Origami, Linkages, & Polyhedra: Folding with Algorithms

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New Book

- www.gfalop.org
- Appearing this month
- Cambridge University Press
- ~600 pages
Geometric Folding

- Linkages (1D)
- Paper (2D)
- Polyhedra (3D)
Folding is Everywhere: Linkages

Mechanics

Robotics

Graphics

Biology

[Leclercq, Akkouche, Galin 2001]

HIV protease
Folding is Everywhere: Paper

Origami

Deployable structures

Airbags
Folding is Everywhere: Polyhedra

Sheet-metal manufacturing

Reconfigurable robotics, self-assembly, nanomanufacturing
Geometric Folding

- Linkages (1D)
- Paper (2D)
- Polyhedra (3D)
Linkages

How to Draw a Straight Line;
A Lecture on Linkages.

London:
Macmillan and Co.
1877.

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Basic Questions about Linkages

Given a linkage...

- **Rigidity:**
  - Does the linkage move at all?

- **Universality:**
  - Into what configurations can it fold?

- **Motion planning:**
  - How do we get there?
Rigidity

- What linkages can move at all?
- **Rigid frameworks**: buildings, bridges, etc.

[Big Dig in Boston]

[Chubynsky, Hespenheide, Jacobs, Kuhn, Lei, Menor, Rader, Thorpe, Whiteley, Zavodszky 2003]
Rigidity

• What linkages can move at all?
  - **Known**: Characterization in 2D [Laman 1970]
  - **Unsolved**: Most problems in 3D
Universality

- Can a linkage move universally between any two configurations?

- Wire bending
- Hydraulic tube bending
- Robotic arm folding
Universality

- Can a chain linkage move universally between any two configurations?
  - Yes in 2D
    [Connelly, Demaine, Rote — FOCS 2000]
  - No in 3D
    [Cantarella & Johnston 1998]
  - Yes in 4D
    [Cocan & O’Rourke 2002]
Universality: Unfolding 2D Chains
[Cantarella, Demaine, Iben, O’Brien — SoCG 2004]
Motion Planning
[Iben, O’Brien, Demaine — SIGGRAPH 2004, SoCG 2006]

- Find short motion from A to B (if possible)
- Universality results give new insight into cases of interest, e.g., polygon morphing
Challenge in 3D

- **3D chains can be locked:**
  [Cantarella & Johnston 1998]

- **Unsolved:** Which 3D chains are locked?

- **Known:** Motion planning of 3D chains is computationally intractable
  [Alt, Knauer, Rote, Whitesides 2004]
Proteins

- DNA encodes proteins in genetic code
- Proteins are “fundamental building blocks of life”
Protein Folding

**Primary protein structure**
is a sequence of a chain of amino acids

**Secondary protein structure**
occur when the sequence of amino acids
are linked by hydrogen bonds

**Tertiary protein structure**
occur when certain attractions are present
between alpha helices and pleated sheets.

**Quaternary protein structure**
is a protein consisting of more than one
amino acid chain.
Importance of Protein Folding

- Geometry of a protein folding is an important aspect of its behavior.
- Prediction of protein folding, and synthesis of proteins with desired foldings, are central problems in computational biology.
  - Drug design
  - Preventing diseases (e.g., Alzheimer’s, mad-cow disease, cystic fibrosis, some forms of cancer)
Mechanics of Protein Folding

- Protein backbone is roughly a 3D chain with fixed-angle constraints

- Natural question:
  - 3D chains are hard to fold
  - How does nature fold proteins so easily?
Mechanics of Protein Folding

Why do proteins fold easily?

