An Embedded Boundary Adaptive Mesh Refinement Method for Highly Nonlinear Internal Waves

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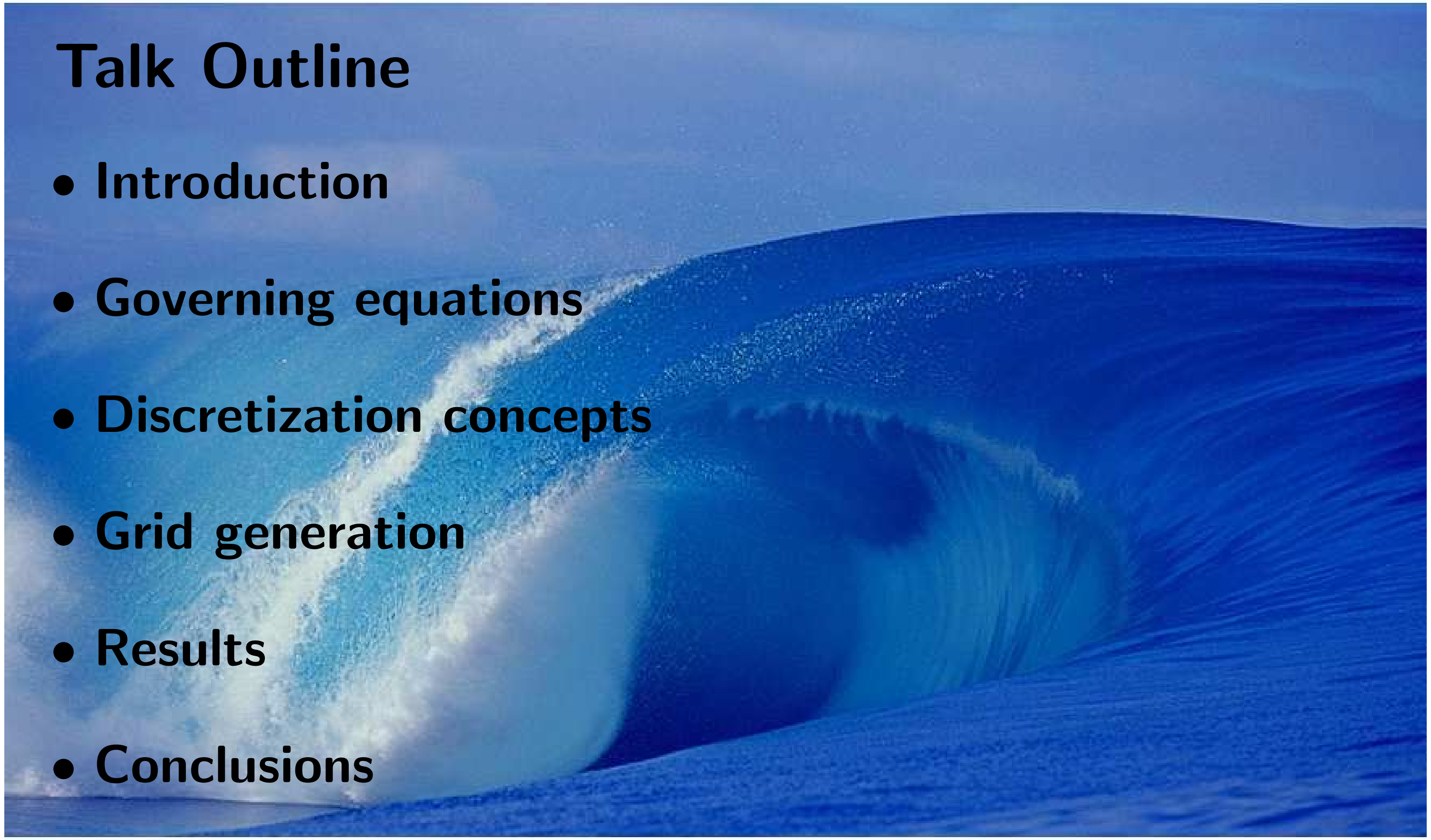
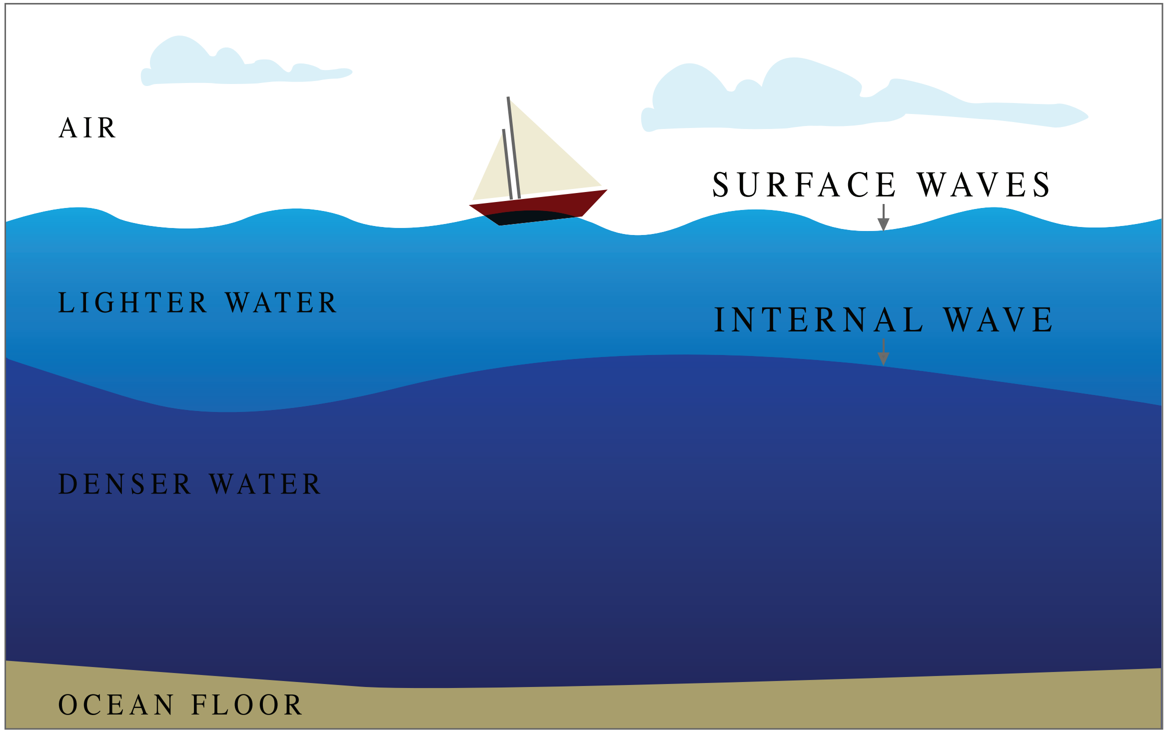


Talk Outline

- **Introduction**
- **Governing equations**
- **Discretization concepts**
- **Grid generation**
- **Results**
- **Conclusions**

What is an internal wave?



- Internal waves are ubiquitous in the world's oceans.
- Internal waves affect: tidal energy dissipation, sediment transport, acoustics, ocean's food web, and the transport of pollutants.
- Oceanic internal wave amplitudes can be larger than 100 meters, and their associated currents can be seen from space!

