The Spin-Hall Effect in Quantum Wires

A.W. Cummings, R. Akis, and D.K. Ferry

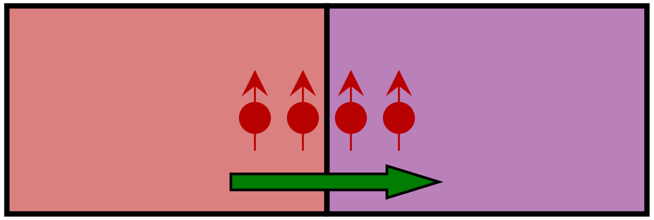
Department of Electrical Engineering and Center for Solid State Electronics Research
Arizona State University
Tempe, AZ 85287-8706, USA

Motivation - spintronics

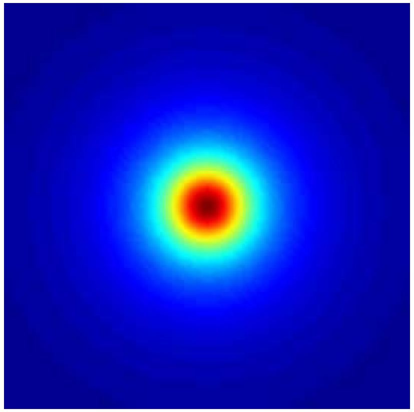
Devices based on the manipulation of an electron's spin

Common approaches include:

Injection of spin-polarized carriers into semiconductors with ferromagnetic contacts



Manipulation of spin via localized magnetic fields



However, an all-electrical approach to spintronics would be nice.

Spin-orbit coupling – free space

The Dirac equation describes a relativistic particle:

$$\left(c\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m_0 c^2+V\right)\psi=E\psi$$

Expansion of this equation in yields the single-particle Schrödinger equation, plus some spin-dependent terms

$$\left(\frac{p^2}{2m_0}+V+\frac{e\hbar}{2m_0}\boldsymbol{\sigma}\cdot\mathbf{B}-\frac{\hbar}{4m_0^2c^2}\boldsymbol{\sigma}\cdot\left(\mathbf{p}\times\nabla V\right)\right)\psi=E\psi$$

The last term on the right hand side couples the spin to the momentum in the presence of an electric field, hence the name "spin-orbit coupling"

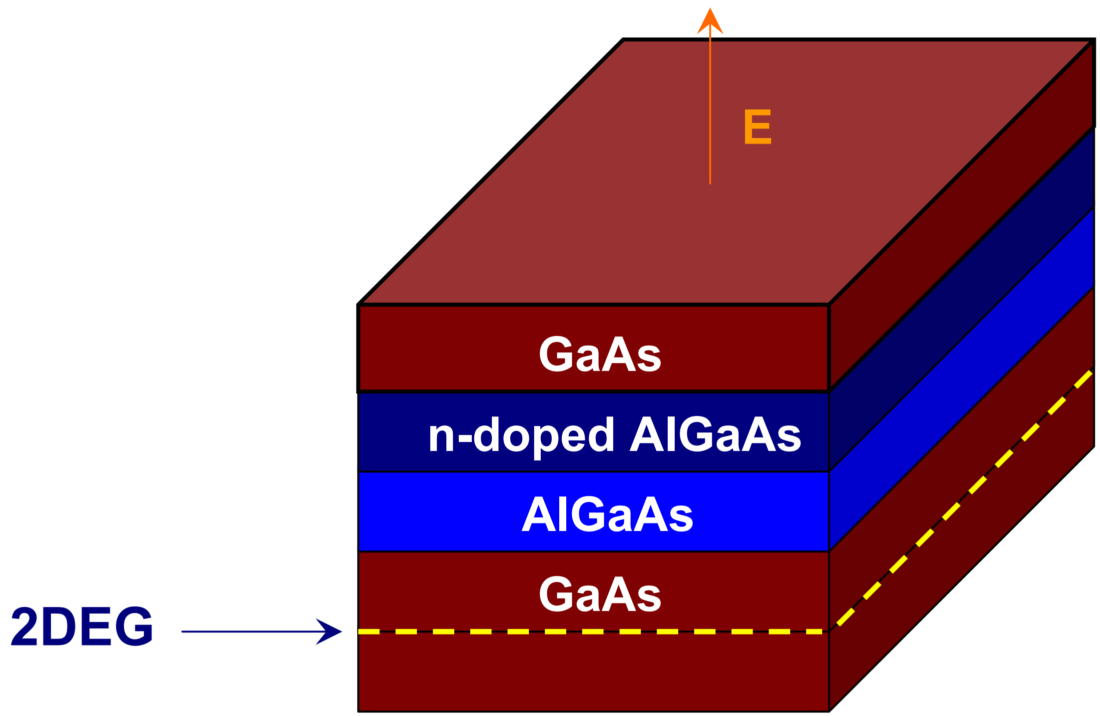
This effect is small in free space, but can be rather large in semiconductors

Spin-orbit coupling – semiconductors

In a semiconductor heterostructures the spin-orbit Hamiltonian is usually described by the Rashba form:

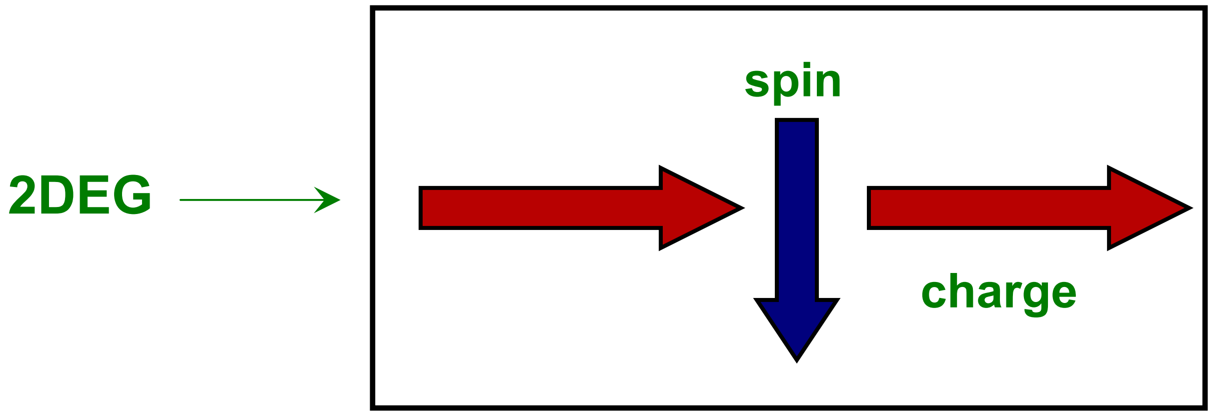
$$H_R = \boldsymbol{\alpha} \cdot (\boldsymbol{\sigma} \times \mathbf{k})$$

