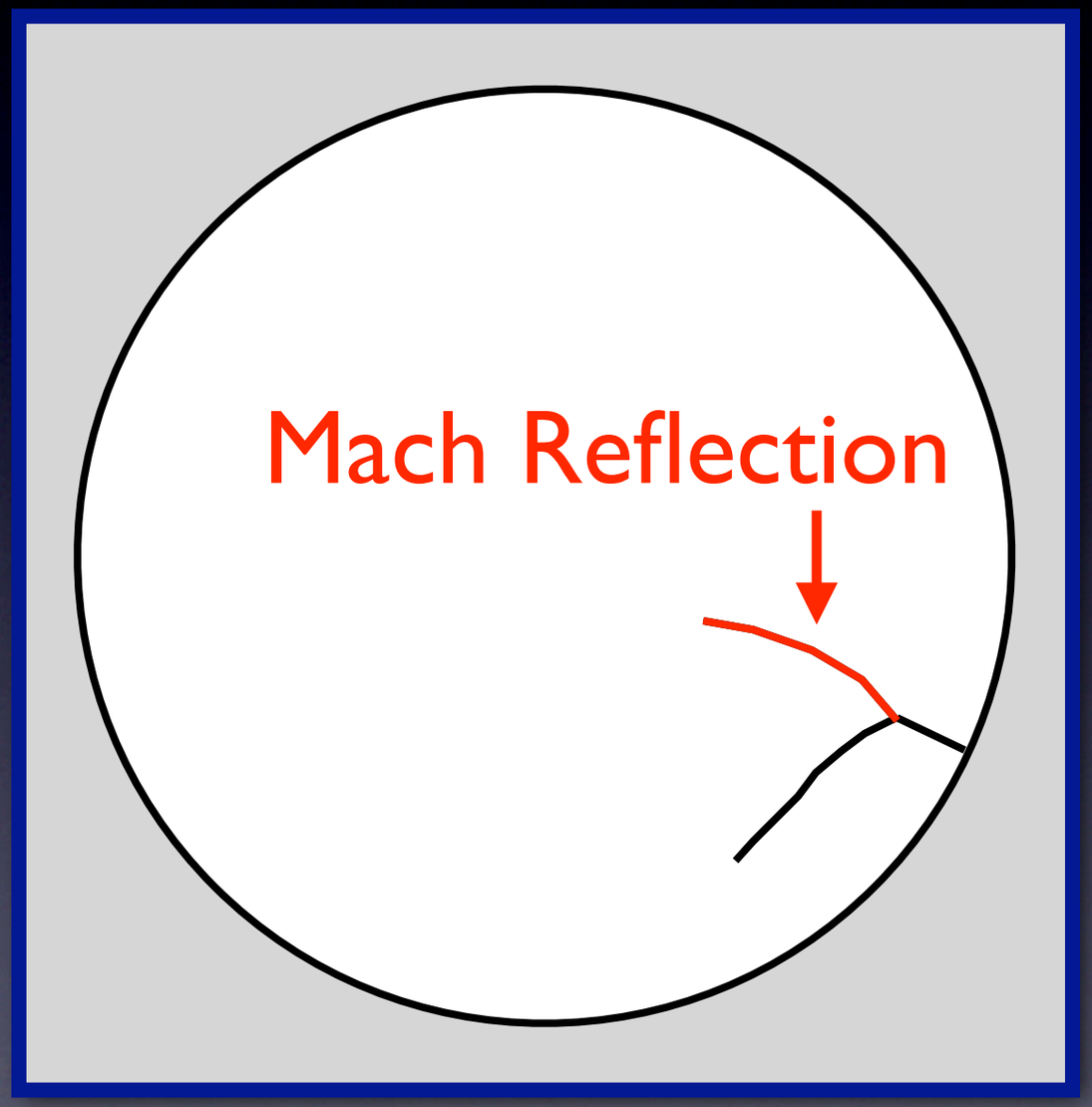




Source: R. E. Duff. *Physics of Fluids* 4:1432, 1961

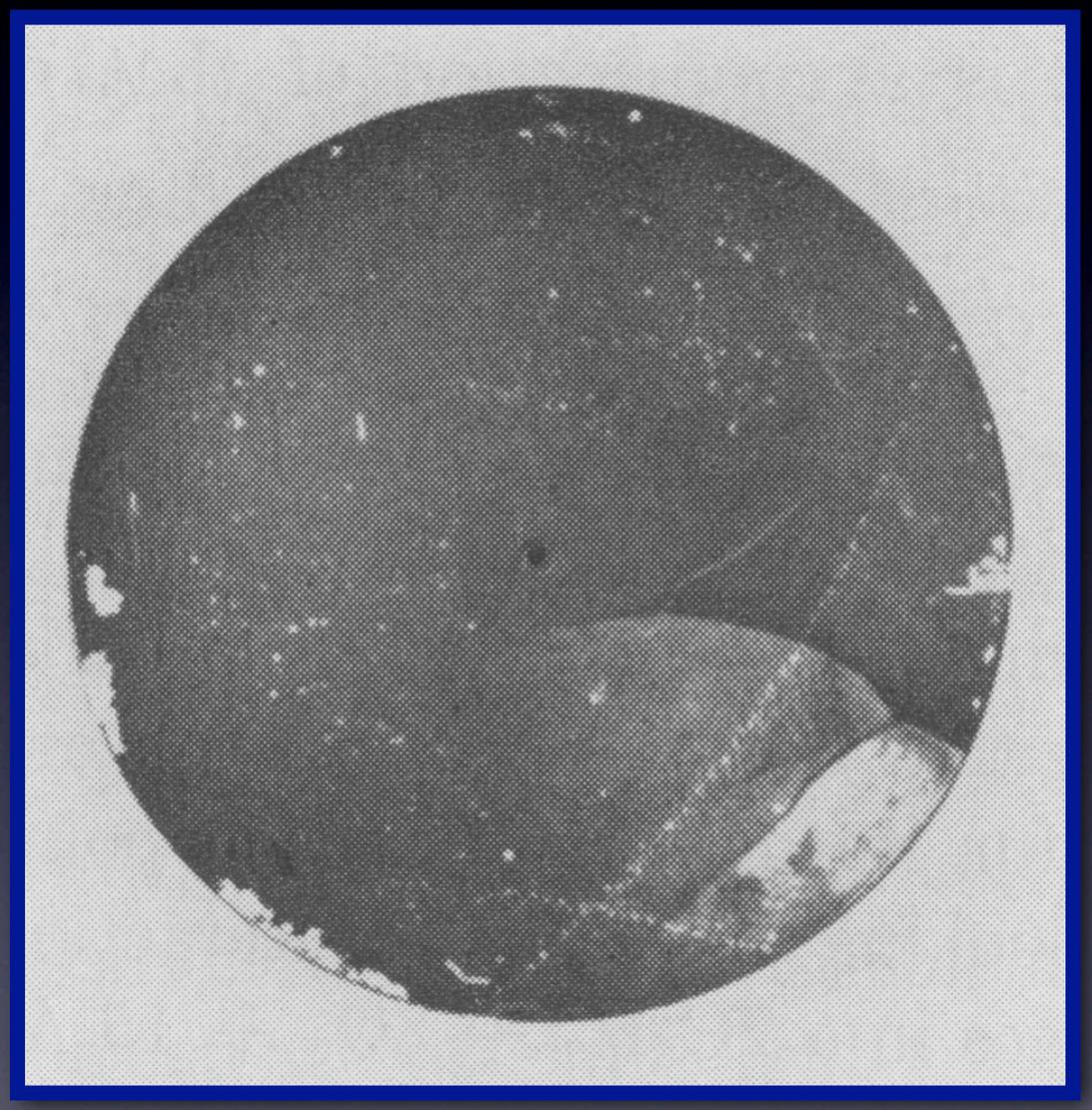
Spin Structure

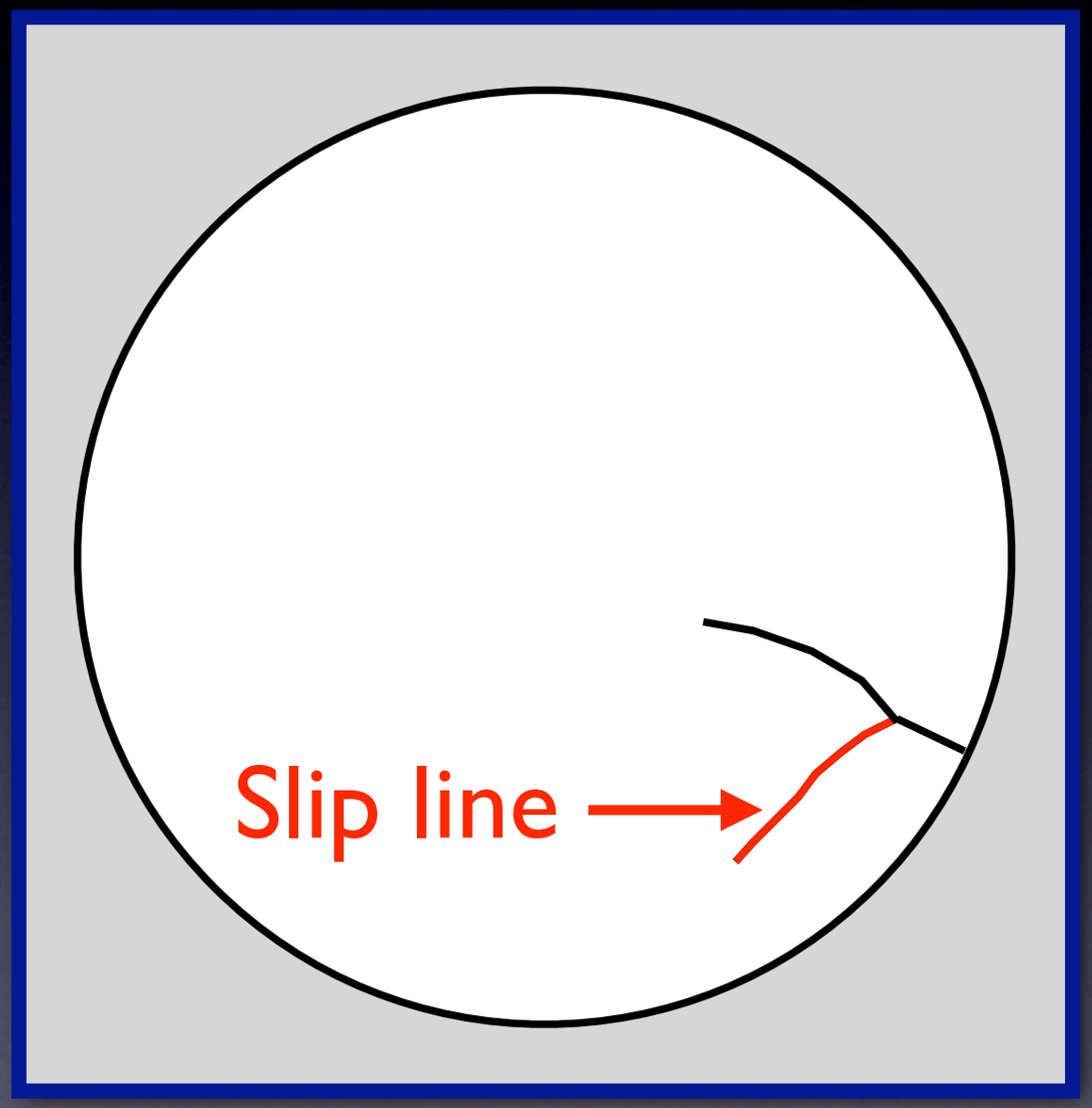




Source: R. E. Duff. *Physics of Fluids* 4:1432, 1961

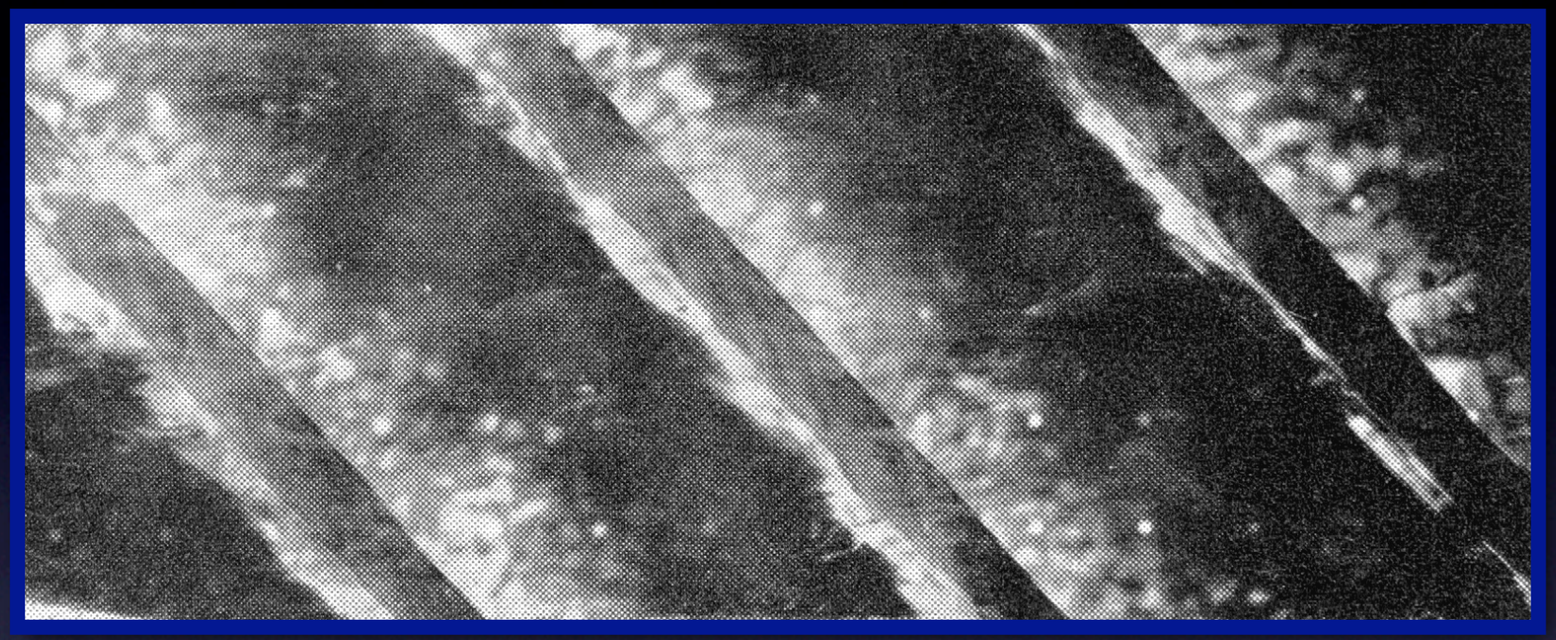
Spin Structure

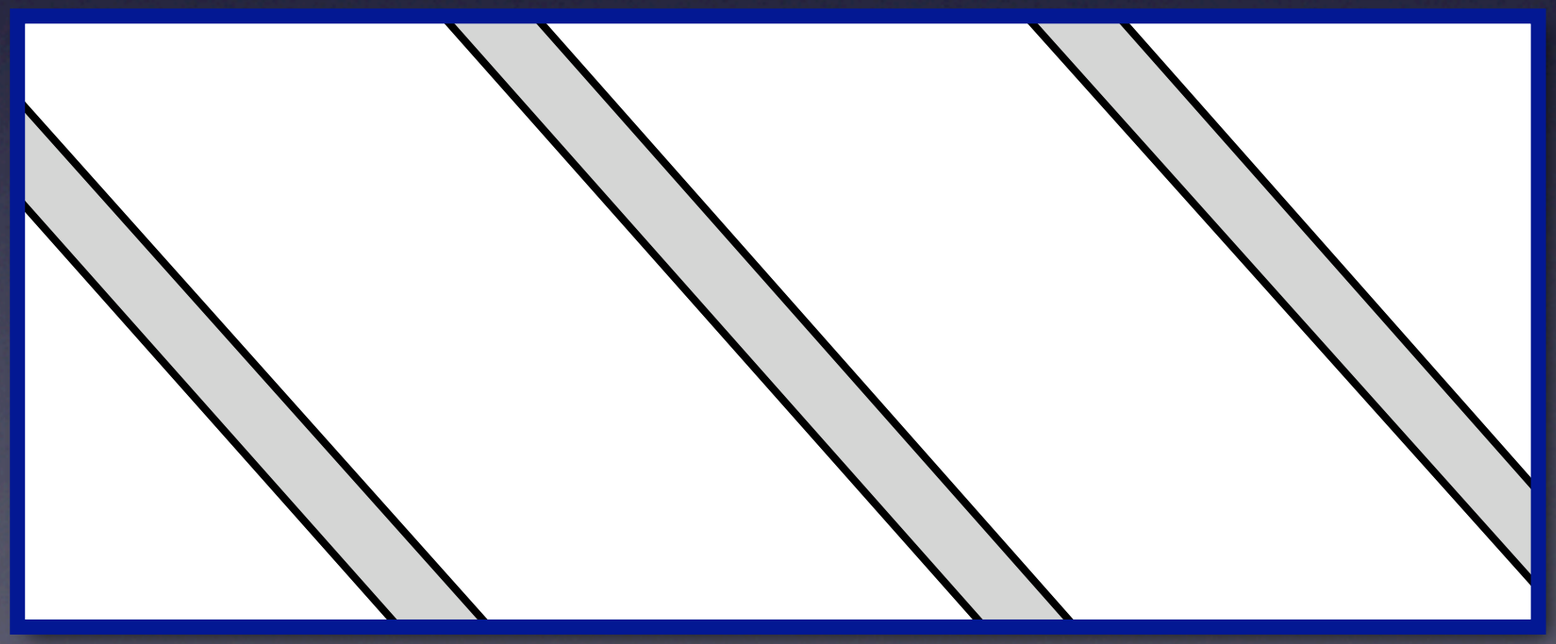




Source: R. E. Duff. *Physics of Fluids* 4:1432, 1961