Satellite Image of Oceanic Internal Waves:

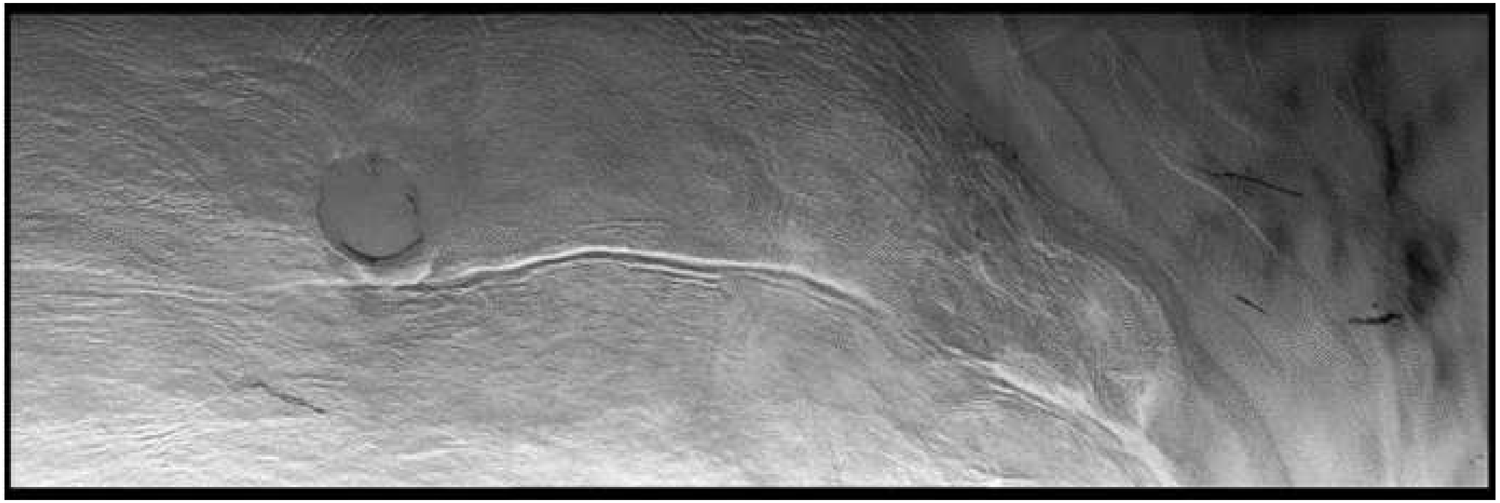


Image is 300 by 100 km, and ©ESA 2000.

Introduction

What are the key issues for modeling multiscale highly nonlinear internal waves?

- Need to capture generation, propagation, and dissipation
- Simplified equation sets won't work, need to solve incompressible Navier-Stokes equations
- Large ranges in spatial and temporal scales
- Internal waves interact with complex bathymetry

What do we hope to provide with this method?

- An enhanced ability to interpret and extend the results of field and laboratory studies
- A predictive tool for both engineering and science

Incompressible Navier-Stokes Equations

- Momentum balance

$$\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \vec{g} + \nu \Delta \vec{u}$$

- Divergence free constraint

$$\nabla \cdot \vec{u} = 0$$

- Density conservation

$$\rho_t + \vec{u} \cdot \nabla \rho = 0$$

- Passive scalar transport

$$c_t + \vec{u} \cdot \nabla c = \nabla \cdot (k_c \nabla c) + H_c$$

Temporal Discretization: Projection Method

We build on a classic second-order accurate projection method (Bell, Colella, Glaz, JCP 1989). We split the momentum equations into three pieces:

- Hyperbolic: $\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = H$

  where we compute the advective term explicitly.

- Parabolic: $\vec{u}_t = \nu \Delta \vec{u} + S$

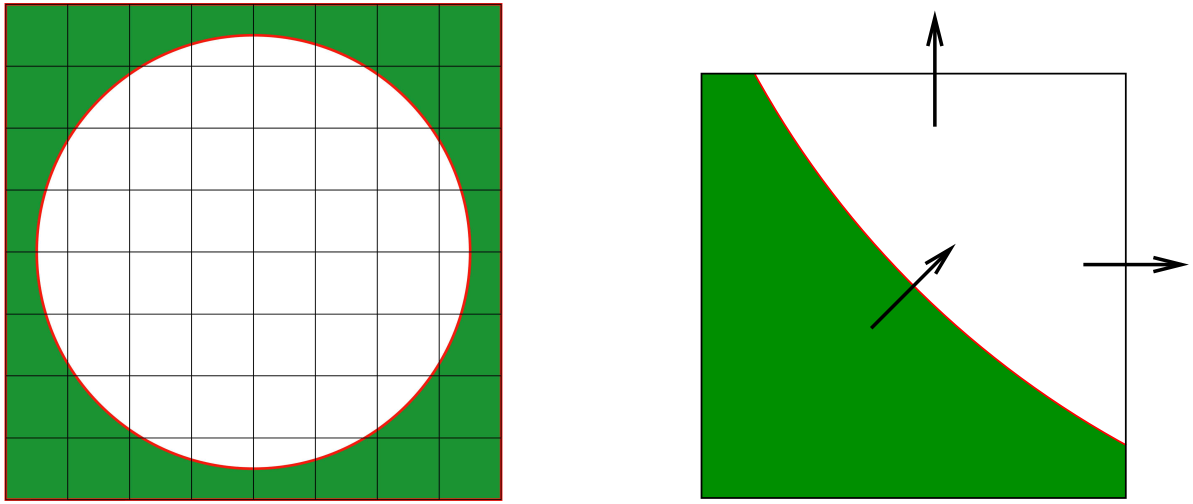
  which we solve implicitly for a predictor velocity.

- Elliptic: $\nabla \cdot \frac{1}{\rho} \nabla p = \nabla \cdot (-(\vec{u} \cdot \nabla)\vec{u} + \nu \Delta \vec{u})$

  which we solve implicitly for pressure, then correct the predictor velocity.

Spatial Discretization: Embedded Boundaries (EB)

For the bulk of the flow, $O(n^3)$ cells in 3D, we compute on a regular Cartesian grid. We use an embedded boundary description for the $O(n^2)$ control-volumes (in 3D) that intersect the boundary.



