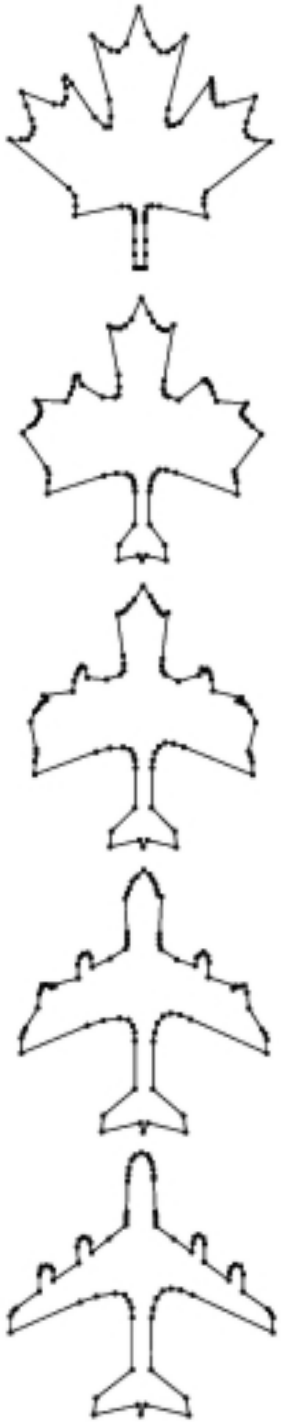
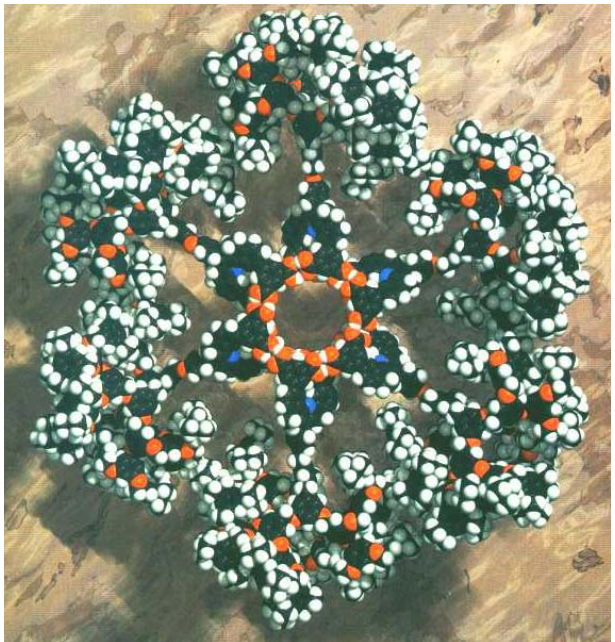


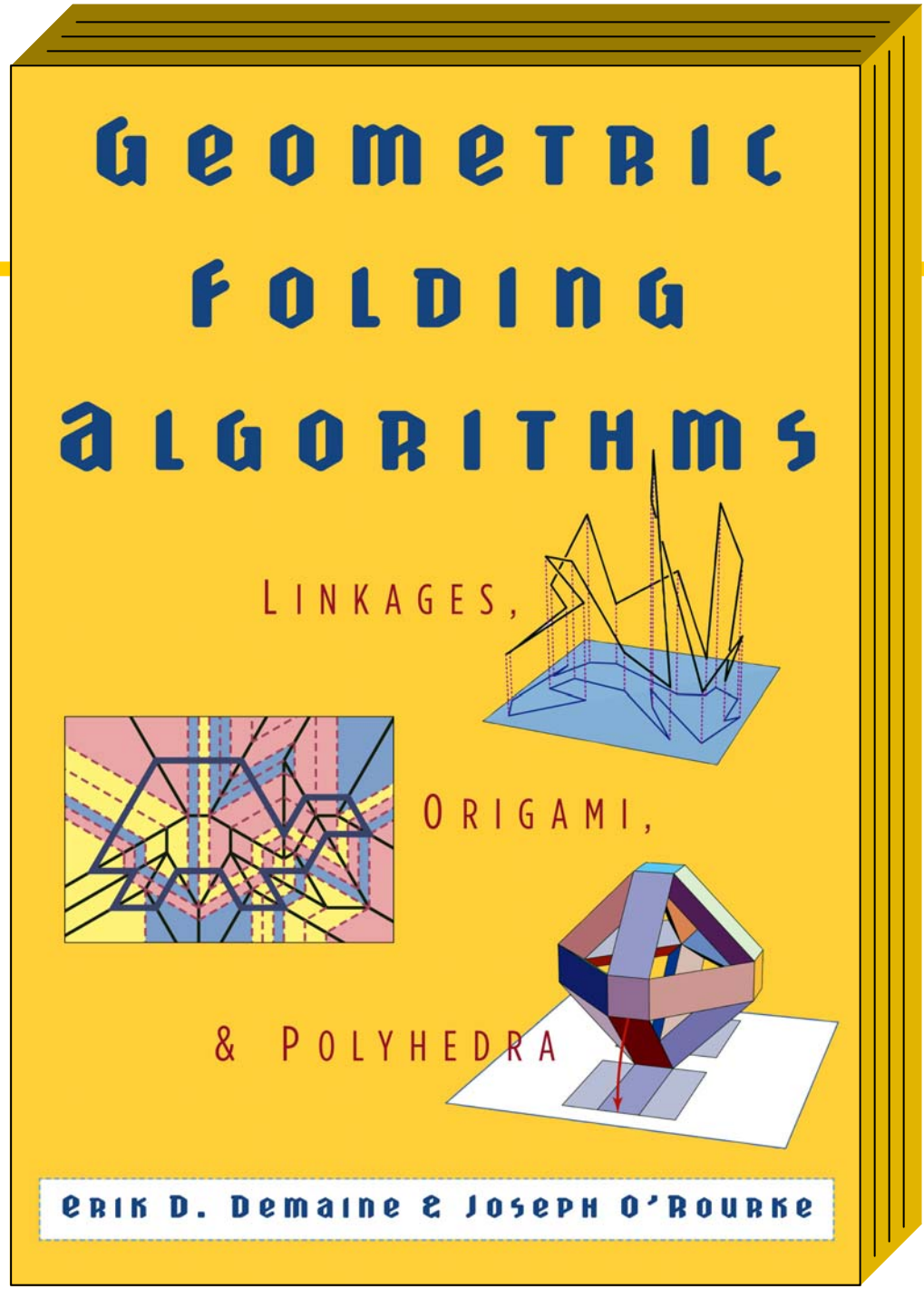
Origami, Linkages, & Polyhedra: Folding with Algorithms

Erik Demaine, MIT

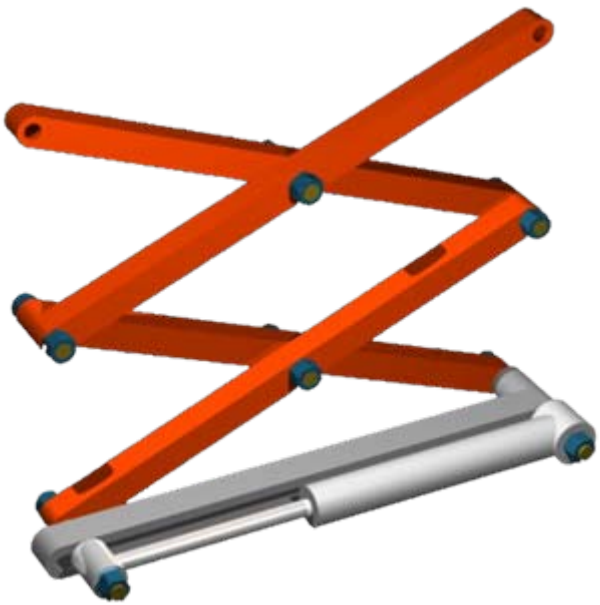


New Book

- www.gfalop.org
- Appearing this month
- Cambridge University Press
- ~600 pages

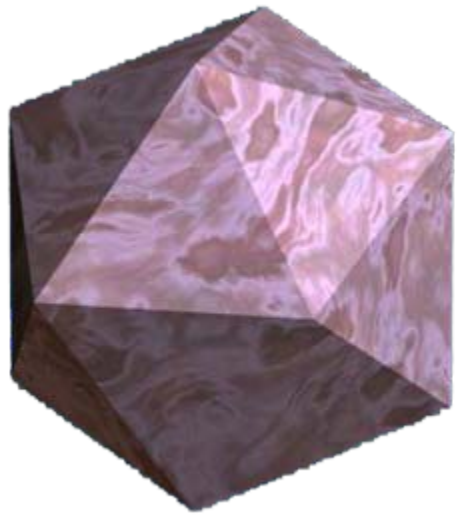


Geometric Folding



Linkages
(1D)

Paper
(2D)

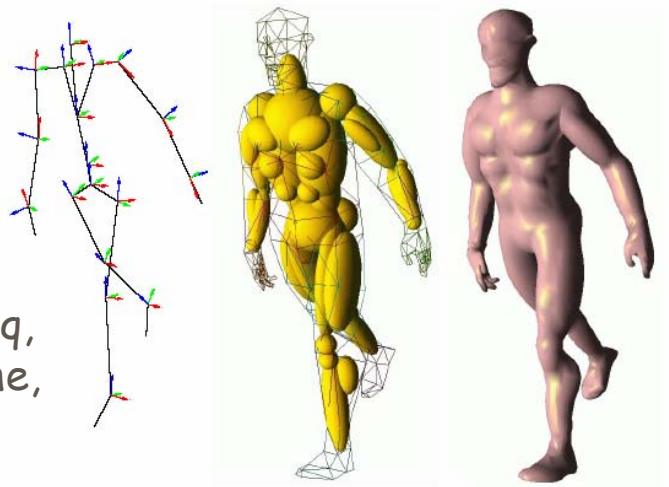


Polyhedra
(3D)

Folding is Everywhere: Linkages

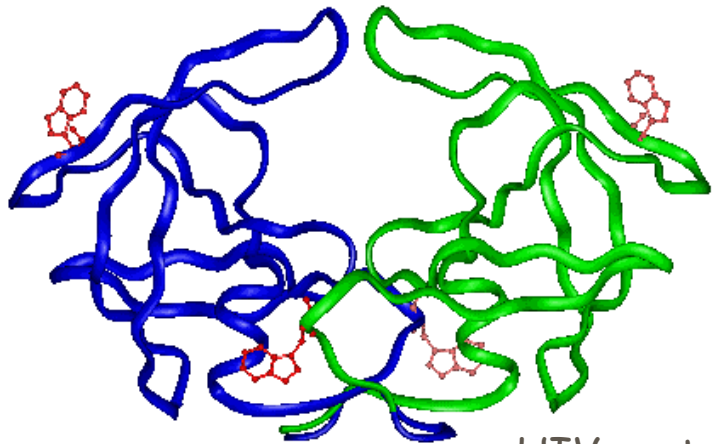
Mechanics

Robotics



[Leclercq, Akkouche, Galin 2001]

Graphics



HIV protease

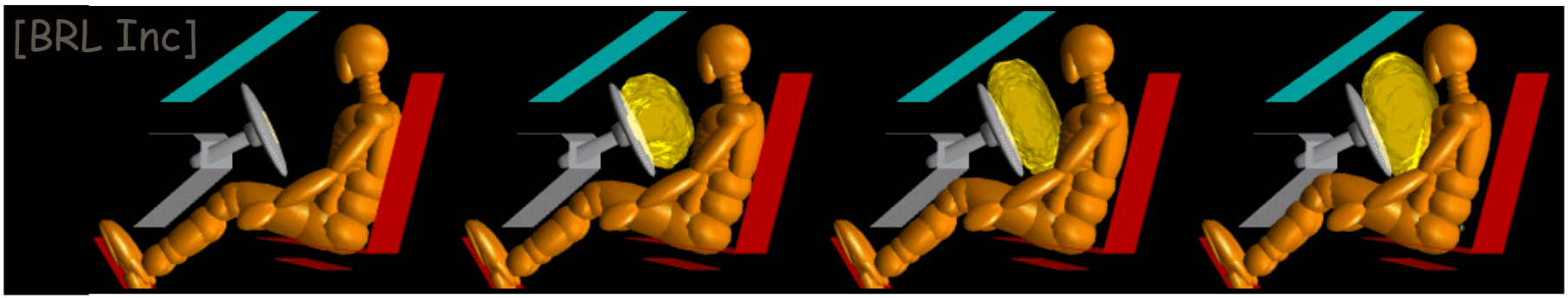
Biology

Folding is Everywhere: Paper

Origami

[You & Kuriba-yashi 2003]

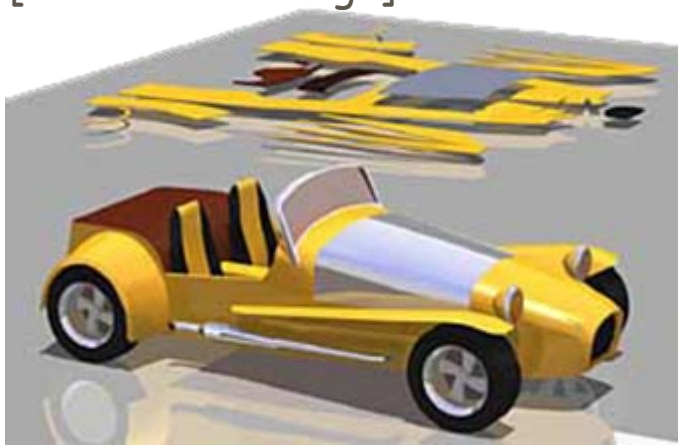
Deployable structures



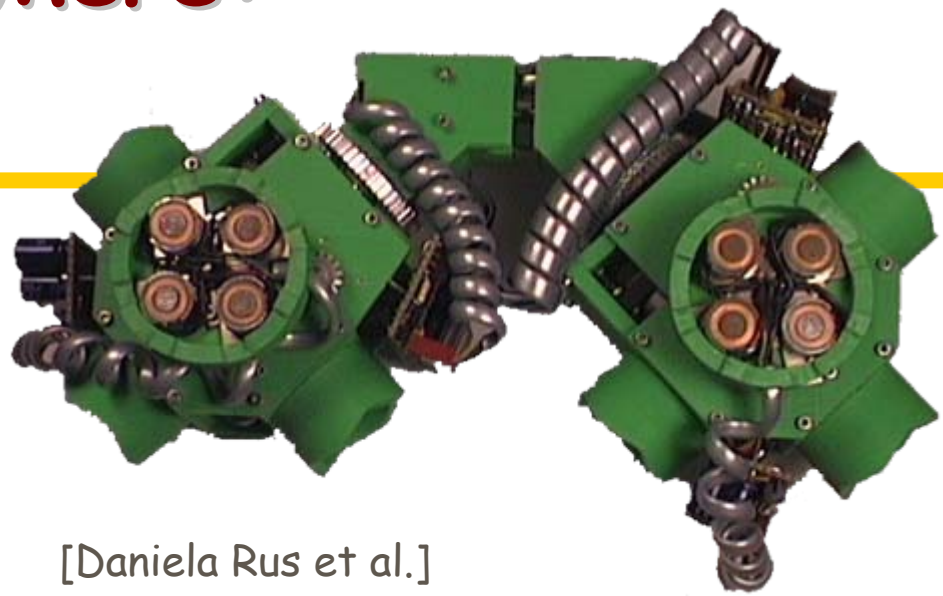
Airbags

Folding is Everywhere: Polyhedra

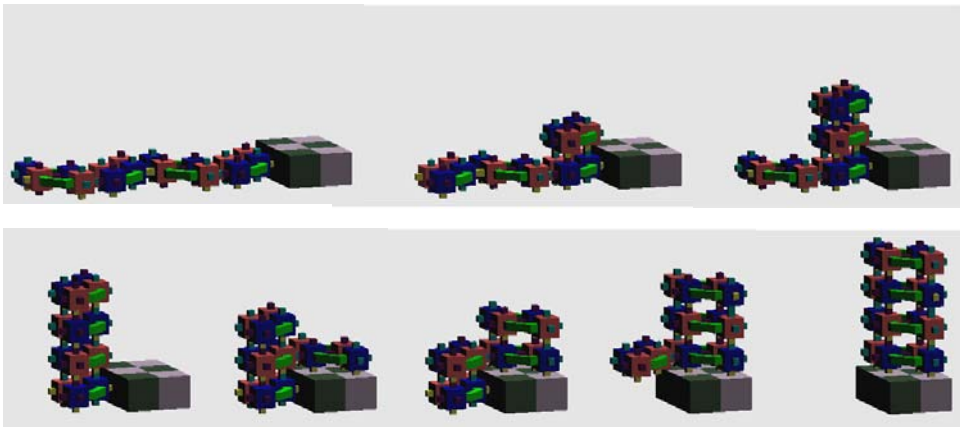
[Lundström Design]



Sheet-metal manufacturing

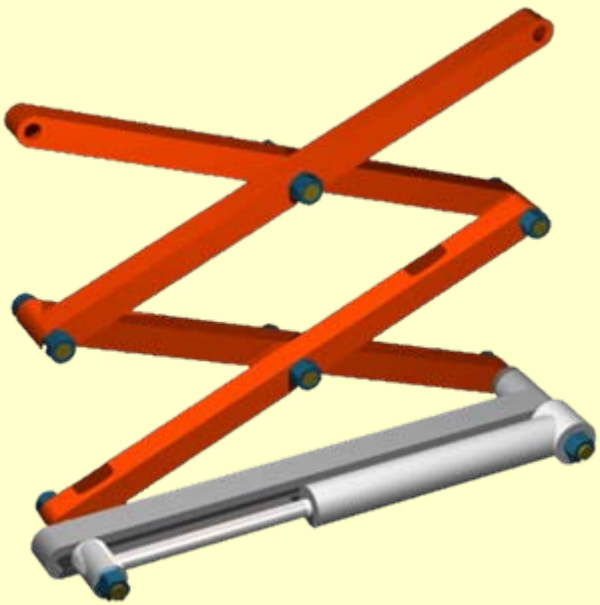


[Daniela Rus et al.]



