A NEW LIQUID-VAPOR PHASE-TRANSITION TECHNIQUE FOR THE LEVEL SET METHOD

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Research Objective

- Improve the liquid-vapor phase-transition capabilities of the level set method
  - Account for different properties in the respective phases
  - Capture the Latent heat absorbed or released
  - Capture the different temperature gradients across the interface
  - Handle temperature gradients in both phases
Level Set Method

- Use a higher-dimensional function to represent the interface [Osher (1988)]
  \[ \phi < 0 \text{ is phase 1} \]
  \[ \phi > 0 \text{ is phase 2} \]
  \[ \phi = 0 \text{ is interface} \]

- The interface is advected according to:
  \[ \frac{\partial \phi}{\partial t} + \vec{V}_{\text{INT}} \cdot \nabla \phi = 0 \]
Capturing the Interface

The interface can be implicitly captured

\[ H_\epsilon(\phi) = \begin{cases} 0 & \text{if } \phi < -\epsilon \\ \frac{1}{2} \left(1 + \phi \frac{1}{\epsilon} \sin \left(\frac{\pi \phi}{\epsilon}\right)\right) & \text{if } |\phi| \leq \epsilon \\ 1 & \text{if } \phi > \epsilon \end{cases} \]

\[ \delta_\epsilon(\phi) = \begin{cases} 0 & \text{if } \phi < -\epsilon \\ \frac{1}{2\epsilon} \left(1 + \cos \left(\frac{\pi \phi}{\epsilon}\right)\right) & \text{if } |\phi| \leq \epsilon \\ 0 & \text{if } \phi > \epsilon \end{cases} \]
Properties and Geometric Quantities

- The properties of each respective phase can be represented using the Heaviside function
  \[ \gamma = \gamma_1 H(\phi) + \gamma_2 (1 - H(\phi)) \]

- Interface geometry
  \[ \vec{n} = \frac{\nabla \phi}{\|\nabla \phi\|_2} \quad \kappa = \nabla \cdot \vec{n} = \nabla \cdot \frac{\nabla \phi}{\|\nabla \phi\|_2} \]
Governing Equations

- Conservation of Mass

\[
\frac{d\rho}{dt} + \nabla \cdot (\rho \vec{V}) = 0
\]

- The respective phases are incompressible

\[
\nabla \cdot \vec{V}_l = 0 \quad \nabla \cdot \vec{V}_v = 0
\]

- The continuity equation corresponding to phase transition [Juric (1998)]

\[
\vec{V} = \vec{V}_l H + \vec{V}_v (1 - H)
\]

\[
\nabla \cdot \vec{V} = (\vec{V}_l - \vec{V}_v) \cdot \nabla H \quad \text{or} \quad \nabla \cdot \vec{V} = \Gamma_{MASS}
\]
Governing Equations

- **Mass**
  \[ \nabla \cdot \vec{V} = \Gamma_{\text{MASS}} \]

- **Momentum [Brackbill (1992)]**
  \[
  \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = -\frac{\nabla P}{\rho} + \vec{g} + \frac{\nabla \cdot \tau}{\rho} \equiv -\sigma k \nabla H
  \]
  \[\tau = \mu \left( \nabla \vec{V} + (\nabla \vec{V})^T \right)\]

- **Energy**
  \[
  \frac{\partial cT}{\partial t} + \vec{V} \cdot \nabla cT = \frac{\nabla \cdot (k \nabla T)}{\rho} + \frac{\Gamma_{\text{ENERGY}}}{\rho}
  \]
Interface Physics

- The heat flux is continuous across the interface, but the properties and the gradient are not continuous.
Interface Jump Conditions

- Mass and energy conservation across the interface [Welch (2000)]

\[
[[\zeta]] = \zeta_v - \zeta_l
\]

- Mass

\[
[[\rho(\vec{V} - \vec{V}_{INT})]] \cdot \vec{n} = 0
\]

- Energy

\[
[[\rho h(\vec{V} - \vec{V}_{INT})]] \cdot \vec{n} = -[[\vec{q}]] \cdot \vec{n}
\]
Interface Jump Conditions

- **Interface velocity**
  \[
  \vec{V}_{INT} = \vec{V} + \left( -k_v \nabla T + k_l \nabla T \right) \frac{1}{\rho} \frac{\rho}{(h_v - h_l)}
  \]

- **Mass source term**
  \[
  \Gamma_{MASS} = \left( \frac{1}{\rho_v} - \frac{1}{\rho_l} \right) \left( -k_v \nabla T + k_l \nabla T \right) \cdot \nabla H(\phi) \frac{1}{h_v - h_l}
  \]
Energy Source Term

- **Discrete energy equation**

\[
T^{n+1} = \left(\frac{c(\phi)T}{c(\phi)^{n+1}}\right) + \frac{\Delta t}{c(\phi)^{n+1}} \left(-\vec{V} \cdot \nabla (c(\phi)T)^n + \nabla \cdot (k(\phi)\nabla T)^n + \frac{\Gamma_{\text{ENERGY}}^{n+1}}{\rho(\phi)^n} + \frac{\Gamma_{\text{ENERGY}}^{n+1}}{\rho(\phi)^n}\right)
\]

