The Magnetothermal Instability and its Application to Clusters of Galaxies

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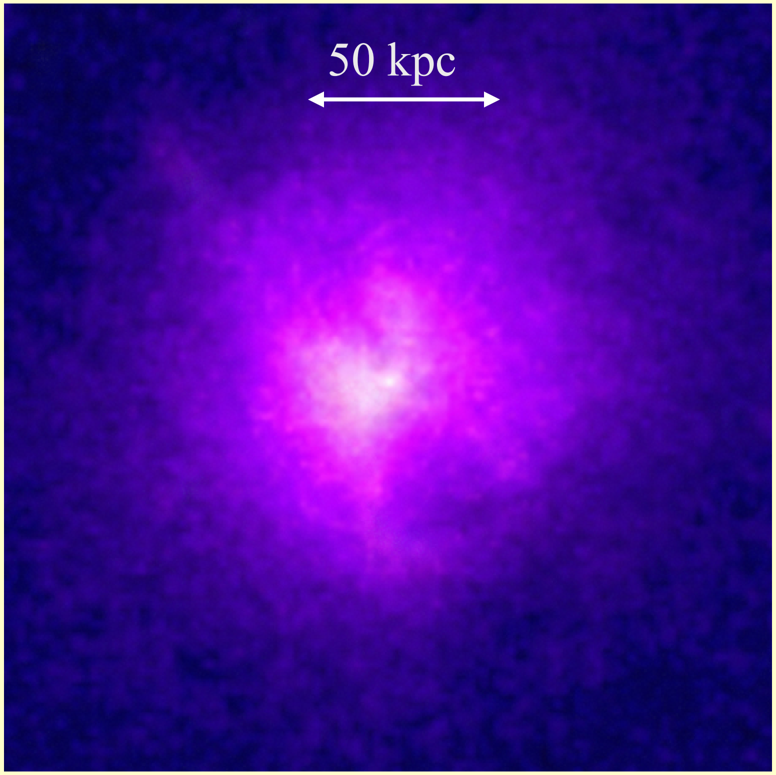
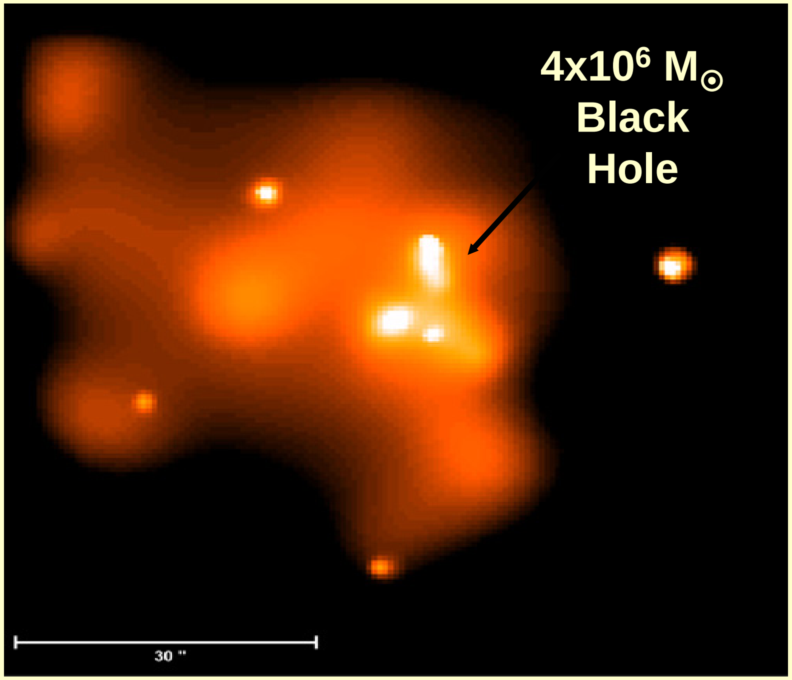
June 19, 2007

Motivation





Hydra A Cluster (Chandra)

T ~ 4.5 keV   n ~ 10^{-3}-10^{-4}

$$\lambda_{mfp} \sim 0.05 R_V >> \rho$$

Collisionless Transport

Sgr A*

$T \sim 1$ keV, $n \sim 10$ cm^{-3}

$$R_s \sim 10^{12} \text{ cm}$$

$$\lambda_{mfp} \sim 10^{17} \text{ cm} >> R_S$$

Motivating example suggested by E. Quataert

A Computational Science Success Story—Talk Outline

•**<u>Idea</u>**:

Stability, Instability, and "Backward" Transport in Stratified Fluids, Steve Balbus, 2000.

Physics of the Magnetothermal Instability (MTI).

•**<u>Algorithm:</u>**

Athena: State of the art, massively parallel MHD solver.

Anisotropic thermal conduction module.

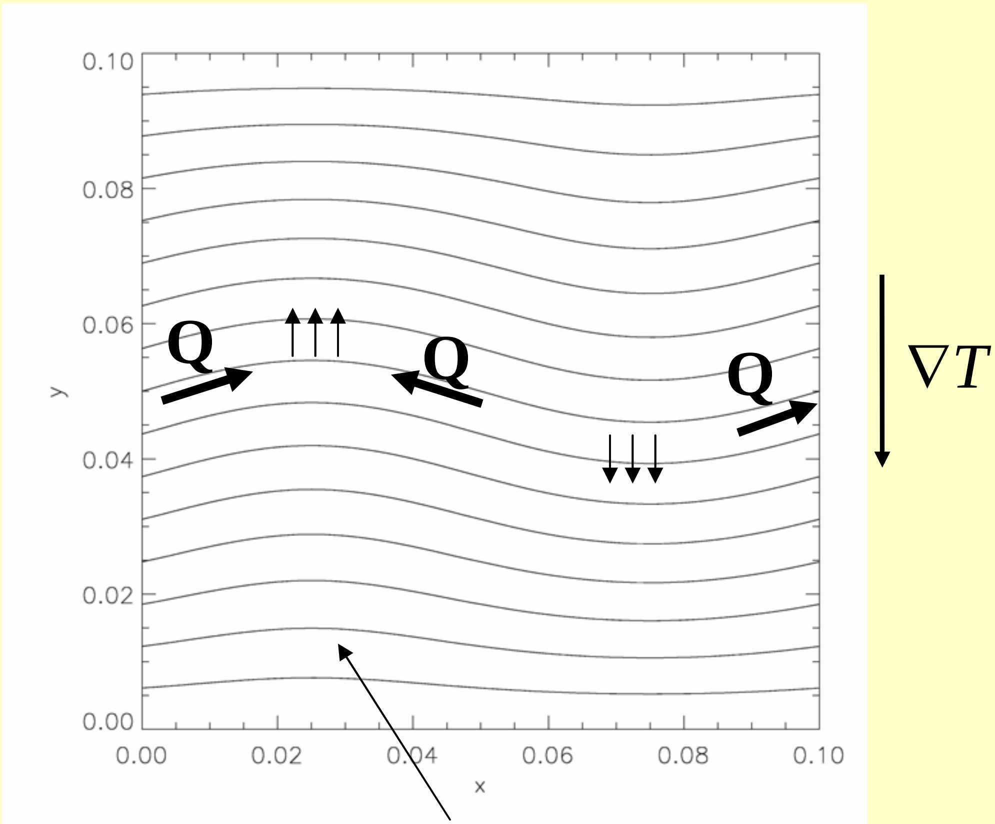
•**<u>Verification and Exploration</u>**

Verification of linear growth rates.