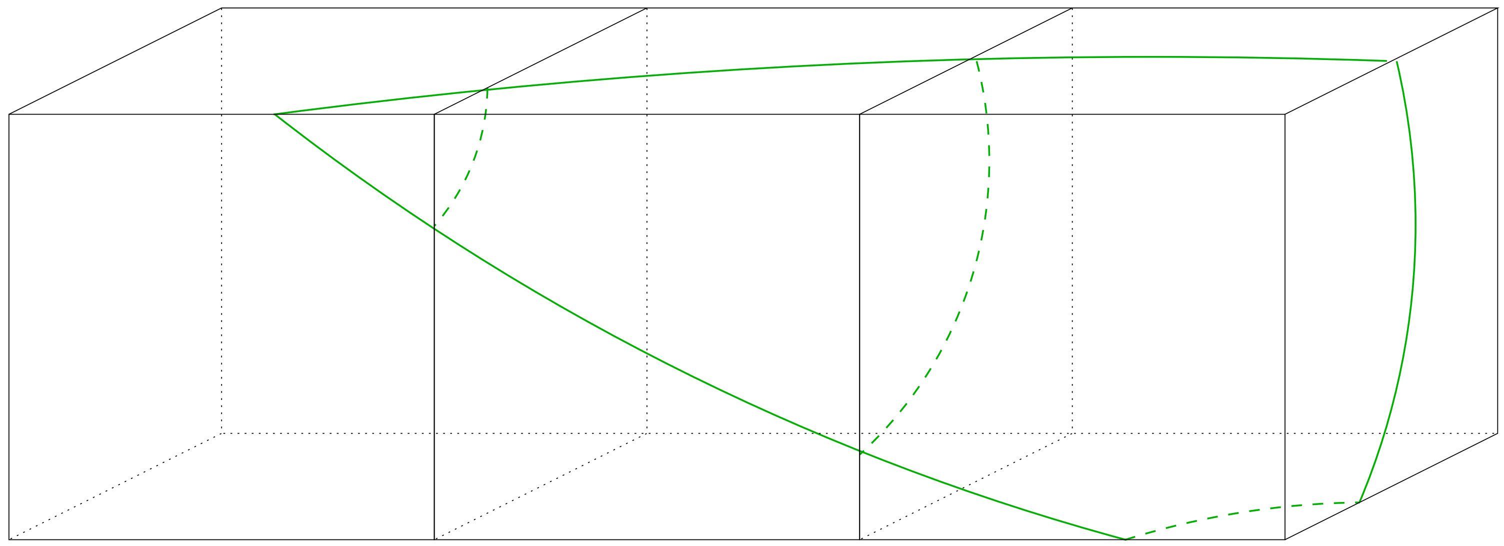
Advantages of underlying rectangular grid:

• Grid generation is tractable, with a straightforward coupling to block-structured adaptive mesh refinement (AMR)

• Good discretization technology, e.g. well-understood consistency theory for conservative finite differences, geometric multigrid for elliptic solvers.

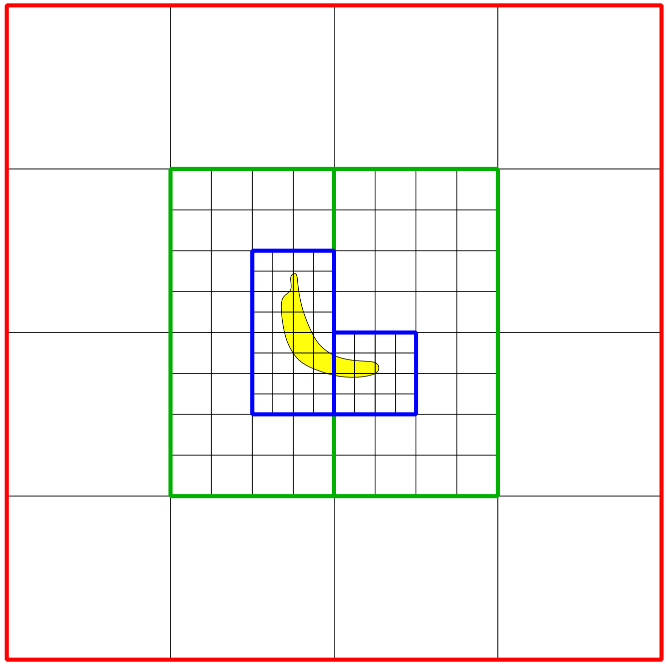
Finite-Volume EB Control Volumes

Three example irregular control volumes are shown below. Green curves indicate the intersection of the exact boundary with a Cartesian cell. We approximate face intersections using quadratic interpolants.



For each control volume we compute: volume fractions, area fractions, centroids, boundary areas, and boundary normals. These are all we need for second-order accurate discretizations of our conservation laws.

Block-Structured Adaptive Mesh Refinement (AMR)

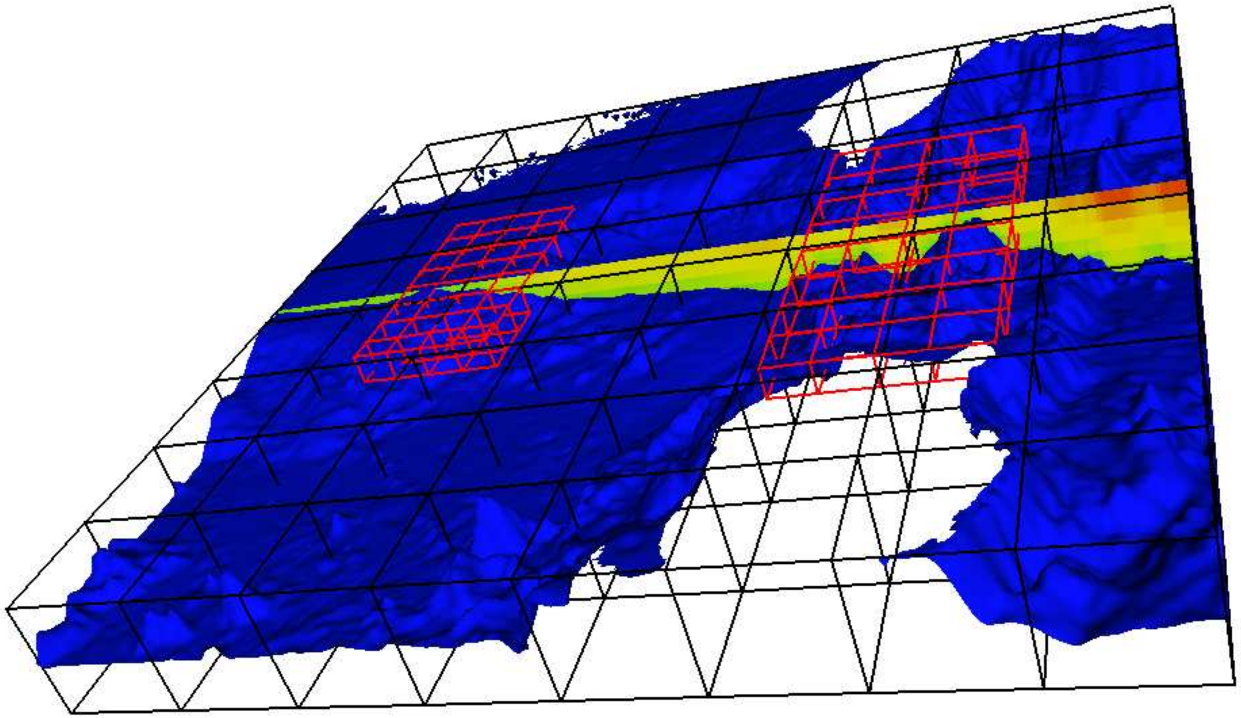


In adaptive methods, one adjusts the computational effort locally to maintain a uniform level of accuracy throughout the problem domain.

- Refined regions are organized into rectangular patches. Refinement is possible in both space and time.
- Using EB AMR finite-volume methods we maintain conservation and second-order accuracy.