- **The source term corrects the temperature field to satisfy the interface boundary condition**

\[
T^* = \left(\frac{c(\phi)T}{c(\phi)^{n+1}}\right) + \frac{\Delta t}{c(\phi)^{n+1}} \left(-\vec{V} \cdot \nabla (c(\phi)T)^n + \nabla \cdot (k(\phi)\nabla T)^n + \frac{\Gamma_{\text{ENERGY}}^{n+1}}{\rho(\phi)^n}\right)
\]

\[
T^{n+1} = T^* + \Delta t \left(\frac{\Gamma_{\text{ENERGY}}^{n+1}}{\rho(\phi)^n c(\phi)^{n+1}}\right) \quad \text{so} \quad T^{n+1} = T^* + \Delta T_{PC}
\]
Interface Boundary Condition

- Extrapolation equation

\[ T_{\text{sat}} = T_{i,j} - \phi_{i,j} \n \cdot \nabla T |_{i,j} \]

\[ \phi = 0 \quad \phi > 0 \quad \phi < 0 \]
Interface Boundary Condition

- Extrapolation equation

\[ T_{i,j} = \frac{T_{sat} - B}{A} \]

\[ A = 1 + \left( -\frac{\min(0, -\phi_{i,j}, n_{i,j}^x)}{\Delta x} + \frac{\max(0, -\phi_{i,j}, n_{i,j}^x)}{\Delta x} - \frac{\min(0, -\phi_{i,j}, n_{i,j}^y)}{\Delta y} + \frac{\max(0, -\phi_{i,j}, n_{i,j}^y)}{\Delta y} \right) \]

\[ B = \left( \frac{\min(0, -\phi_{i,j}, n_{i,j}^x)}{\Delta x} T_{i+1,j} - \frac{\max(0, -\phi_{i,j}, n_{i,j}^x)}{\Delta x} T_{i-1,j} + \frac{\min(0, -\phi_{i,j}, n_{i,j}^y)}{\Delta y} T_{i,j+1} - \frac{\max(0, -\phi_{i,j}, n_{i,j}^y)}{\Delta y} T_{i,j-1} \right) \]
Interface Temperature Gradients (I)

- Use ghost nodes
  - Extrapolate temperature values to the ghost nodes in the other respective phase along the interface

Red nodes = vapor ghost nodes
Blue nodes = liquid ghost nodes
Interface Temperature Gradients (II)

- Derivative stencil

![Diagram showing liquid and vapor phases with derivative stencil and vapor ghost nodes.](image-url)
Ghost Node Construction

- Ghost node construction equation

\[ T_{GHOST_{i,j}} = \frac{T_{sat} - B}{A} \]

\[ A = 1 + \left( -\frac{\min(0, \phi_{i,j} n^x_{i,j})}{\Delta x} + \frac{\max(0, \phi_{i,j} n^x_{i,j})}{\Delta x} - \frac{\min(0, \phi_{i,j} n^y_{i,j})}{\Delta y} + \frac{\max(0, \phi_{i,j} n^y_{i,j})}{\Delta y} \right) \]

\[ B = \left( \frac{\min(0, \phi_{i,j} n^x_{i,j})}{\Delta x} T_{i+1,j} - \frac{\max(0, \phi_{i,j} n^x_{i,j})}{\Delta x} T_{i-1,j} + \frac{\min(0, \phi_{i,j} n^y_{i,j})}{\Delta y} T_{i,j+1} - \frac{\max(0, \phi_{i,j} n^y_{i,j})}{\Delta y} T_{i,j-1} \right) \]
Solution Strategy

- Calculate the projected velocity field (Chorin 1968)
- Calculate the new level set field
- Calculate the projected temperature field
- Apply the interface temperature condition
- Construct the new ghost node values
- Evaluate the continuity equation source term
- Calculate the new pressure field
- Calculate the new velocity
- Now, go back to step one
Test Problems (I)

- 1-D phase-change [Özişik (1993)]

\[ \frac{\partial T_v}{\partial t} = \alpha_v \frac{\partial^2 T_v}{\partial x^2} \]

\[ \rho_v h_{fg} \left( \frac{d\delta}{dt} \right) = -k_v \frac{\partial T_v}{\partial x} \bigg|_{x=\delta(t)} \]

\[ T_v(x = \delta(t), t) = T_{sat} \]

\[ T_v(x = 0, t) = T_{wall} \]

\[ L_2 = \sqrt{\sum_{i,j} \left( T_{i,j} - T_{Exact} \right)^2} \]

\[ \frac{N_x N_y}{(T_{wall} - T_{sat})^2} \]
Test Problems (II)

- 2-D phase-change [Özişik (1993)]

\[
\frac{\partial T_v}{\partial t} = \alpha_v \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial T_v}{\partial r} \right)
\]

\[
\rho_v h_{fg} \frac{d\delta}{dt} = -k_v \left. \frac{\partial T_v}{\partial r} \right|_{x=\delta(t)}
\]

\[
T_v(x = \delta(t), t) = T_{sat}
\]

\[
\lim_{r \to 0} \left( 2\pi k_v \frac{\partial T_v}{\partial r} \right) = Q
\]

\[
L_2 = \sqrt{\frac{\sum_{i,j} (T_{i,j} - T_{\text{Exact}})^2}{N_x N_y}}
\]
1D Test Problem Results (I)

- \( Ja = 9 \)
- \( \alpha_v = 10^{-3} \)
1D Test Problem Results (II)
1D Test Problem Results (III)
2D Test Results (I)

2 sec

3 sec
2D Test Results (II)
2-D Test Results (III)
Film Boiling Movie
Conclusions

- The new liquid-vapor phase-transition technique extends the modeling capabilities of the level set method
  - (1) It handles different properties in each respective phase
  - (2) It captures the liberation or absorption of latent heat
  - (3) It captures the discontinuous temperature gradients across the interface
  - (4) It can handle temperature gradients in both phases