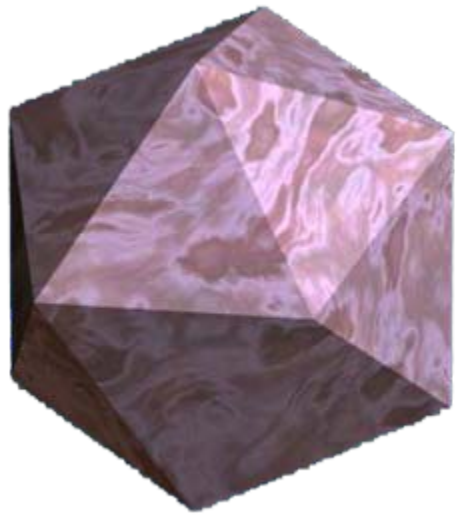
Reconfigurable robotics, self-assembly, nanomanufacturing

Geometric Folding



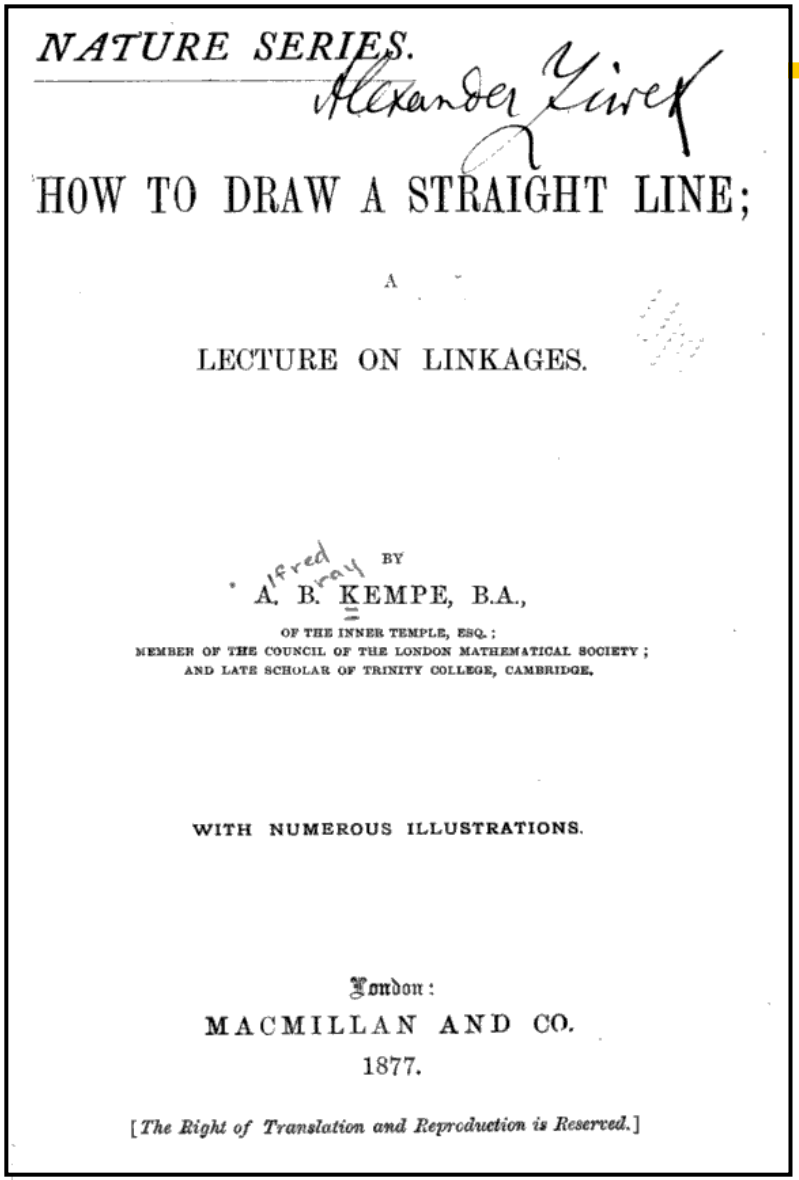
Linkages
(1D)

Paper
(2D)



Polyhedra
(3D)

Linkages



NATURE SERIES.

Alexander Ziwet

HOW TO DRAW A STRAIGHT LINE;

A

LECTURE ON LINKAGES.

BY

A. B. KEMPE, B.A.,

OF THE INNER TEMPLE, ESQ.;
MEMBER OF THE COUNCIL OF THE LONDON MATHEMATICAL SOCIETY;
AND LATE SCHOLAR OF TRINITY COLLEGE, CAMBRIDGE.

WITH NUMEROUS ILLUSTRATIONS.

London:
MACMILLAN AND CO.
1877.

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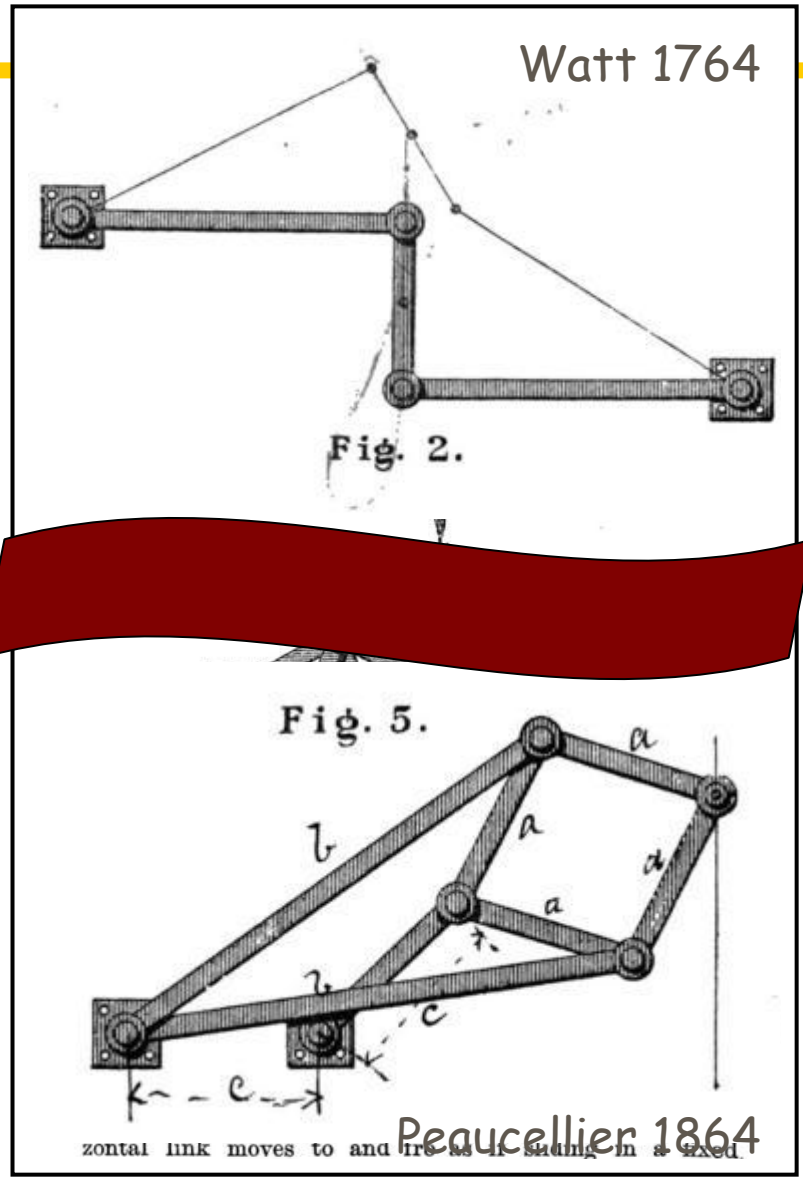
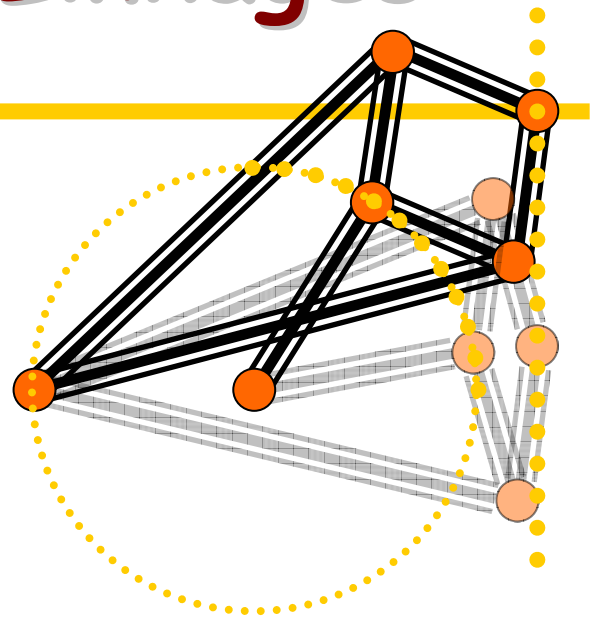


Basic Questions about Linkages



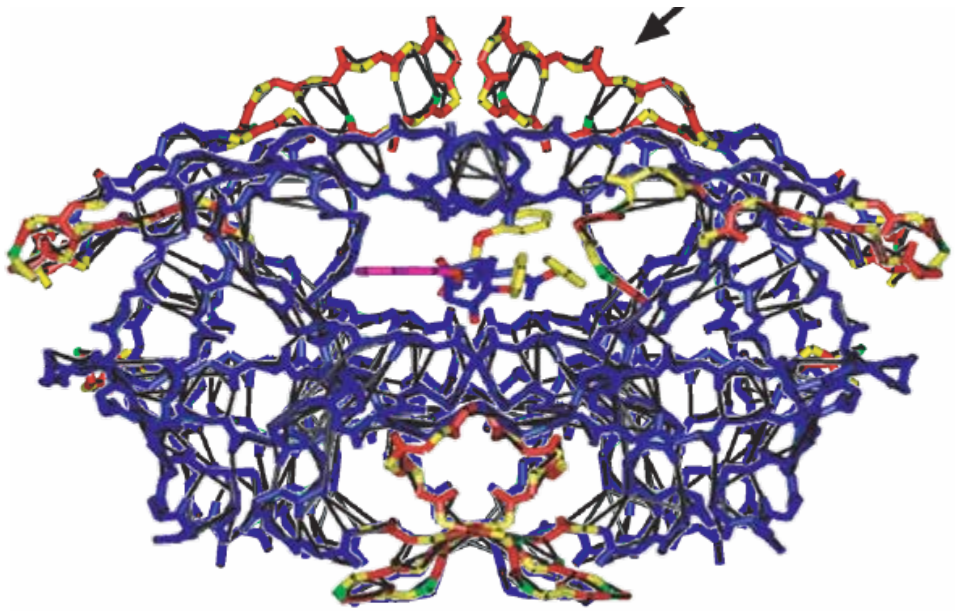
Given a linkage…

- **Rigidity:**
  - Does the linkage move at all?
- **Universality:**
  - Into what configurations can it fold?
- **Motion planning:**
  - How do we get there?

Rigidity

- What linkages can move at all?
- **Rigid** frameworks: buildings, bridges, etc.

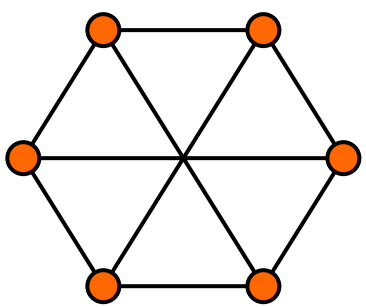
[Big Dig in Boston]



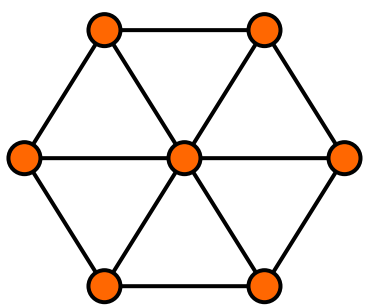
[Chubynsky, Hespenheide, Jacobs, Kuhn, Lei, Menor, Rader, Thorpe, Whiteley, Zavodszky 2003]

Rigidity

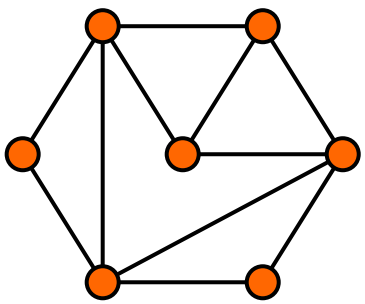
- What linkages can move at all?
  - **Known:** Characterization in 2D [Laman 1970]
  - **Unsolved:** Most problems in 3D



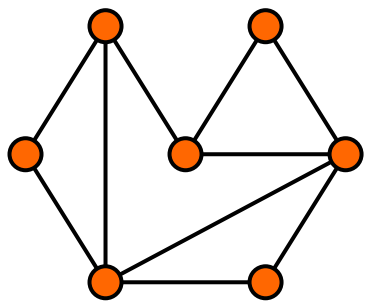
Rigid



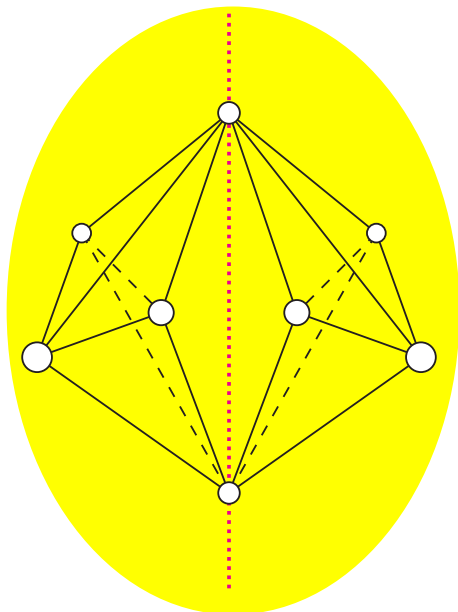
Rigid



Rigid

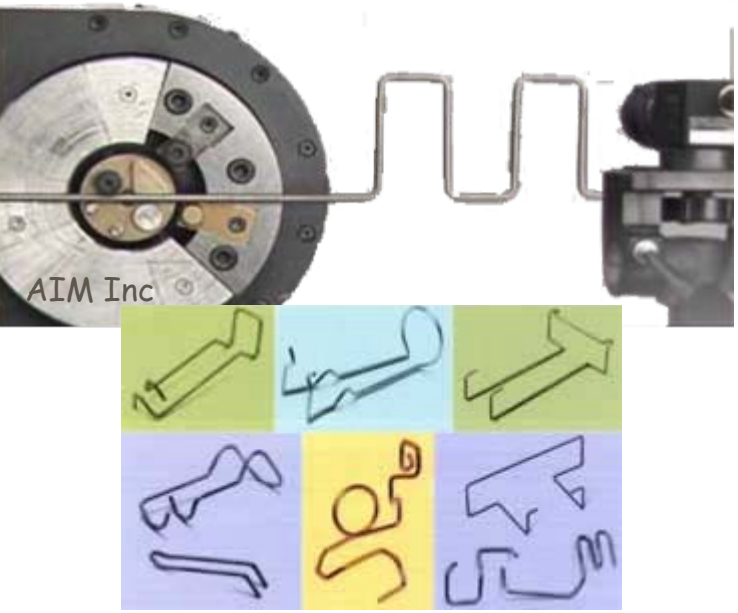


Flexible



Universality

- Can a linkage move universally between any two configurations?