Possible answer:
- **Ribosome** constructs proteins, enforcing geometric constraints

[Nissen, Hansen, Ban, Moore, Steitz — Science 2000]
Mechanics of Protein Folding

- Why do proteins fold easily?
- Possible answer:
  - Ribosome constructs proteins, enforcing geometric constraints
- Cone model: [Demaine, Langerman, O’Rourke 2006]
  - All producible states can reach each other
  - Flattenable
  - Helical canonical state
Geometric Folding

Linkages
(1D)

Paper
(2D)

Polyhedra
(3D)
Origami

- Perhaps as old as paper itself (105 AD)
- Revolution in complex origami design over past ~25 years
Black Forest Cuckoo Clock by Robert Lang

Mask by Eric Joisel

Bat by Michael LaFosse

Pangolin by Eric Joisel

Photos from Origamido: Masterworks of Paper Folding
Roosevelt Elk, opus 358

Mt Diablo Tarantula, opus 481

Dancers, opp. 457 & 458

Hermit Crab

Tree Frog, opus 280

Koi, opus 425

Models & photos by Robert Lang
Explosion in technical origami thanks in part to growing mathematical and computational understanding of origami.
**Theorem:** Any 2D or 3D shape can be folded from a square of paper

[Demaine, Demaine, Mitchell 1999]
• **Algorithm** to fold optimal origami “base” with desired stick-figure projection

[Lang 1996–2006; Lang & Demaine 2006]
Folded States vs. Folding Motions

- **Folded state**: origami model
- **Folding motion**: how you got there

Every folded state can be made by a folding motion: No "locked origami"  
[Demaine, Devadoss, Mitchell, O’Rourke 2004]
Paper Thickness
[Galivan 2001]

- Analysis of paper “loss” from repeatedly folding in half
- \( \frac{3}{4} \)-mile long paper folds in half 12 times!
Fold-and-Cut Problem

- Fold a sheet of paper flat
- Make one complete straight cut
- Unfold the pieces

- What shapes can result?
Fold-and-Cut Result

- Any collection of straight cuts can be made by folding flat & one straight cut

[Demaine, Demaine, Lubiw 1998]
[Bern, Demaine, Eppstein, Hayes 1999]
Deployable Structures

- Existing design ad hoc
- Unsolved: techniques & algorithms to design

[You & Kuribayashi 2003]

[Lang & LLNL 2002]
Origami Flashers [Jeremy Shafer]
Origami Flashers

[Jeremy Shafer]
Self-Folding Origami: Pleated Hyperbolic Paraboloid
Circular Variation from Bauhaus
Circular Variations [Demaine & Demaine]
Simulating Paper Folding
[Demaine, Demaine, Fizel, Ochsendorf 2006]

- Particle-spring simulation of forces in paper: elasticity and crease “failure”
Geometric Folding

Linkages (1D)

Paper (2D)

Polyhedra (3D)
Unfolding Polyhedra

- Given 3D polyhedron
- Cut surface & unfold
- No overlap

**Goals:**
- Minimum cutting (⇒ minimum gluing)
- Efficient layout

[Lundström Design]
Theory of Unfolding Polyhedra

- Focus on one-piece unfoldings
- Convex polyhedra (no “dents”)
  - Known: Always have a one-piece unfolding
    [Sharir & Schorr 1986; Aronov & O’Rourke 1991]
  - Unsolved: By cutting only along edges?
    [Dürer 1525]
Theory of Unfolding Polyhedra

• Focus on one-piece unfoldings
• Nonconvex polyhedra
  - Unsolved: Always have a one-piece unfolding?
  - Known: Not possible just by edge cuts [Bern, Demaine, Eppstein, Kuo, Mantler, Snoeyink 2003]
Folding Polygons into Polyhedra

- Given polygon of paper
- Fold arbitrarily
- Glue boundary together
- What **convex** polyhedra can be made?
Folding Polygons into Polyhedra

- Efficient algorithms to find all gluings into convex polyhedra
  [Demaine, Demaine, Lubiw, O'Rourke 2002]
  - **Unsolved:** Efficient algorithms to find actual polyhedra formed
Reconfigurable Robotics

- Crux of many reconfigurable robots is the attach/detach mechanism
- Hinged polyhedra suggest that components can remain connected [O’Rourke]
Self-Assembly & Nanomanufacturing

- Millimeter-scale “self-working” 2D hinged polygons
  [Mao, Thalladi, Wolfe, Whitesides, Whitesides 2002]
Self-Assembly & Nanomanufacturing

- Generalization to arbitrary desired 3D shapes via hinged polyhedra (currently at macro level) [Demaine, Griffith, Jacobson 2007]
DNA Folding

- Synthetic DNA to fold into desired polygon [Rothemund — Nature 2006]