α is proportional to the electric field, which is perpendicular to the 2D electron gas formed in semiconductor heterostructures



The spin-Hall effect

It turns out that the Rashba Hamiltonian gives rise to a pure transverse spin current in response to a charge current

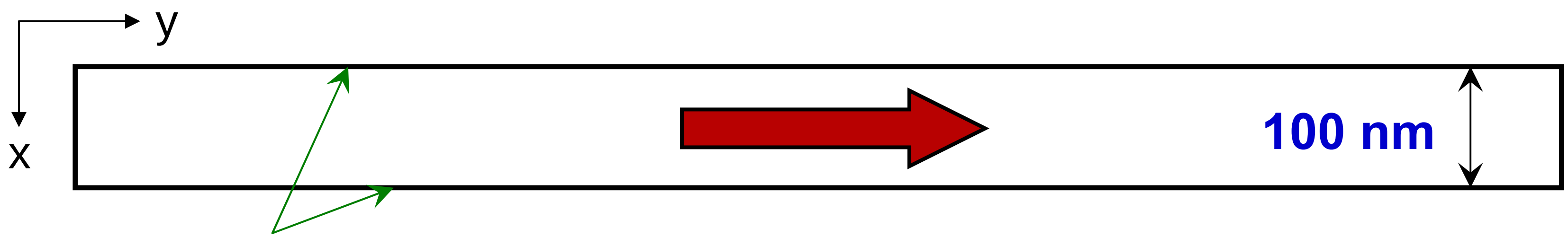


The associated spin-Hall conductivity has a universal value in the 2D plane, and is of much interest to the spintronics community.

How does this effect manifest itself in quasi-1D quantum wires?

The spin-Hall effect – 1D calculations

Assume a quantum wire with confinement along the *x*-axis and ballistic transport along the *y*-axis:



Also assume hard-wall boundary conditions along x

With an electric field along the *z*-axis, the Schrödinger equation becomes

$$H\psi = \left[\frac{\mathbf{p}^2}{2m^*} + \frac{\alpha_z}{\hbar}\left(\sigma_x p_y - \sigma_y p_x\right)\right]\psi = E\psi$$

Write the wave function as

$$\psi = e^{iky}\begin{bmatrix}\phi_\uparrow(x)\\ \phi_\downarrow(x)\end{bmatrix}$$

The spin-Hall effect – 1D calculations

We end up with an eigenvalue problem:

$$\begin{bmatrix} \dfrac{d^2}{dx^2} - k^2 & -k_\alpha \dfrac{d}{dx} - kk_\alpha \\ k_\alpha \dfrac{d}{dx} - kk_\alpha & \dfrac{d^2}{dx^2} - k^2 \end{bmatrix} \begin{bmatrix} \phi_\uparrow \\ \phi_\downarrow \end{bmatrix} = -\frac{2m^* E}{\hbar^2} \begin{bmatrix} \phi_\uparrow \\ \phi_\downarrow \end{bmatrix}$$

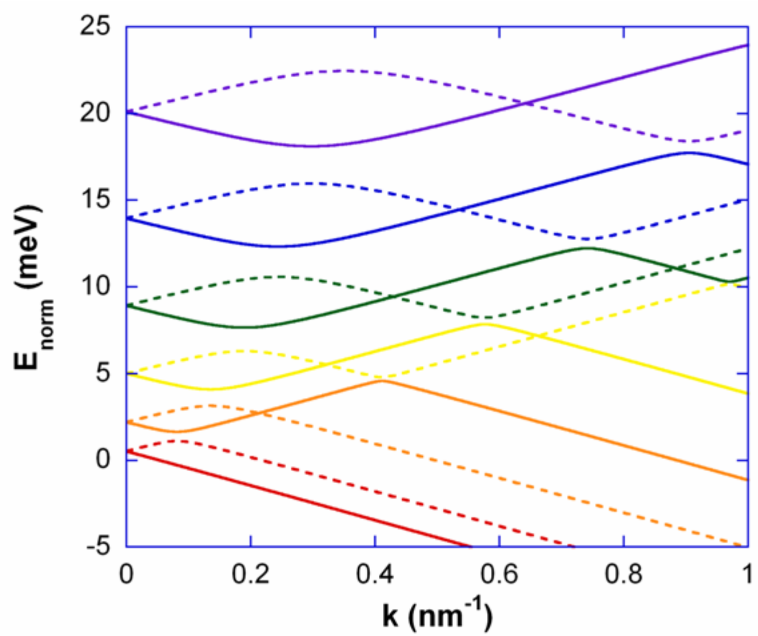
In the infinite 2D plane, the eigenvalues of the Rashba Hamiltonian are given by