Exploration of nonlinear consequences.

•**Application to Galaxy Clusters**

Idea: Magnetothermal Instability Qualitative Mechanism



Magnetic Field Lines

Anisotropic heat flux given by Braginskii conductivity.

Convective Stability in a Gravitational Field

- Clasically: Schwarzschild Criterion

$$\frac{dS}{dz} > 0$$

- Long MFP: Balbus Criterion

$$\frac{dT}{dz} > 0$$

New Stability Criterion!

Algorithm: MHD with Athena

$$\frac{\partial(\rho)}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + \left(p + \frac{B^2}{8\pi}\right)\mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{4\pi}\right] = \rho \mathbf{g}, \quad (2)$$

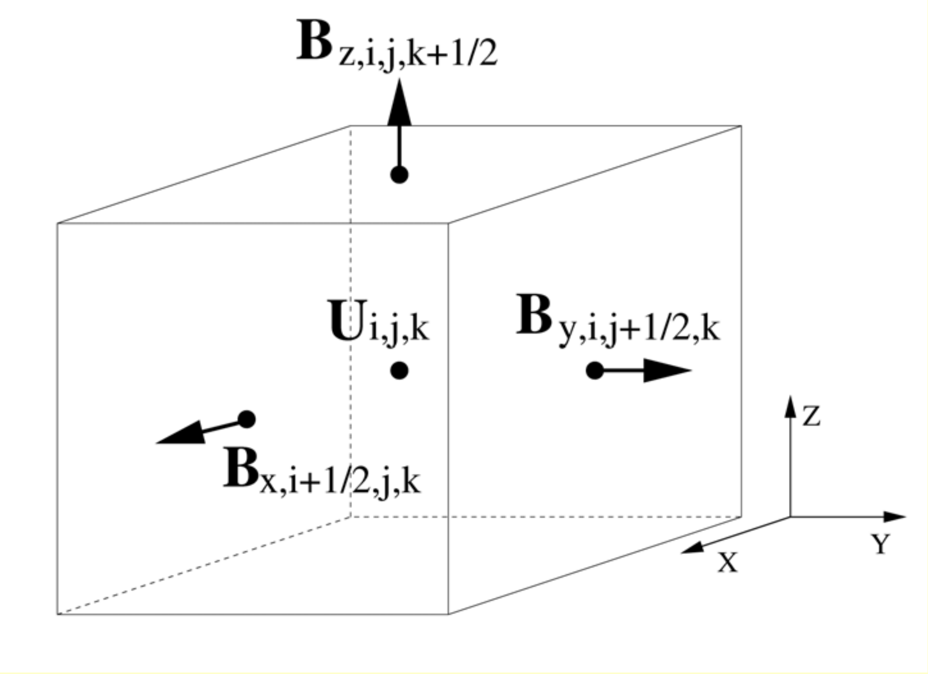
$$\frac{\partial E}{\partial t} + \nabla \cdot \left[\mathbf{v}\left(E + p + \frac{B^2}{8\pi}\right) - \frac{\mathbf{B}(\mathbf{B}\cdot\mathbf{v})}{4\pi}\right] = \rho \mathbf{g}\cdot\mathbf{v} - \nabla \cdot \mathbf{Q}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \quad (4)$$

Athena: Higher order Godunov Scheme

•Constrained Transport for preserving divergence free.

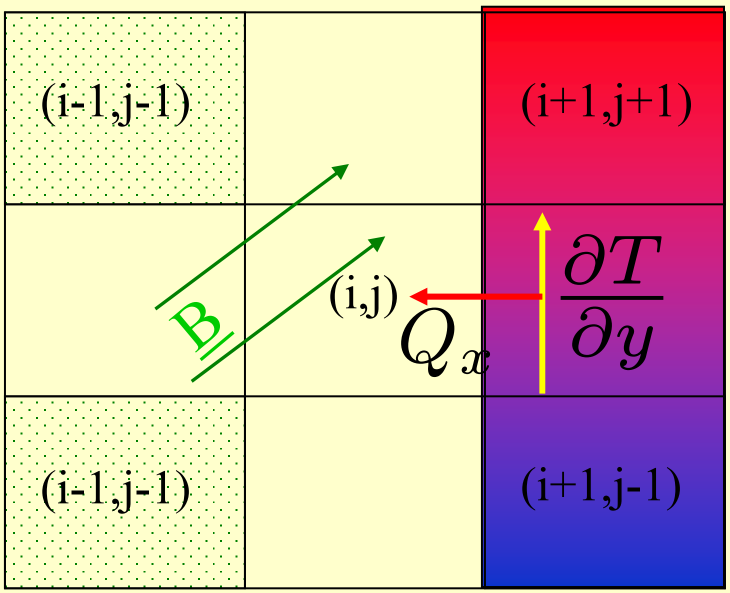
•Unsplit CTU integrator



Algorithm: Heat Conduction

$$\frac{3}{2}\frac{P\,\mathrm{d}\ln P\rho^{-5/3}}{\mathrm{d}t} = -\nabla\cdot Q = \nabla\cdot\left[\hat{b}\left(\chi_C\hat{b}\cdot\nabla T\right)\right]$$

$$Q_x = -\chi_C\left(\hat{b}_x^2\frac{\partial T}{\partial x} + \hat{b}_x\hat{b}_y\frac{\partial T}{\partial y}\right)$$