EB AMR Grid Generation Example: South China Sea

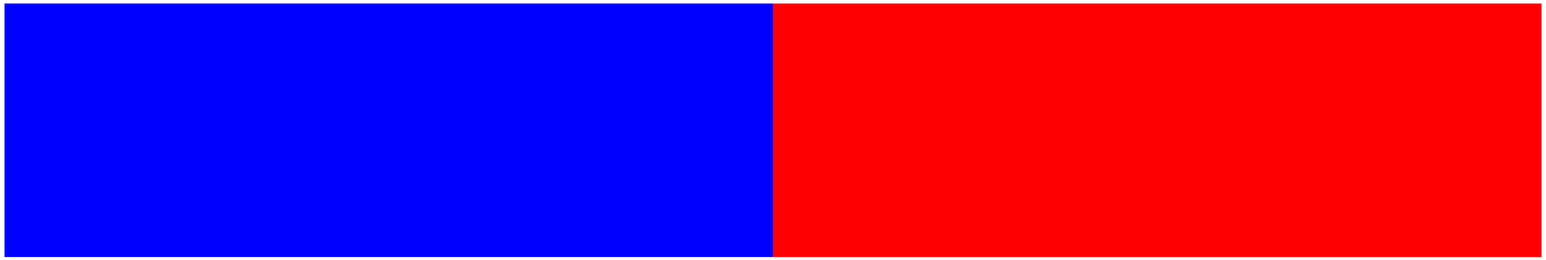
- Black boxes are a decomposition of the coarsest level, red boxes are finer grids.
- Each box is further sub-divided into individual control volumes.
- Upper right is Taiwan, lower right is the Philippines, mainland China is upper left.



Why is AMR important?

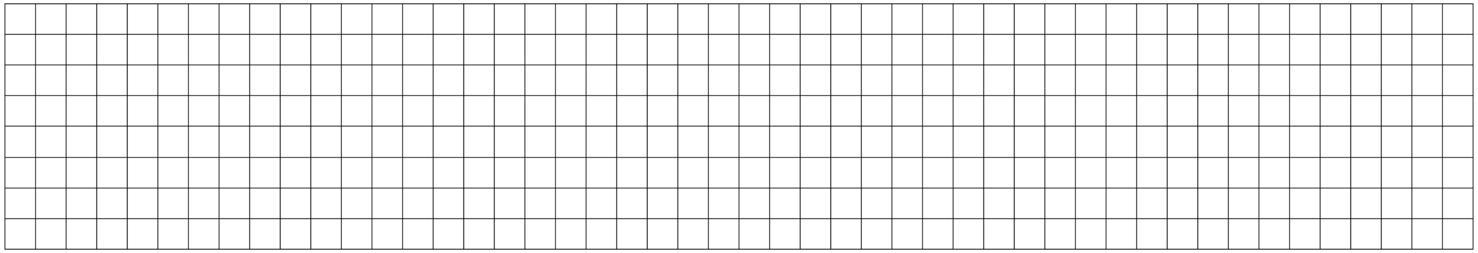
The "classic" lock-exchange test problem with AMR will show why...

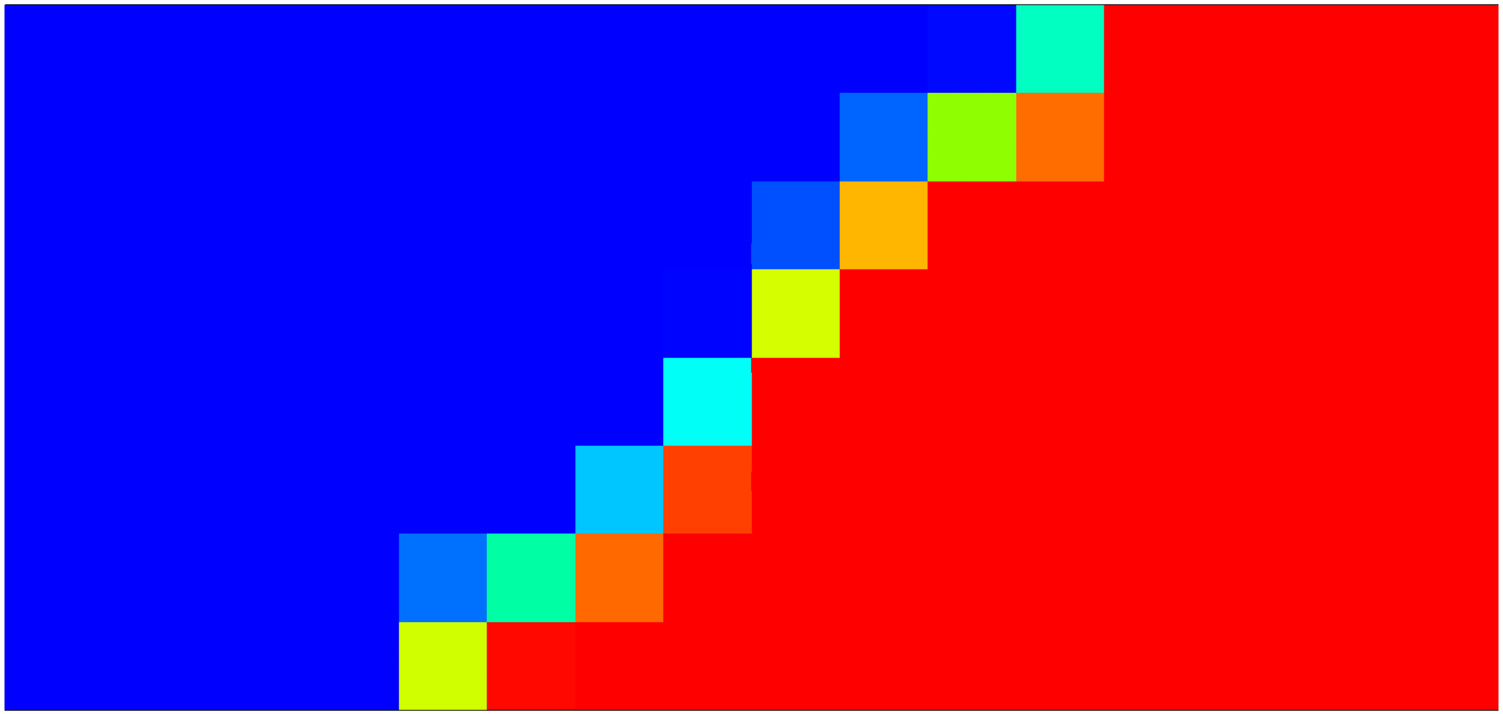
- Flow is inside a 0.5m tall, by 3m wide tank.
- On the left side of the tank we start with light freshwater, on the right is heavy saltwater. The density ratio of light fluid to heavy fluid is 1000/1030.



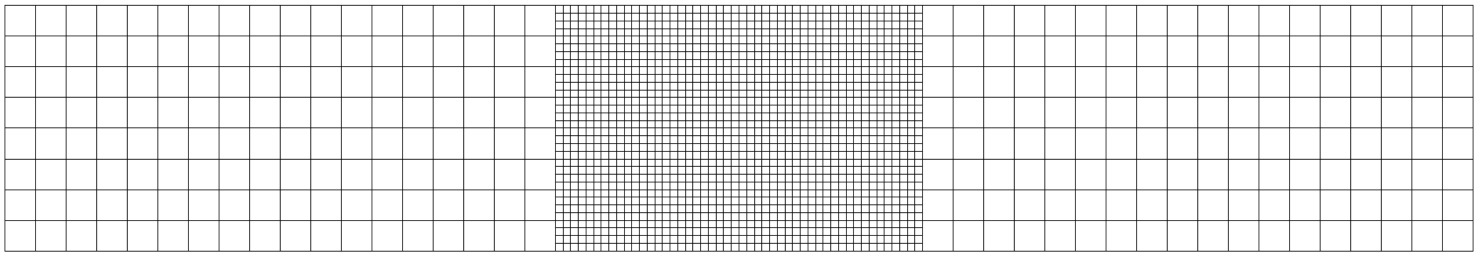
- On the following lock-exchange slides, the lower figure is a zoom in on the center region of the tank.