$$E_\pm^R = \pm\alpha|\mathbf{k}|$$

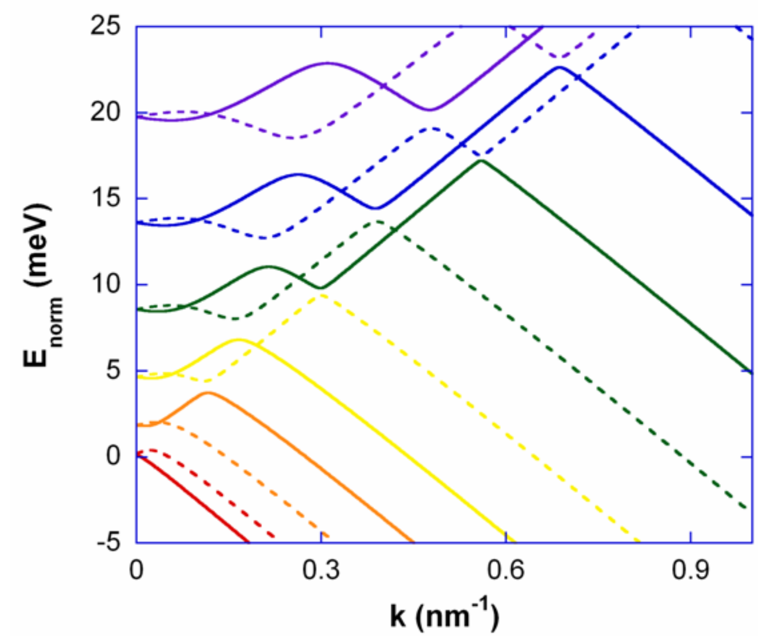
With quantum confinement, the picture is a little different…

1D calculations – band structure

α_z = 10 meV-nm



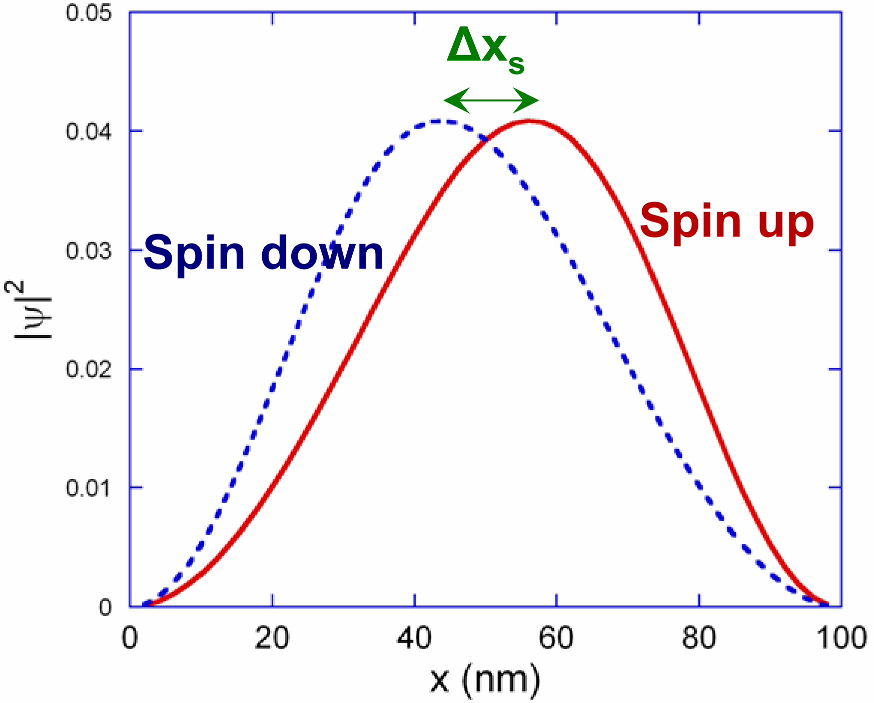
α_z = 30 meV-nm



Linear spin splitting only seen for small values of wave number

Band structure characterized by crossing and anti-crossing behavior

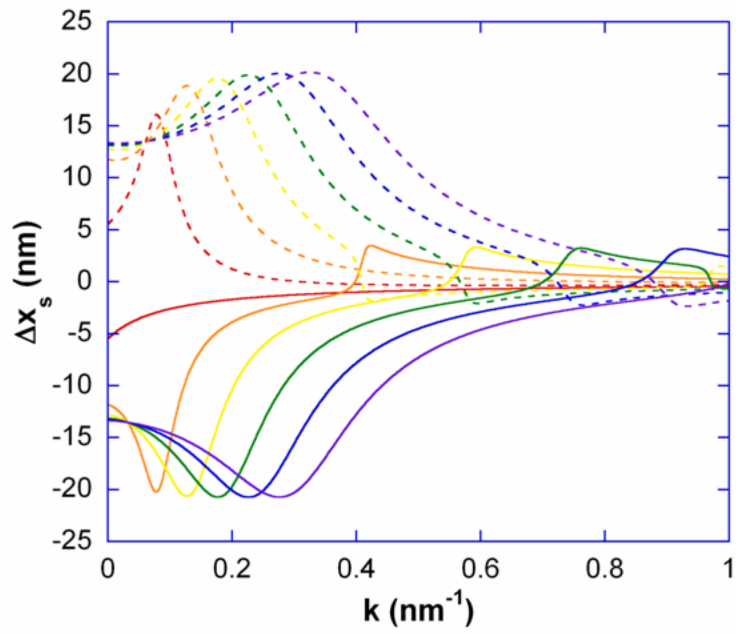
1D calculations – spin separation



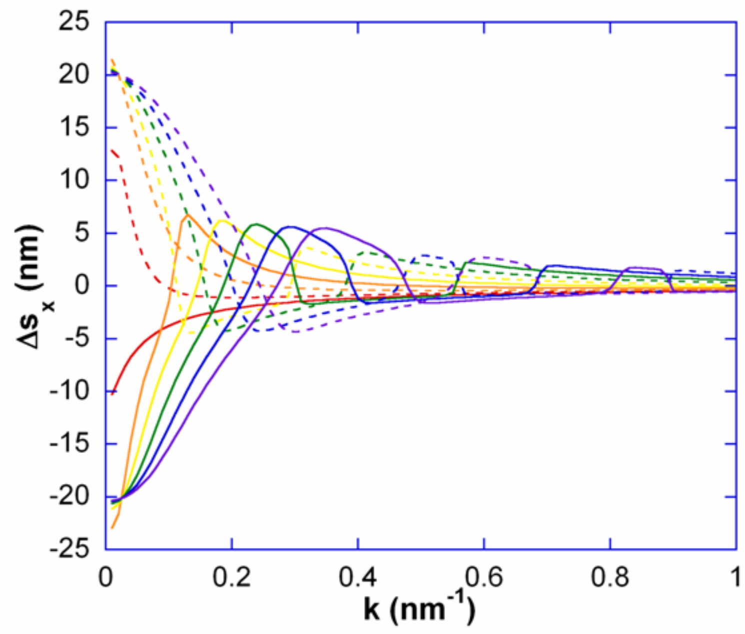
The spin separation is calculated as the difference of the expectation values of position of the spin-up and spin-down wave functions.