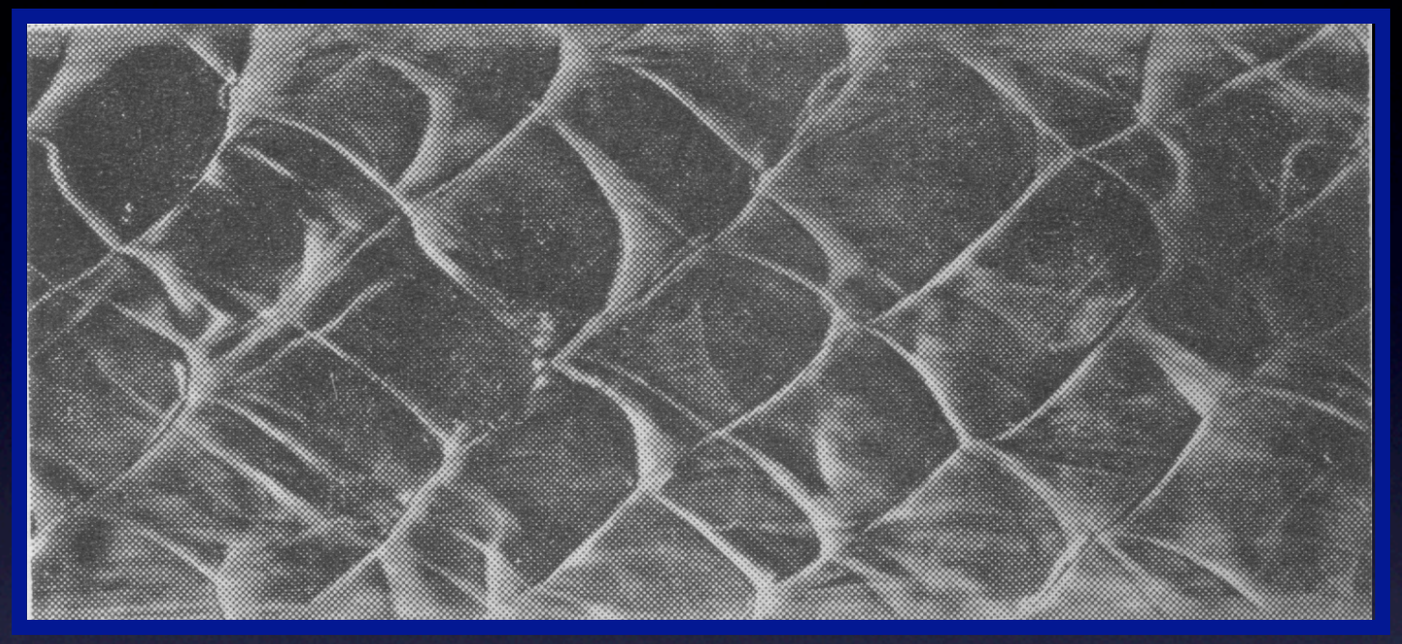
Helical Path - Single Spin

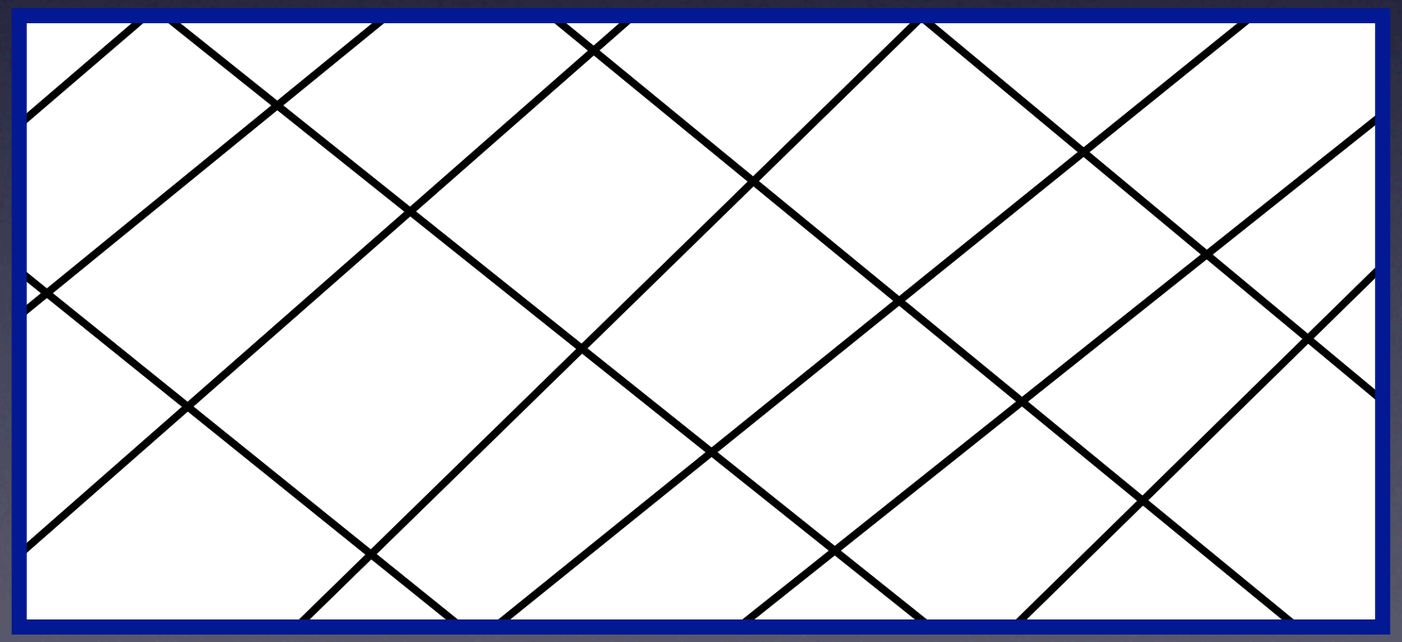




Source: G. Schott. *Physics of Fluids* 8:855, 1965

Complex Path - 2x3 Spin

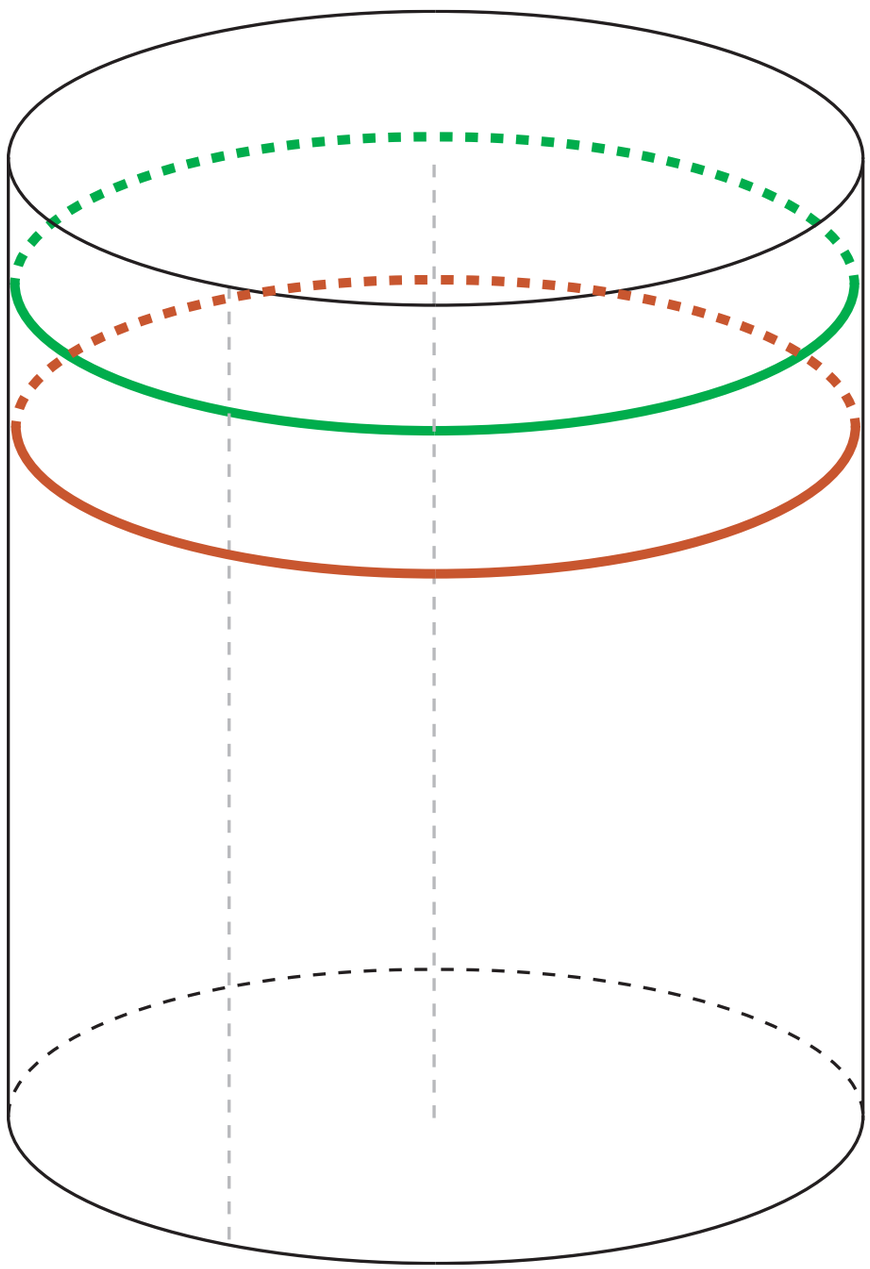


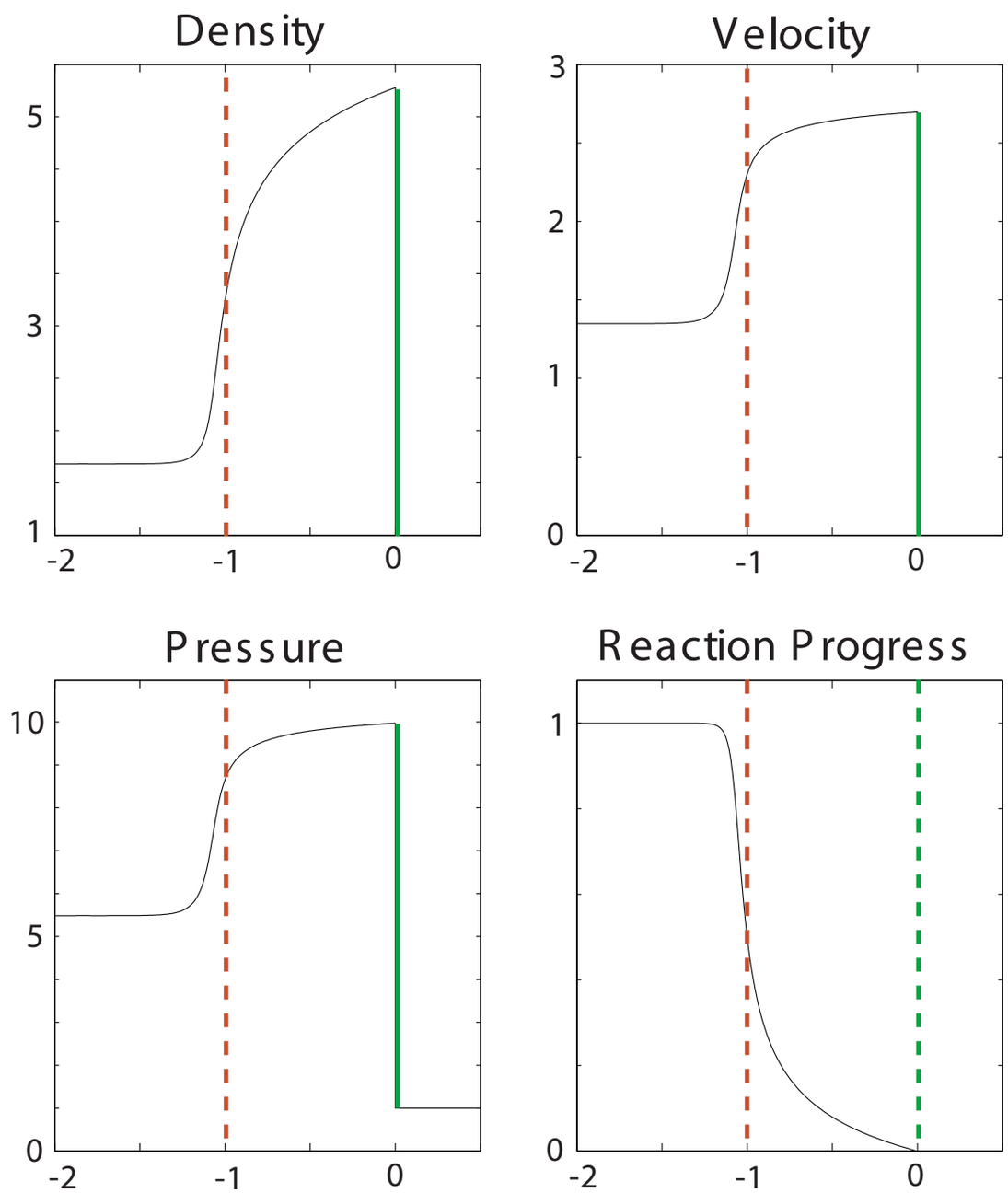


Source: R. E. Duff. *Physics of Fluids* 4:1430, 1961

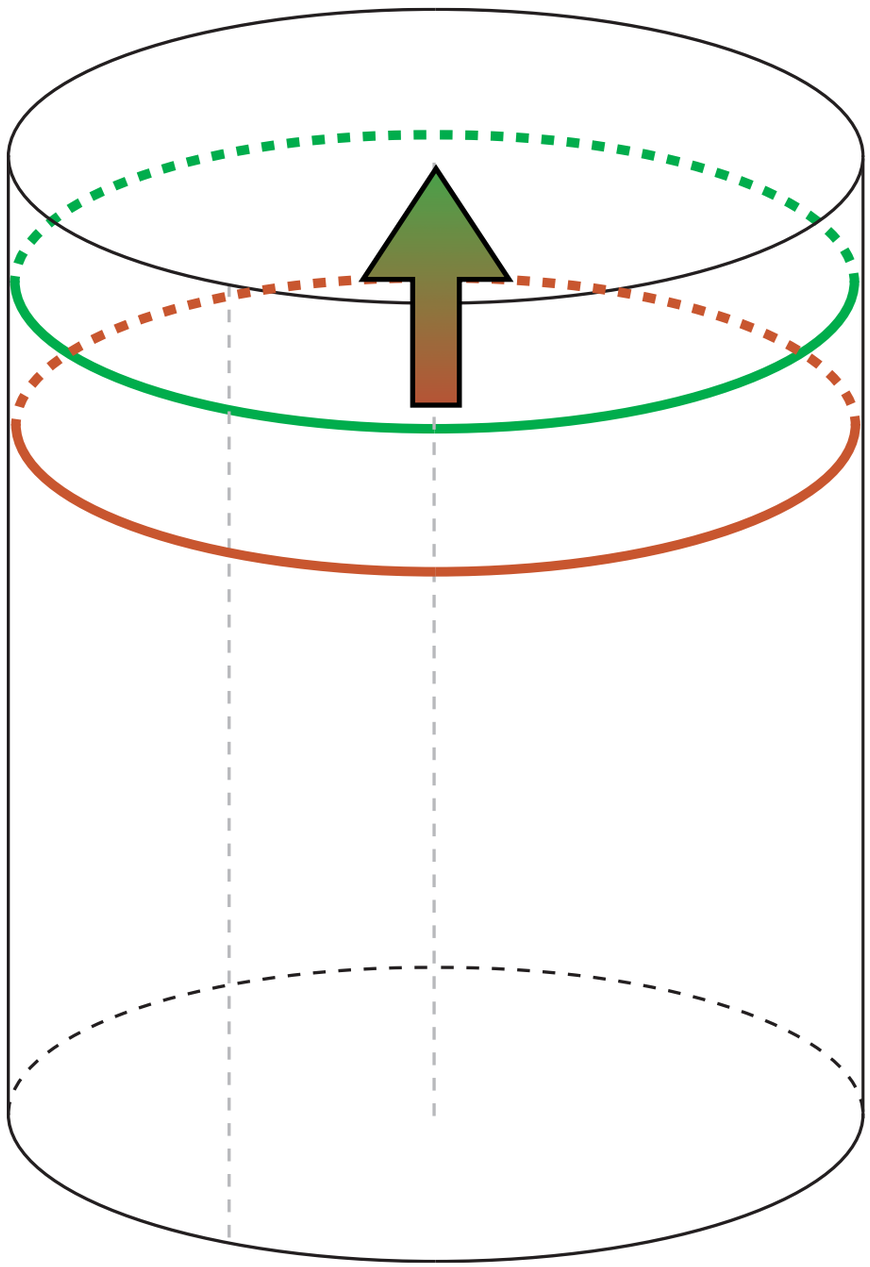
Linear Stability Analysis

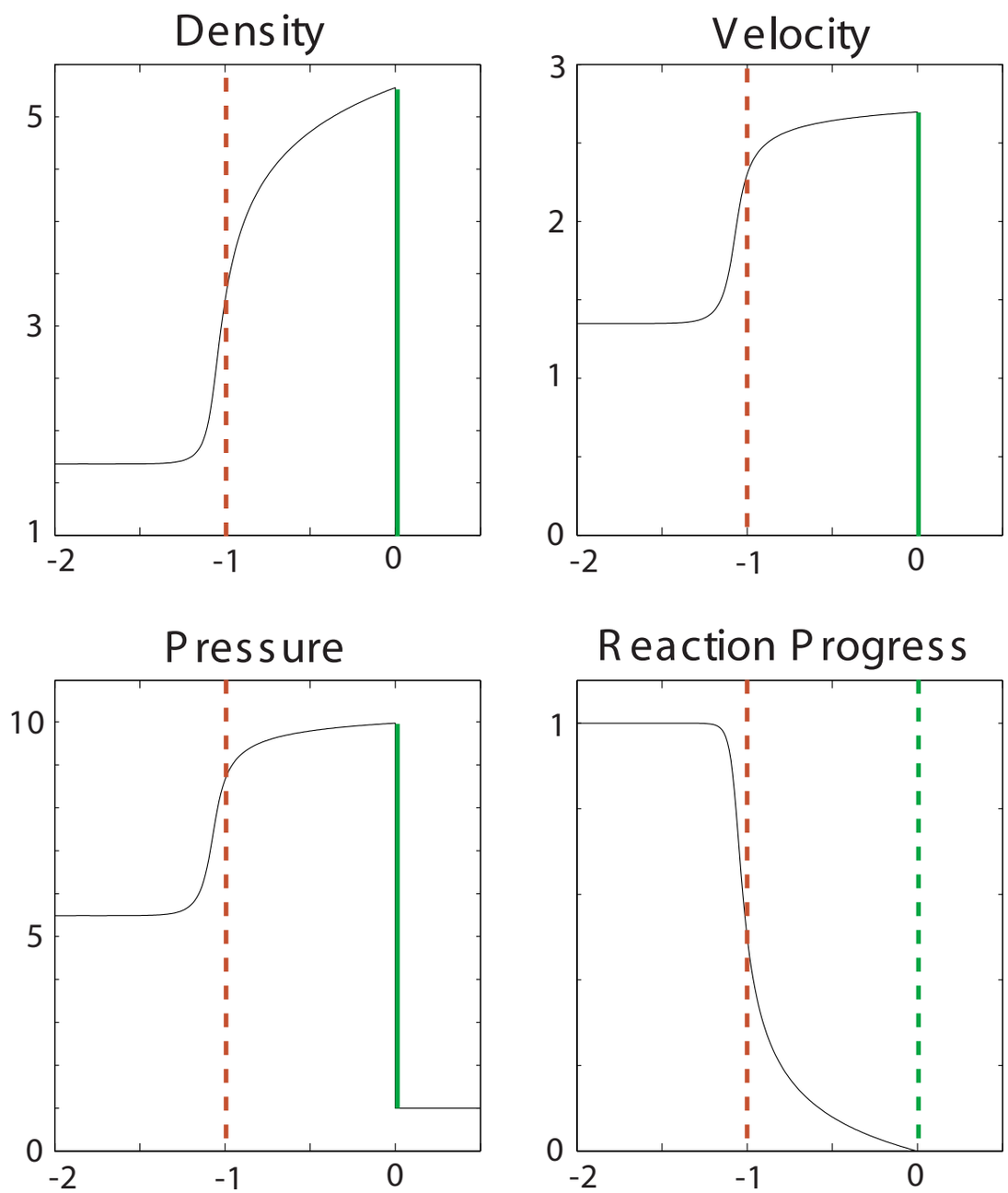