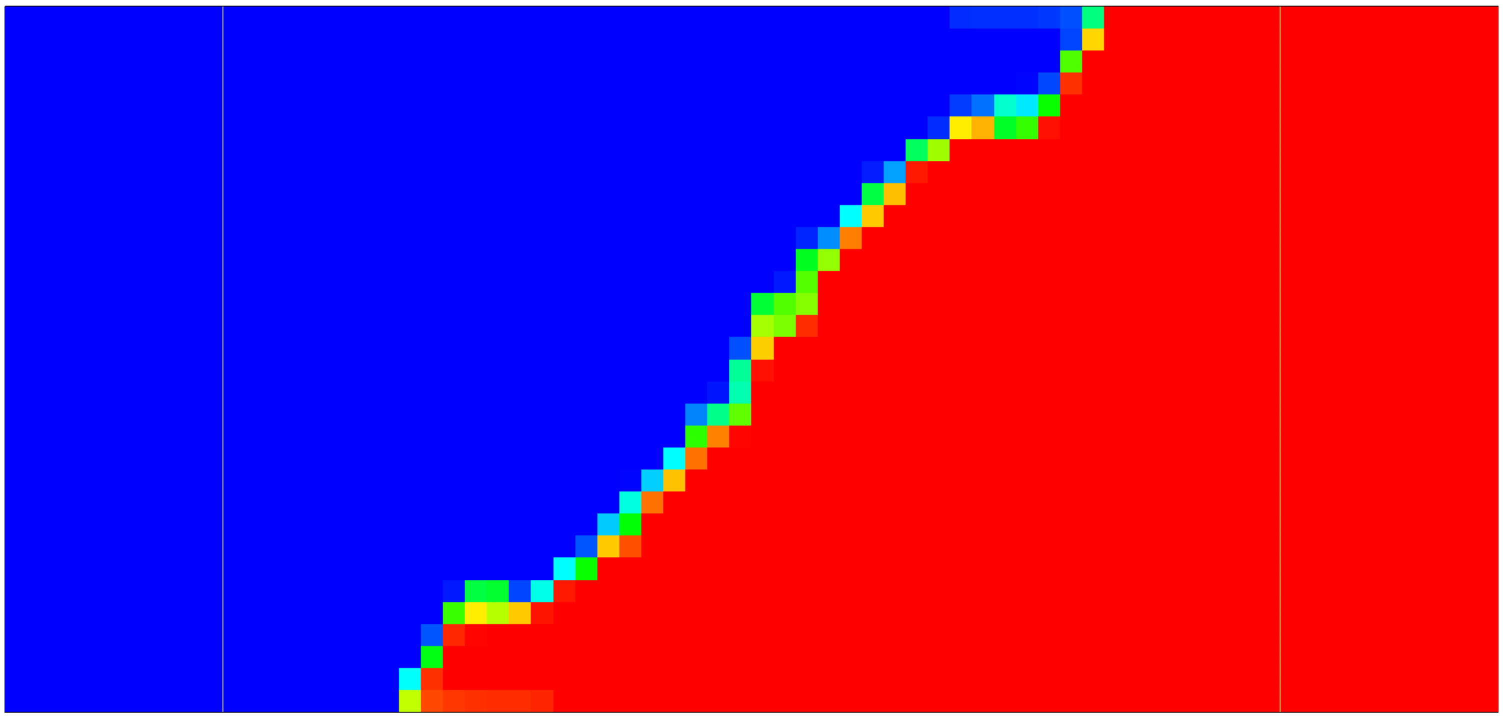
Single Level...



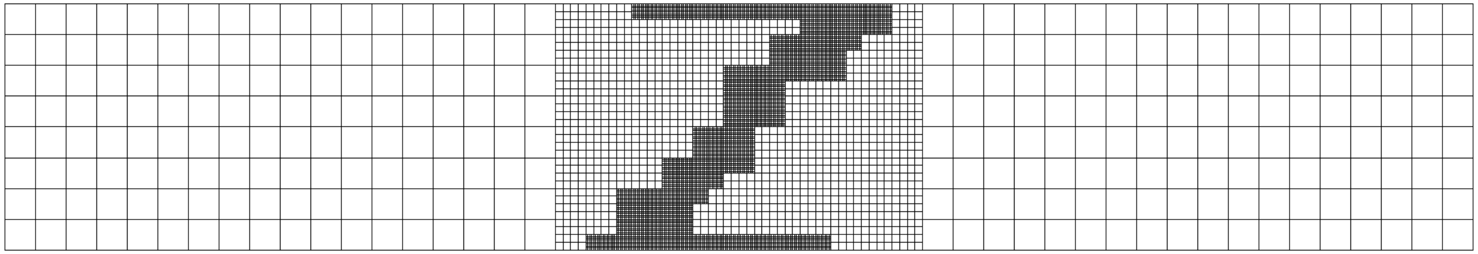


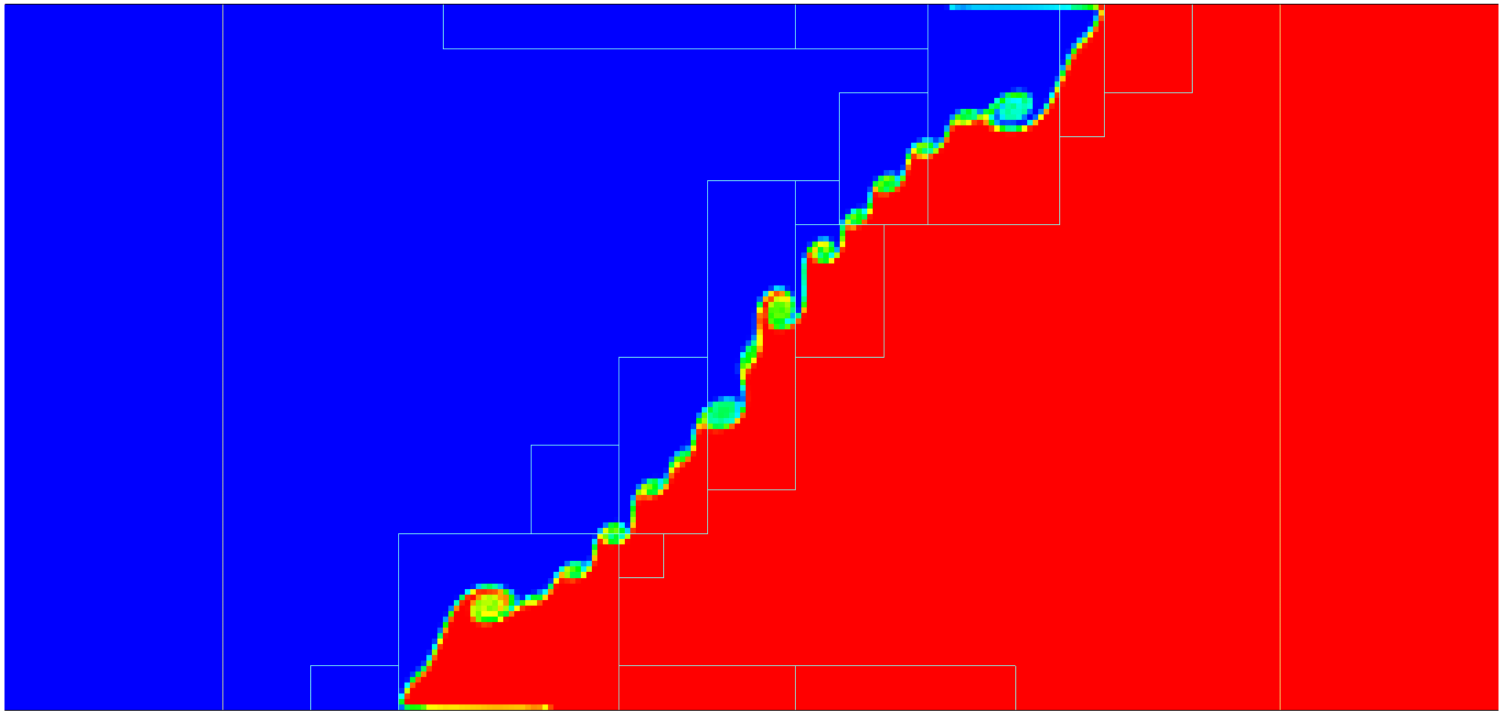
Two Levels...



