A NEW LIQUID-VAPOR PHASE-TRANSITION TECHNIQUE FOR THE LEVEL SET METHOD

Nathaniel R. Morgan
George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

- Dr. Smith (Advisor)
- Dr. Ghiaasiaan
- Dr. Sotiropoulos
- Dr. Mucha
- Dr. Dolbow [CSGF Alumni]

Acknowledgments

- This work was supported by the Department of Energy Computational Science Graduate Fellowship Program of the Office of Scientific Computing and Office of Defense Programs in the Department of Energy under contract DE-FG02-97ER25308.

Research Objective

- Improve the liquid-vapor phase-transition capabilities of the level set method
  - Account for different properties in the respective phases
  - Capture the Latent heat absorbed or released
  - Capture the different temperature gradients across the interface
  - Handle temperature gradients in both phases

Level Set Method

- Use a higher-dimensional function to represent the interface [Osher (1988)]

  $\phi<0$ is phase 1

  $\phi>0$ is phase 2

  $\phi=0$ is interface

- The interface is advected according to:

  $$\frac{\partial \phi}{\partial t} + \vec{V}_{INT} \bullet \nabla \phi = 0$$

Capturing the Interface

- The interface can be implicitly captured

$$H_\varepsilon(\phi) = \begin{cases} 0 & if \quad \phi < -\varepsilon \\ \frac{1}{2}\left(1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin\left(\frac{\pi\phi}{\varepsilon}\right)\right), & if \quad |\phi| <= \varepsilon \\ 1 & if \quad \phi > \varepsilon \end{cases}$$

$$\delta_\varepsilon(\phi) = \begin{cases} 0 & if \quad \phi < -\varepsilon \\ \frac{1}{2\varepsilon}\left(1 + \cos\left(\frac{\pi\phi}{\varepsilon}\right)\right), & if \quad |\phi| <= \varepsilon \\ 0 & if \quad \phi > \varepsilon \end{cases}$$

Properties and Geometric Quantities

- The properties of each respective phase can be represented using the Heaviside function

$$\gamma = \gamma_1 H(\phi) + \gamma_2 \left(1 - H(\phi)\right)$$

- Interface geometry

$$\vec{n} = \frac{\nabla \phi}{\left\|\nabla \phi\right\|_2} \qquad \kappa = \nabla \bullet \vec{n} = \nabla \bullet \frac{\nabla \phi}{\left\|\nabla \phi\right\|_2}$$

Governing Equations

- Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \vec{V}\right) = 0$$

- The respective phases are incompressible

$$\nabla \bullet \vec{V}_l = 0 \qquad \nabla \bullet \vec{V}_v = 0$$

- The continuity equation corresponding to phase transition [Juric (1998)]

$$\vec{V} = \vec{V}_l H + \vec{V}_v (1 - H)$$

$$\nabla \bullet \vec{V} = (\vec{V}_l - \vec{V}_v) \bullet \nabla H \qquad \text{or} \qquad \nabla \bullet \vec{V} = \Gamma_{MASS}$$

Governing Equations

- Mass

$$\nabla \bullet \vec{V} = \Gamma_{MASS}$$

- Momentum [Brackbill (1992)]

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \bullet \nabla\right)\vec{V} = -\frac{\nabla P}{\rho} + \vec{g} + \frac{\nabla \bullet \underline{\underline{\tau}}}{\rho} - \frac{\sigma\kappa\nabla H}{\rho}$$