Verification

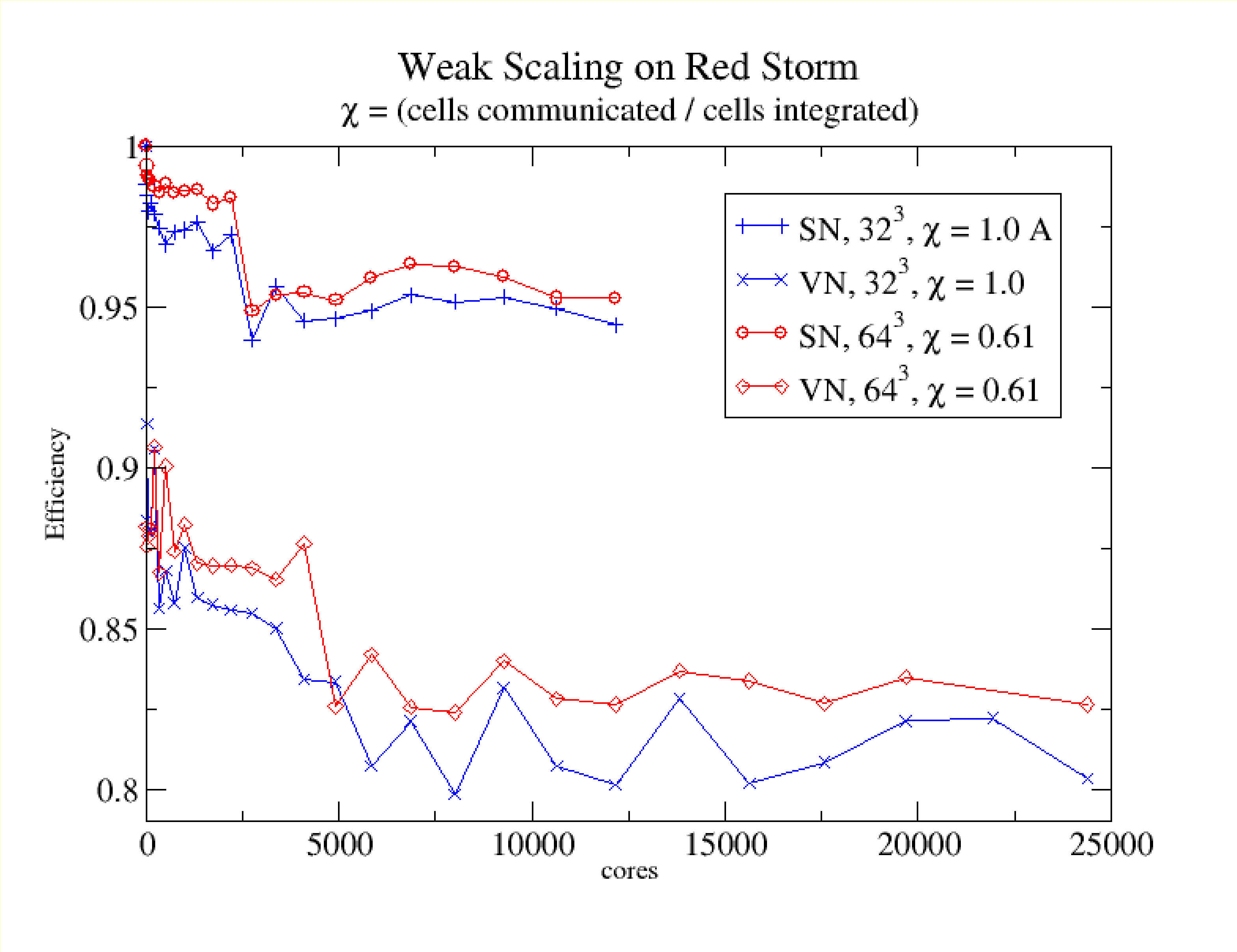
•Gaussian Diffusion: 2nd order accurate.

•Circular Field Lines.

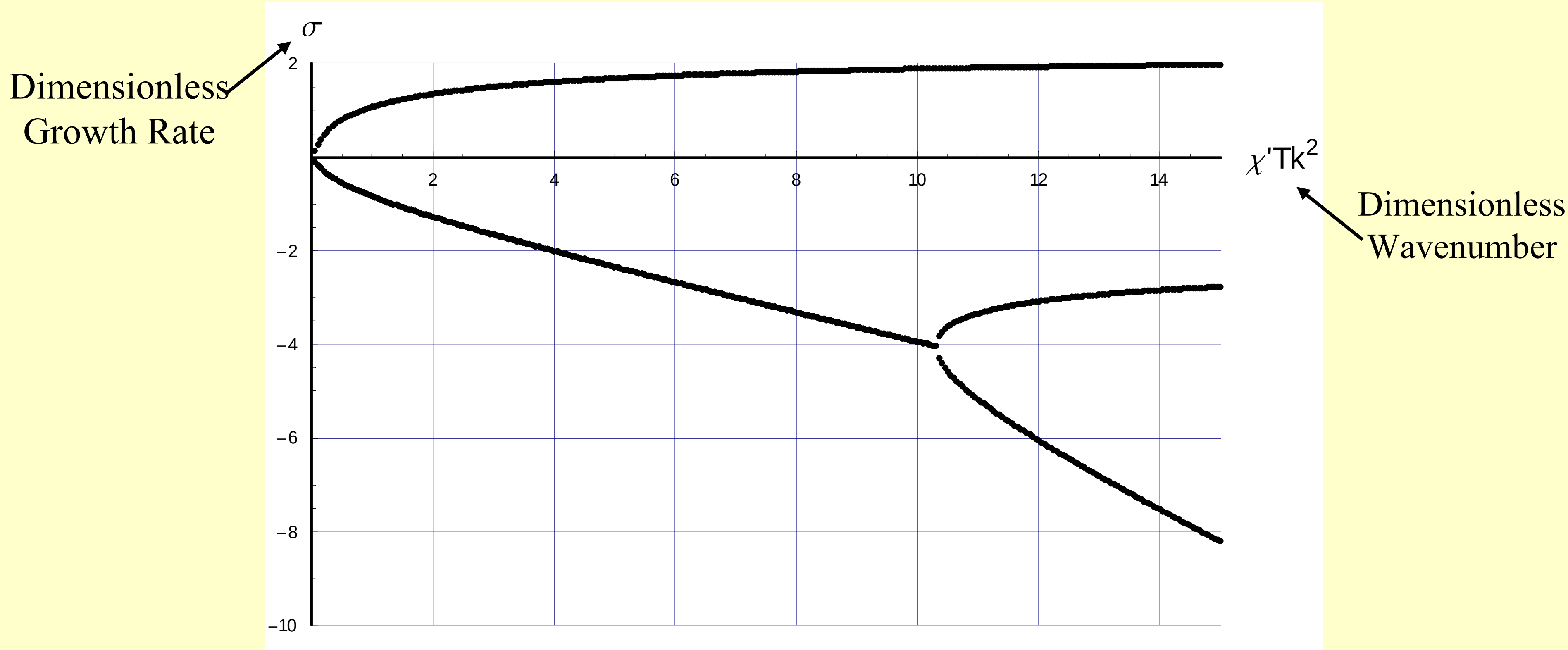
$$\frac{\chi_\perp}{\chi_\parallel} \le 10^{-4}$$

Implemented through sub-cycling diffusion routine.