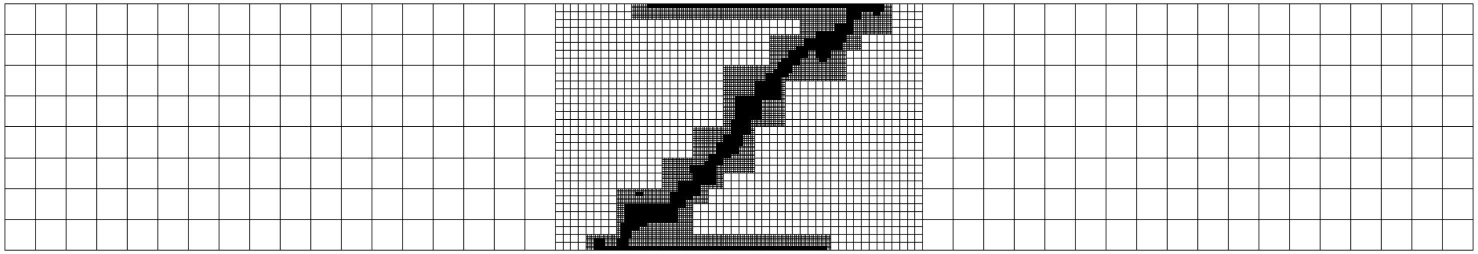


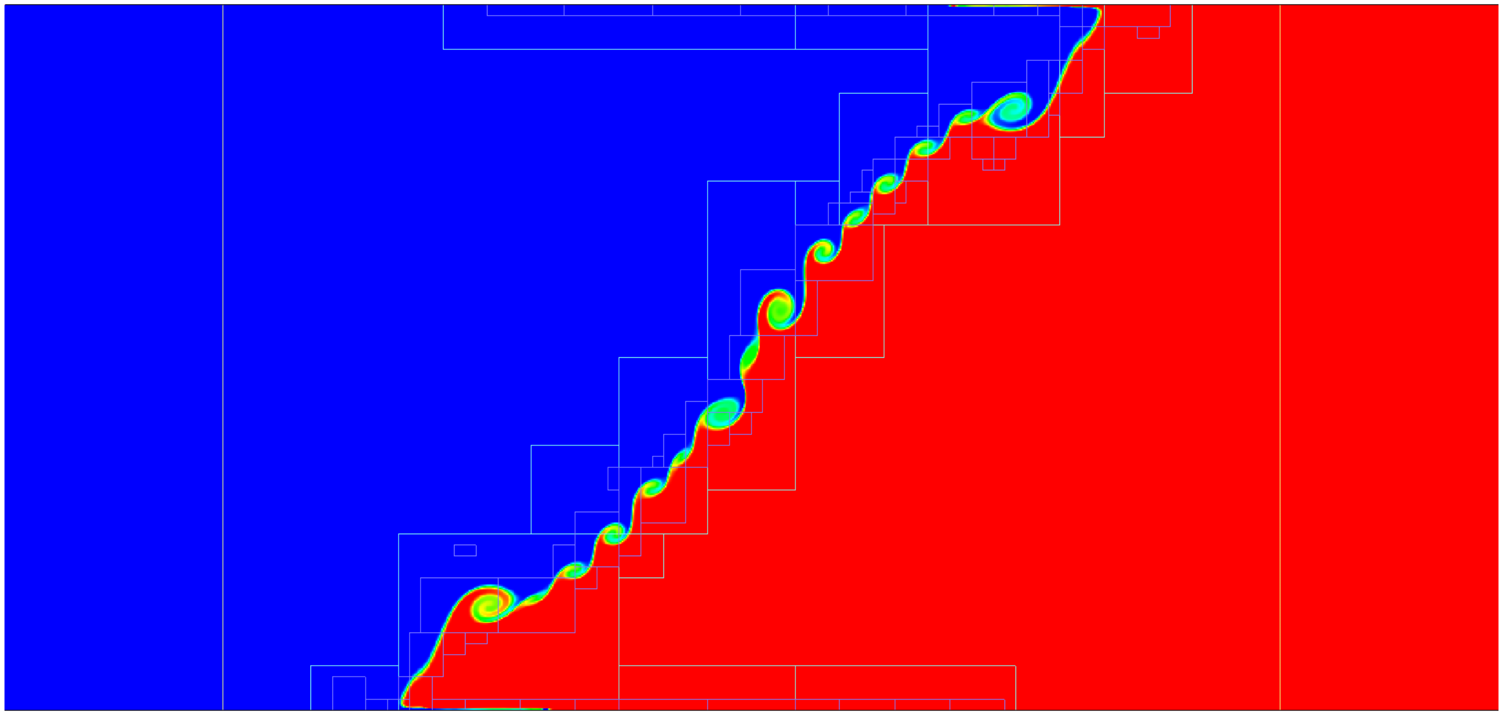
Three Levels...





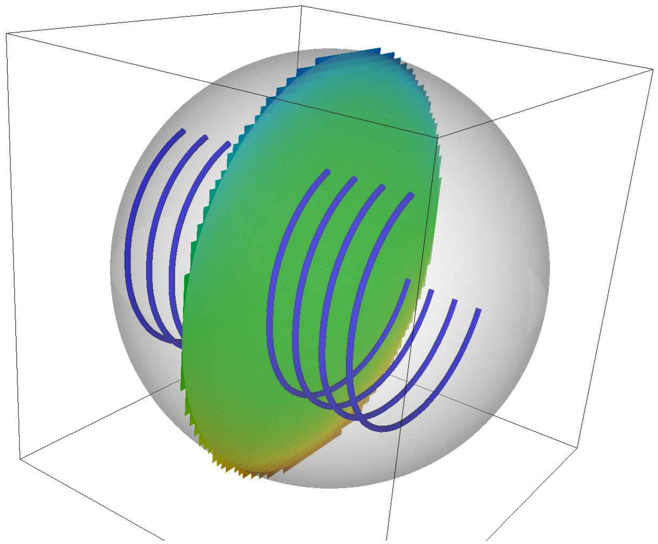
Why is AMR important? Answer: With AMR, one can "zoom in" on moving regions, and accurately capture the important flow physics at multiple scales.