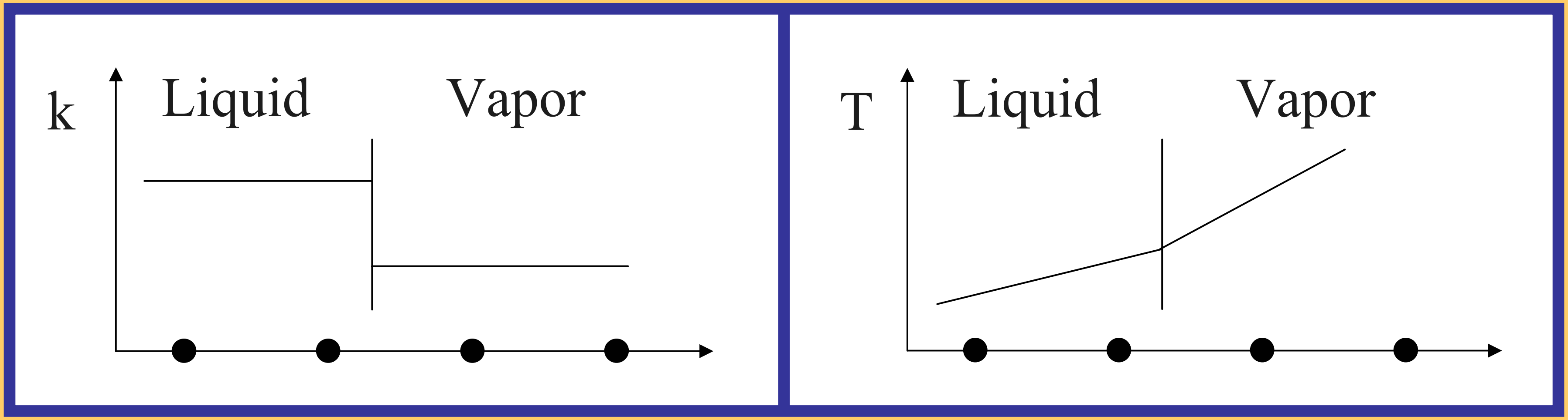
$$\underline{\underline{\tau}} = \mu\left(\nabla\vec{V} + \left(\nabla\vec{V}\right)^{T}\right)$$

- Energy

$$\frac{\partial cT}{\partial t} + \vec{V} \bullet \nabla cT = \frac{\nabla \bullet \left(k\nabla T\right)}{\rho} + \frac{\Gamma_{ENERGY}}{\rho}$$

Interface Physics

- The heat flux is continuous across the interface, but the properties and the gradient are not continuous



Thermal Conductivity

Temperature

Interface Jump Conditions

- Mass and energy conservation across the interface [Welch (2000)]

$$[[\zeta]] = \zeta_v - \zeta_l$$

- Mass $$[[\rho\left(\vec{V} - \vec{V}_{INT}\right)]] \bullet \vec{n} = 0$$

- Energy $$[[\rho h\left(\vec{V} - \vec{V}_{INT}\right)]] \bullet \vec{n} = -[[\vec{q}]] \bullet \vec{n}$$

Interface Jump Conditions

- Interface velocity

$$\vec{V}_{INT} = \vec{V} + \frac{\left(-k_v \nabla T + k_l \nabla T\right)}{\rho\left(h_v - h_l\right)}$$

- Mass source term

$$\Gamma_{MASS} = \left(\frac{1}{\rho_v} - \frac{1}{\rho_l}\right)\frac{\left(-k_v \nabla T + k_l \nabla T\right) \bullet \nabla H(\phi)}{h_v - h_l}$$

Energy Source Term

- Discrete energy equation

$$T^{n+1} = \frac{(c(\phi)T)^n}{c(\phi)^{n+1}} + \frac{\Delta t}{c(\phi)^{n+1}}\left(-\vec{V}\bullet\nabla\left(c(\phi)T\right)^n + \right.$$

$$\left. + \frac{\nabla\bullet(k(\phi)\nabla T)^n}{\rho(\phi)^n} + \frac{\Gamma_{ENERGY}^{n+1}}{\rho(\phi)^n}\right)$$

- The source term corrects the temperature field to satisfy the interface boundary condition