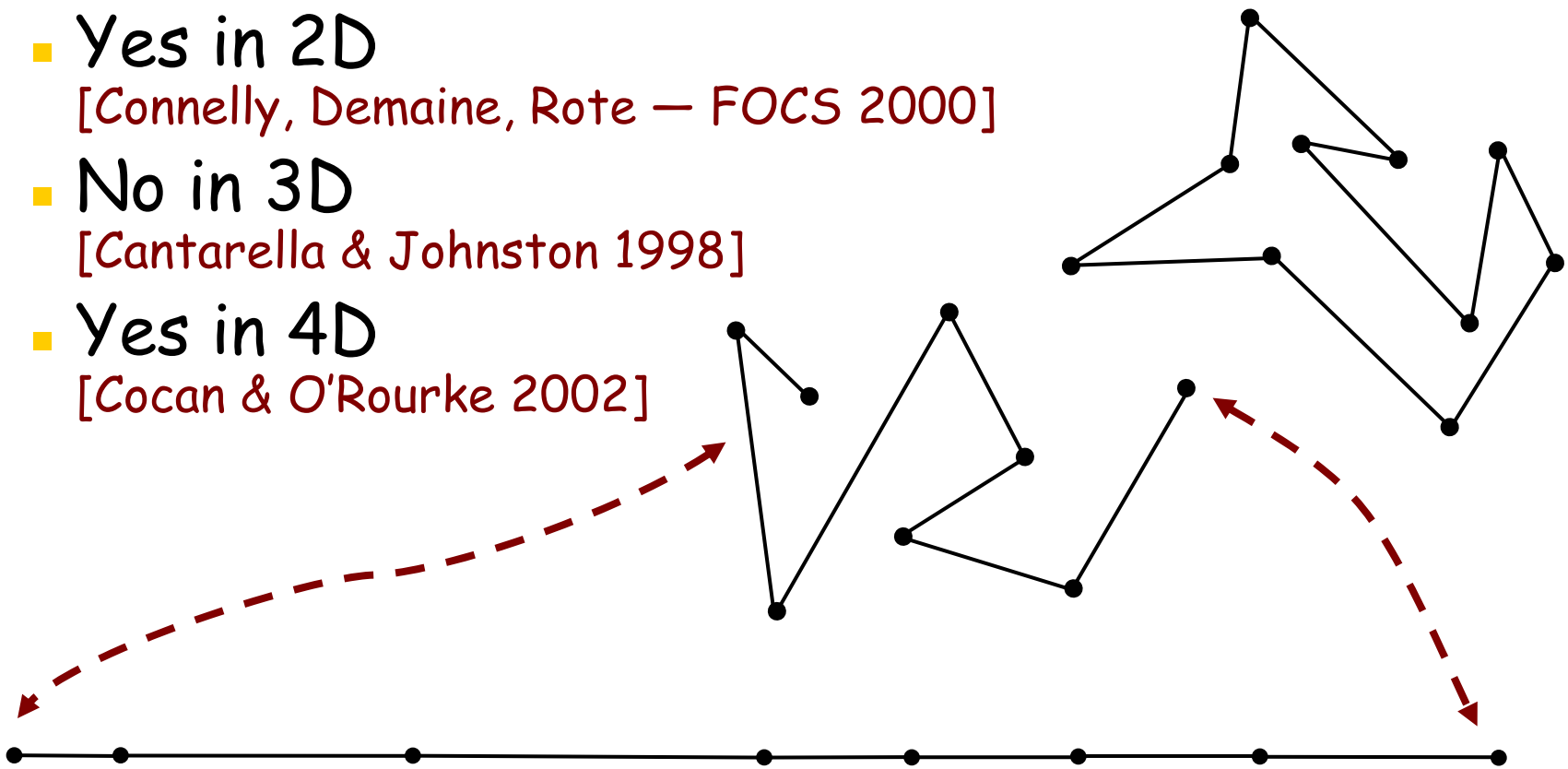
Wire bending

Hydraulic tube bending

Robotic arm folding

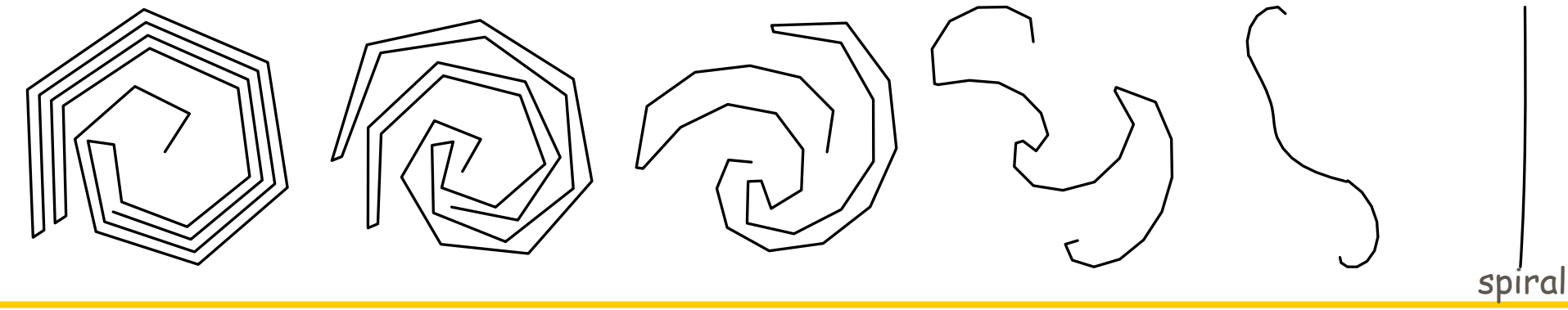
Universality

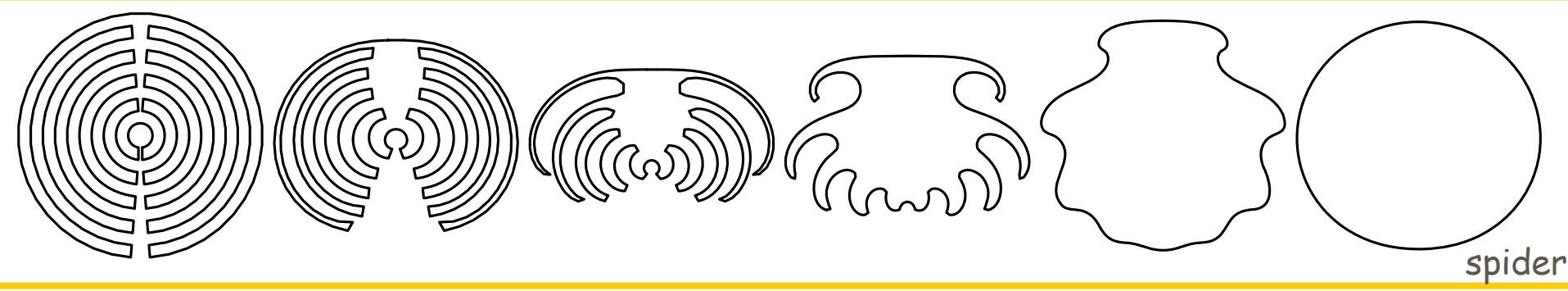
- Can a **chain** linkage move universally between any two configurations?
  - Yes in 2D
    [Connelly, Demaine, Rote — FOCS 2000]
  - No in 3D
    [Cantarella & Johnston 1998]
  - Yes in 4D
    [Cocan & O'Rourke 2002]



Universality: Unfolding 2D Chains

[Cantarella, Demaine, Iben, O'Brien — SoCG 2004]

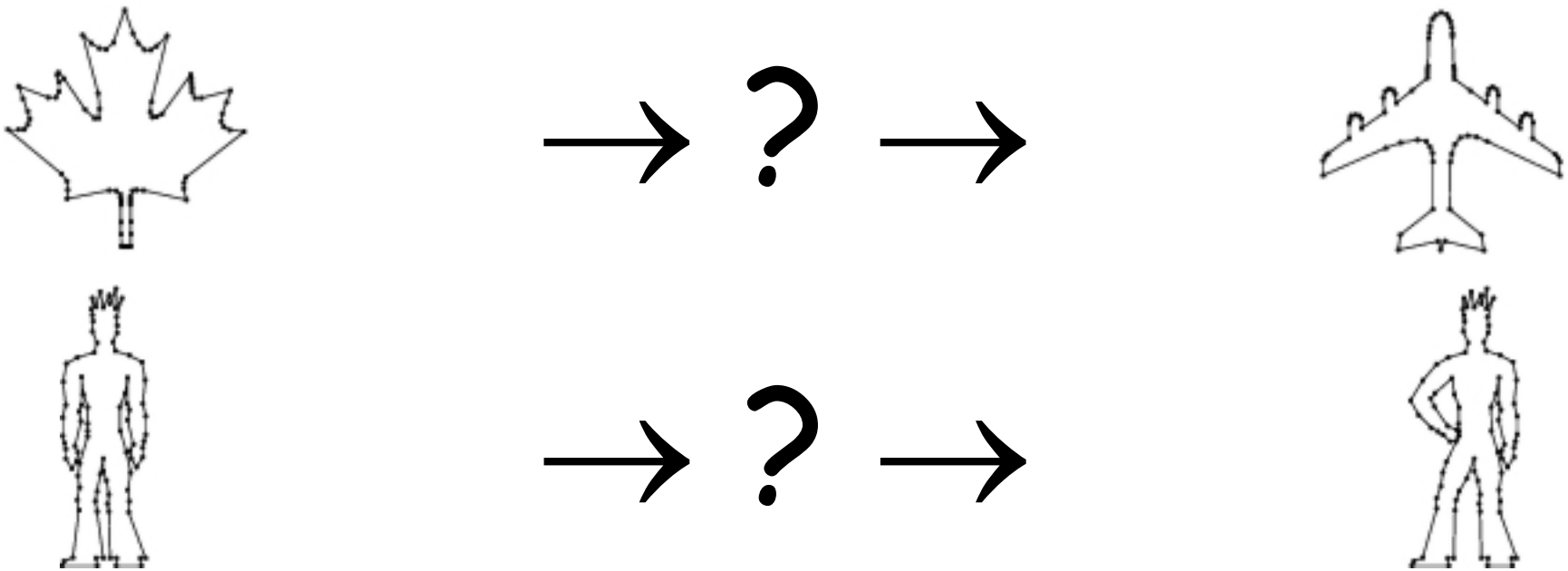




Motion Planning

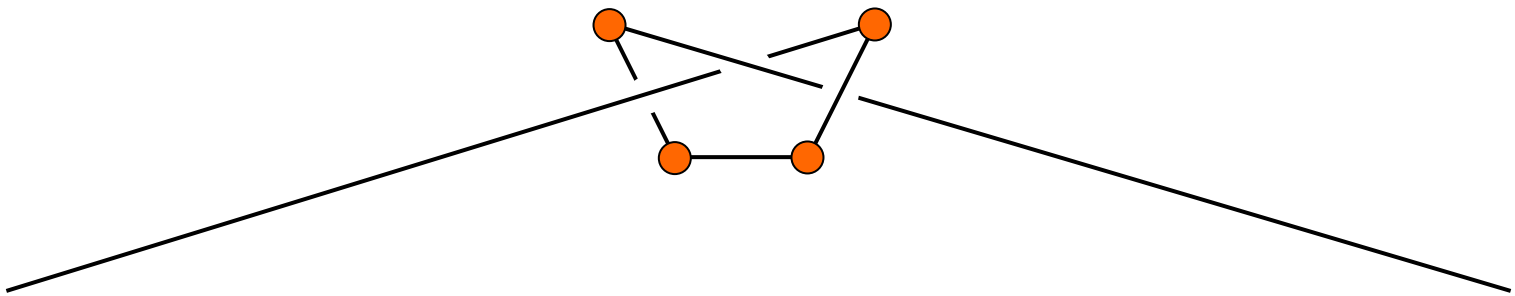
[Iben, O'Brien, Demaine – SIGGRAPH 2004, SoCG 2006]

- Find short motion from A to B (if possible)
- Universality results give new insight into cases of interest, e.g., polygon morphing



Challenge in 3D

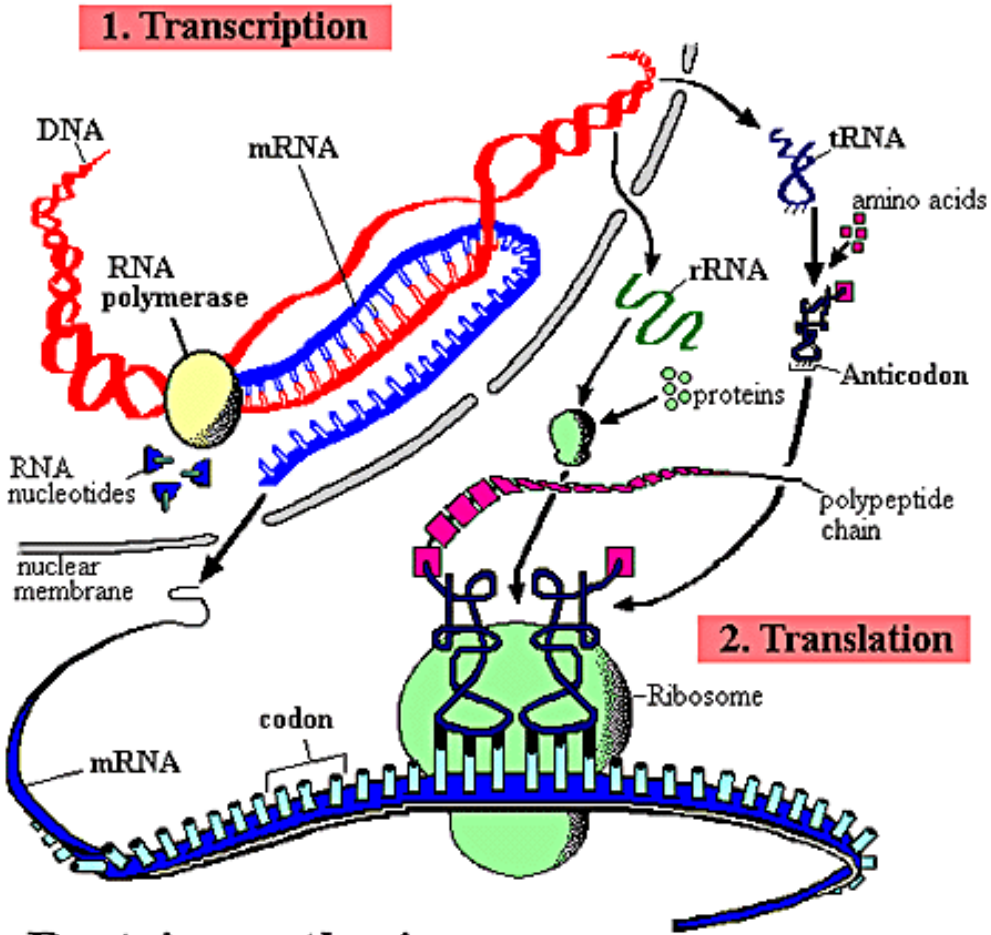
- 3D chains can be **locked**:
  [Cantarella & Johnston 1998]



- **Unsolved:** Which 3D chains are locked?
- **Known:** Motion planning of 3D chains is computationally intractable
  [Alt, Knauer, Rote, Whitesides 2004]

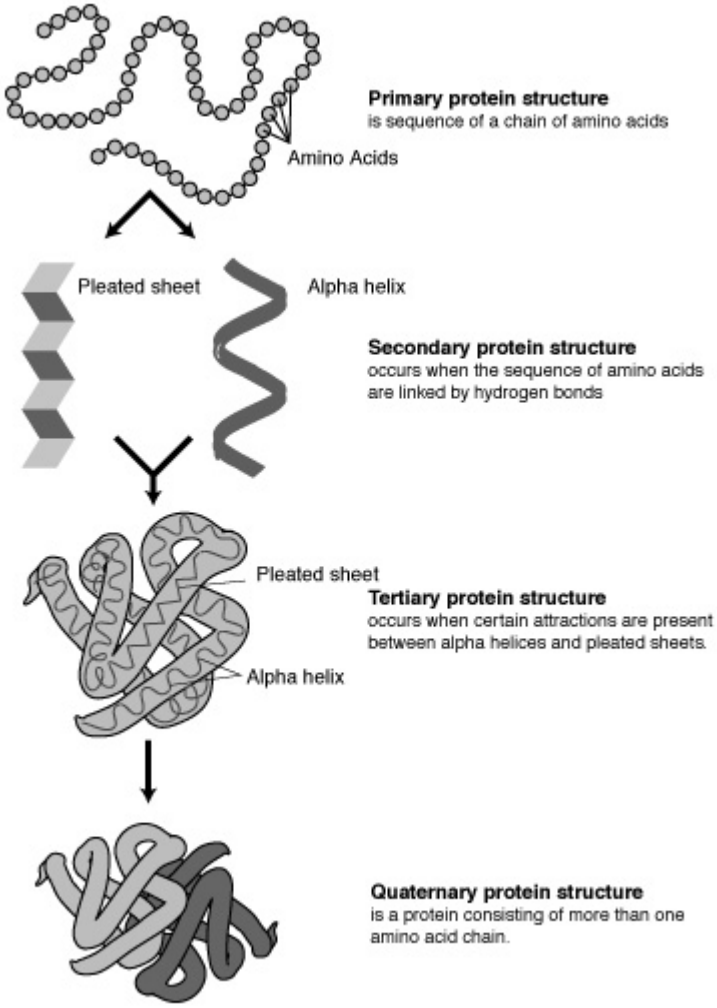
Proteins

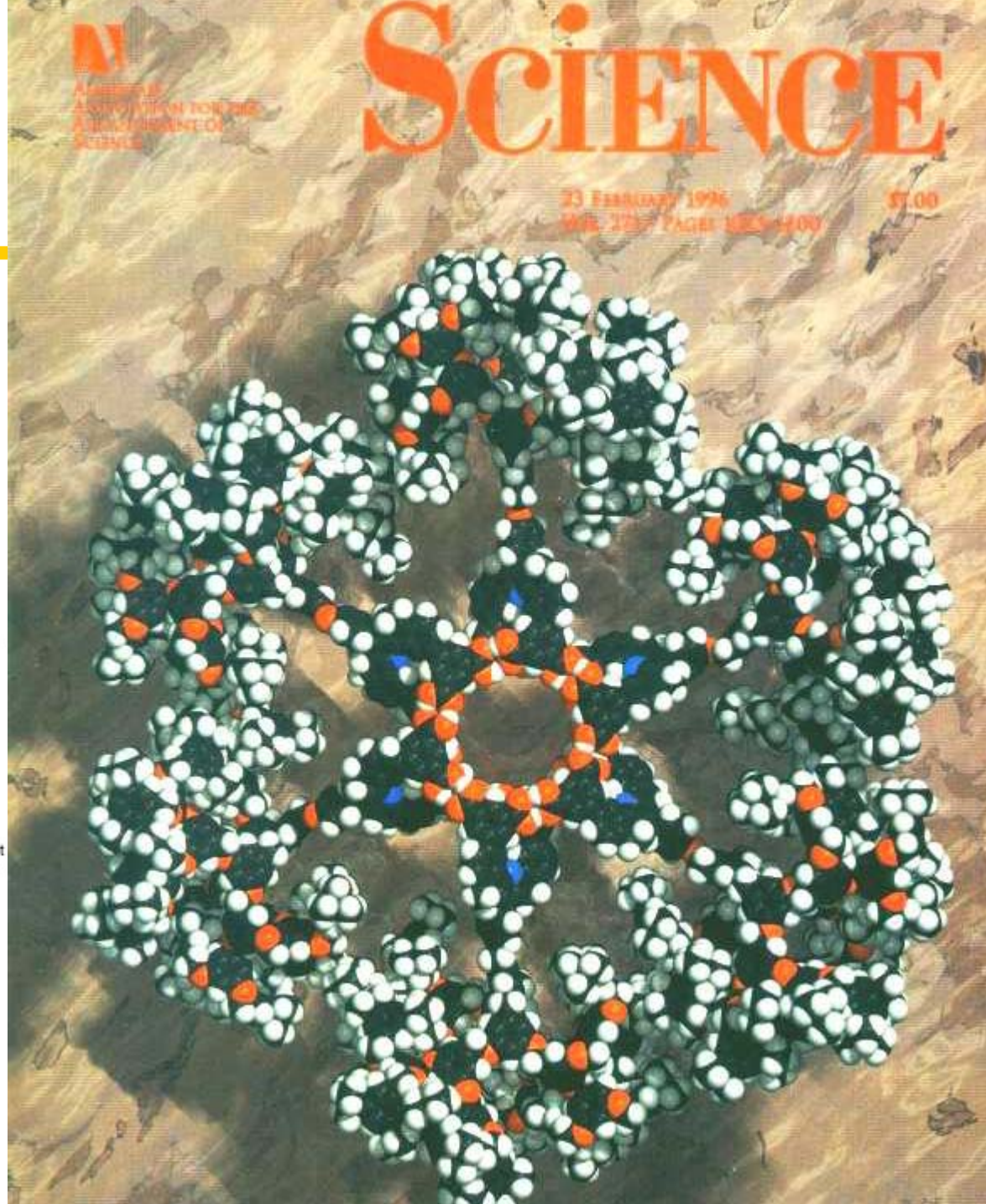
- DNA encodes proteins in genetic code
- Proteins are "fundamental building blocks of life"