1D calculations – spin separation vs. k

α_z = 10 meV-nm



α_z = 30 meV-nm

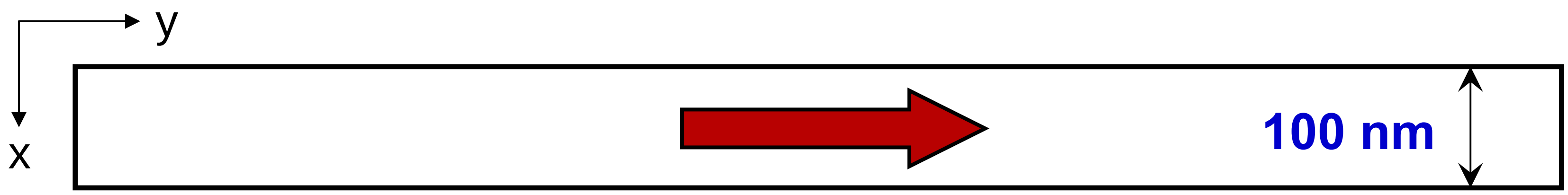


The lowest (non-interacting) subband behaves as expected

For the higher bands, the spin-Hall effect is enhanced by hybridization

The spin-Hall effect – 2D transport

Assume a quantum wire with confinement along the *x*-axis and ballistic transport along the *y*-axis:



Also assume a vector potential in the Landau gauge, A = (0, A_y, 0), in the Hamiltonian to include an applied magnetic field

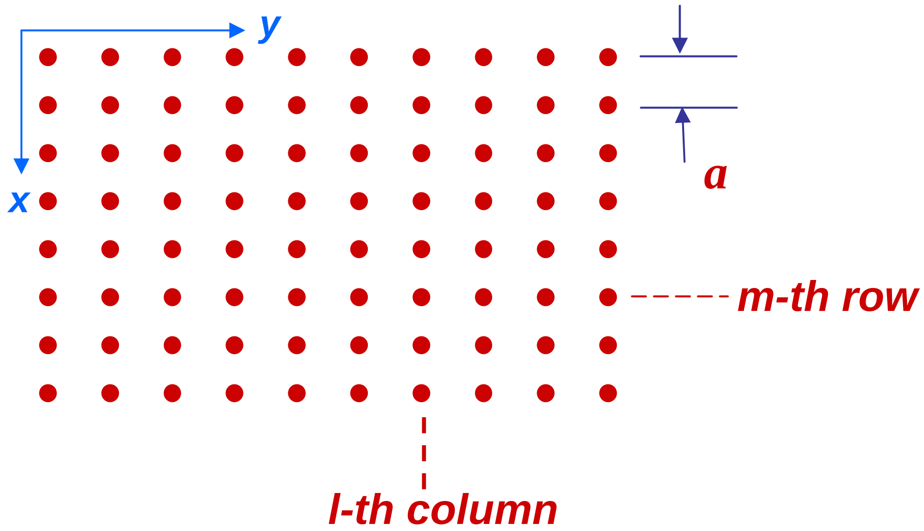
$$H\psi = \left[\frac{(\mathbf{p}+e\mathbf{A})^2}{2m^*} + \frac{\alpha_z}{\hbar}\left(\sigma_x\left(p_y + eA_y\right) - \sigma_y p_x\right) \right]\psi = E\psi \qquad (*)$$

Similar to before, write the wave function as

$$\psi = \begin{bmatrix} \psi^{\uparrow}(x,y) \\ \psi^{\downarrow}(x,y) \end{bmatrix}$$

The spin-Hall effect – 2D transport

The Schrödinger equation (*) can be discretized on a 2D grid:



After some manipulation, we arrive at a transfer equation for the wave function at slice *l+1* along the wire

$$\psi_{l+1} = \left(H_{l,l+1}\right)^{-1} H_l \psi_l + \left(H_{l,l+1}\right)^{-1} H_{l,l-1} \psi_{l-1} \qquad (**)$$

$$\psi_l = \begin{bmatrix} \psi_l^{\uparrow}(x,y) \\ \psi_l^{\downarrow}(x,y) \end{bmatrix}$$

The spin-Hall effect – 2D transport

Equation (**) can be recast as a transfer matrix equation which can be used to iterate through the structure

$$\begin{bmatrix} \psi_l \\ \psi_{l+1} \end{bmatrix} = \begin{bmatrix} 0 & I \\ \left(H_{l,l+1}\right)^{-1} H_{l,l-1} & \left(H_{l,l+1}\right)^{-1} H_l \end{bmatrix} \begin{bmatrix} \psi_{l-1} \\ \psi_l \end{bmatrix}$$

At the input and output of the wire, the incoming and outgoing modes are found via the eigenvalue equation