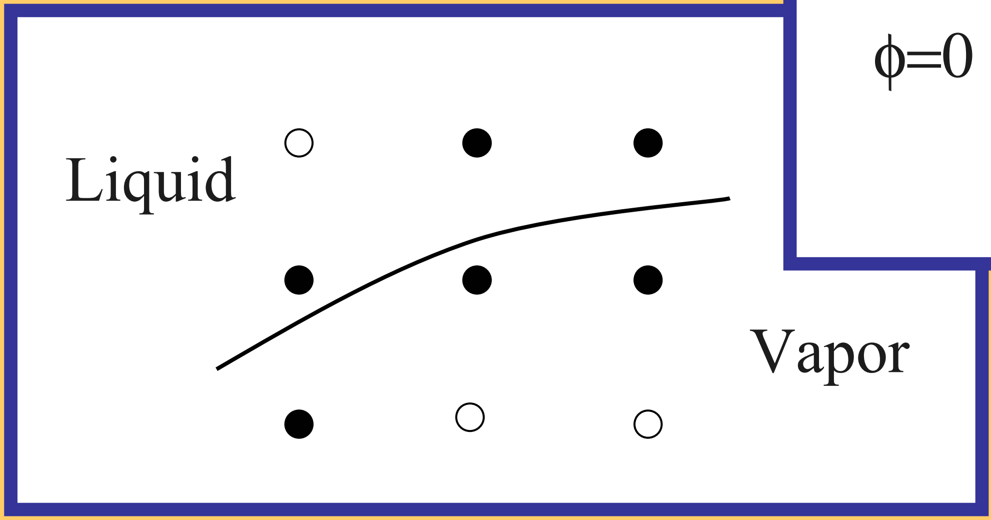
$$T^{*} = \frac{(c(\phi)T)^n}{c(\phi)^{n+1}} + \frac{\Delta t}{c(\phi)^{n+1}}\left(-\vec{V}\bullet\nabla\left(c(\phi)T\right)^n + \frac{\nabla\bullet(k(\phi)\nabla T)^n}{\rho(\phi)^n}\right)$$

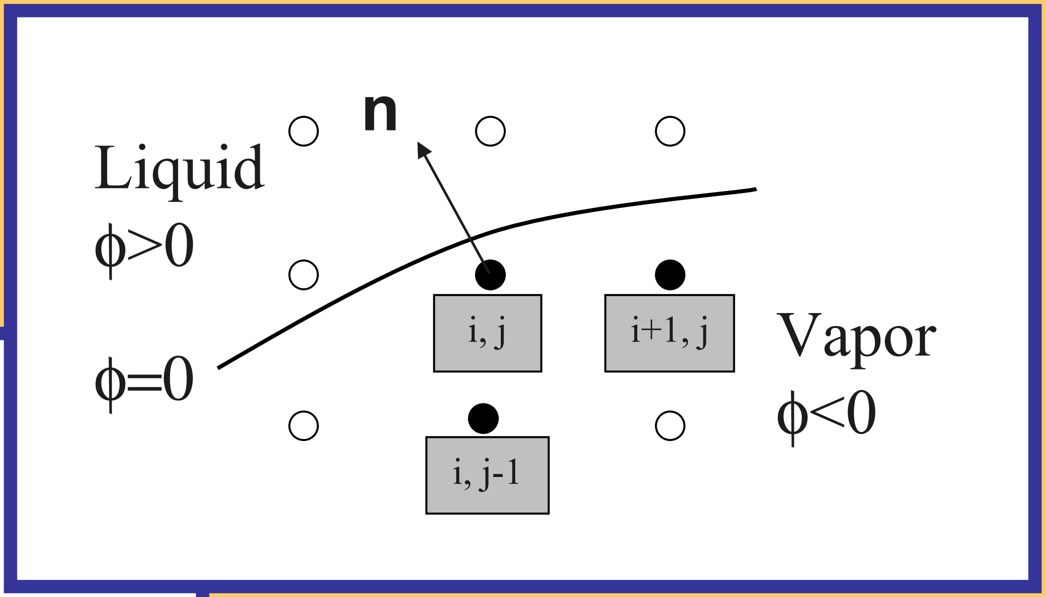
$$T^{n+1} = T^{*} + \Delta t\left(\frac{\Gamma_{ENERGY}^{n+1}}{\rho(\phi)^n\, c(\phi)^{n+1}}\right) \quad \text{so} \quad T^{n+1} = T^{*} + \Delta T_{PC}$$