Protein synthesis

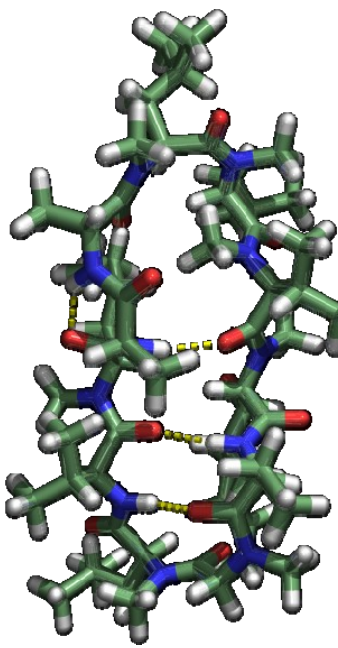
Protein Folding





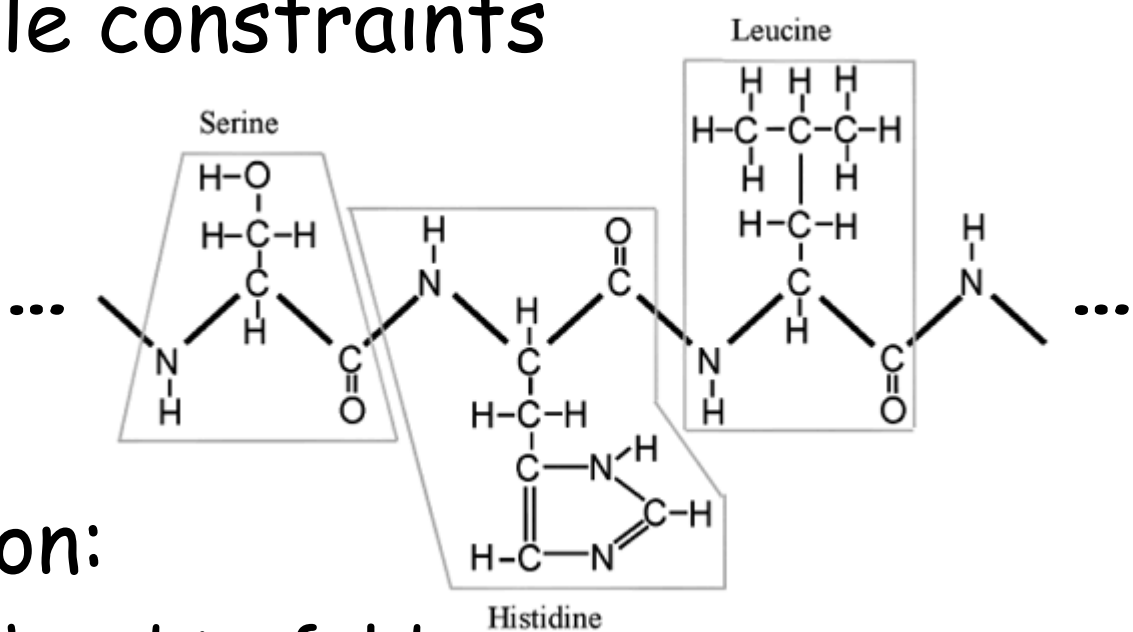
Importance of Protein Folding

- Geometry of a protein folding is an important aspect of its behavior
- Prediction of protein folding, and synthesis of proteins with desired foldings, are central problems in computational biology
  - Drug design
  - Preventing diseases (e.g., Alzheimer's, mad-cow disease, cystic fibrosis, some forms of cancer)

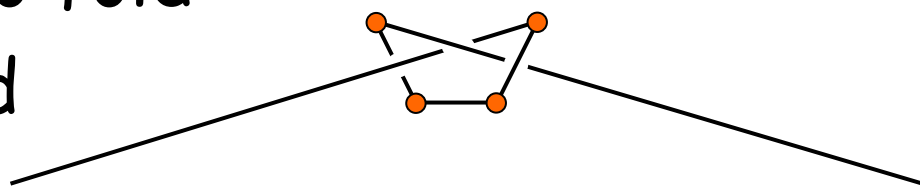


Mechanics of Protein Folding

- Protein backbone is roughly a 3D chain with fixed-angle constraints

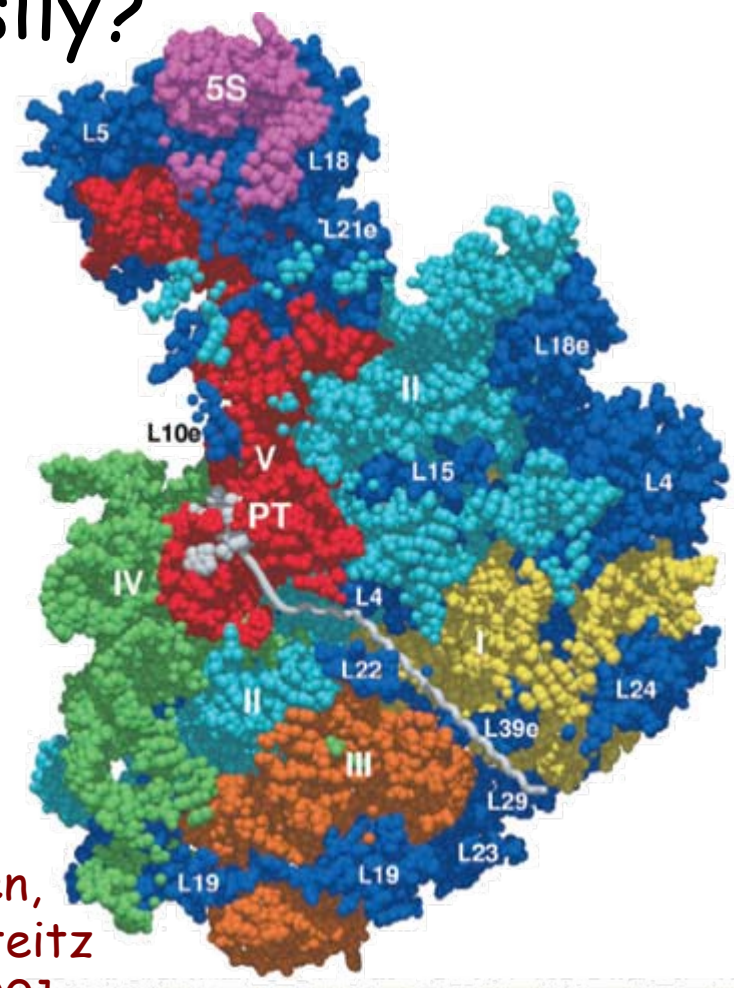


- Natural question:
  - 3D chains are hard to fold
  - How does nature fold proteins so easily?



Mechanics of Protein Folding

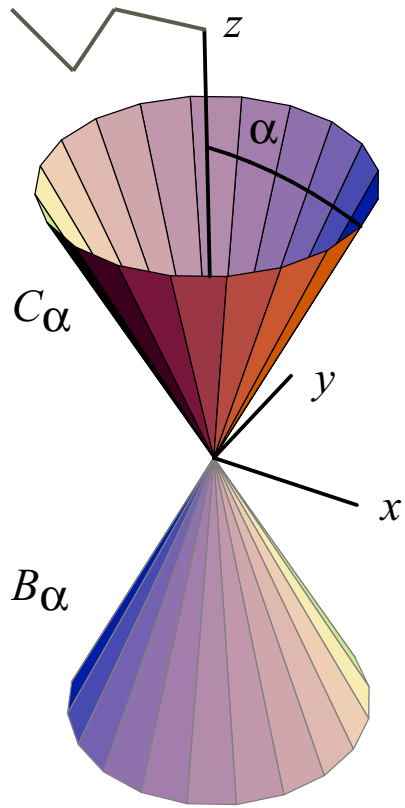
- Why do proteins fold easily?
- Possible answer:
  - **Ribosome** constructs proteins, enforcing geometric constraints



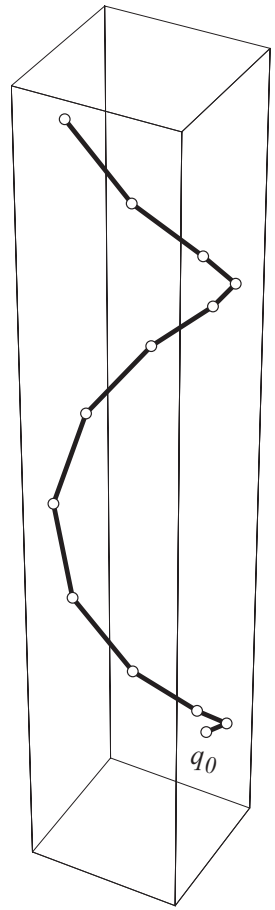
[Nissen, Hansen, Ban, Moore, Steitz — Science 2000]

Mechanics of Protein Folding

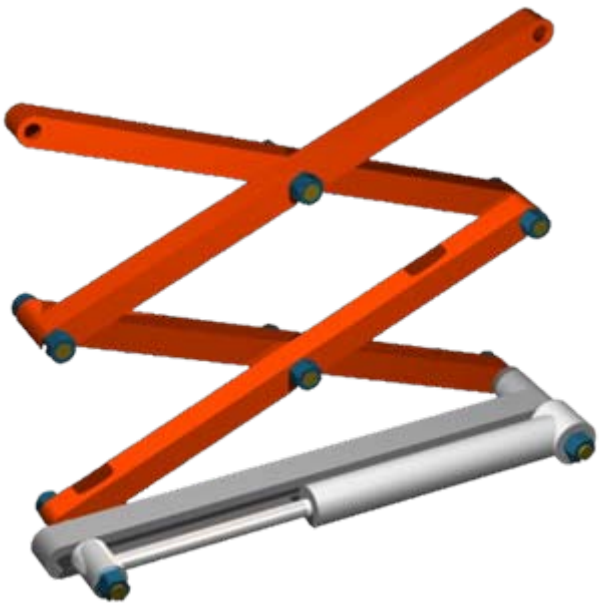
- Why do proteins fold easily?
- Possible answer:
  - **Ribosome** constructs proteins, enforcing geometric constraints
- **Cone model:** [Demaine, Langerman, O'Rourke 2006]
  - All producible states can reach each other
  - Flattenable
  - Helical canonical state



ribosome

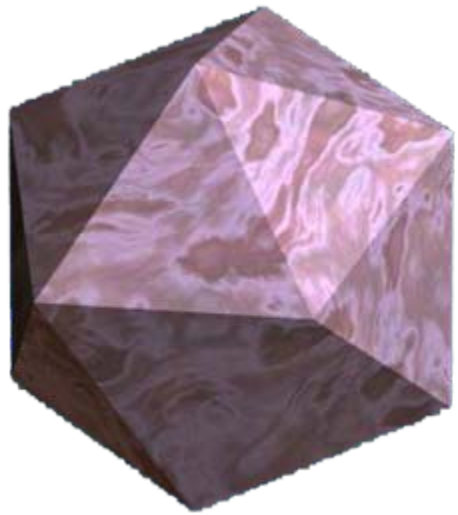


Geometric Folding



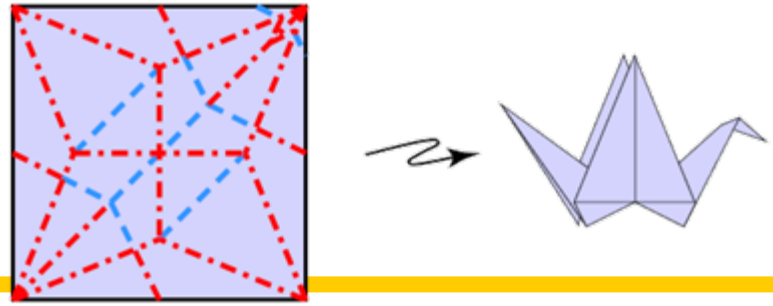
Linkages
(1D)

Paper
(2D)



Polyhedra
(3D)

Origami

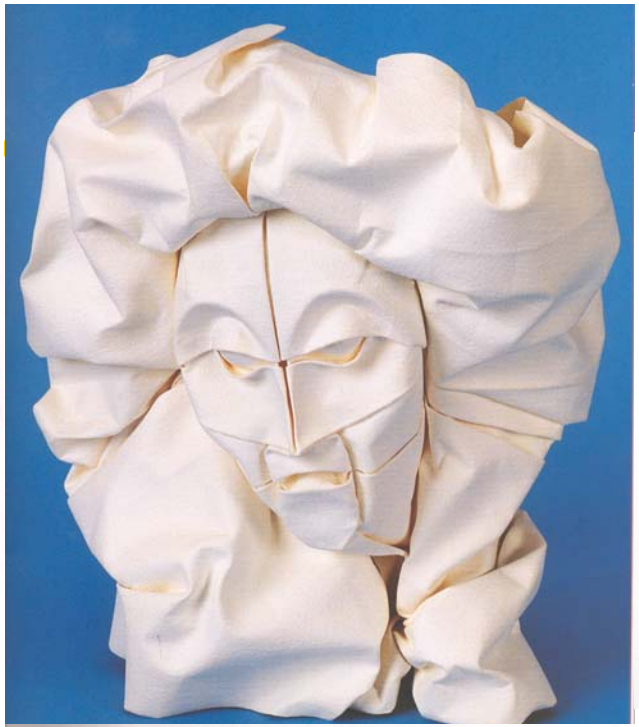


- Perhaps as old as paper itself (105 AD)
- Revolution in complex origami design over past ~25 years

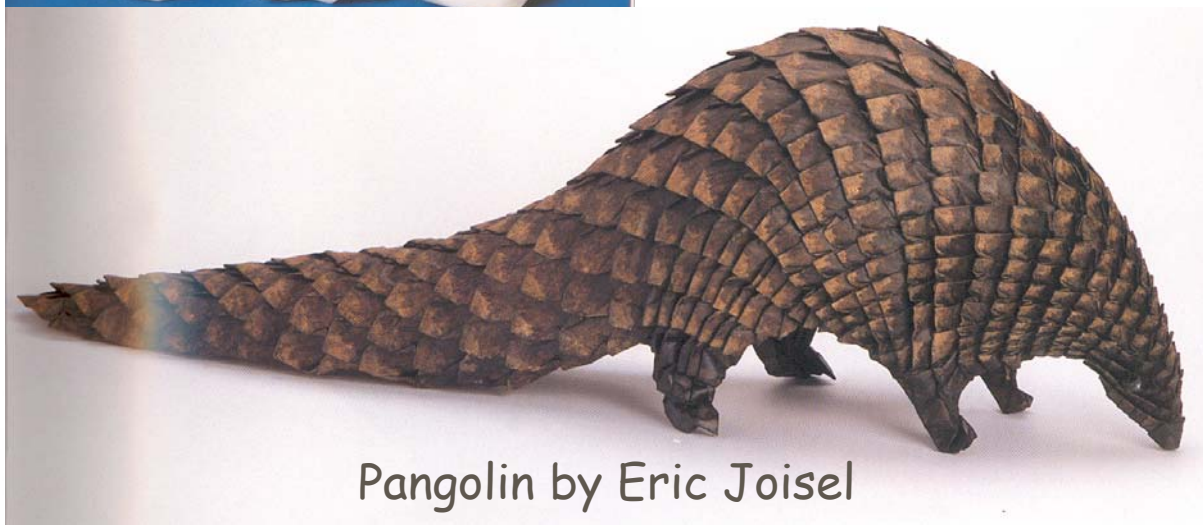
Photos from *Origamido: Masterworks of Paper Folding*