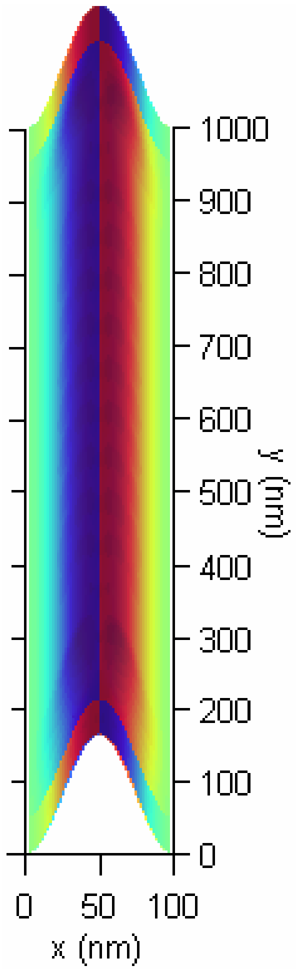
$$\lambda \begin{bmatrix} \psi_{-1} \\ \psi_0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ H_{0,1}^{-1} H_{0,-1} & H_{0,1}^{-1} H_0 \end{bmatrix} \begin{bmatrix} \psi_{-1} \\ \psi_0 \end{bmatrix}$$

where we assume the wave functions at slices *l* and *l+1* are related by some phase factor