Interface Boundary Condition

- Extrapolation equation

$$T_{sat} = T_{i,j} - \phi_{i,j}\vec{n} \bullet \nabla T\big|_{i,j}$$





Interface Boundary Condition

- Extrapolation equation

$$T_{i,j} = \frac{T_{sat} - B}{A}$$

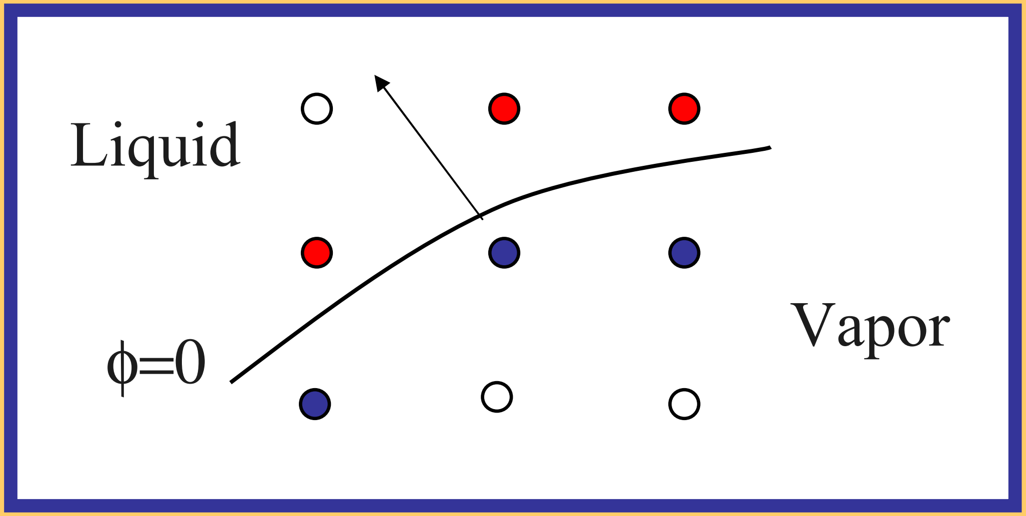
$$A = 1 + \left(-\frac{\min(0,-\phi_{i,j} n^{x}_{i,j})}{\Delta x} + \frac{\max(0,-\phi_{i,j} n^{x}_{i,j})}{\Delta x} - \frac{\min(0,-\phi_{i,j} n^{y}_{i,j})}{\Delta y} + \frac{\max(0,-\phi_{i,j} n^{y}_{i,j})}{\Delta y} \right)$$

$$B = \left(\frac{\min(0,-\phi_{i,j} n^{x}_{i,j})}{\Delta x} T_{i+1,j} - \frac{\max(0,-\phi_{i,j} n^{x}_{i,j})}{\Delta x} T_{i-1,j} + \right.$$

$$\left. + \frac{\min(0,-\phi_{i,j} n^{y}_{i,j})}{\Delta y} T_{i,j+1} - \frac{\max(0,-\phi_{i,j} n^{y}_{i,j})}{\Delta y} T_{i,j-1} \right)$$

Interface Temperature Gradients (I)