Black Forest Cuckoo Clock by Robert Lang

Mask by Eric Joisel



Bat by Michael LaFosse



Pangolin by Eric Joisel

Models & photos by Robert Lang

Roosevelt Elk, opus 358

Dancers, opp. 457 & 458

Mt Diablo Tarantula, opus 481

Hermit Crab

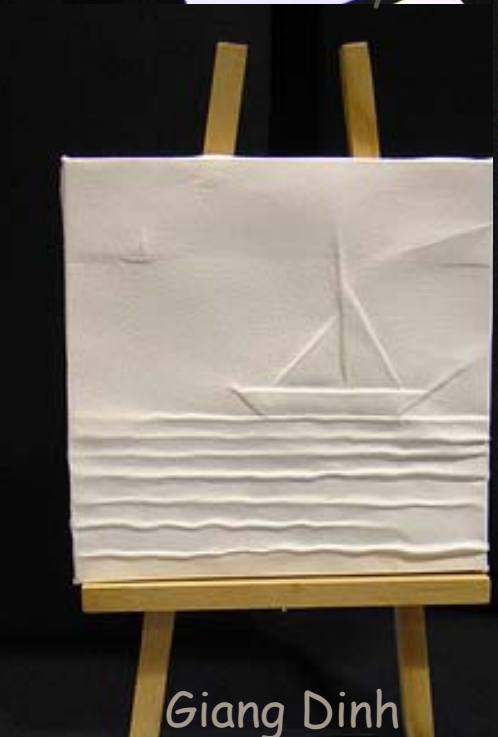
Tree Frog, opus 280

Koi, opus 425

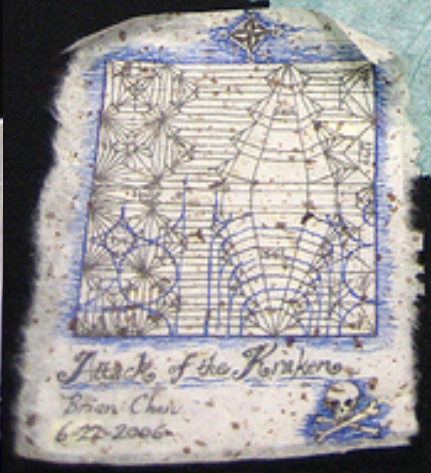
2006 Design Challenge
Photos by Brian Chan

Andrea Hawksley

Jason Ku



Giang Dinh

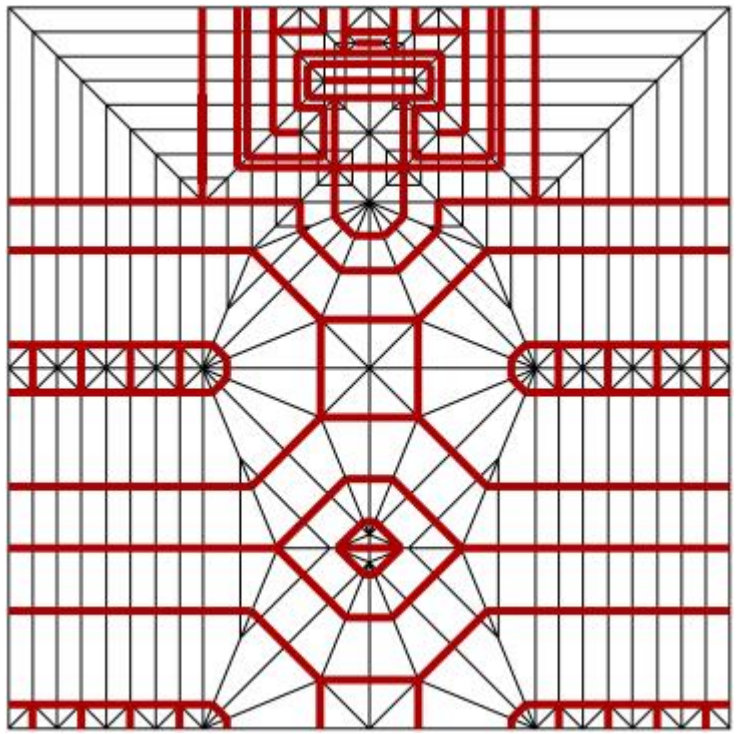


Brian Chan

Robert Lang

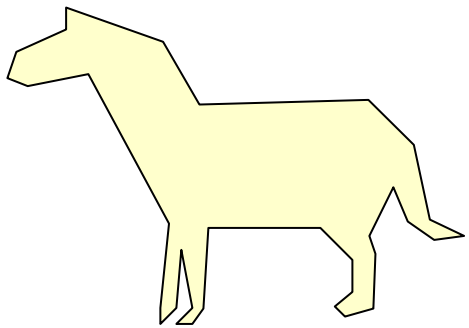
Origami Mathematics & Algorithms

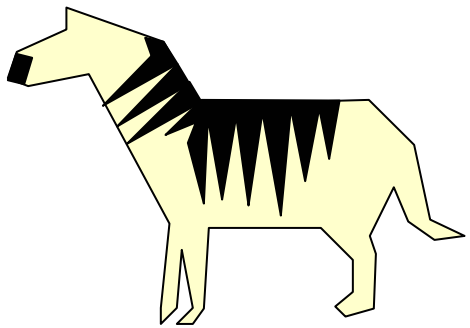
- Explosion in technical origami thanks in part to growing mathematical and computational understanding of origami

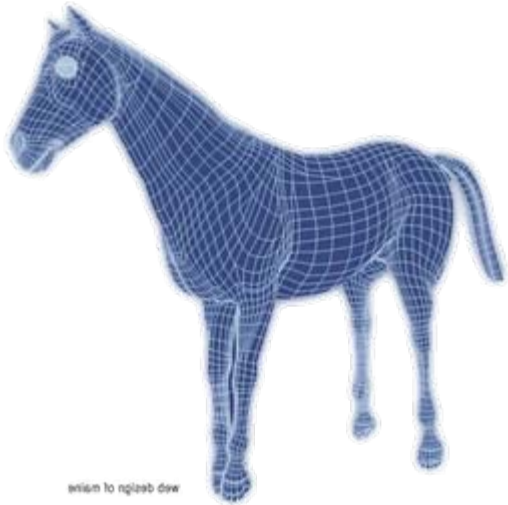


Origami Mathematics & Algorithms

- **Theorem:** Any 2D or 3D shape can be folded from a square of paper [Demaine, Demaine, Mitchell 1999]

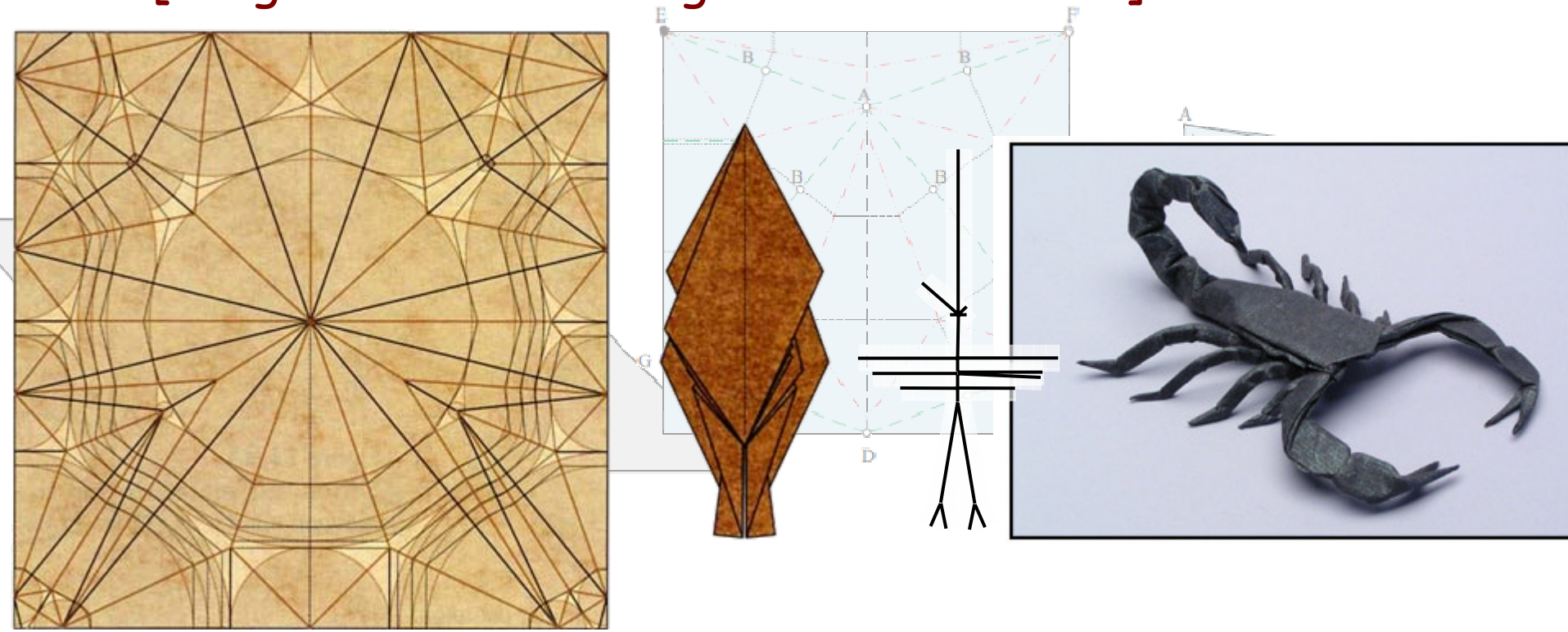






Origami Mathematics & Algorithms

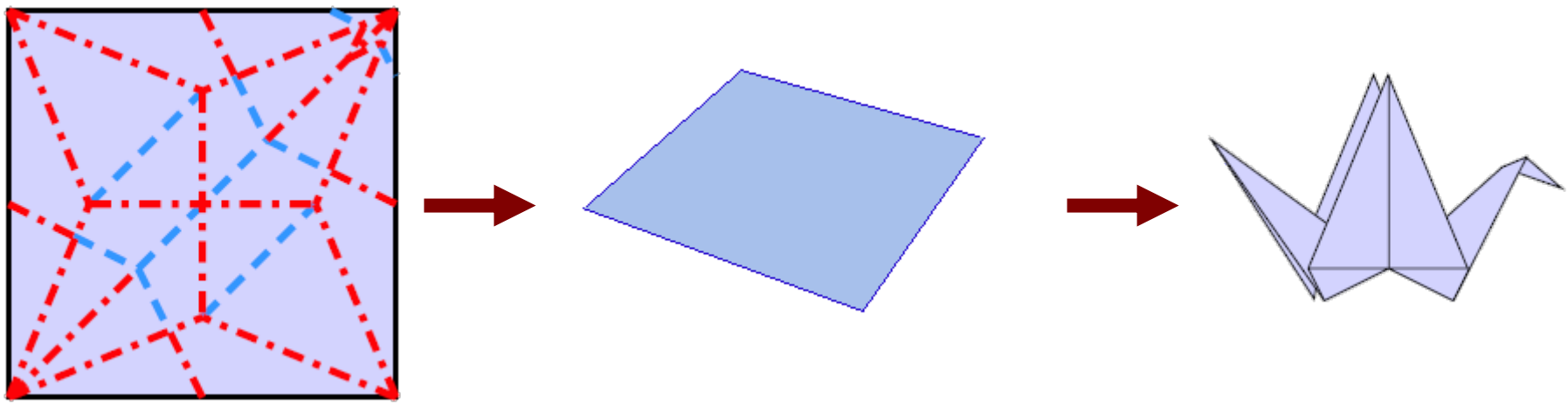
- **Algorithm** to fold optimal origami "base" with desired stick-figure projection
  [Lang 1996–2006; Lang & Demaine 2006]



Folded States vs. Folding Motions

- Folded state = origami model
- Folding motion = How you got there

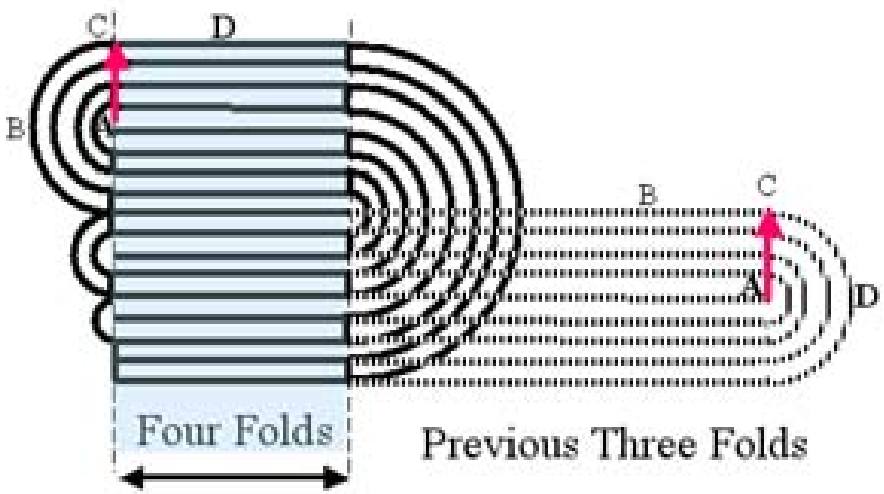
Every folded state can be made by a folding motion: No "locked origami"
[Demaine, Devadoss, Mitchell, O'Rourke 2004]



crease pattern folding motion folded state

Paper Thickness
[Galivan 2001]

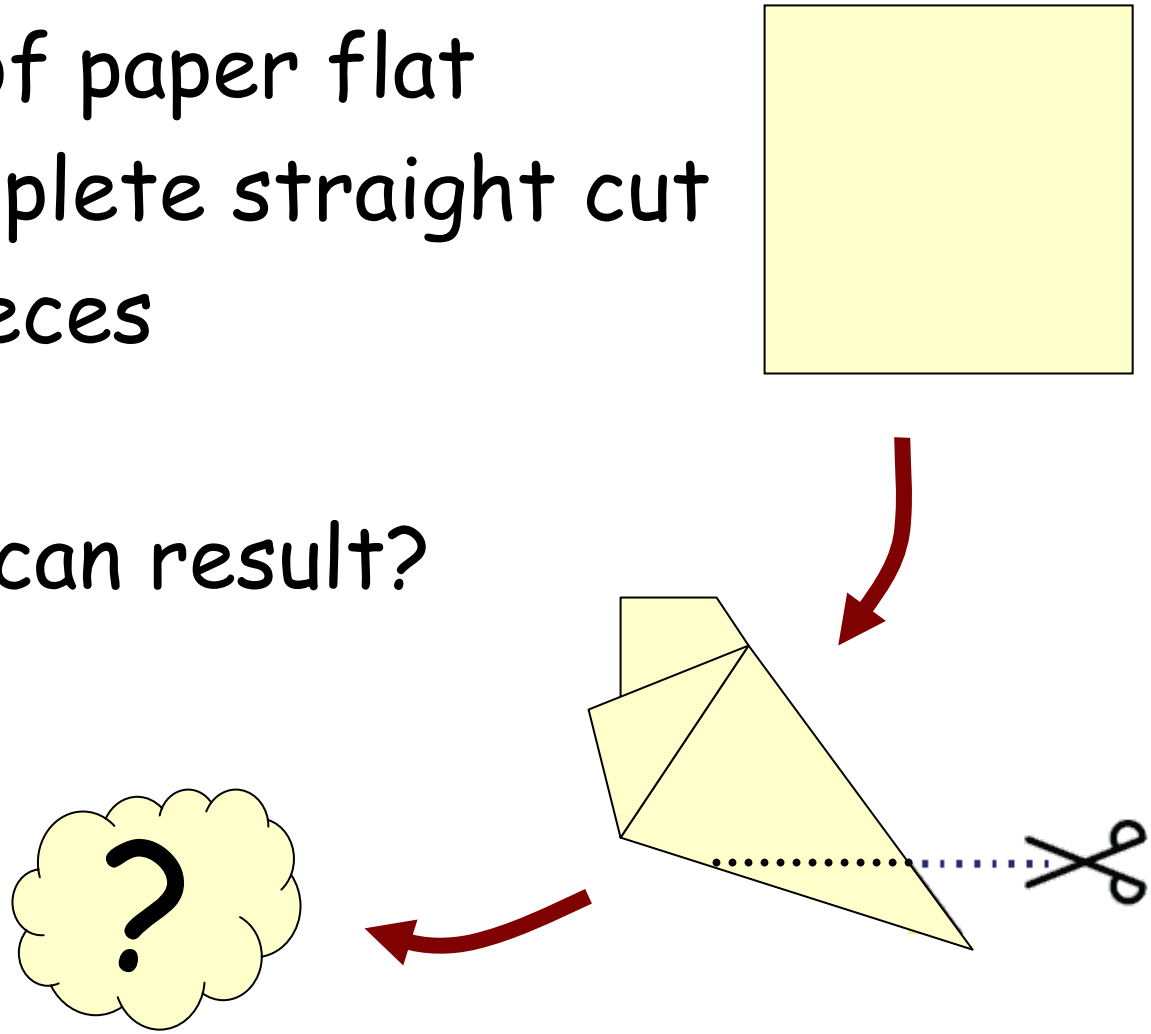
- Analysis of paper "loss" from repeatedly folding in half
- $\frac{3}{4}$-mile long paper folds in half 12 times!