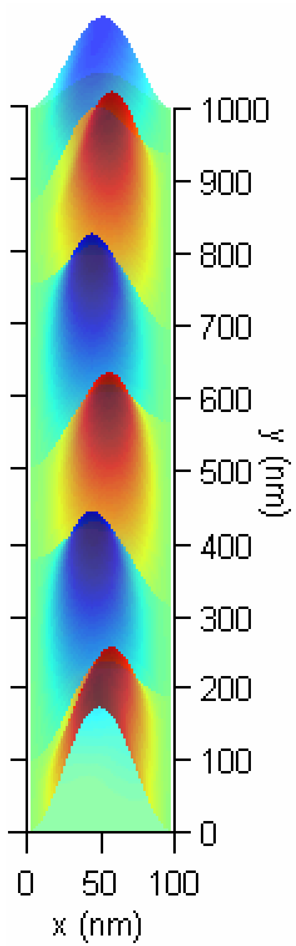
$$\psi_{l+1} = \lambda \psi_l$$

2D transport – single wire

Input both spin modes



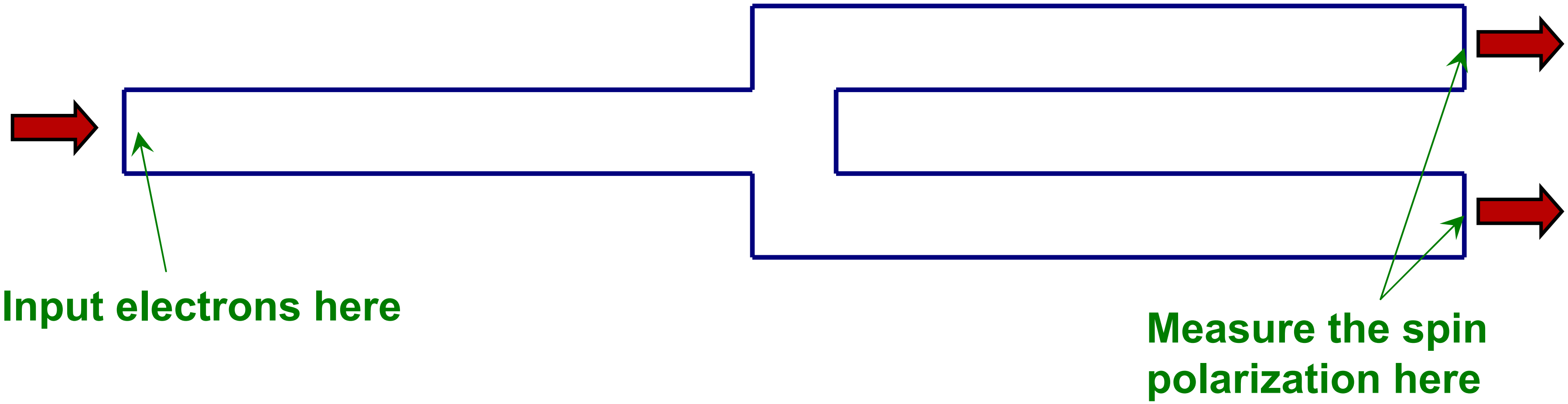
Input only spin up

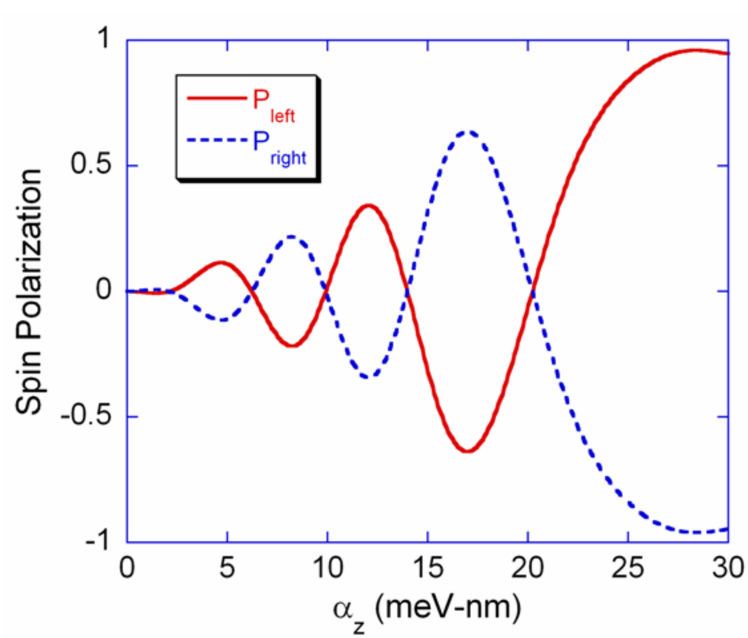


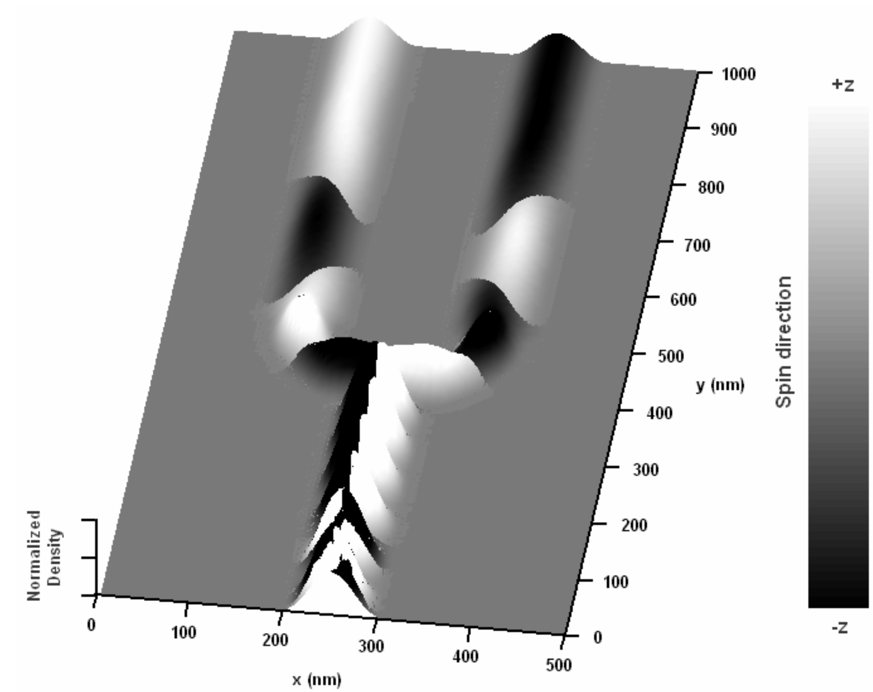
There is a connection between the spin-Hall effect and *zitterbewung* (jitter) in mobile electrons

