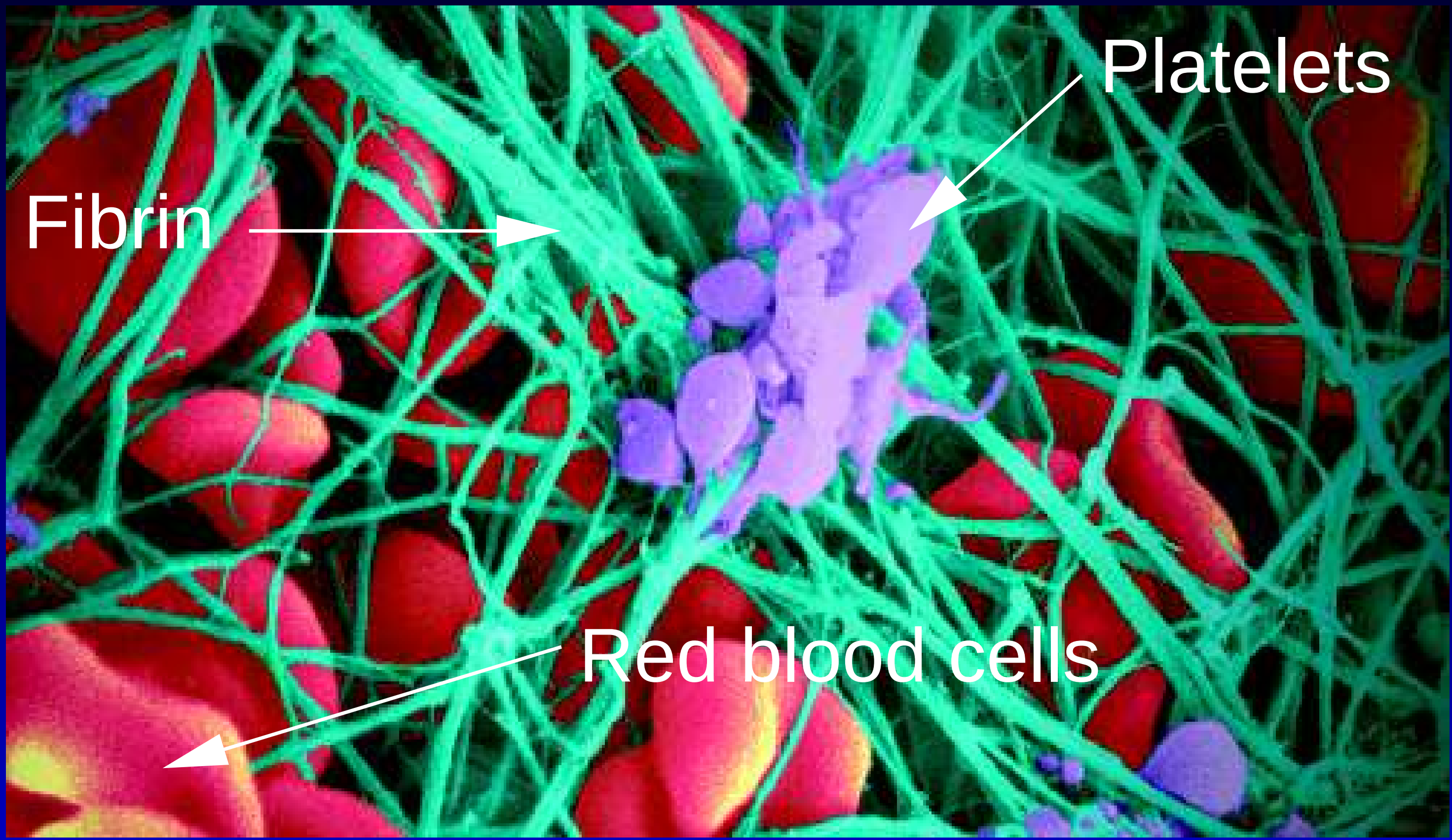
# A Computational Method for Simulating the Interaction between Fluid and Elastic Structures

Elijah Newren

Department of Mathematics
University of Utah

# Motivation




A colorized scanning electron micrograph of a blood clot formed *in vitro* without flow (Image by J. Weisel)