Fold-and-Cut Problem

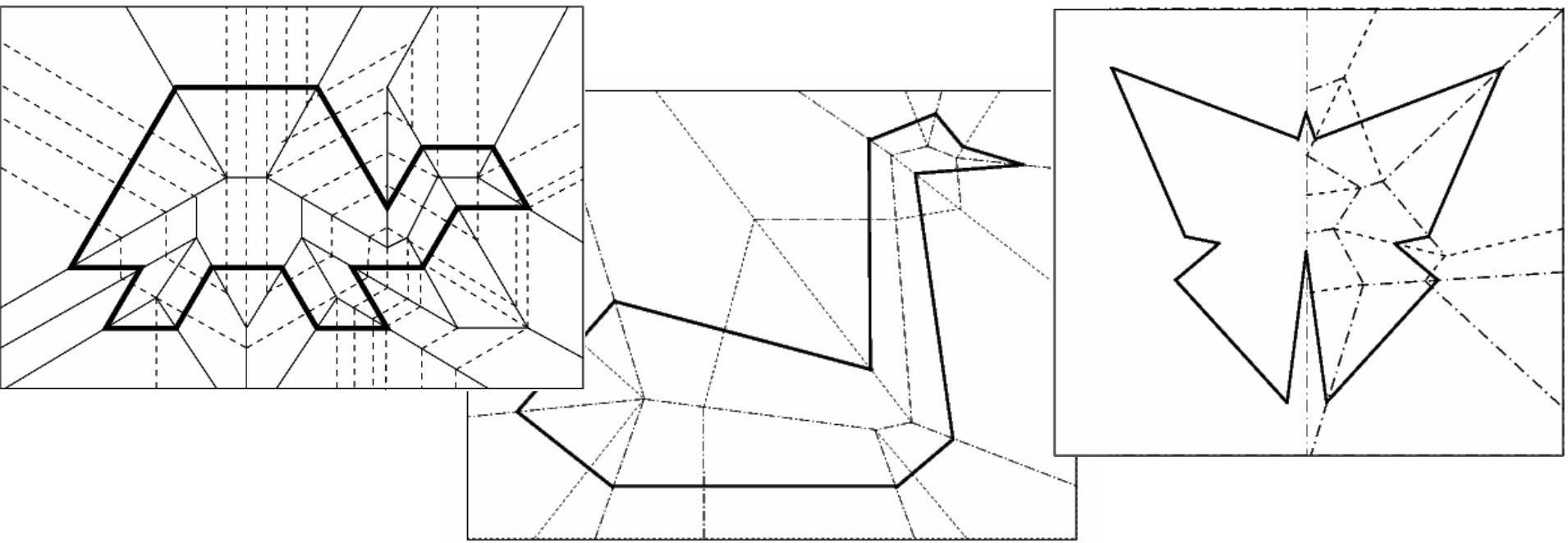
- Fold a sheet of paper flat
- Make one complete straight cut
- Unfold the pieces

- What shapes can result?



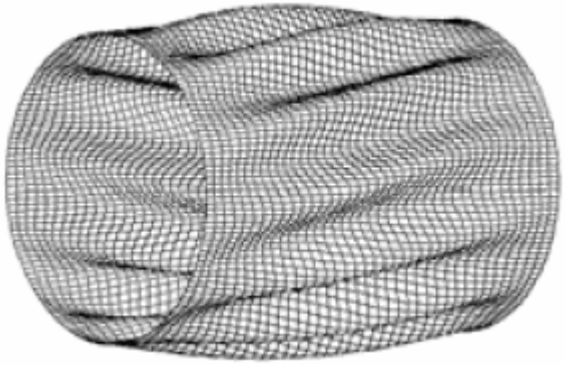
Fold-and-Cut Result

- Any collection of straight cuts can be made by folding flat & one straight cut
  [Demaine, Demaine, Lubiw 1998]
  [Bern, Demaine, Eppstein, Hayes 1999]



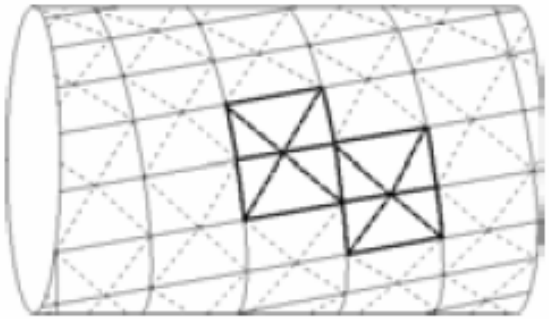
Deployable Structures

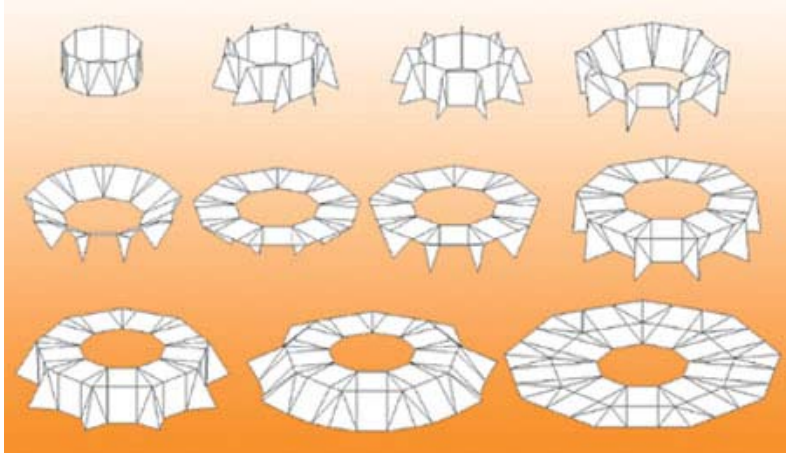
- Existing design ad hoc
- **Unsolved:** techniques & algorithms to design



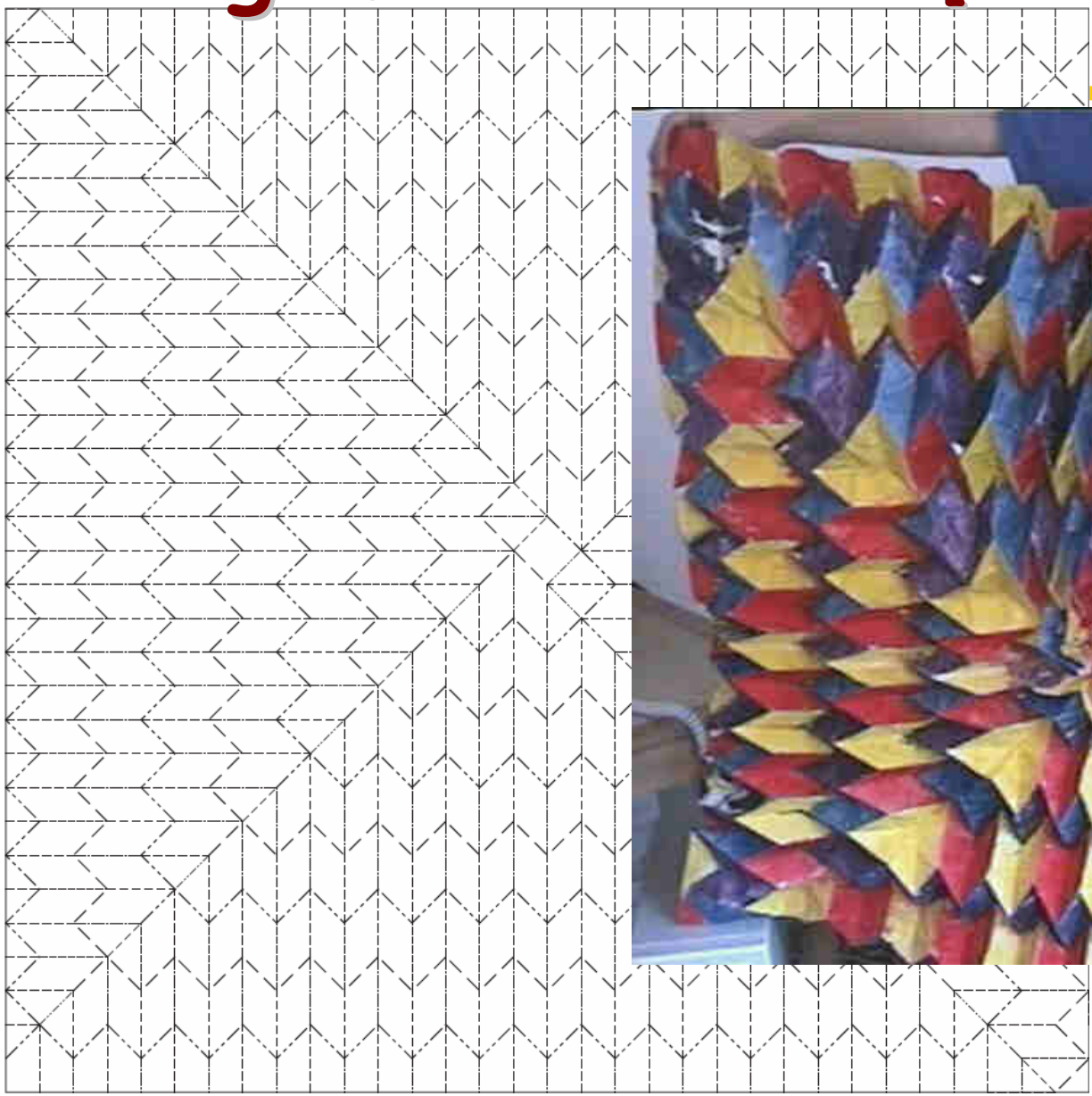
[You & Kuribayashi 2003]

[Lang & LLNL 2002]

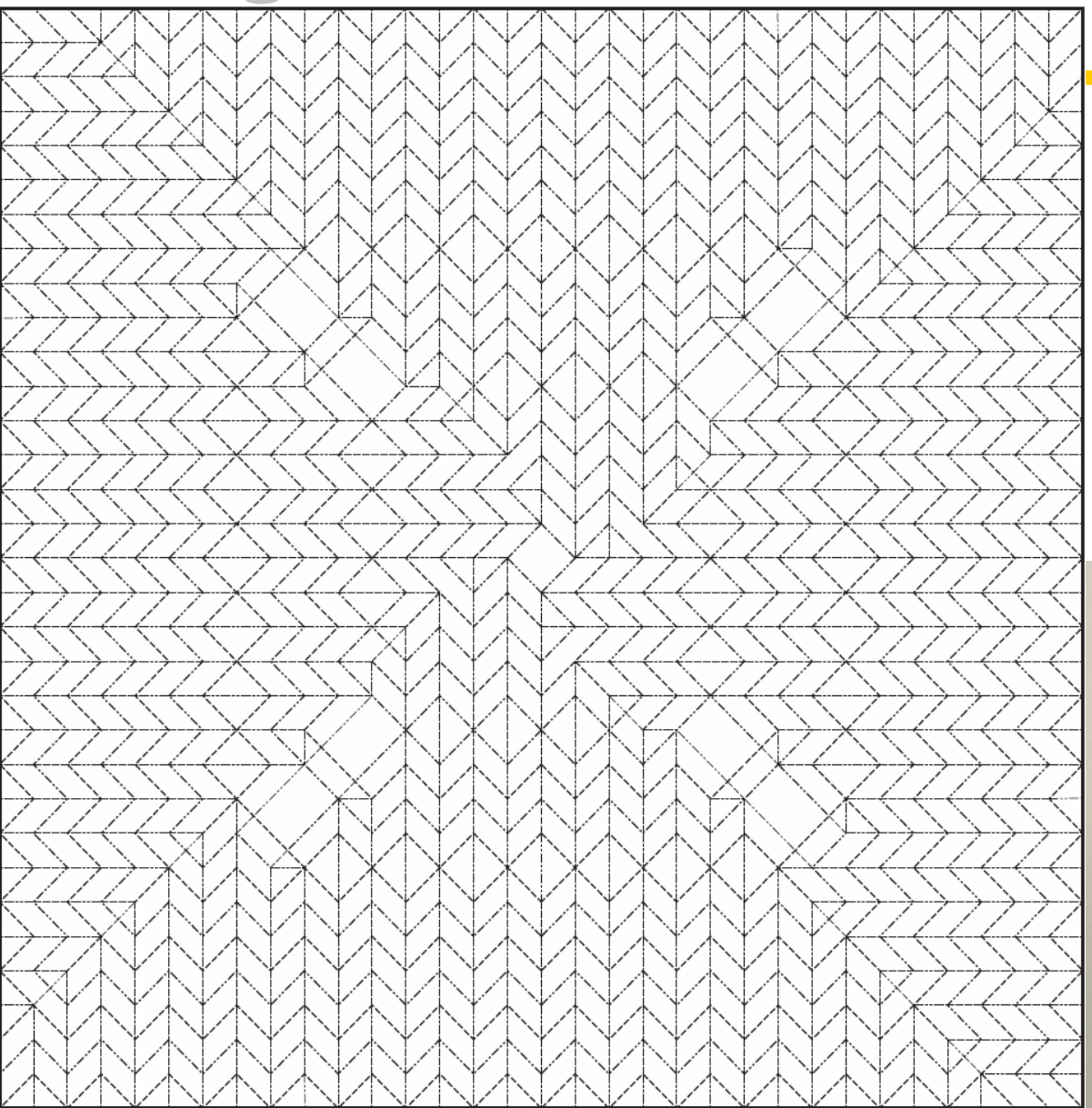




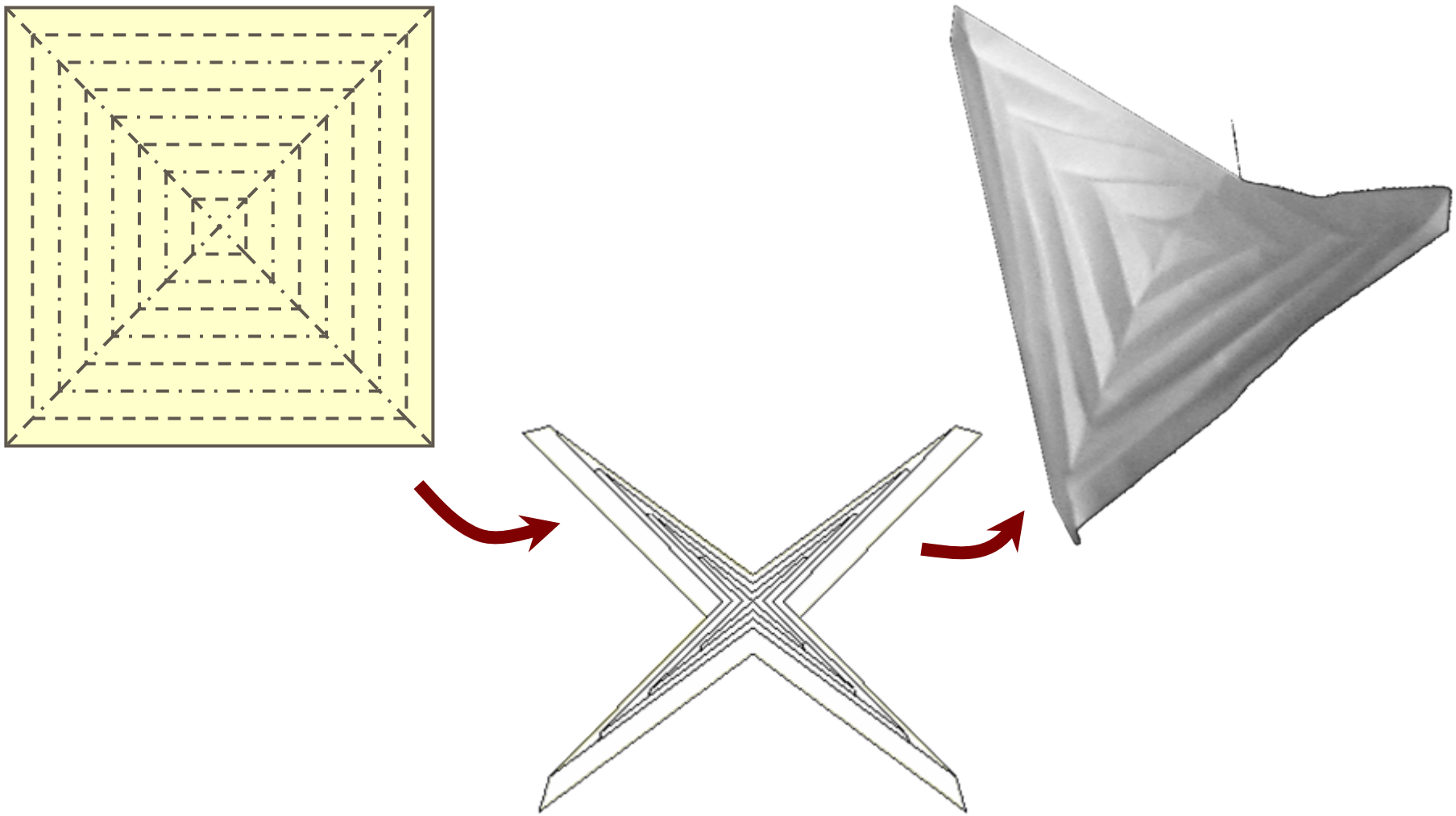
Origami Flashers [Jeremy Shafer]