α_z = 10 meV-nm

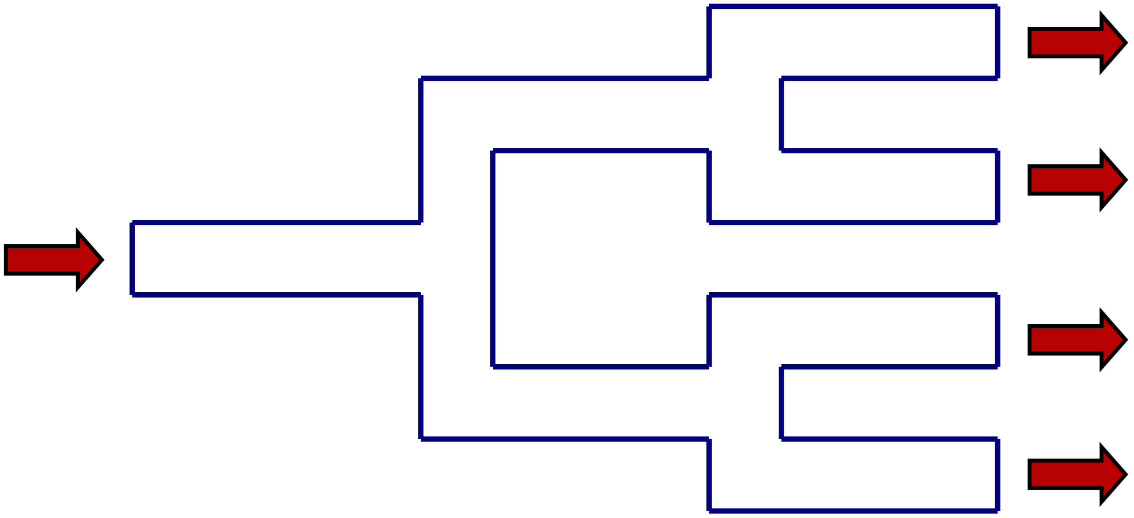
2D transport – branching structure

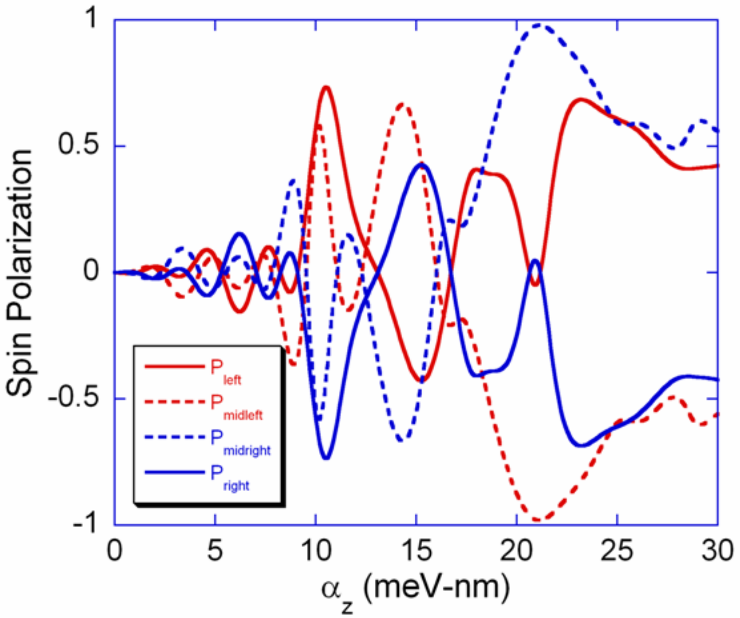






2D transport – double branching structure

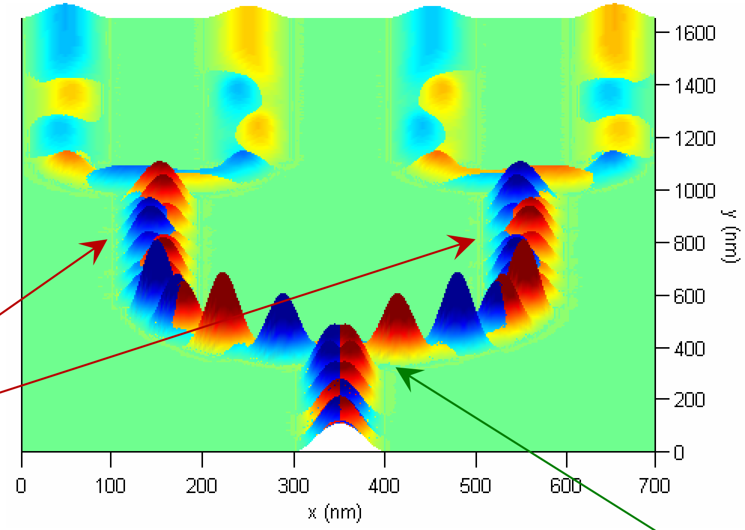




Not much improvement over the single-stage spin filter

2D transport – double branching structure

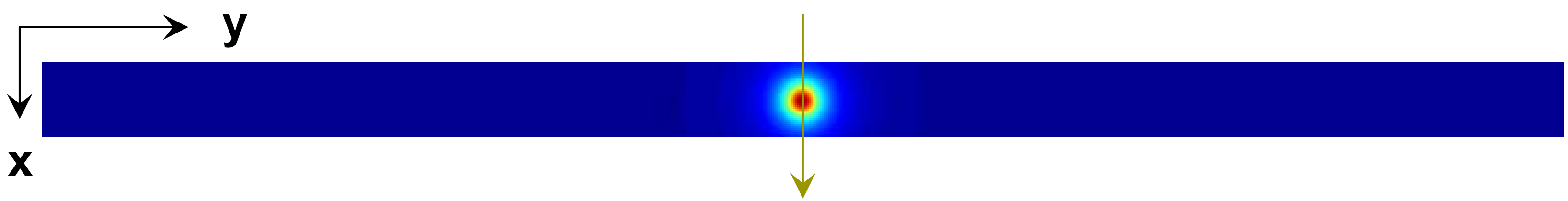
Spin precession in the intermediate stage means electrons reach the second stage in a less than ideal state for spin filtering



Spin precession here

Spin polarization here

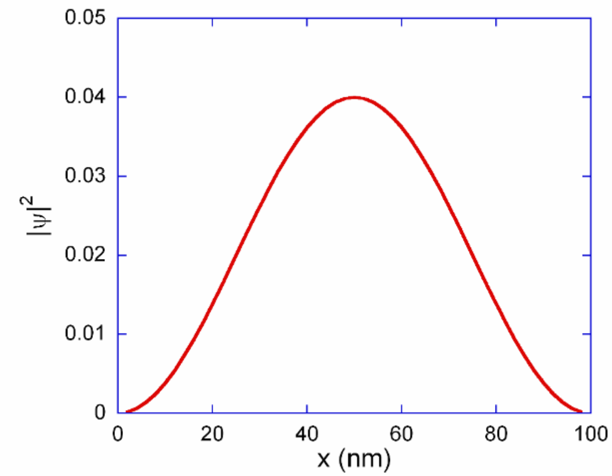
2D transport – probing the spin-Hall effect



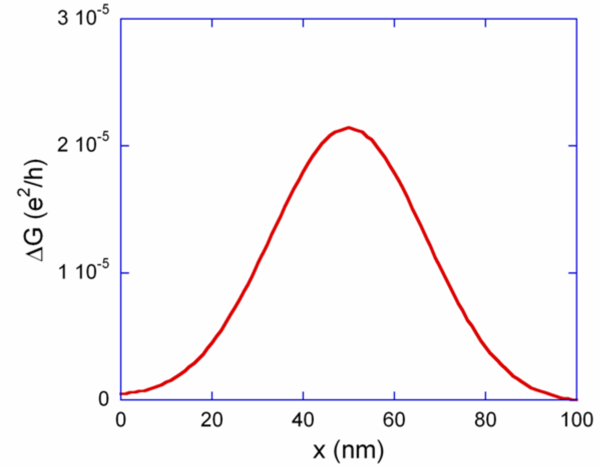
Sweep a magnetic perturbation along the transverse axis, and measure the change in conductance at each point.