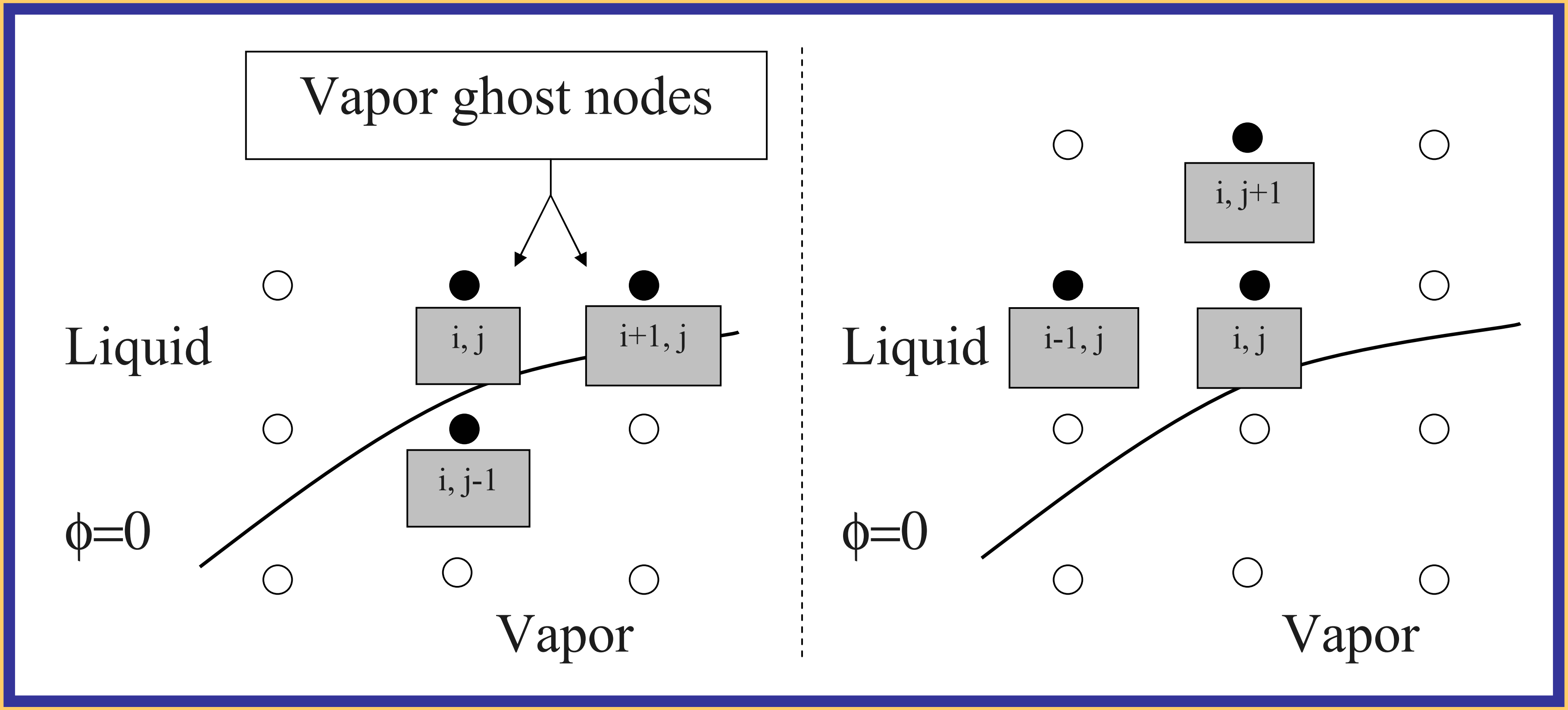
- Use ghost nodes
  - Extrapolate temperature values to the ghost nodes in the other respective phase along the interface



Red nodes = vapor ghost nodes
Blue nodes = liquid ghost nodes

Interface Temperature Gradients (II)

- Derivative stencil



Ghost Node Construction

- Ghost node construction equation

$$T_{GHOST\,i,j} = \frac{T_{sat} - B}{A}$$

$$A = 1 + \left(-\frac{\min(0, \phi_{i,j} n^x_{i,j})}{\Delta x} + \frac{\max(0, \phi_{i,j} n^x_{i,j})}{\Delta x} - \frac{\min(0, \phi_{i,j} n^y_{i,j})}{\Delta y} + \frac{\max(0, \phi_{i,j} n^y_{i,j})}{\Delta y} \right)$$

$$B = \left(\frac{\min(0, \phi_{i,j} n^x_{i,j})}{\Delta x} T_{i+1,j} - \frac{\max(0, \phi_{i,j} n^x_{i,j})}{\Delta x} T_{i-1,j} + \right.$$

$$\left. + \frac{\min(0, \phi_{i,j} n^y_{i,j})}{\Delta y} T_{i,j+1} - \frac{\max(0, \phi_{i,j} n^y_{i,j})}{\Delta y} T_{i,j-1} \right)$$