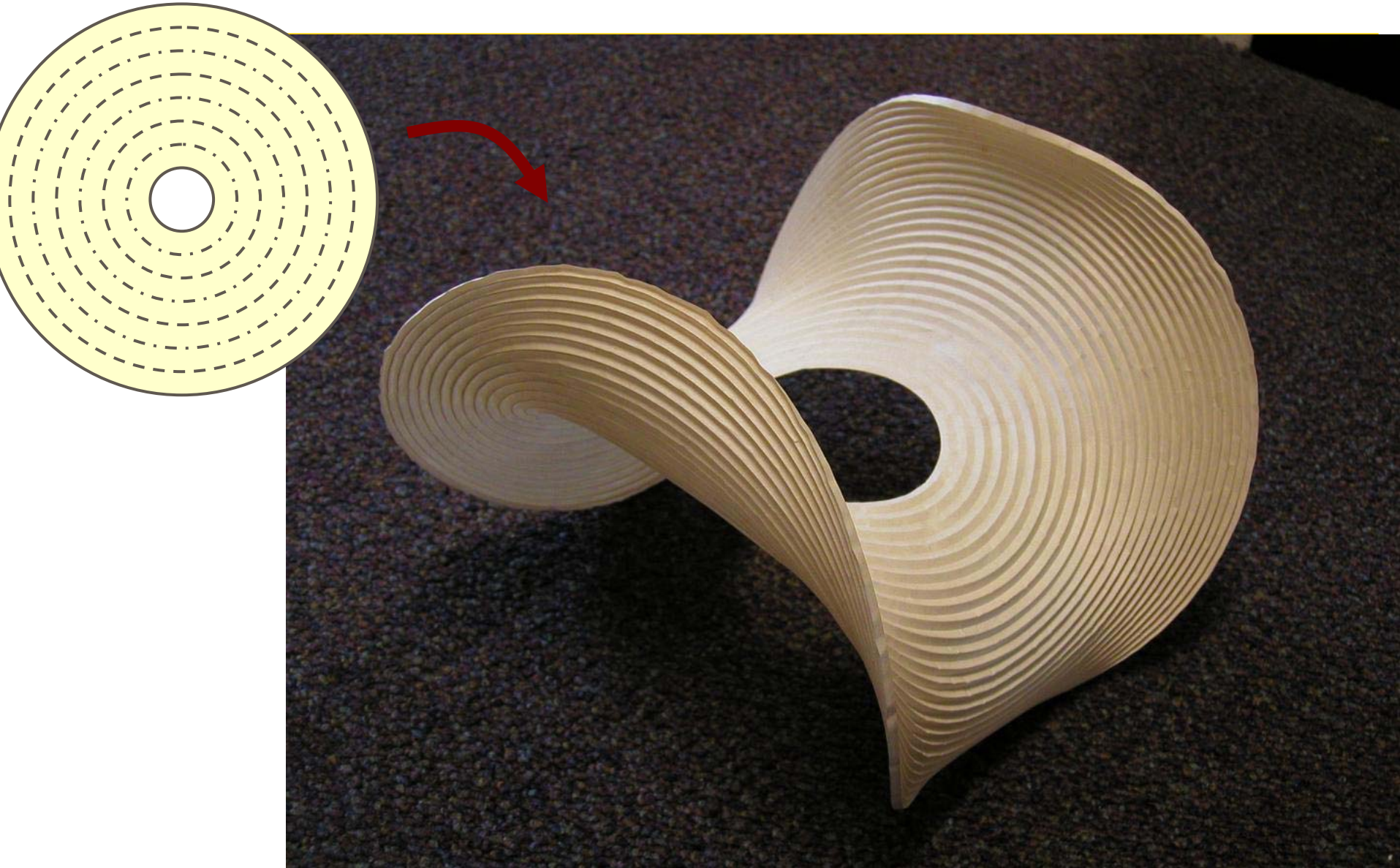
Origami Flashers [Jeremy Shafer]



Self-Folding Origami: Pleated Hyperbolic Paraboloid



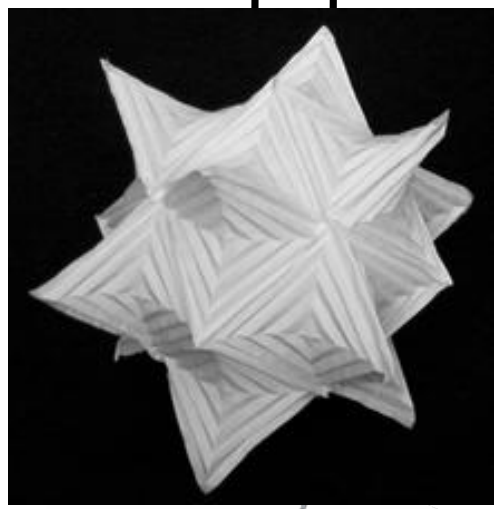
Circular Variation from Bauhaus

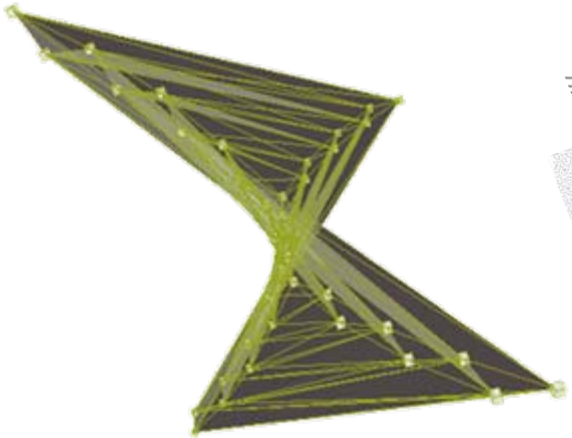


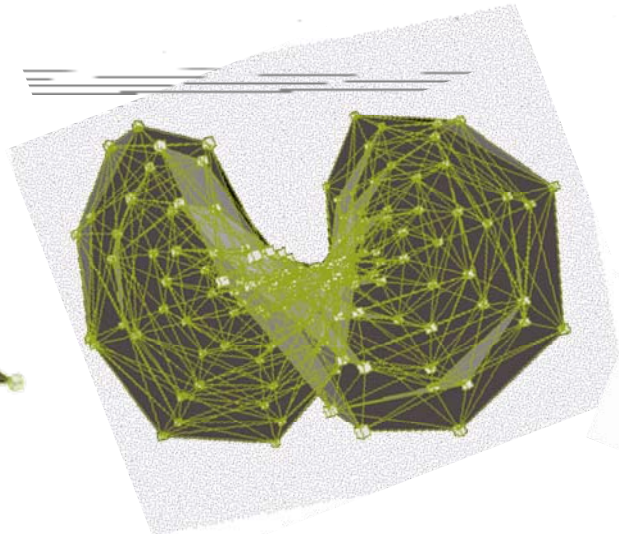
Circular Variations [Demaine & Demaine]

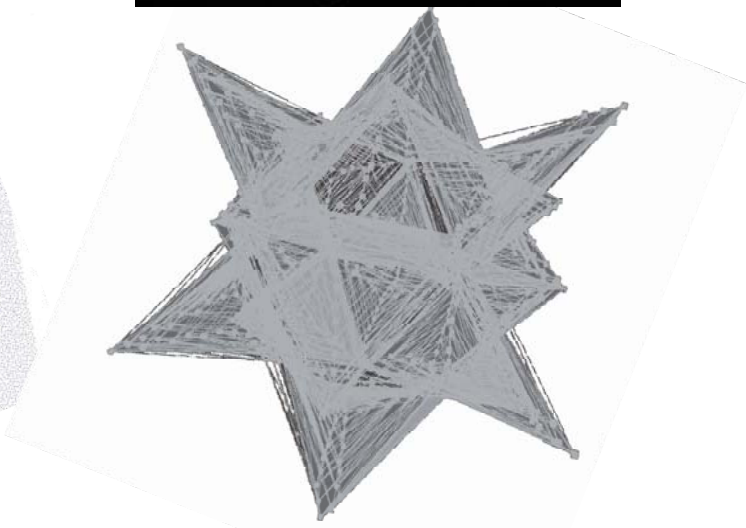
Simulating Paper Folding
[Demaine, Demaine, Fizel, Ochsendorf 2006]

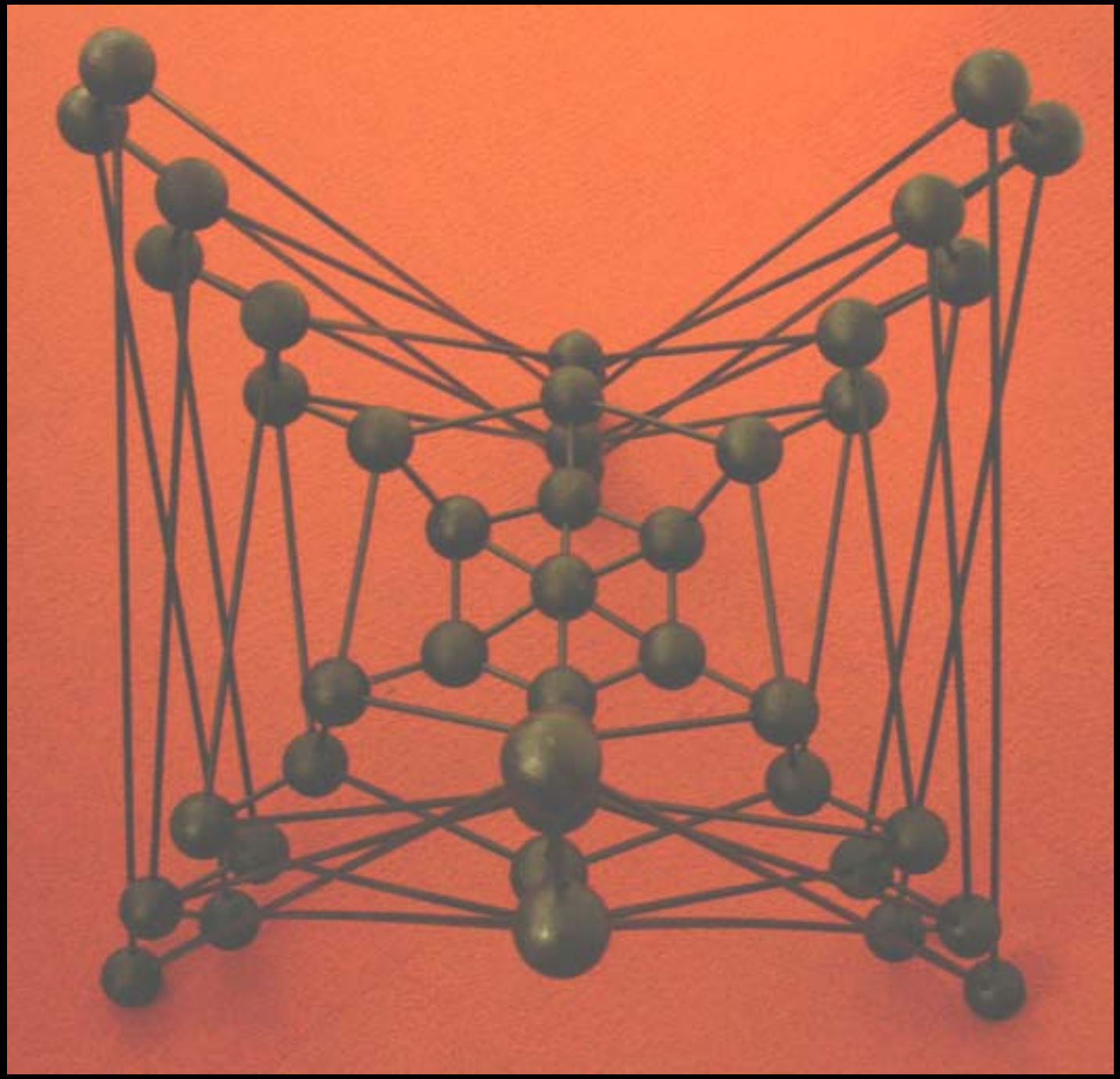
- Particle-spring simulation of forces in paper: elasticity and crease "failure"



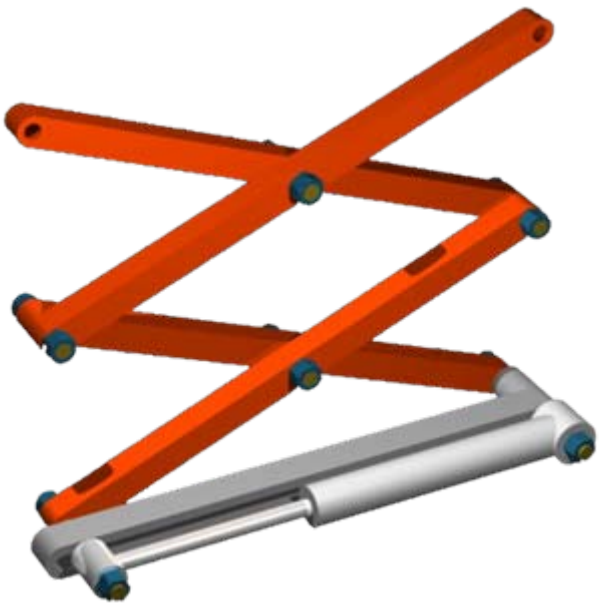






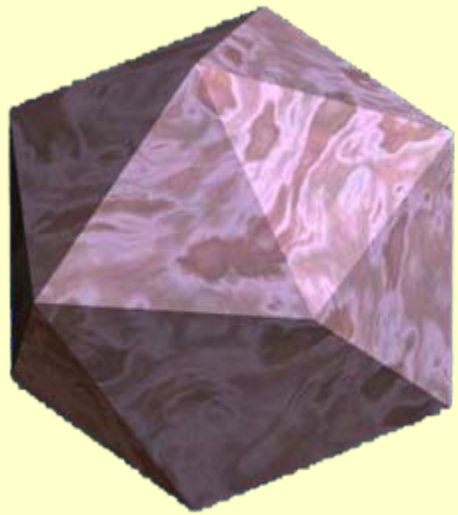


Geometric Folding



Linkages
(1D)

Paper
(2D)

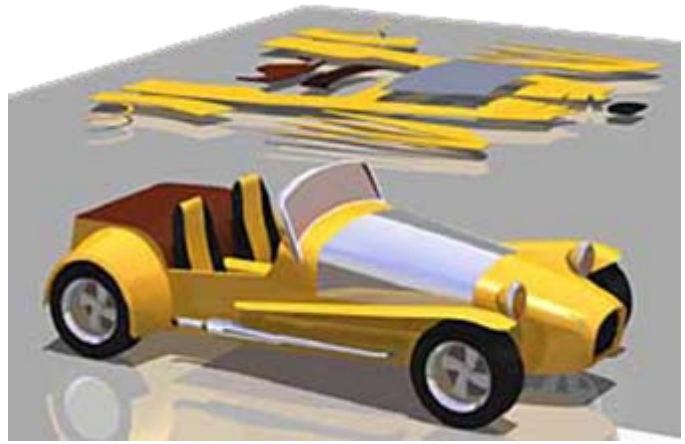


Polyhedra
(3D)

Unfolding Polyhedra

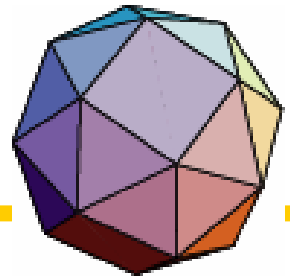
- Given 3D polyhedron
- Cut surface & unfold
- No overlap

- **Goals:**
  - Minimum cutting ($\Rightarrow$ minimum gluing)
  - Efficient layout

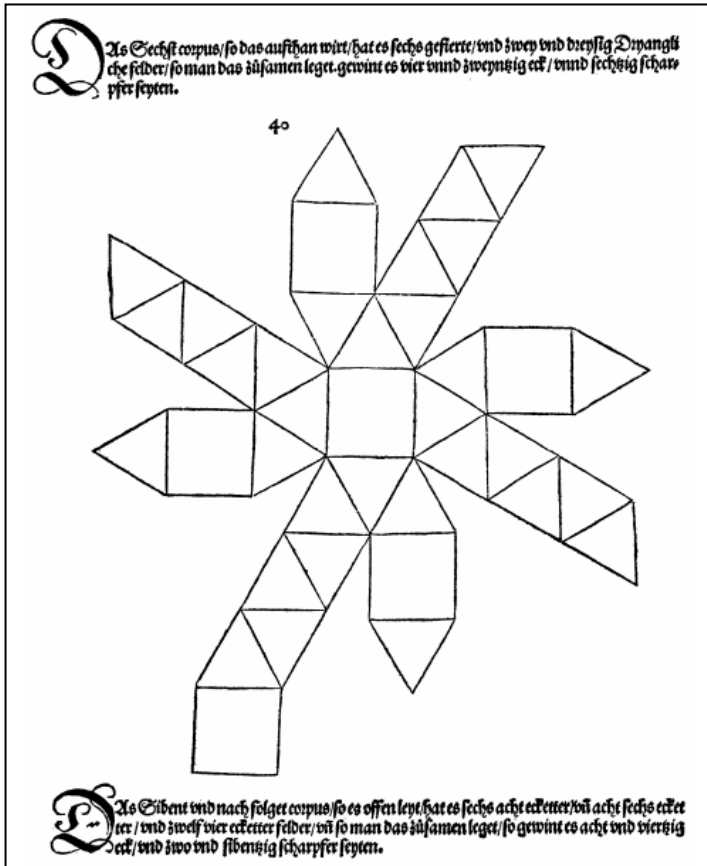


[Lundström Design]