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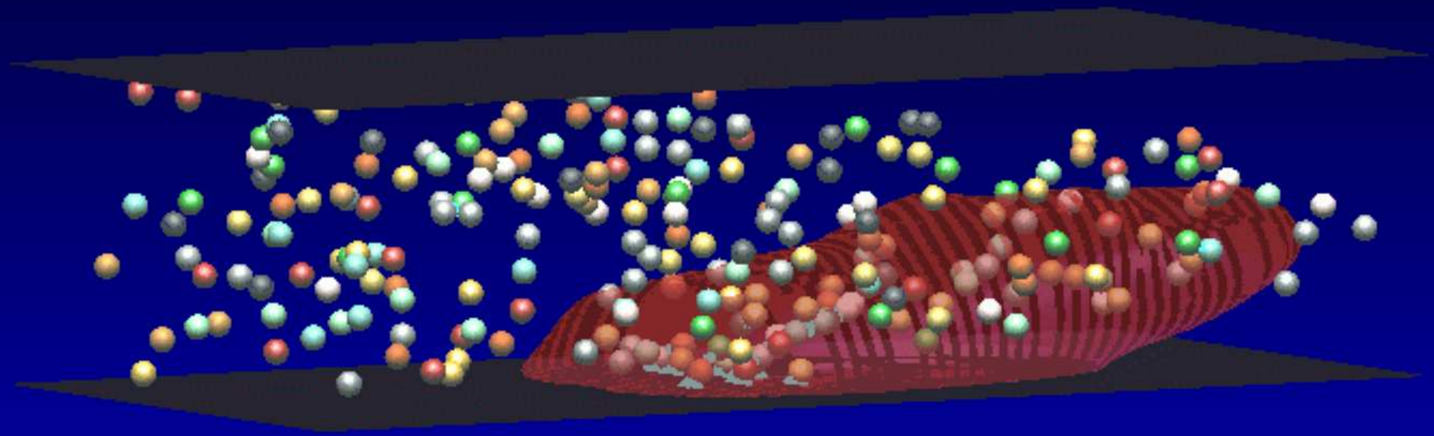
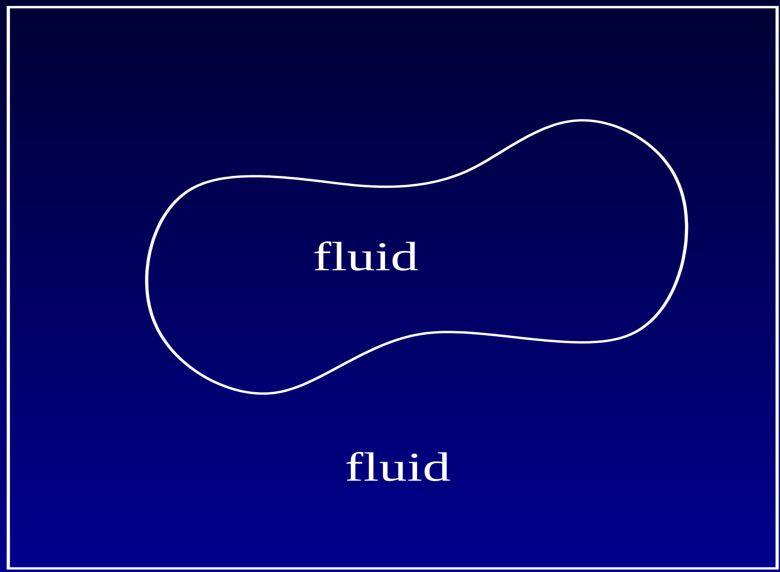


Simulation of Platelet Aggregation by H. Yu and A. Fogelson

# Simple Model Problem

Think “2D water balloon”