The peak magnetic field is 10 mT, with a Lorentzian decay width of 20 nm

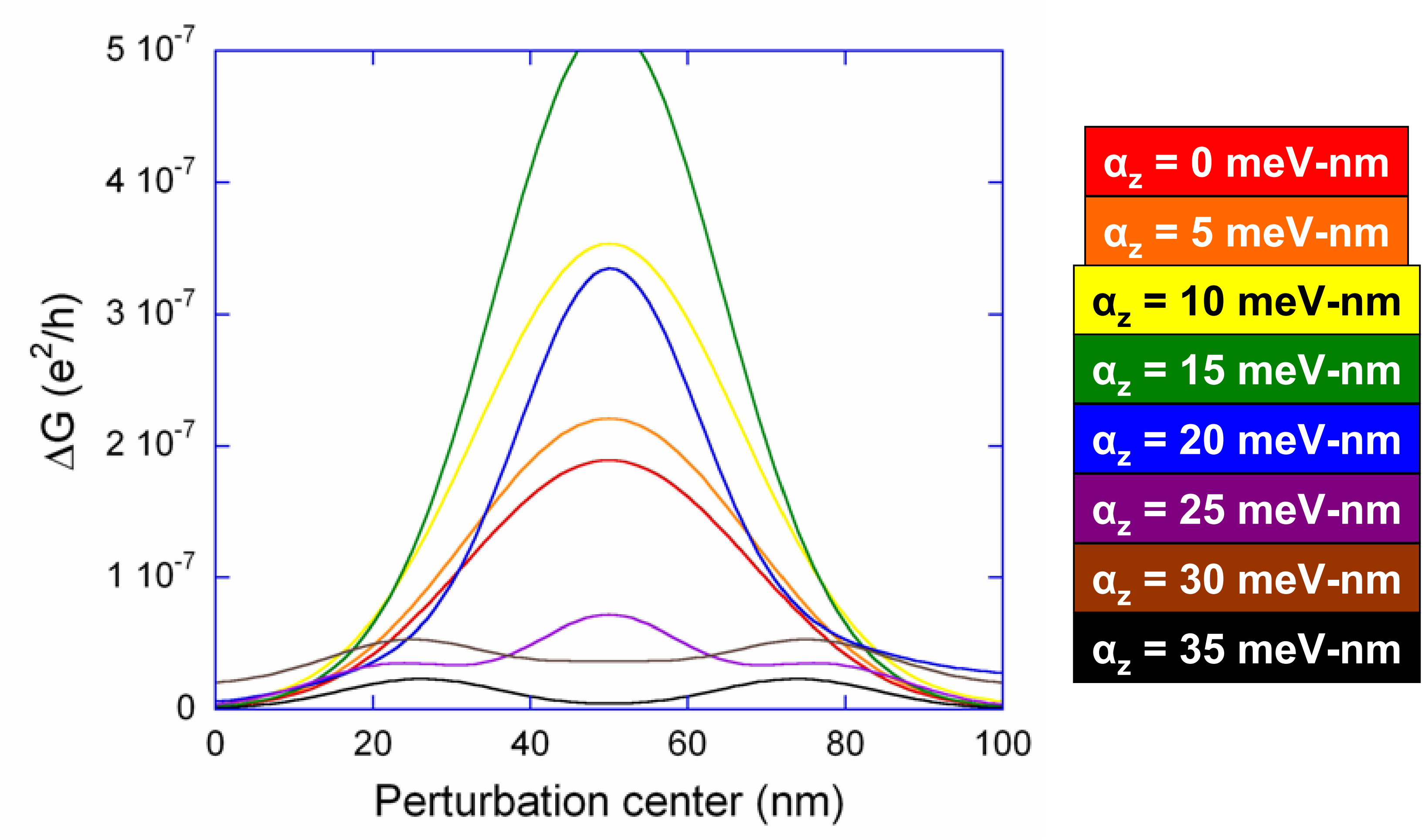
$|\psi|^2$



ΔG

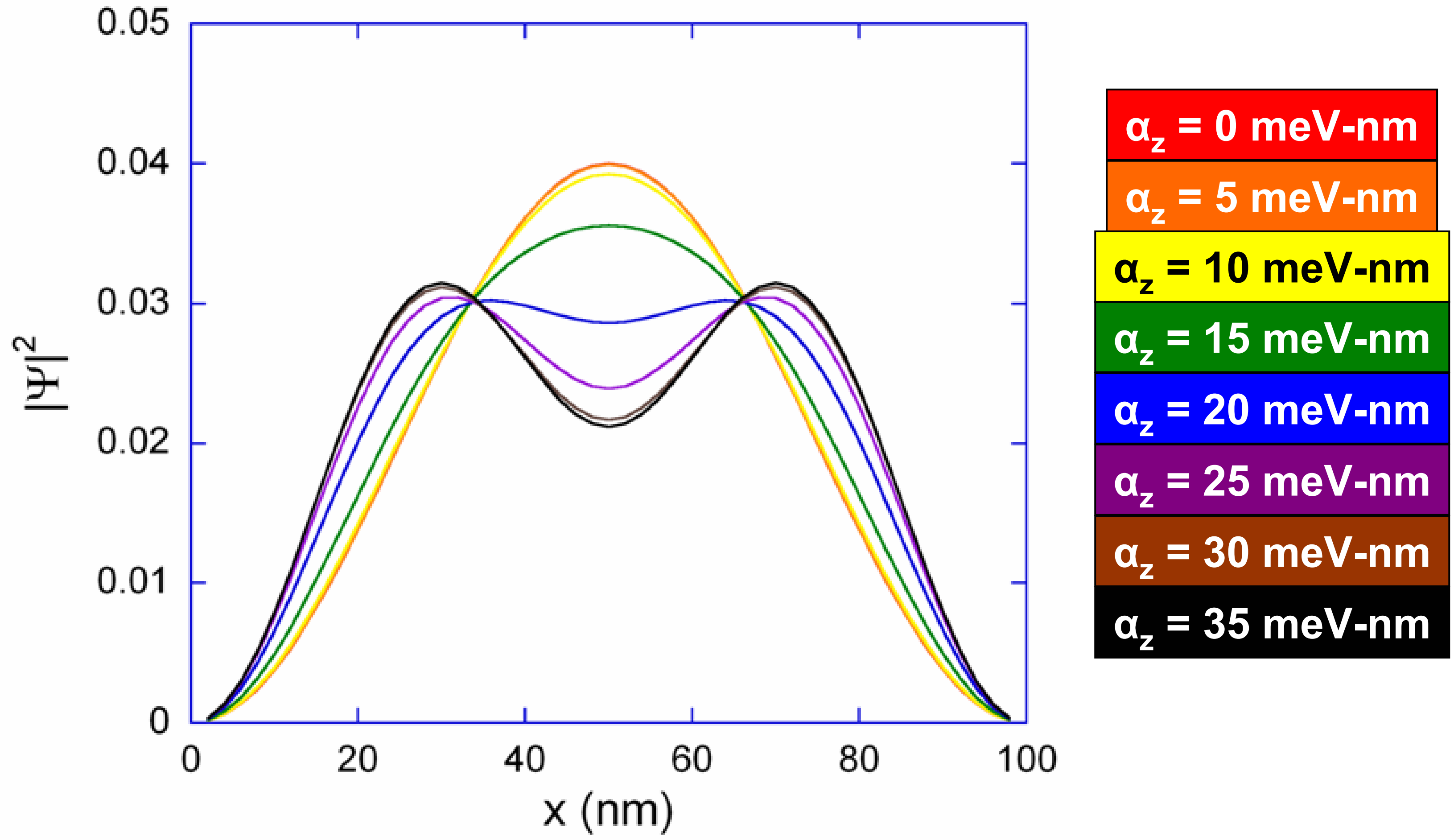


2D transport – probing the spin-Hall effect



A double peak develops in the conductance change, but no asymmetry is observed.

2D transport – probing the spin-Hall effect



The double peak is a result of hybridization between the second-lowest and third-lowest subbands, as seen in the transverse wave function profiles above.

Acknowledgments

My advisor and research compatriots.

The Krell folks.

My fellow fellows.