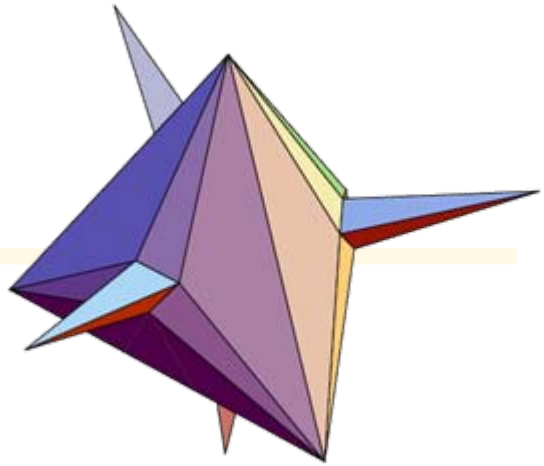
Theory of Unfolding Polyhedra



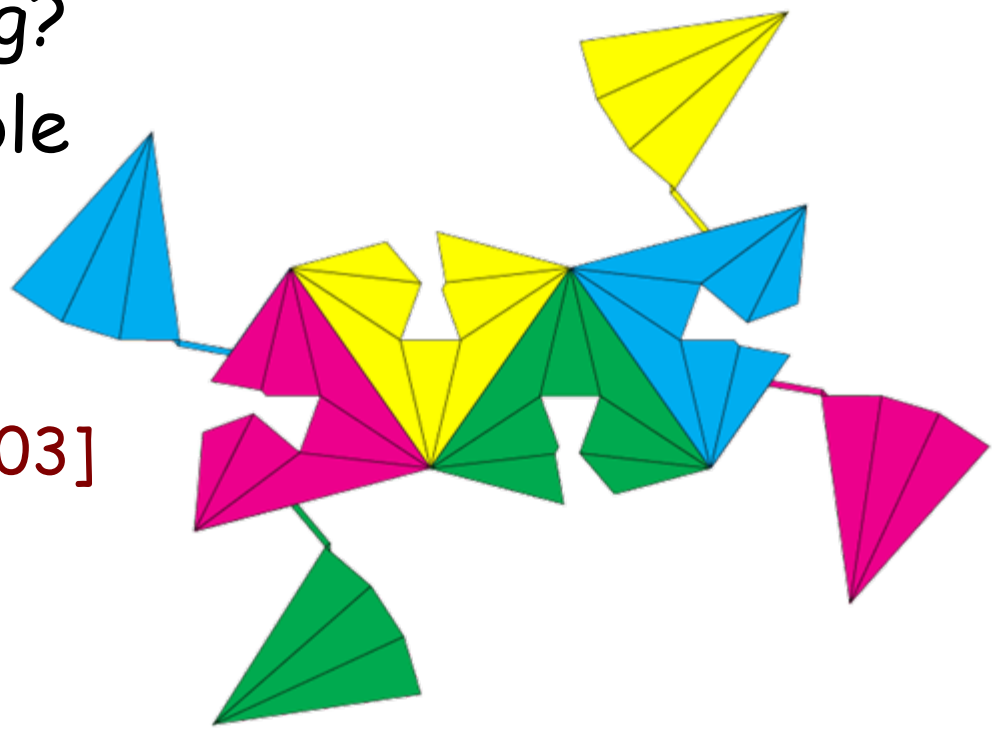
- Focus on one-piece unfoldings
- Convex polyhedra (no "dents")
  - **Known:** Always have a one-piece unfolding [Sharir & Schorr 1986; Aronov & O'Rourke 1991]
  - **Unsolved:** By cutting only along edges? [Dürer 1525]



Theory of Unfolding Polyhedra

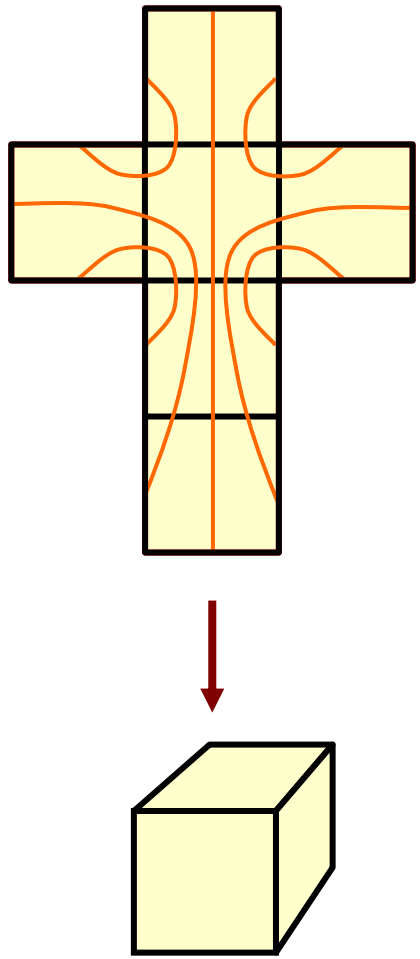


- Focus on one-piece unfoldings
- Nonconvex polyhedra
  - **Unsolved:** Always have a one-piece unfolding?
  - **Known:** Not possible just by edge cuts [Bern, Demaine, Eppstein, Kuo, Mantler, Snoeyink 2003]



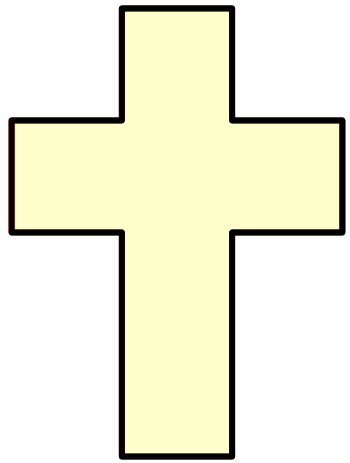
Folding Polygons into Polyhedra

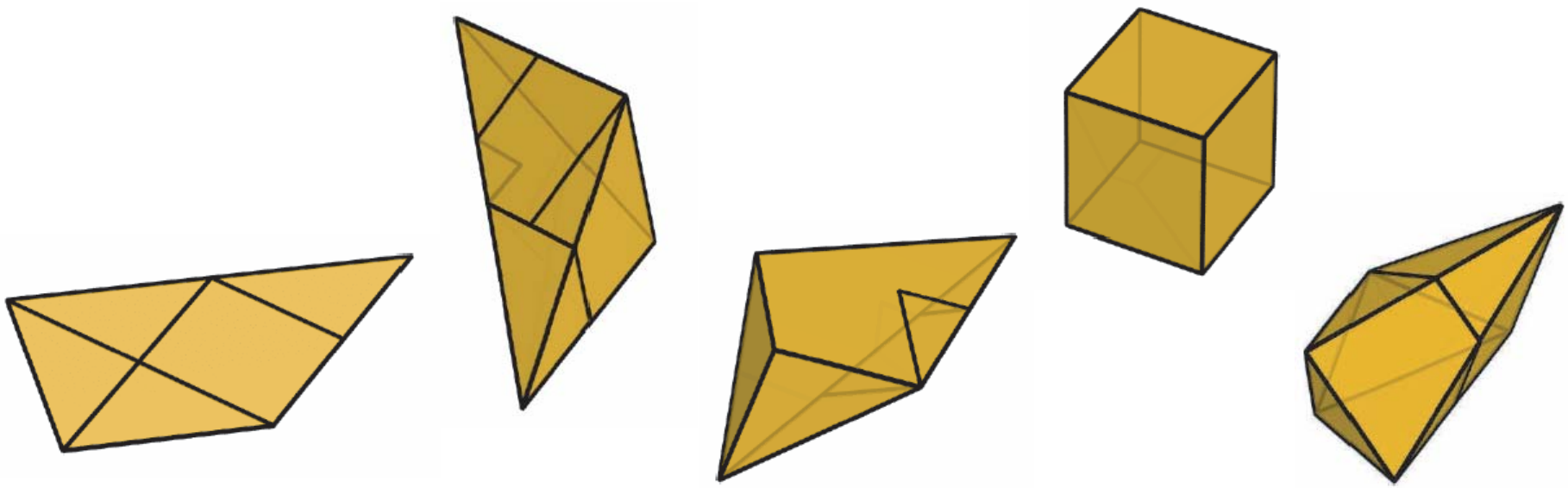
- Given polygon of paper
- Fold arbitrarily
- Glue boundary together
- What **convex** polyhedra can be made?



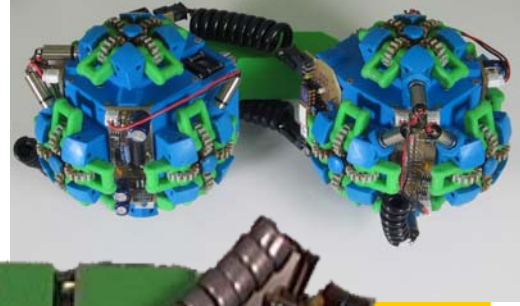
Folding Polygons into Polyhedra

- Efficient algorithms to find all gluings into convex polyhedra [Demaine, Demaine, Lubiw, O'Rourke 2002]
  - **Unsolved:** Efficient algorithms to find actual polyhedra formed

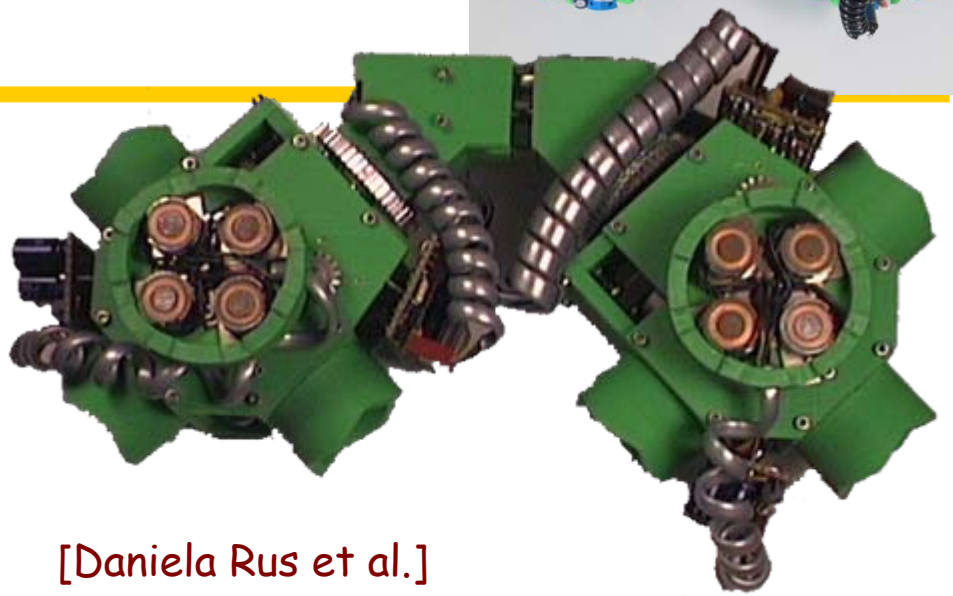




Reconfigurable Robotics



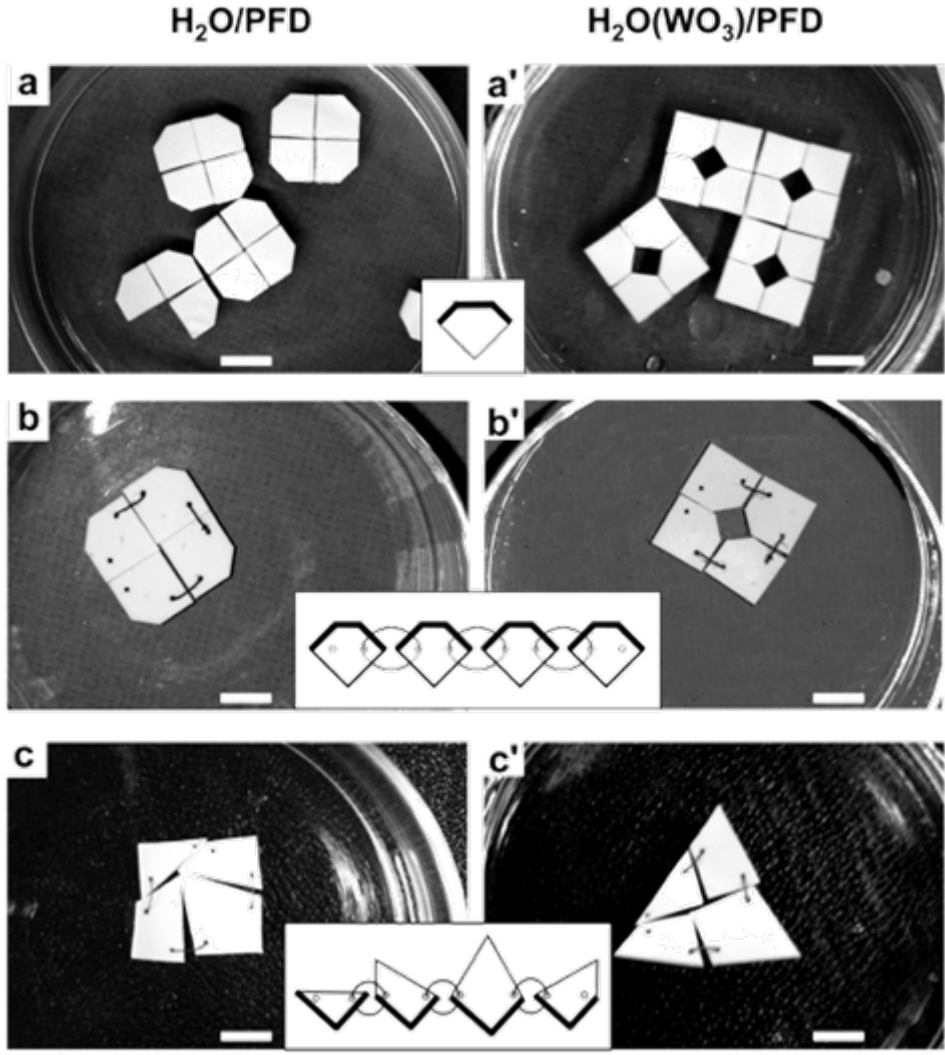
- Crux of many reconfigurable robots is the **attach/detach** mechanism
- Hinged polyhedra suggest that components can remain connected [O'Rourke]



[Daniela Rus et al.]

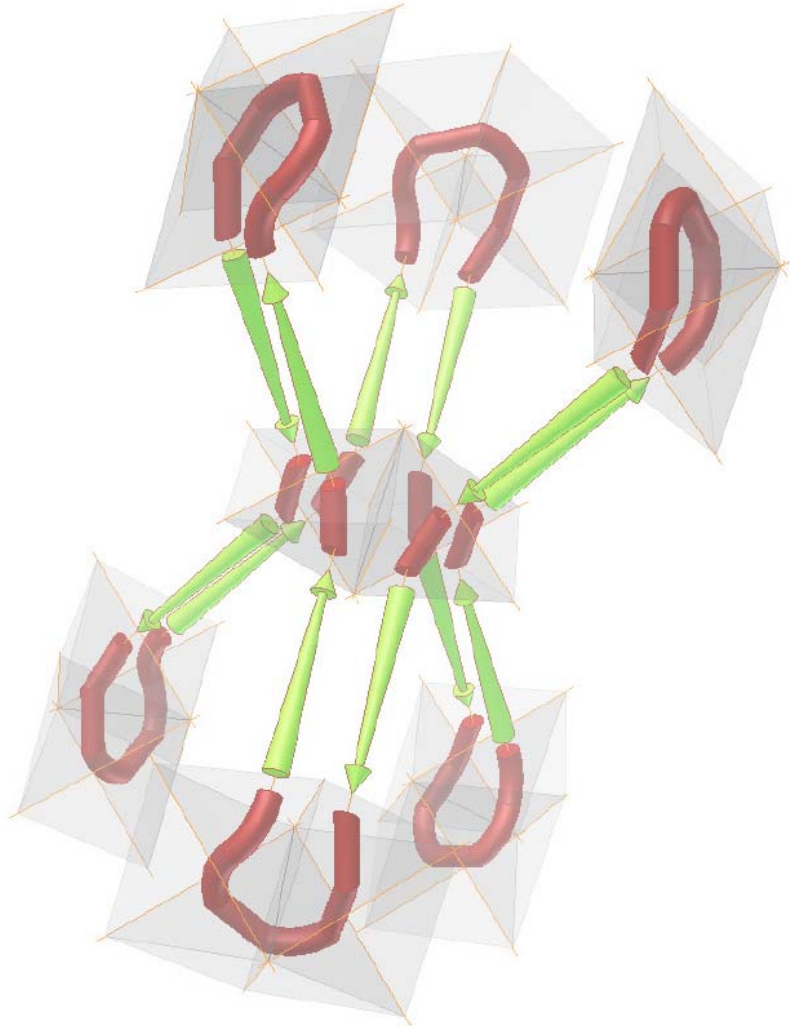
Self-Assembly & Nanomanufacturing

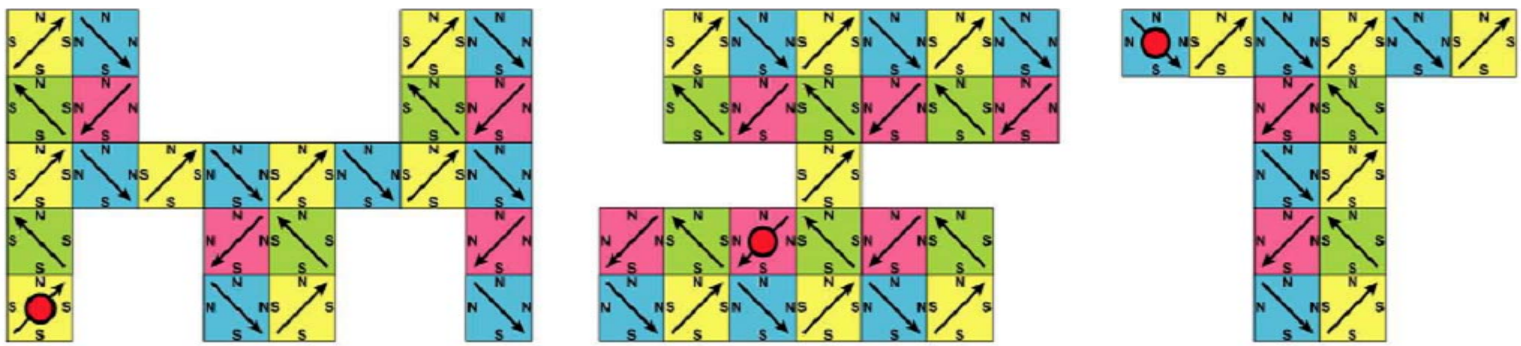
- Millimeter-scale "self-working" 2D hinged polygons
  [Mao, Thalladi, Wolfe, Whitesides, Whitesides 2002]



Self-Assembly & Nanomanufacturing

- Generalization to arbitrary desired 3D shapes via hinged polyhedra (currently at macro level) [Demaine, Griffith, Jacobson 2007]





DNA Folding

- Synthetic DNA to fold into desired polygon [Rothemund — Nature 2006]