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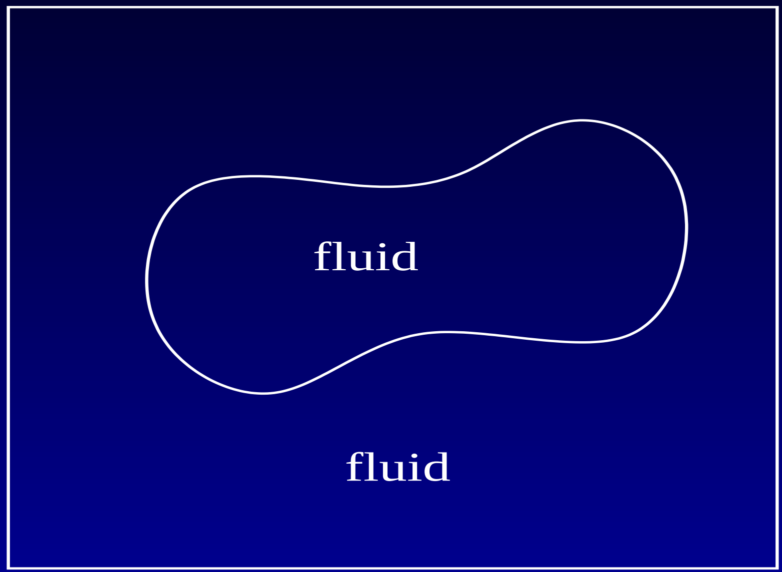

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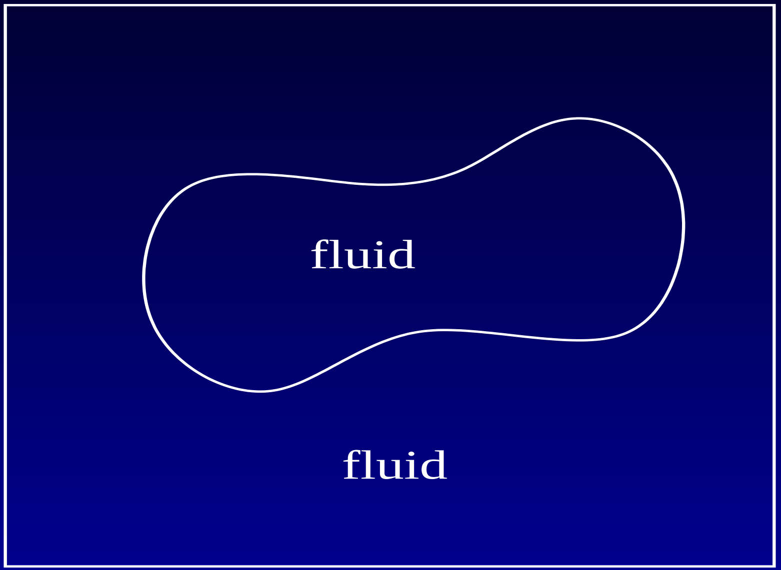

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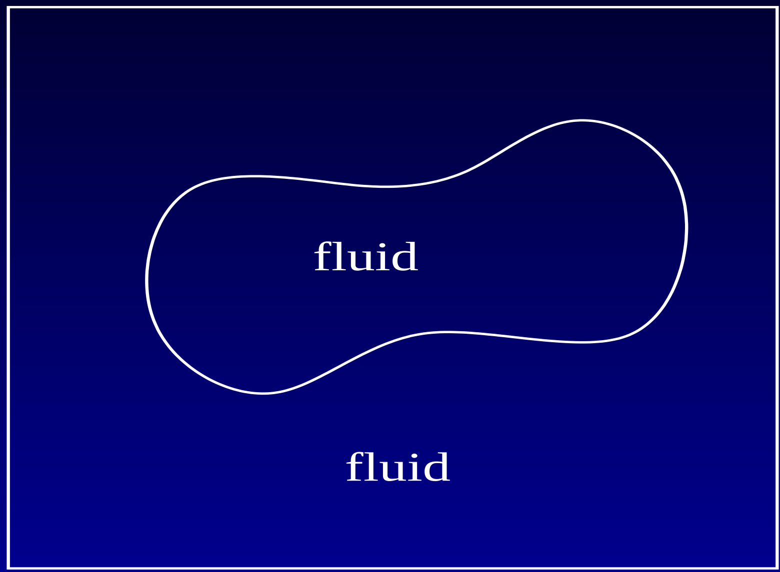


- Flow past multiple, moving, deformable objects
- One possiblity: curve fitting grids
- Much simpler alternatives: Immersed Boundary and Immersed Interface Methods