Algorithm: Performance



Dispersion Relation



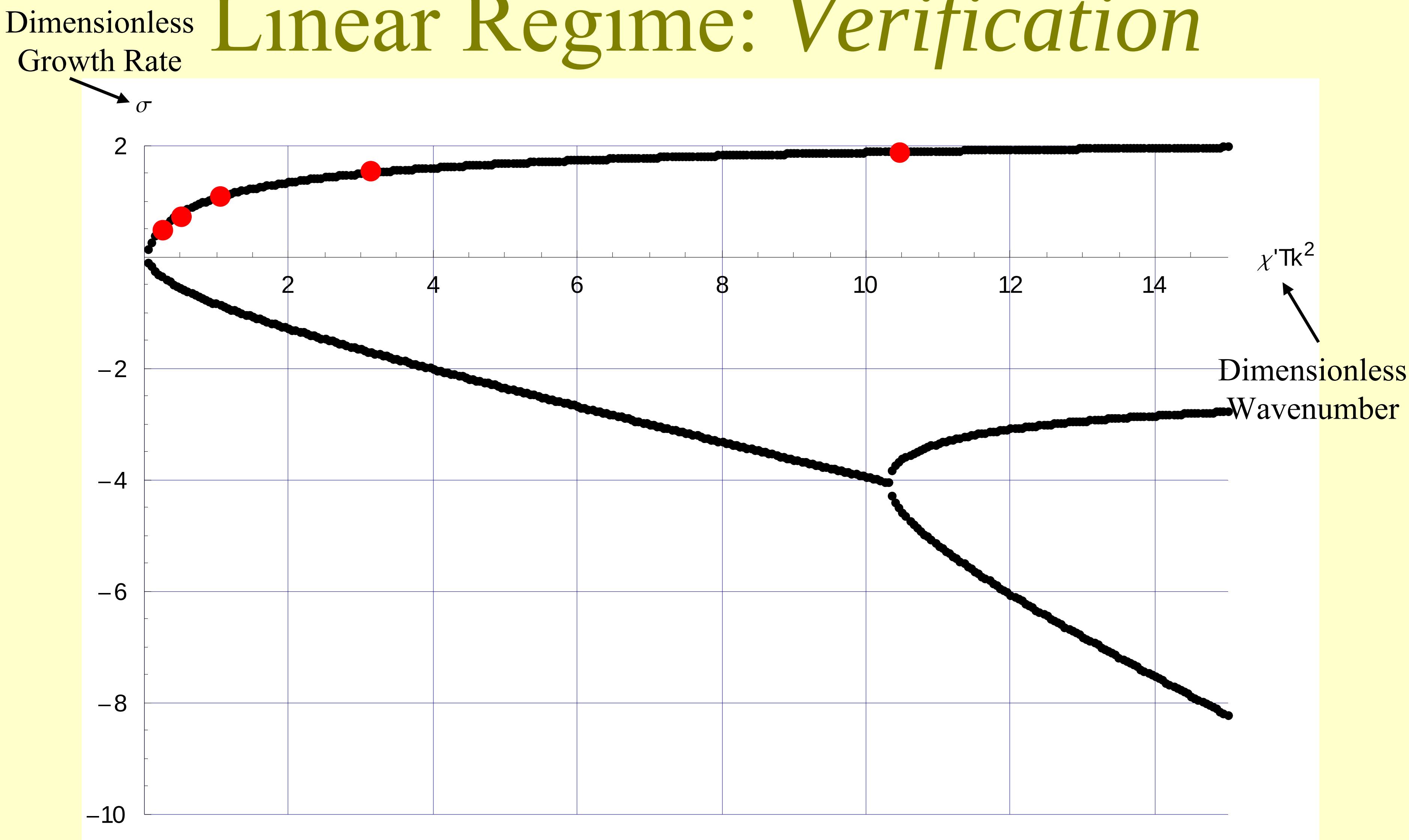
Weak Field Limit

$$\sigma^3 + \frac{\sigma^2}{\gamma}\chi' T k^2 + N^2\sigma - \frac{k^2}{\gamma}\frac{\chi_C'}{\rho}\frac{\partial P}{\partial z}\frac{\partial T}{\partial z} = 0$$

