An Embedded Boundary Adaptive Mesh Refinement Method for Highly Nonlinear Internal Waves

Mike Barad
Civil and Environmental Engineering
University of California, Davis

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Talk Outline

• Introduction
• Governing equations
• Discretization concepts
• Grid generation
• Results
• Conclusions
What is an internal wave?

- Internal waves are ubiquitous in the world’s oceans.
- Internal waves affect: tidal energy dissipation, sediment transport, acoustics, ocean’s food web, and the transport of pollutants.
- Oceanic internal wave amplitudes can be larger than 100 meters, and their associated currents can be seen from space!
Satellite Image of Oceanic Internal Waves:

Image is 300 by 100 km, and ©ESA 2000.
Introduction

What are the key issues for modeling multiscale highly nonlinear internal waves?

- Need to capture generation, propagation, and dissipation
- Simplified equation sets won’t work, need to solve incompressible Navier-Stokes equations
- Large ranges in spatial and temporal scales
- Internal waves interact with complex bathymetry

What do we hope to provide with this method?

- An enhanced ability to interpret and extend the results of field and laboratory studies
- A predictive tool for both engineering and science
Incompressible Navier-Stokes Equations

- Momentum balance
  \[ \ddot{\vec{u}}_t + (\vec{u} \cdot \nabla) \vec{u} = -\frac{\nabla p}{\rho} + \vec{g} + \nu \Delta \vec{u} \]

- Divergence free constraint
  \[ \nabla \cdot \vec{u} = 0 \]

- Density conservation
  \[ \rho_t + \vec{u} \cdot \nabla \rho = 0 \]

- Passive scalar transport
  \[ c_t + \vec{u} \cdot \nabla c = \nabla \cdot (k_c \nabla c) + H_c \]
Temporal Discretization: Projection Method

We build on a classic second-order accurate projection method (Bell, Colella, Glaz, JCP 1989). We split the momentum equations into three pieces:

- **Hyperbolic:** \( \vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = H \)
  
  where we compute the advective term explicitly.

- **Parabolic:** \( \vec{u}_t = \nu \Delta \vec{u} + S \)
  
  which we solve implicitly for a predictor velocity.

- **Elliptic:** \( \nabla \cdot \frac{1}{\rho} \nabla p = \nabla \cdot (- (\vec{u} \cdot \nabla)\vec{u} + \nu \Delta \vec{u}) \)
  
  which we solve implicitly for pressure, then correct the predictor velocity.
Spatial Discretization: Embedded Boundaries (EB)

For the bulk of the flow, $O(n^3)$ cells in 3D, we compute on a regular Cartesian grid. We use an embedded boundary description for the $O(n^2)$ control-volumes (in 3D) that intersect the boundary.

Advantages of underlying rectangular grid:

- Grid generation is tractable, with a straightforward coupling to block-structured adaptive mesh refinement (AMR)
- Good discretization technology, e.g. well-understood consistency theory for conservative finite differences, geometric multigrid for elliptic solvers.
Finite-Volume EB Control Volumes

Three example irregular control volumes are shown below. Green curves indicate the intersection of the exact boundary with a Cartesian cell. We approximate face intersections using quadratic interpolants.

For each control volume we compute: volume fractions, area fractions, centroids, boundary areas, and boundary normals. These are all we need for second-order accurate discretizations of our conservation laws.
In adaptive methods, one adjusts the computational effort locally to maintain a uniform level of accuracy throughout the problem domain.

- Refined regions are organized into rectangular patches. Refinement is possible in both space and time.
- Using EB AMR finite-volume methods we maintain conservation and second-order accuracy.
EB AMR Grid Generation Example: South China Sea

- Black boxes are a decomposition of the coarsest level, red boxes are finer grids.
- Each box is further sub-divided into individual control volumes.
- Upper right is Taiwan, lower right is the Philippines, mainland China is upper left.
Why is AMR important?

The “classic” lock-exchange test problem with AMR will show why...

- Flow is inside a 0.5m tall, by 3m wide tank.

- On the left side of the tank we start with light freshwater, on the right is heavy saltwater. The density ratio of light fluid to heavy fluid is 1000/1030.

- On the following lock-exchange slides, the lower figure is a zoom in on the center region of the tank.
Single Level...
Two Levels...
Three Levels...
Why is AMR important? Answer: With AMR, one can “zoom in” on moving regions, and accurately capture the important flow physics at multiple scales.
## Convergence Study:
### Evolution of the Flow Field Within a Rotating Sphere

<table>
<thead>
<tr>
<th>Base Grids</th>
<th>$64^3$-$128^3$</th>
<th>Rate</th>
<th>$128^3$-$256^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$ Norm of U Velocity Error</td>
<td>2.80e-06</td>
<td>1.81</td>
<td>7.97e-07</td>
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<tr>
<td>$L_1$ Norm of W Velocity Error</td>
<td>4.13e-08</td>
<td>2.23</td>
<td>8.78e-09</td>
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<td>$L_2$ Norm of U Velocity Error</td>
<td>4.61e-06</td>
<td>1.73</td>
<td>1.39e-06</td>
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<td>$L_2$ Norm of W Velocity Error</td>
<td>9.42e-08</td>
<td>2.02</td>
<td>2.32e-08</td>
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</tbody>
</table>

The slice is colored by u-velocity, and streamtubes aid in visualizing the flow.
Internal Wave Generation:

Oceanic Stratified Flow Past A Sill

• Flow is inside a 2D domain that is 256 meters deep by 4096 meters long.

• There is a 196 meter tall Gaussian sill centered 1024 meters from the left side.

• The domain is forced by a stratified inflowing current on the left at 0.2 m/s.

• The stably stratified initial and inflowing density profile is given by
\[ \rho = 1001 - e^{0.0673z} \], where \( z = 0 \) is at the top of the domain.
Internal Wave Generation: Oceanic Stratified Flow Past A Sill

Hydraulic jumps occur in rivers too...
Internal Wave Dissipation:

Breaking Waves on a Slope

• Flow is inside a 0.5m tall, by 3m long tank, with an 8:1 slope starting 1m from the left side

• Below is the initial density distribution (blue is light fluid, red is heavy fluid).

• Density ratio of light fluid to heavy fluid is 1000/1030, and our pycnocline is smoothed over 10 cm. The pycnocline is perturbed on the left side of the tank.

• Thanks to Prof. Fringer (a former CSGF fellow) of Stanford University for this test problem
Breaking Internal Wave on a Slope: 2D Calculation
Breaking Internal Wave on a Slope: 3D Calculation
Conclusions and Future Work

- We now have a second-order accurate adaptive incompressible Navier-Stokes model for 2D or 3D irregular domains.
- We are showing promising results for highly nonlinear internal waves
- Future Work:
  - Multiscale South China Sea internal waves (with Dr. Fringer and Dr. Colella)
  - Fourth-order accuracy (with Dr. Colella)
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- Check out my website: http://seesar.lbl.gov/ANAG/staff/barad/