Steady, Planar Detonation





Steady, Planar Detonation





- Linear stability analysis can tell us:
  - Is the steady solution stable for a given set of parameters?
  - What types of instabilities exist?
    Axial oscillations, radial, spin, etc
  - How does a parameter affect stability?
- Stability analysis can guide simulation:
  Where are single spin modes predicted?

Governing Equations

Reactive Euler Equations

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla P = 0$$

$$\frac{\partial P}{\partial t} + \mathbf{u} \cdot \nabla P + \rho c^2 \nabla \cdot \mathbf{u} = \rho c^2 \sigma \omega$$

$$\frac{\partial \lambda}{\partial t} + \mathbf{u} \cdot \nabla \lambda = \omega$$

Rankine-Hugoniot Relations

$$-\rho_0 \mathbf{D} \cdot \hat{\mathbf{n}} = \rho \left(\mathbf{u}_S - \mathbf{D} \right) \cdot \hat{\mathbf{n}} = \mathcal{M}$$

$$P_S - P_0 = \mathcal{M}^2 \left(v_0 - v_S \right)$$

$$h_0 + \frac{\mathbf{D}^2}{2} = h_S + \frac{\left(\mathbf{u}_S - \mathbf{D} \right)^2}{2}$$

Reaction Rate Law

$$\omega = k \left(1 - \lambda \right)^{\nu} \exp\left(-\frac{E \rho}{P} \right)$$

Equation of State

$$e \left(\rho, P; \lambda \right) = \frac{P}{\rho \left(\gamma - 1 \right)} - \lambda Q$$

Expansion

- Expand governing equations in normal modes

$$\mathbf{q} = \mathbf{q}^* (z) + \epsilon \mathbf{q}' (z, r) \exp (\alpha t + in\theta)$$

- Resulting equations are separable in *r* and z

$$\mathbf{q}' (z, r) = \begin{bmatrix} \rho' (z) J_n (ikr) \\ u_r' (z) \dfrac{\partial J_n (ikr)}{\partial r} \\ u_\theta' (z) \dfrac{J_n (ikr)}{r} \\ u_z' (z) J_n (ikr) \\ P' (z) J_n (ikr) \\ \lambda' (z) J_n (ikr) \end{bmatrix}$$