Instability Criterion

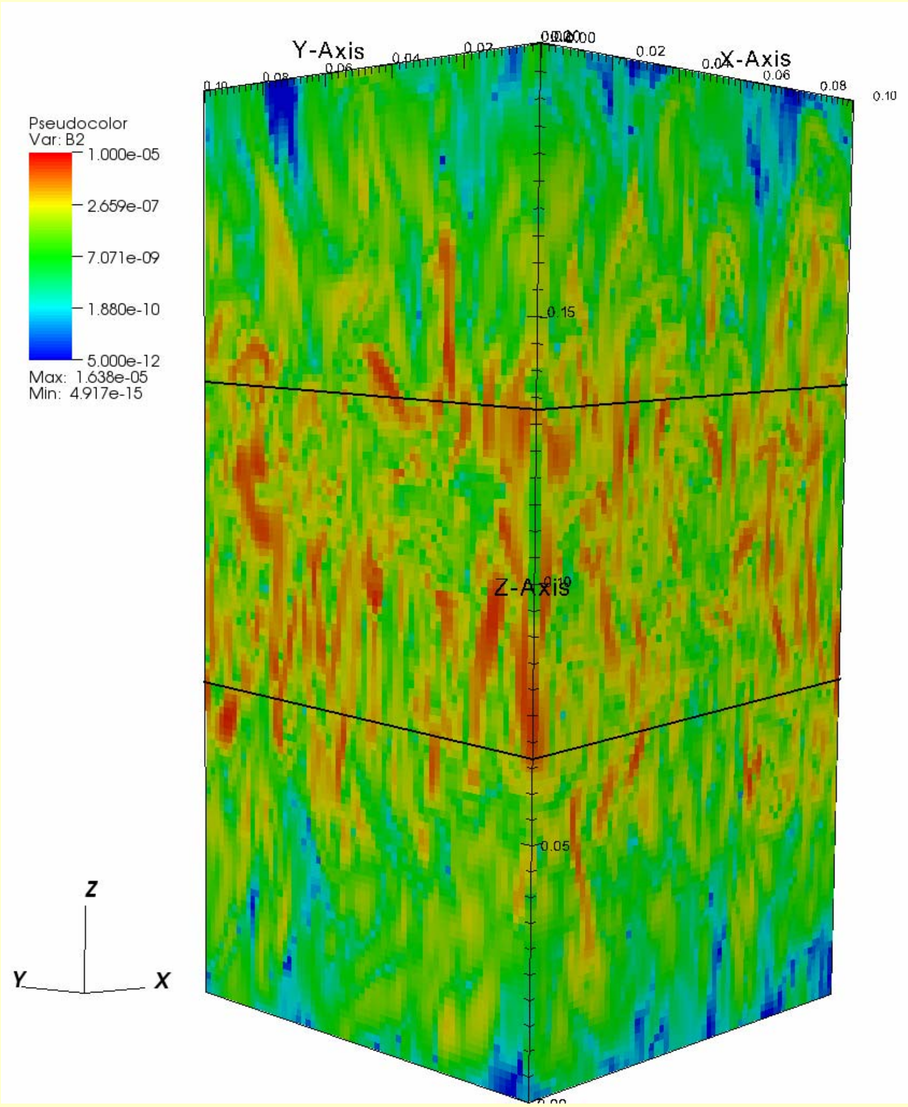
$$k^2 v_A^2 - \frac{\chi_C'}{\rho\chi'}\frac{\partial P}{\partial z}\frac{\partial \ln T}{\partial z} < 0 \xrightarrow{\lim k\to 0} \frac{\partial P}{\partial z}\frac{\partial \ln T}{\partial z} > 0$$

Linear Regime: *Verification*



~3% error

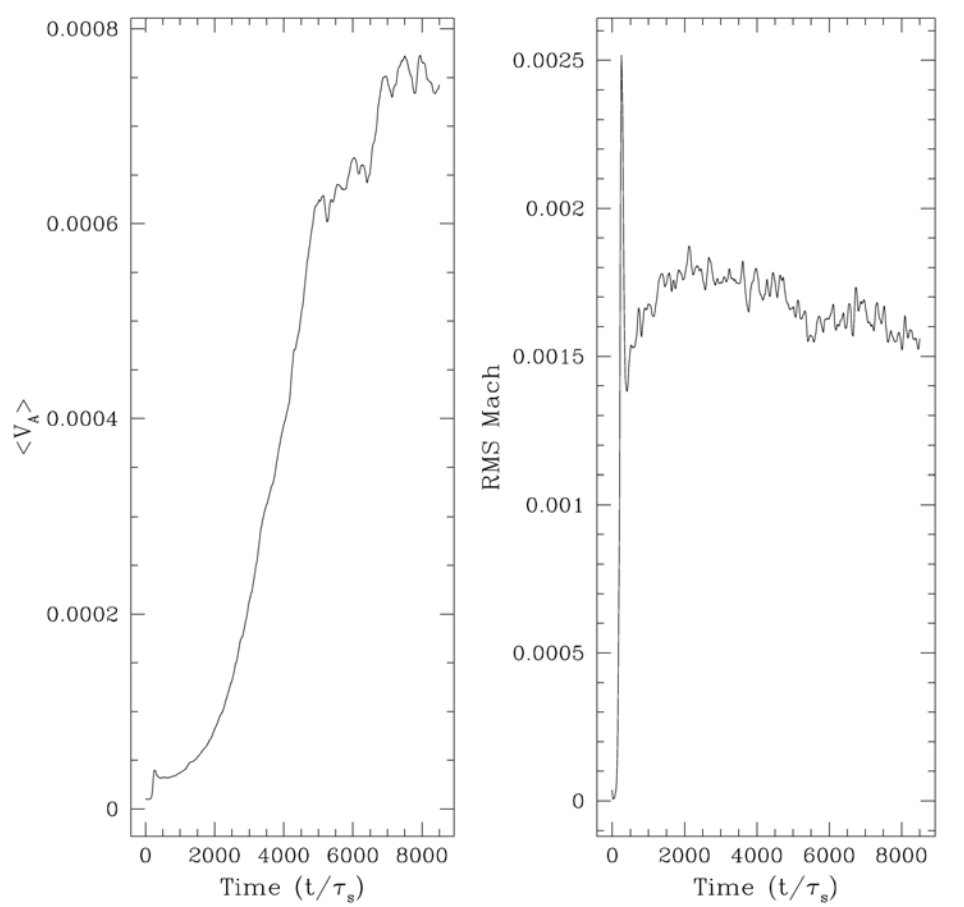
Exploration: 3D Nonlinear Behavior



Magnetic Energy Density = $B^2/2$

- Subsonic convective turbulence, Mach ~ 1.5 x 10^{-3}.
- Magnetic dynamo leads to equipartition with kinetic energy.
- Efficient heat conduction. Steady state heat flux is 1/3 to 1/2 of Spitzer value.