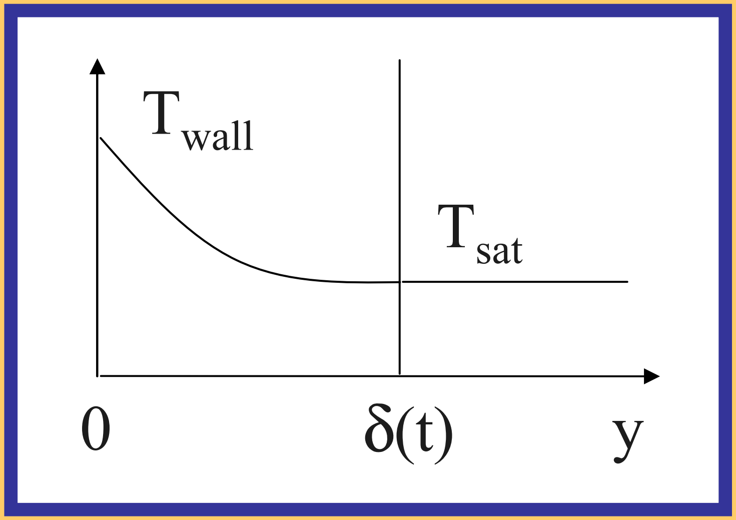
Solution Strategy

- Calculate the projected velocity field (Chorin 1968)
- Calculate the new level set field
- Calculate the projected temperature field
- Apply the interface temperature condition
- Construct the new ghost node values
- Evaluate the continuity equation source term
- Calculate the new pressure field
- Calculate the new velocity
- Now, go back to step one

Test Problems (I)

- 1-D phase-change [Özişik (1993)]

$$\frac{\partial T_v}{\partial t} = \alpha_v \frac{\partial^2 T_v}{\partial x^2}$$



$$\rho_v h_{fg} \frac{d\delta}{dt} = -k_v \left. \frac{\partial T_v}{\partial x} \right|_{x=\delta(t)}$$

$$T_v(x = \delta(t), t) = T_{sat}$$

$$T_v(x = 0, t) = T_{wall}$$

$$L_2 = \sqrt{\frac{\sum_{i,j} \left(T_{i,j} - T_{Exact}\right)^2}{N_x N_y}}$$

Test Problems (II)

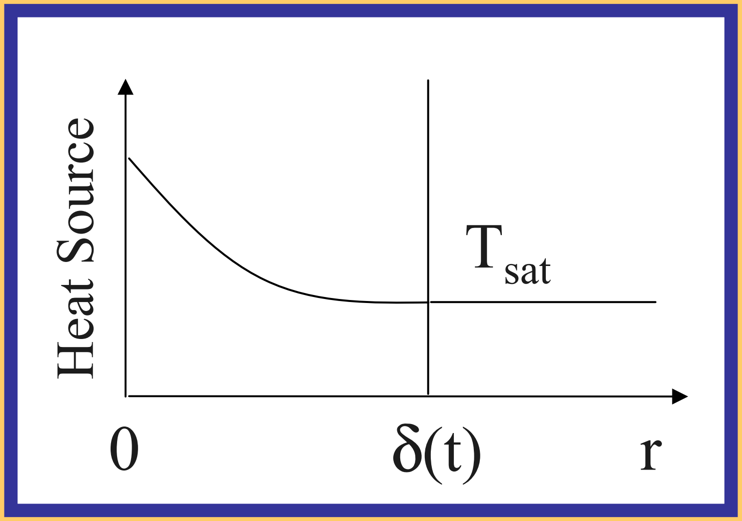
- 2-D phase-change [Özişik (1993)]

$$\frac{\partial T_v}{\partial t} = \alpha_v \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T_v}{\partial r}\right)$$

$$\rho_v h_{fg} \frac{d\delta}{dt} = -k_v \frac{\partial T_v}{\partial r}\bigg|_{x=\delta(t)}$$