Ref: Kasimov & Stewart. *J Fluid Mech* 466:179-203, 2002

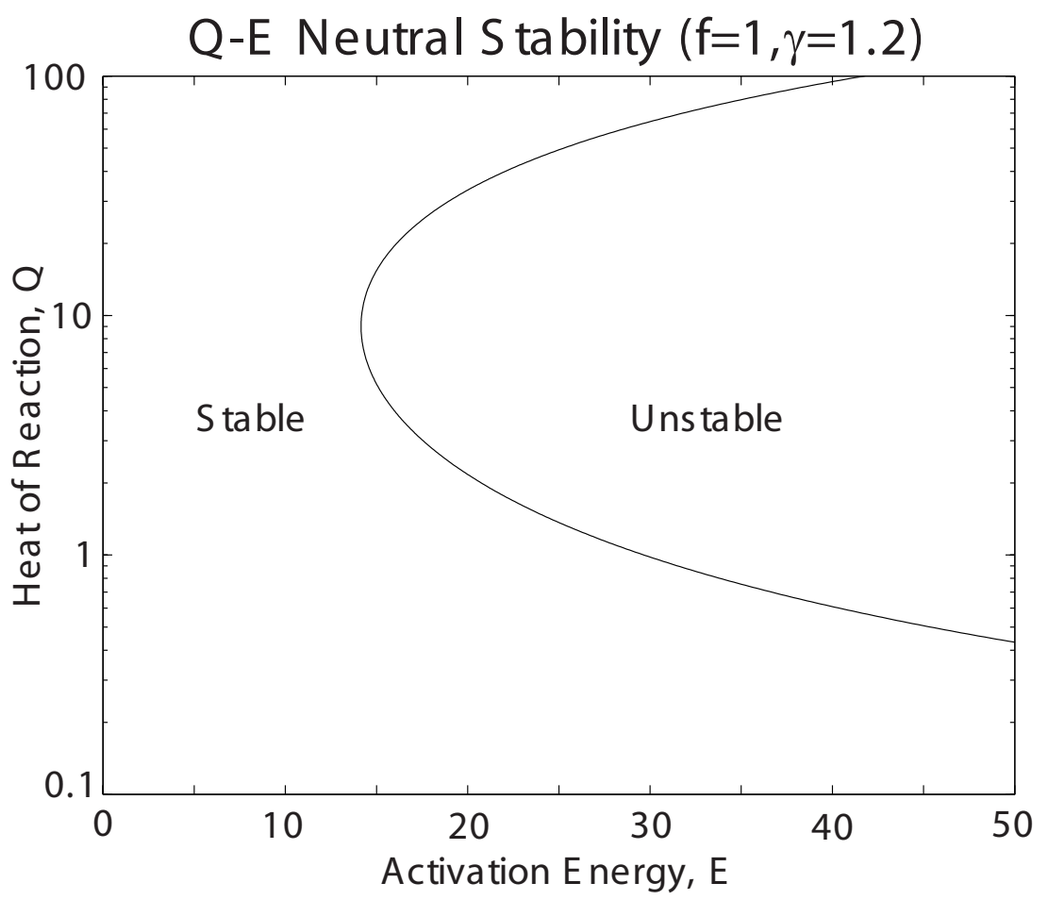
Result of Derivation

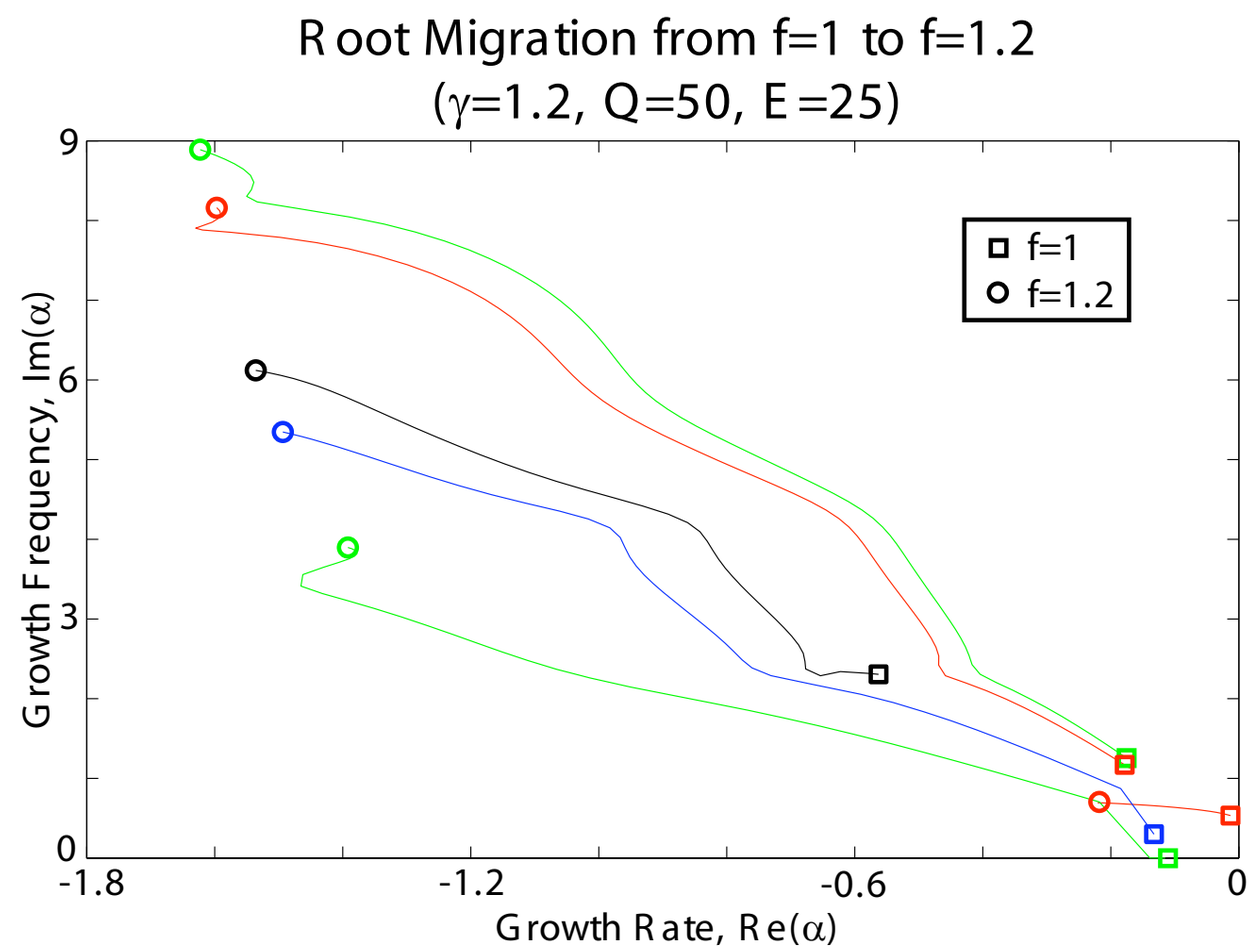
- System of 5 complex-valued ODE's

$$\mathbf{A}^* \frac{\mathrm{d}\mathbf{q}'}{\mathrm{d}z} + \mathbf{C}^* \mathbf{q}' + \alpha \mathbf{q}' + \mathbf{b}^* = 0$$

- Coefficients are functions of steady solution
- Initial conditions from linearization of the Rankine-Hugoniot conditions
- Closure condition specifies forward characteristic in far-field has zero strength
- Solve for eigenvalues using shooting method