Exploration: 3D Nonlinear Behavior



$\mathcal{V}_A^2 \;/\; \frac{B^{\in}}{4\pi\rho}$ RMS Mach

- **Subsonic convective turbulence, Mach ~ 1.5 x 10^{-3}.**
- **Magnetic dynamo leads to equipartition with kinetic energy.**
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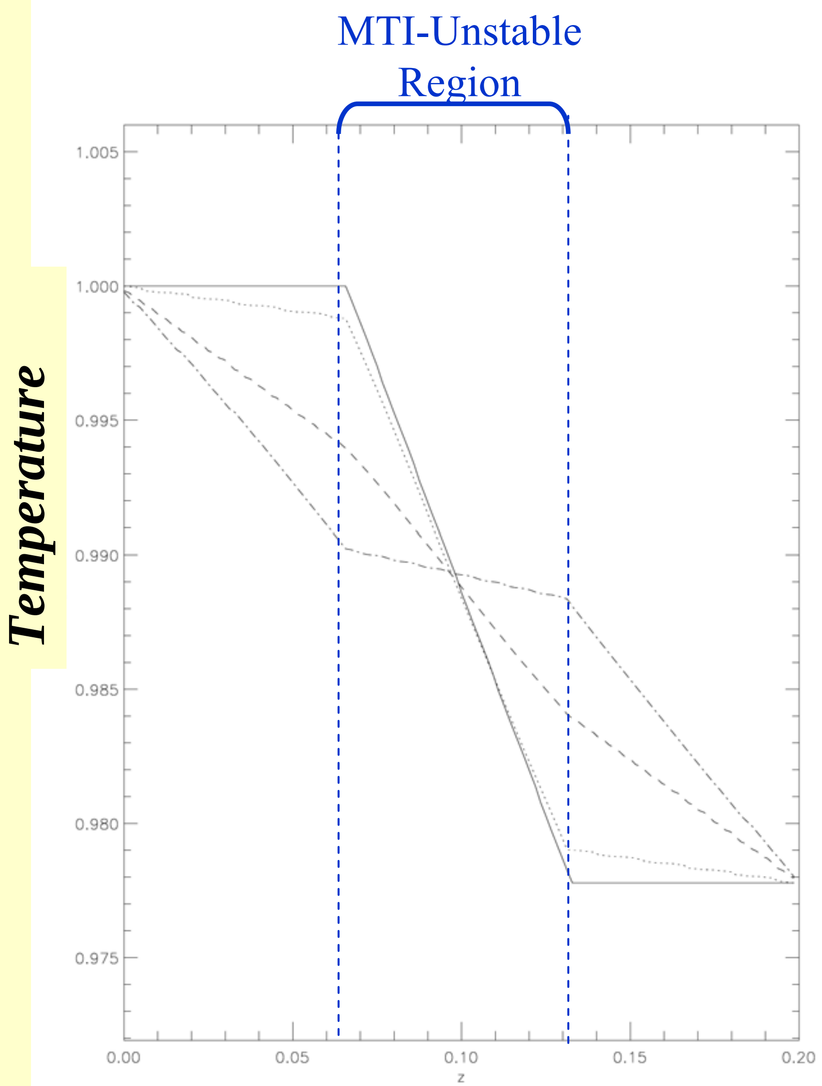
Exploration: 3D Nonlinear Behavior



•Subsonic convective turbulence, Mach ~ 1.5 x 10^{-3}.

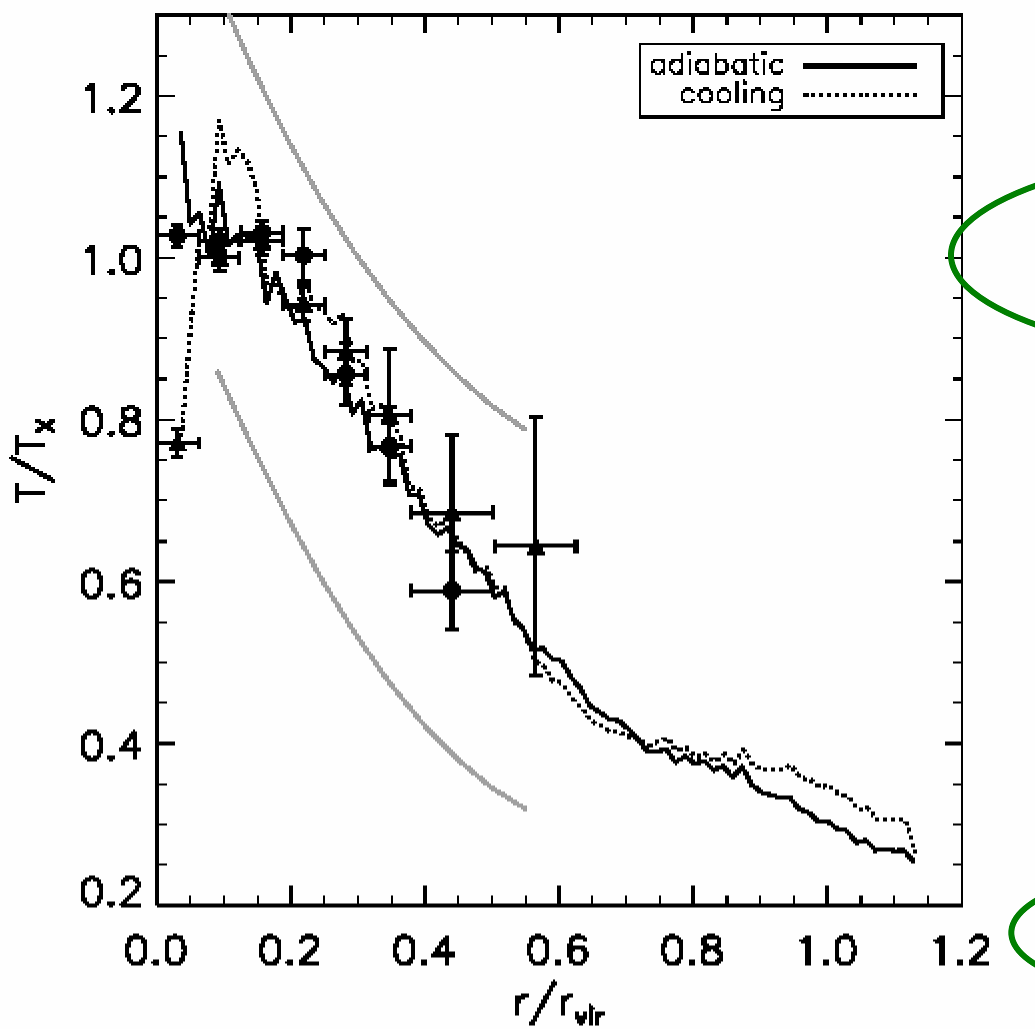
•Magnetic dynamo leads to equipartition with kinetic energy.

•Efficient heat conduction. Steady state heat flux is 1/3 to 1/2 of Spitzer value.

•Temperature profile can be suppressed significantly.