$$T_v(x=\delta(t),t) = T_{sat}$$

$$\lim_{r\to 0}\left(2\pi k_v \frac{\partial T_v}{\partial r}\right) = Q$$



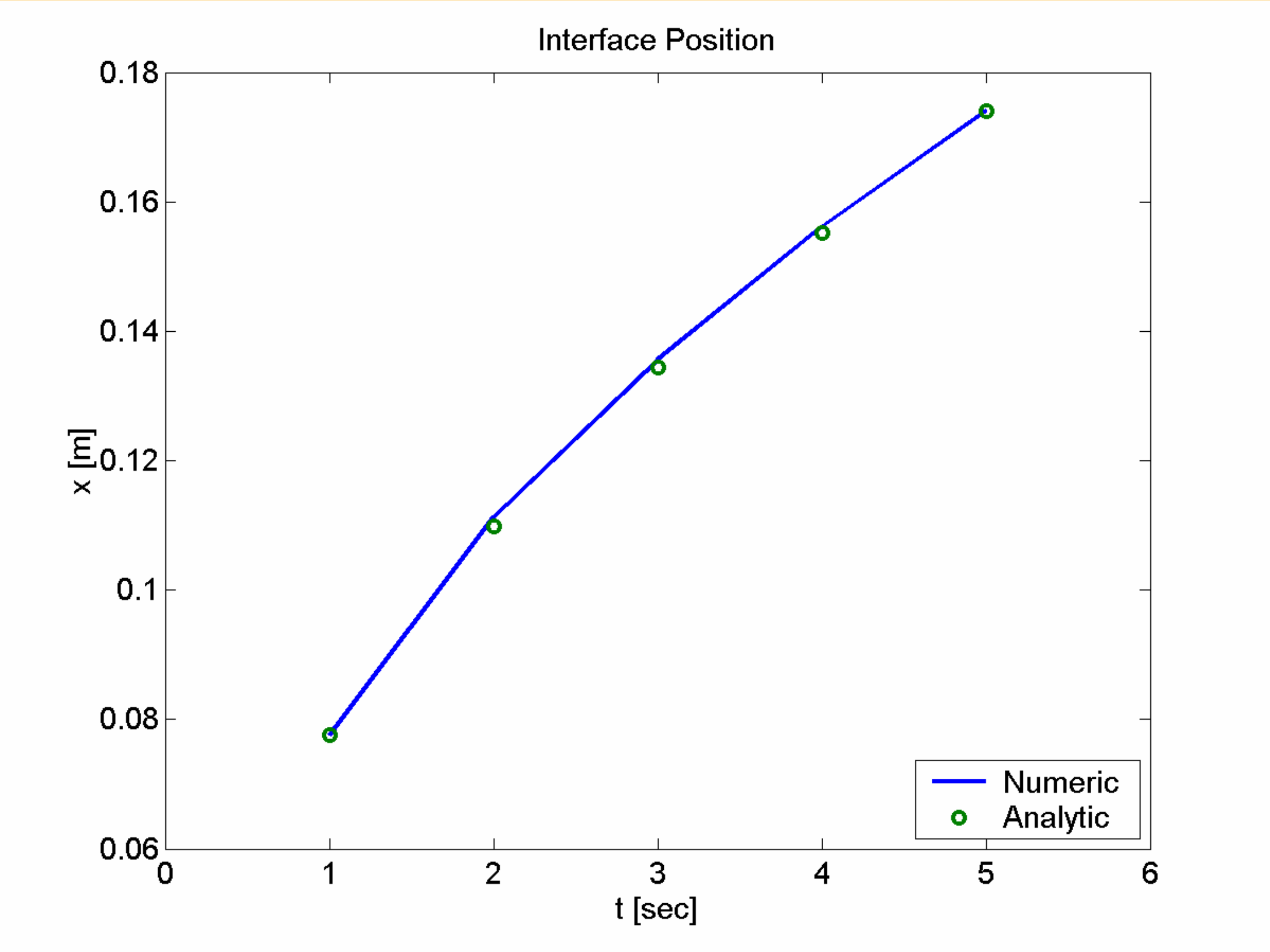
$$L_2 = \sqrt{\frac{\sum_{i,j}\left(T_{i,j} - T_{Exact}\right)^2}{N_x N_y}}$$