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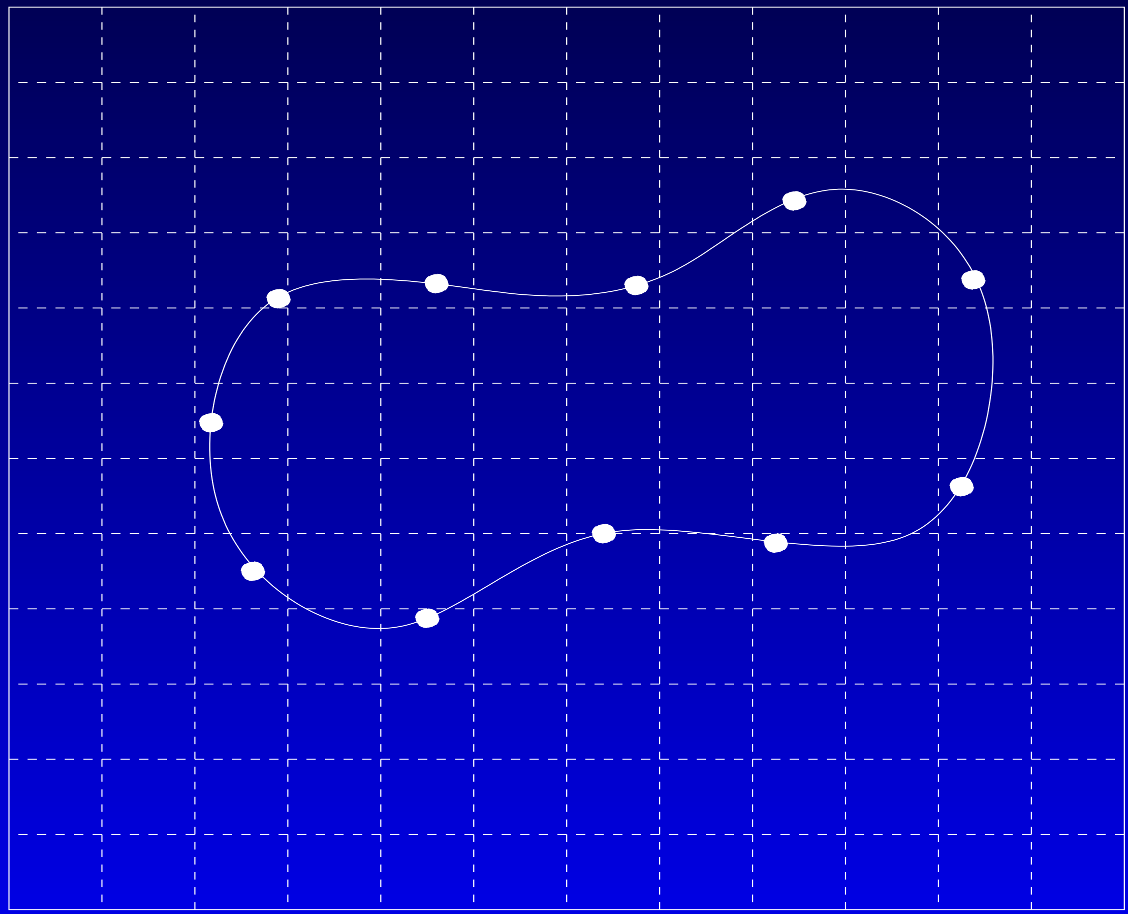
- Results in two grids being used
    - Fluid variables tracked on a structured Cartesian grid
    - Structure variables tracked on a *moving* irregular surface mesh

# Simple Model Problem

Mixed Eulerian-Lagrangian method; fixed Cartesian grid for fluid variables, *moving* irregular grid for the elastic structure.