Convergence Study:
Evolution of the Flow Field Within a Rotating Sphere

Base Grids	64^3-128^3	Rate	128^3-256^3
L_1 Norm of U Velocity Error	2.80e-06	**1.81**	7.97e-07
L_1 Norm of W Velocity Error	4.13e-08	**2.23**	8.78e-09
L_2 Norm of U Velocity Error	4.61e-06	**1.73**	1.39e-06
L_2 Norm of W Velocity Error	9.42e-08	**2.02**	2.32e-08

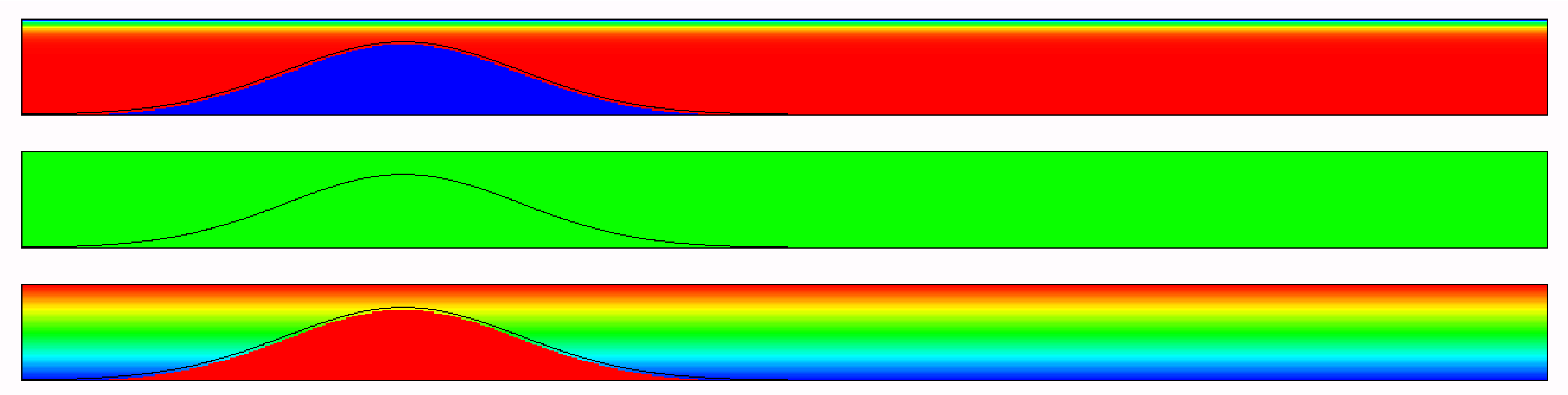


The slice is colored by u-velocity, and streamtubes aid in visualizing the flow.

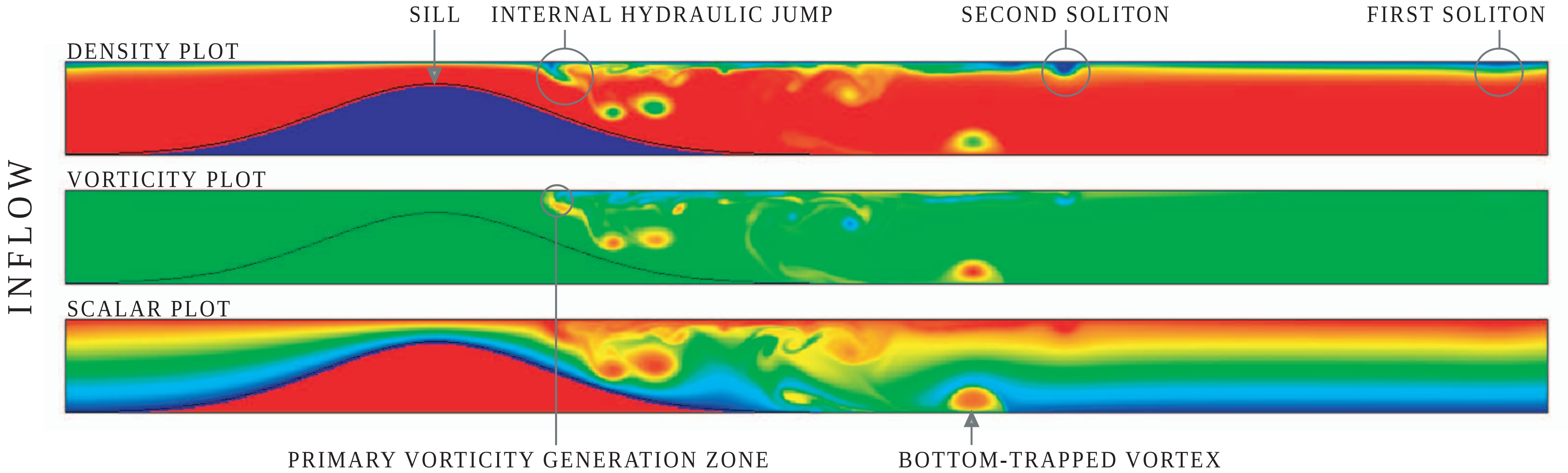
Internal Wave Generation:

Oceanic Stratified Flow Past A Sill

- Flow is inside a 2D domain that is 256 meters deep by 4096 meters long.
- There is a 196 meter tall Gaussian sill centered 1024 meters from the left side.
- The domain is forced by a stratified inflowing current on the left at 0.2 m/s.
- The stably stratified initial and inflowing density profile is given by $\rho = 1001 - e^{0.0673z}$, where $z = 0$ is at the top of the domain.



Internal Wave Generation: Oceanic Stratified Flow Past A Sill

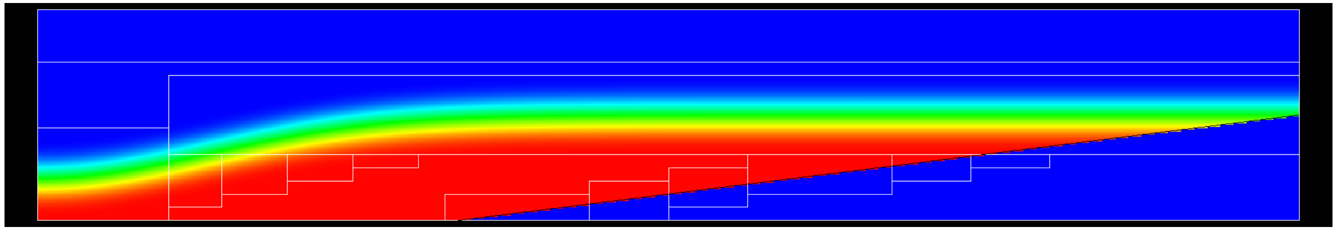


Hydraulic jumps occur in rivers too...