Results of Stability Analysis

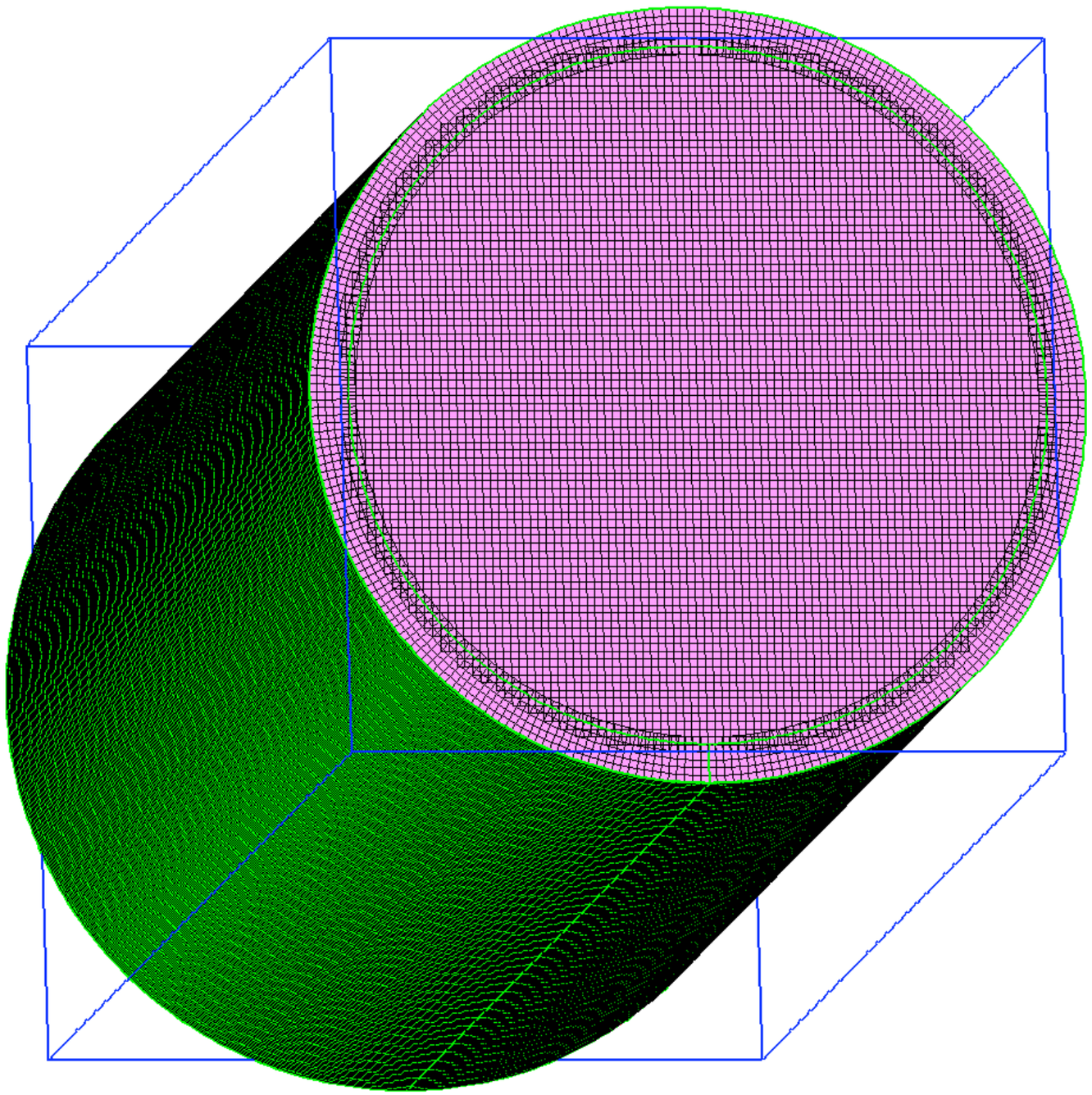




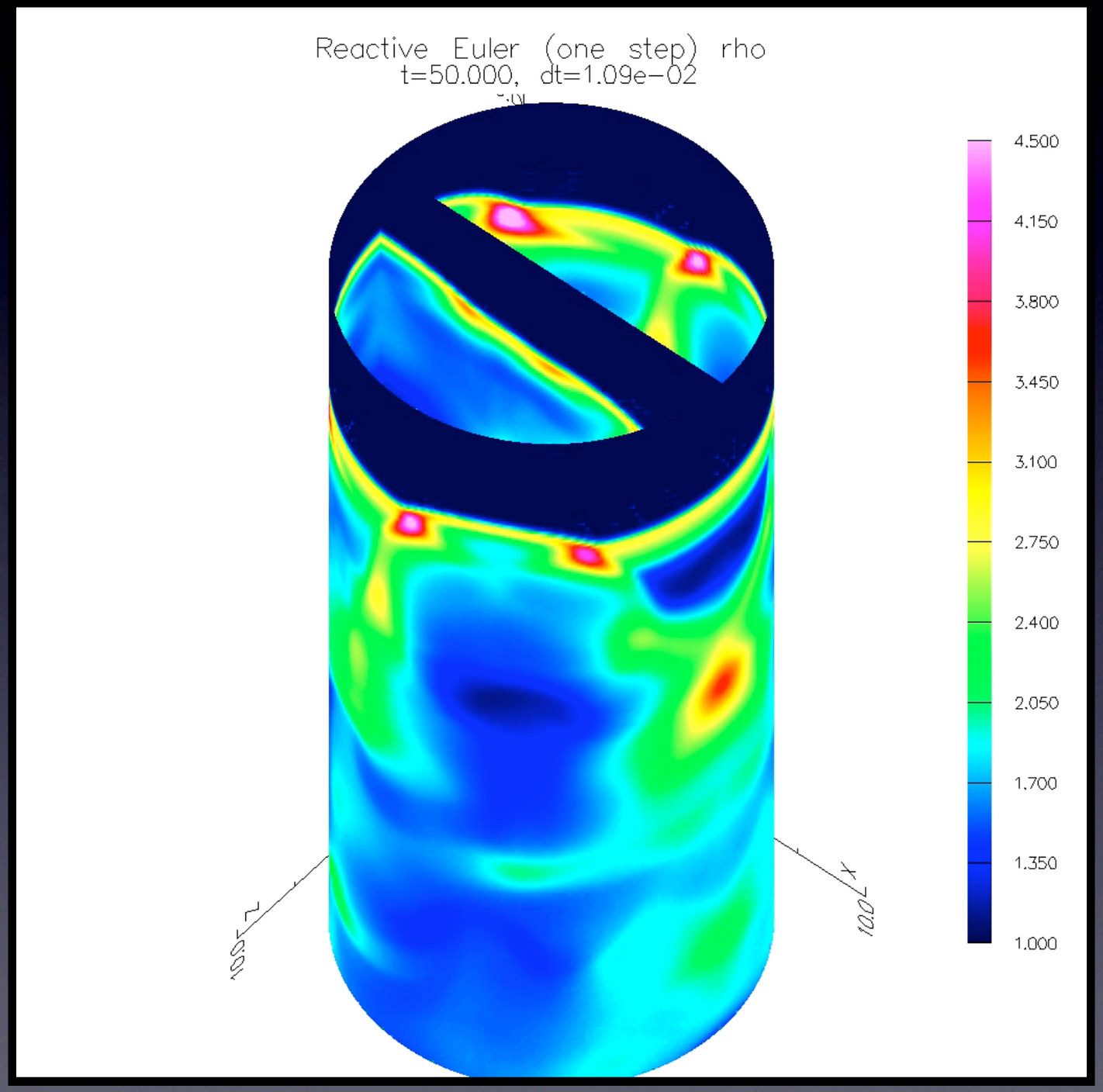
Numerical Simulation

- Numerically solve the reactive Euler equations
- Goals:
  - Verify results of linear stability analysis
  - Observe single and multi-headed spin

Overlapping Grid



Simulation Results



Conclusion

- Extending linear stability analysis of detonation to cylindrical geometry
- Verifying stability analysis through simulation
- Using stability analysis and simulation to understand spin detonation

Acknowledgments

- Aslan Kasimov, MIT
- Bill Henshaw & Kyle Chand, LLNL
- Krell Institute
- DOE CSGF program