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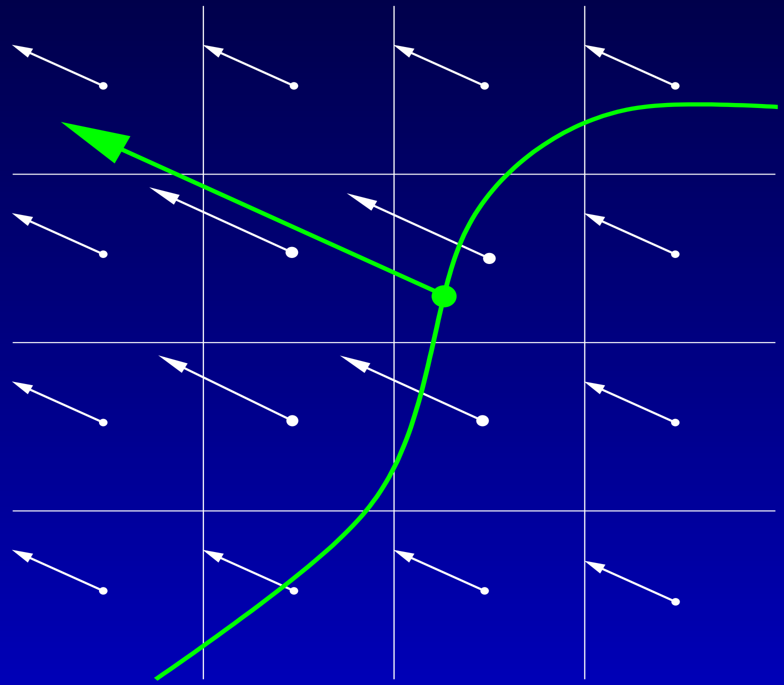
# Setting up grid from forces

Immersed Boundary



Spreading the singular force

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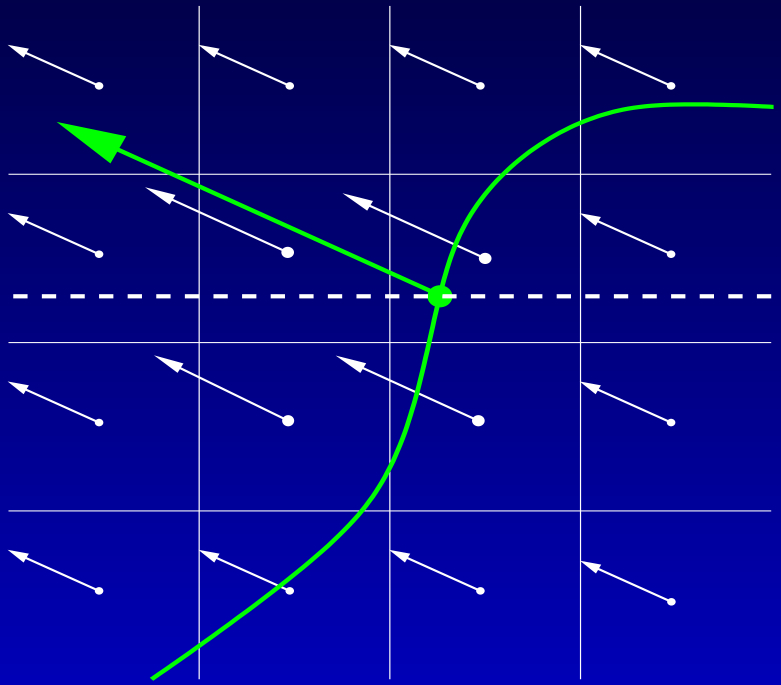