Internal Wave Dissipation:

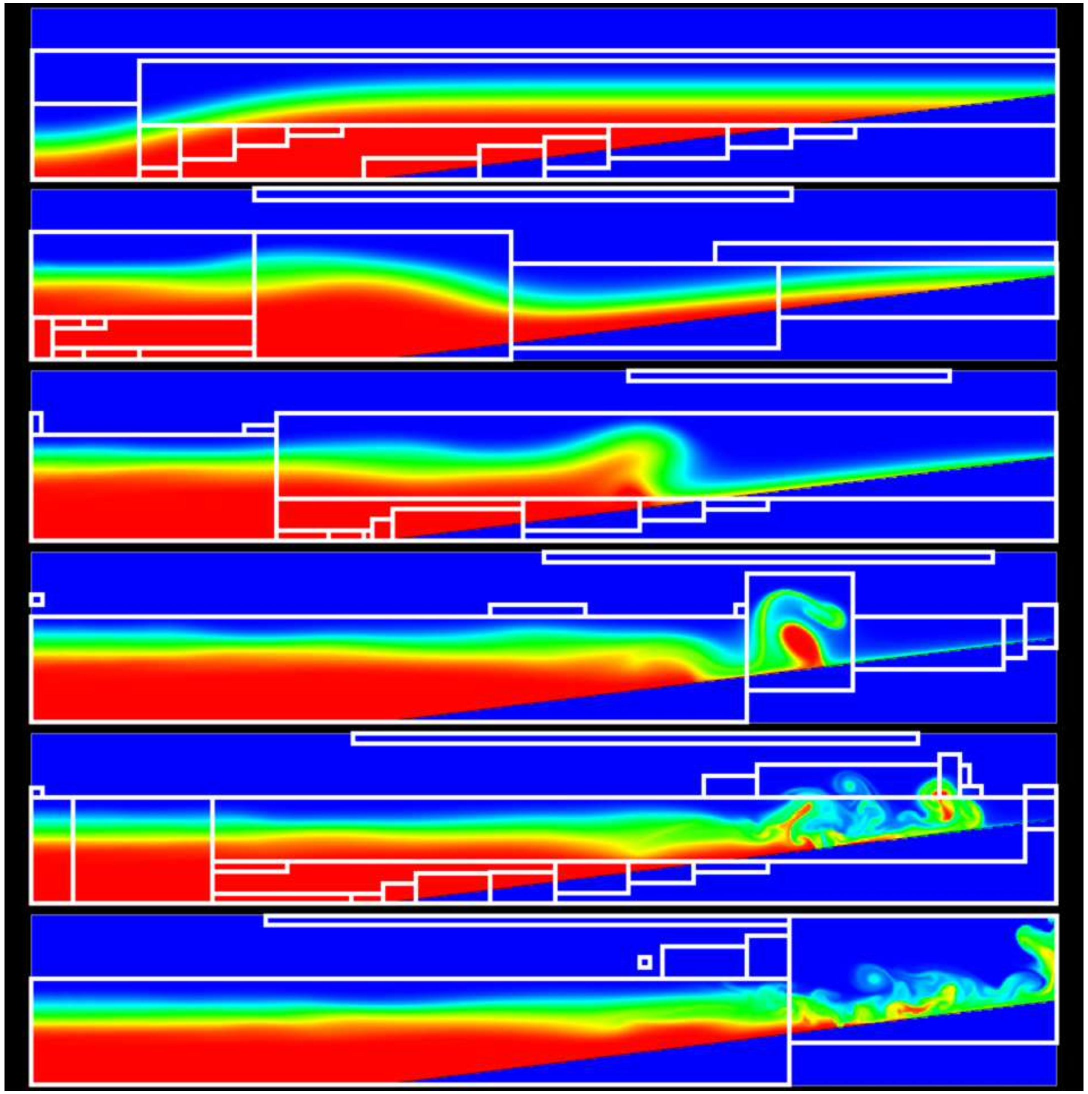
Breaking Waves on a Slope

- Flow is inside a 0.5m tall, by 3m long tank, with an 8:1 slope starting 1m from the left side
- Below is the initial density distribution (blue is light fluid, red is heavy fluid).

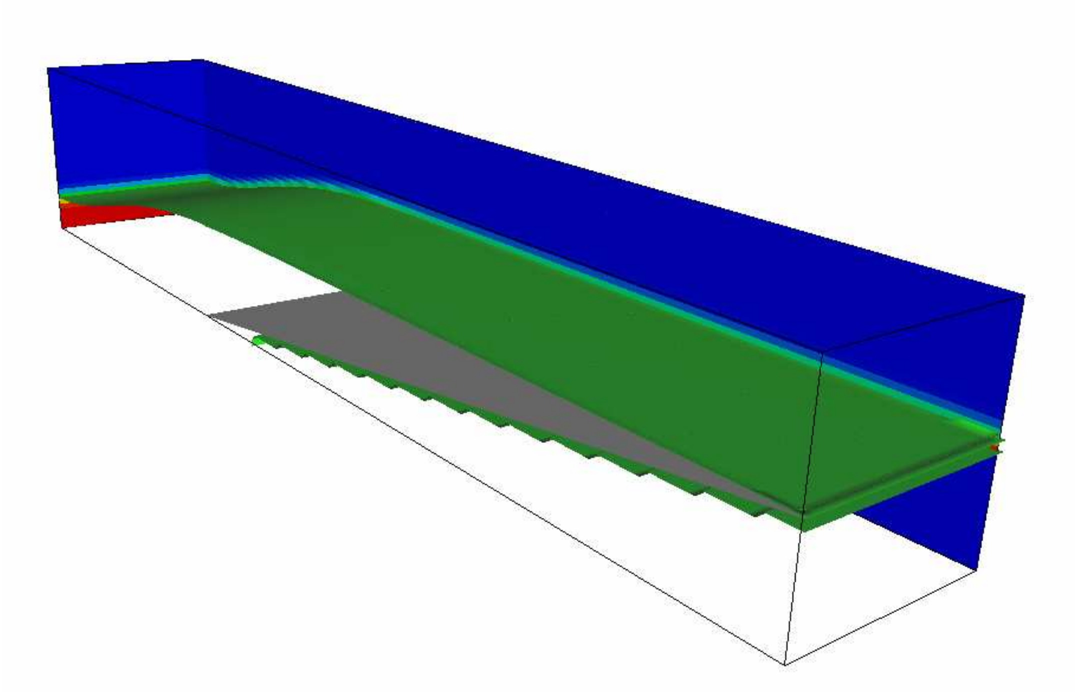


- Density ratio of light fluid to heavy fluid is 1000/1030, and our pycnocline is smoothed over 10 cm. The pycnocline is perturbed on the left side of the tank.
- Thanks to Prof. Fringer (a former CSGF fellow) of Stanford University for this test problem

Breaking Internal Wave on a Slope: 2D Calculation



Breaking Internal Wave on a Slope: 3D Calculation



Conclusions and Future Work

- We now have a second-order accurate adaptive incompressible Navier-Stokes model for 2D or 3D irregular domains.
- We are showing promising results for highly nonlinear internal waves
- Future Work:
  - Multiscale South China Sea internal waves (with Dr. Fringer and Dr. Colella)
  - Fourth-order accuracy (with Dr. Colella)

Acknowledgments

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- U.C. Davis: Professor Schladow.

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- Check out my website: http://seesar.lbl.gov/ANAG/staff/barad/