Application: Clusters of Galaxies

Expectations from Structure Formation



Loken, Norman, et al (2002)

Hydro Simulation:
ΛCDM Cosmology, Eulerian

Expect: steep temperature profile

$R_v \sim$ 1-3 Mpc

$M \sim 10^{14} - 10^{15}$ solar masses
(84% dark matter, 13% ICM, 3% stars)

$T \sim$ 1-15 keV

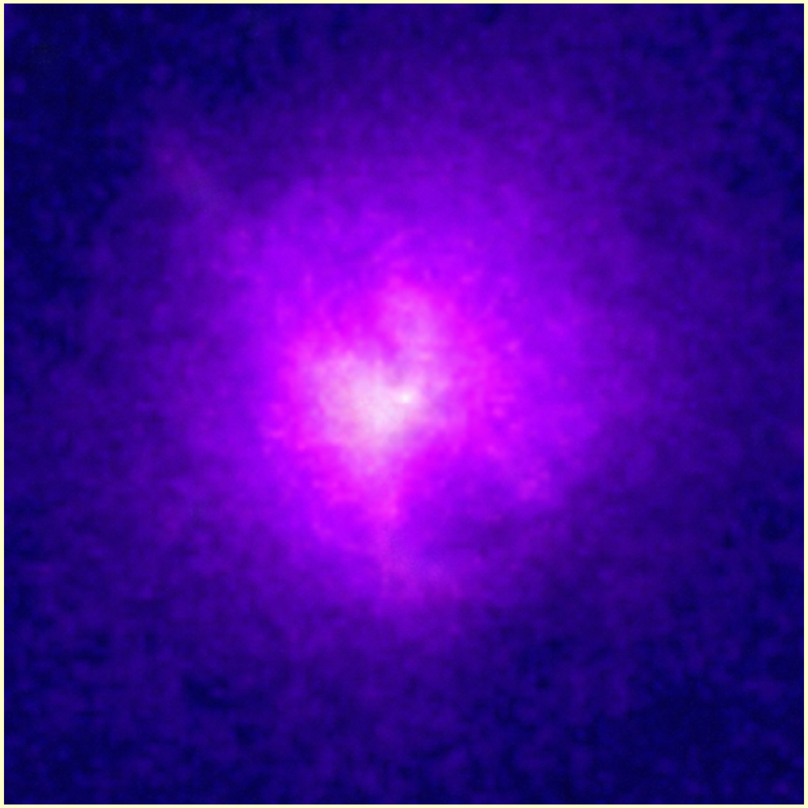
$L_X \sim 10^{43} - 10^{46}$ erg/s

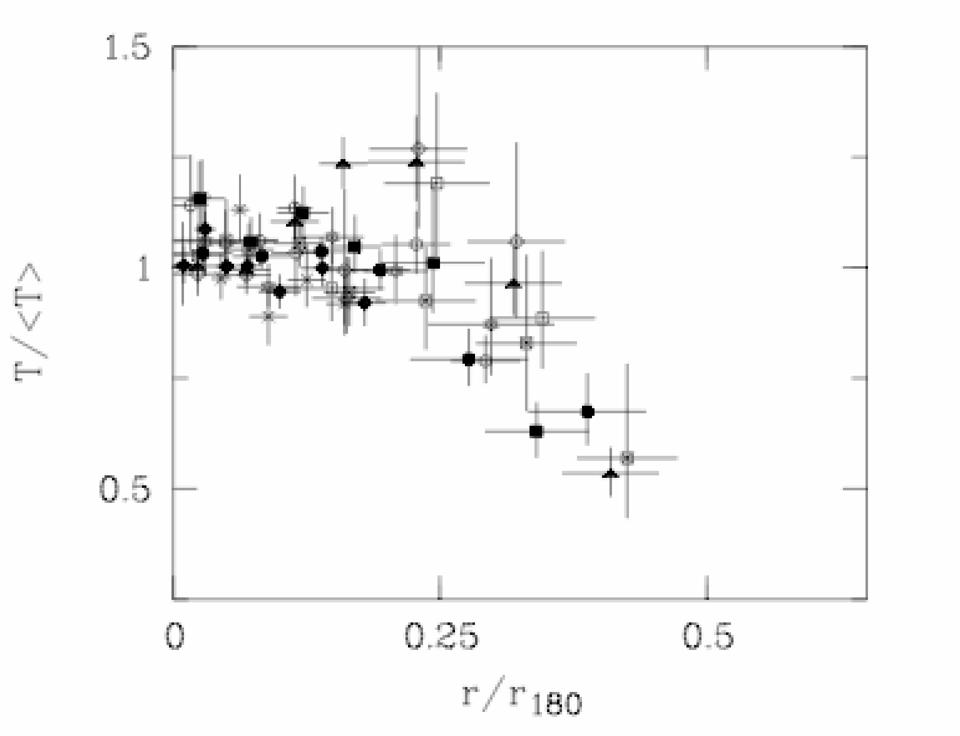
$B \sim 1.0\ \mu G$

Anisotropic Thermal Conduction Dominates

Application: Clusters of Galaxies

Observational Data



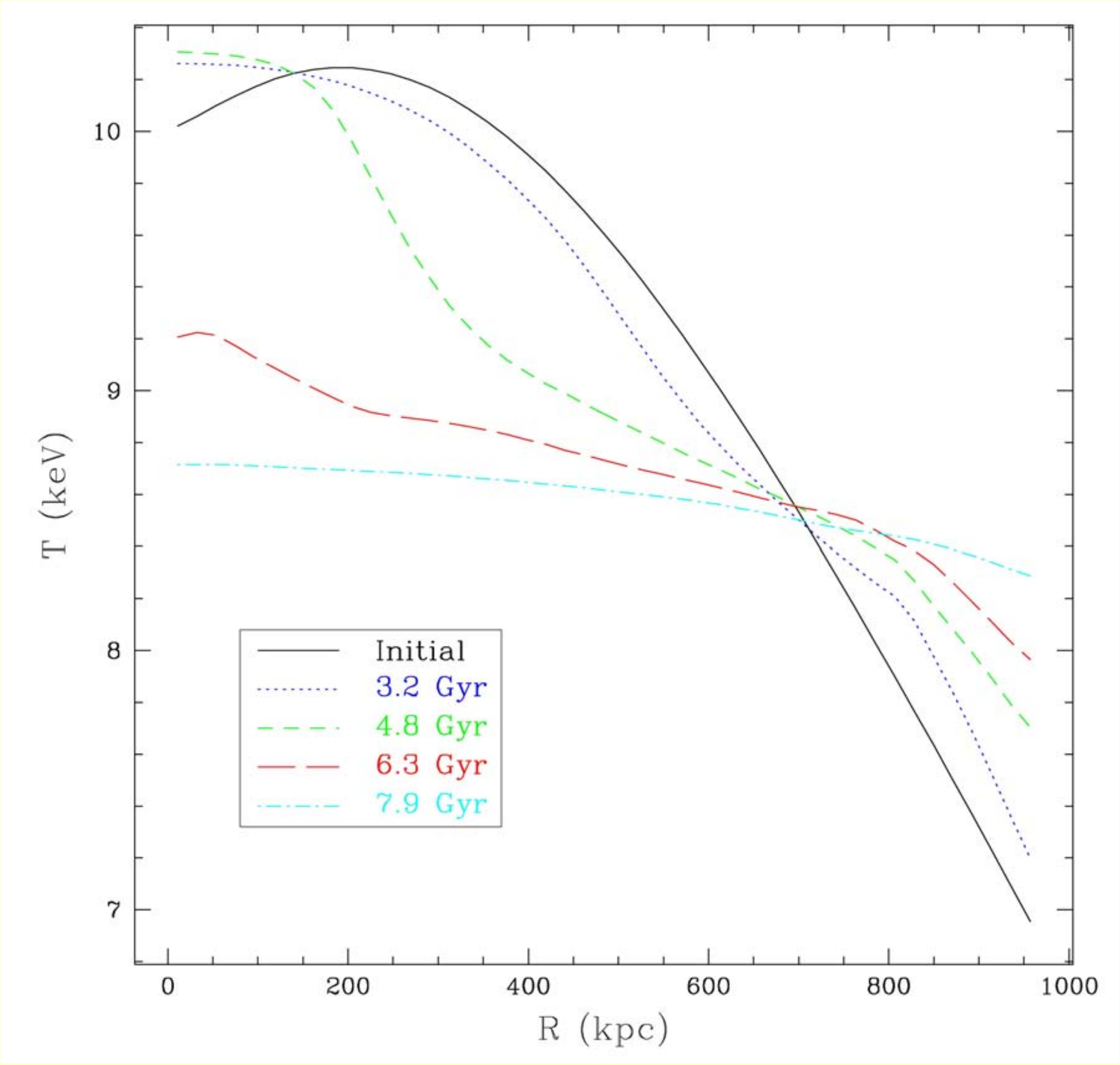


Plot from DeGrandi and Molendi 2002

ICM unstable to the MTI on scales greater than

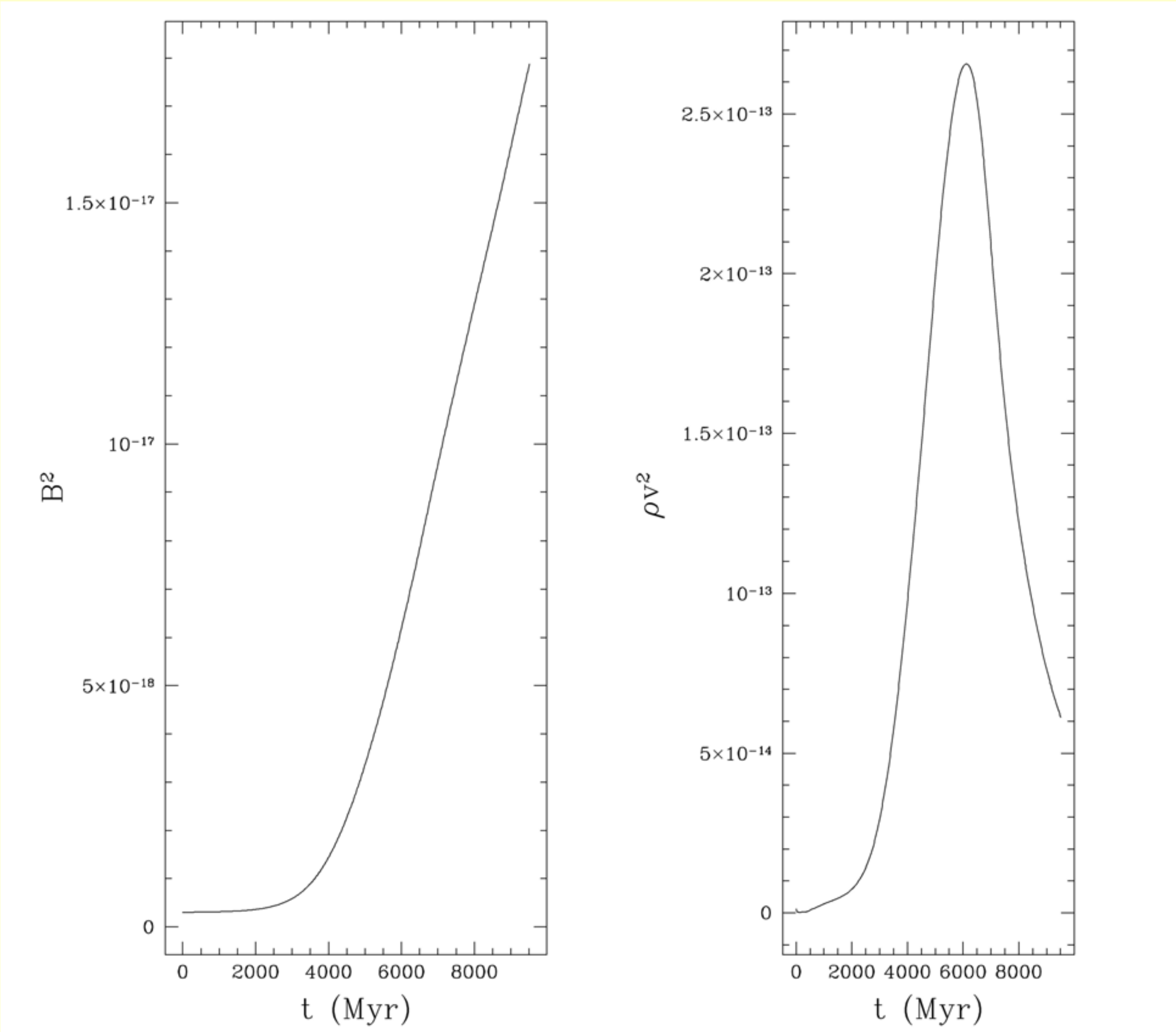
$$\lambda_{\rm crit} = 4.6\,{\rm kpc}\left(\frac{T}{5\,{\rm keV}}\right)^{1/2}\left(\frac{2000}{\beta}\right)^{1/2}$$

Simulation: Clusters of Galaxies



Temperature Profile becomes Isothermal

Simulation: Clusters of Galaxies



Magnetic Dynamo:

B^2 amplified by ~ 60

Vigorous Convection:

Mean Mach: ~ 0.1

Peak Mach: > 0.6

Summary

- Physics of the MTI.
- Verification and validation of MHD + anisotropic thermal conduction.
- Nonlinear behavior of the MTI.
- Application to the thermal structure of clusters of galaxies.

Future Work

- Galaxy cluster heating/cooling mechanisms: jets, bubbles, cosmic rays, etc.
- Application to neutron stars.
- Mergers of galaxy clusters with dark matter.

Acknowledgements

- Krell Institute (especially Rachel & Martin), other DOE grants.
- Many calculations performed on Princeton's Orangena Supercomputer

DOE CSGF Department of Energy Computational Science Graduate Fellowship