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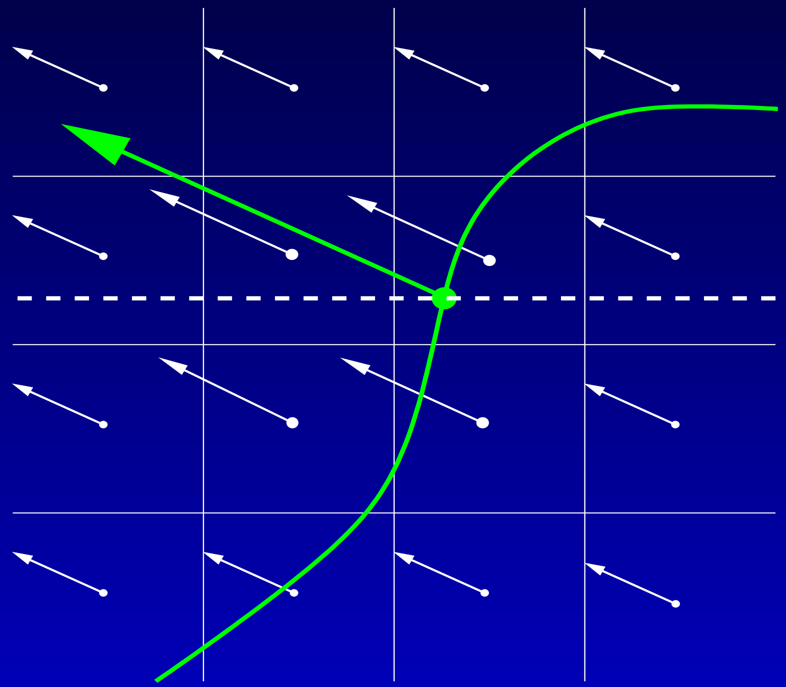
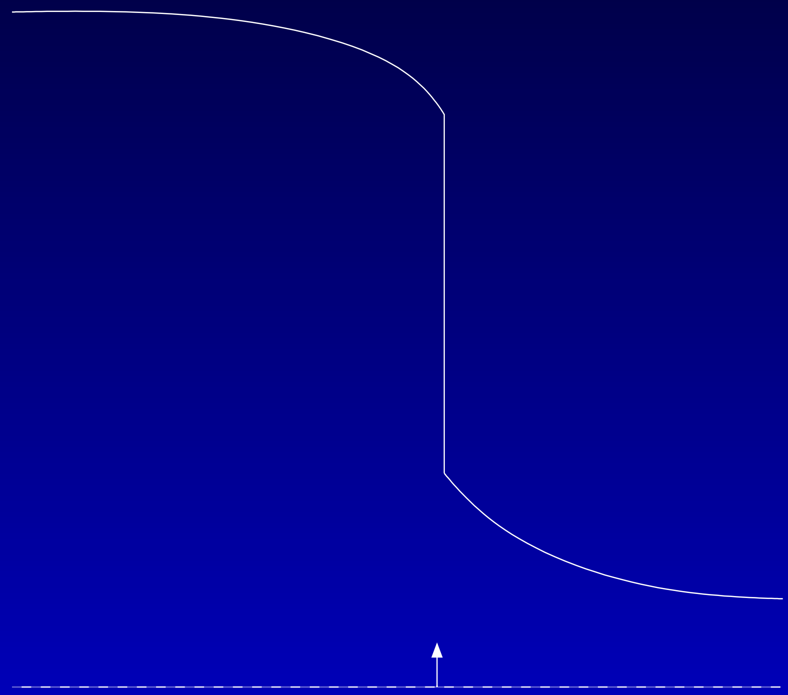


Spreading the singular force

Immersed Interface



Handling known discontinuities (e.g. in pressure) due to forces

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Usage of old positions results in explicit method

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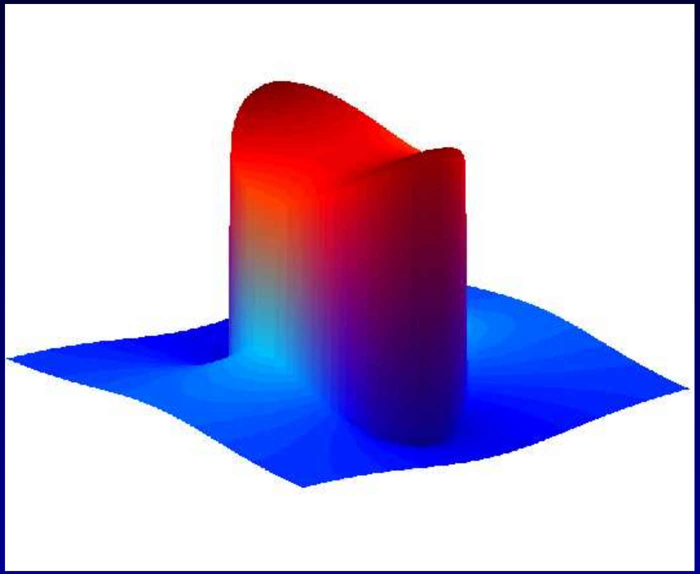
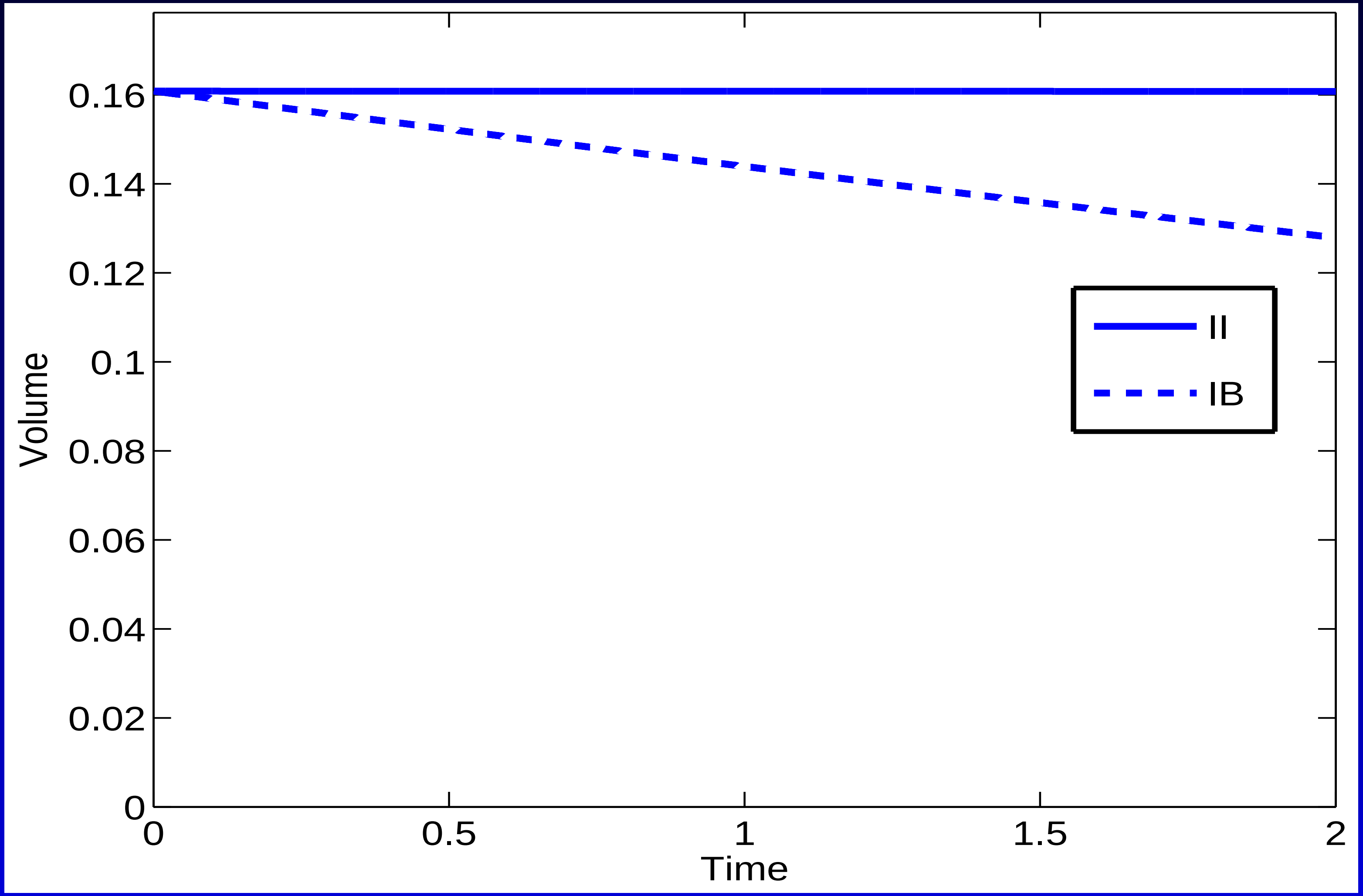
Jacobian is large, dense, and involves nonlinear PDE solve

# Example problem



# Volume Conservation



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  - Doesn't handle links between objects
- Implementation underway using Radial Basis Functions

# Solution components

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- (Approximate) Projection Method
- Multigrid
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- MPI
- Splines → Radial Basis Functions
- Quasi-Newton Solver
- Lapack, Valgrind, others

# Acknowledgements


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