1D Test Problem Results (I)

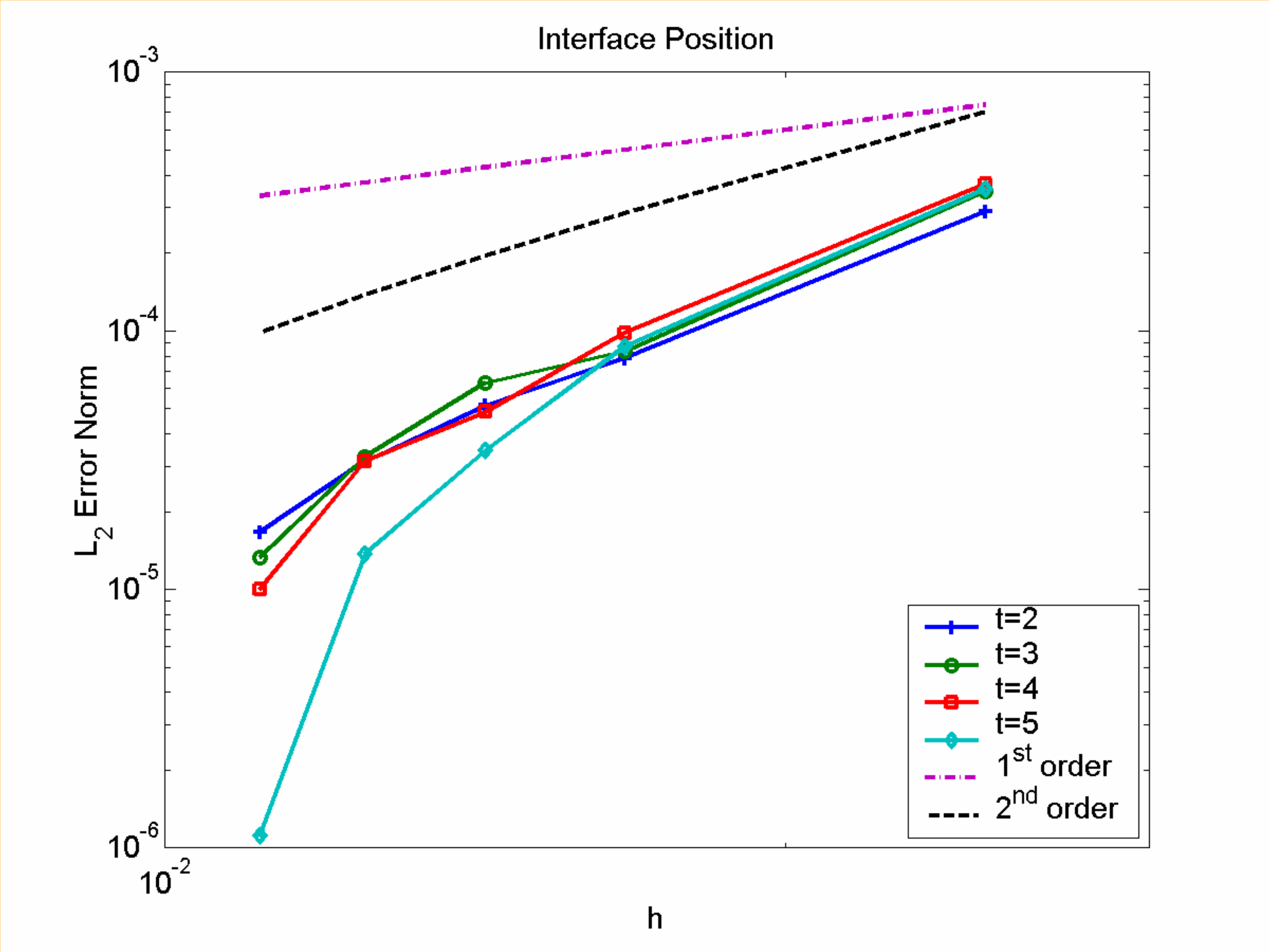
properties

Ja = 9

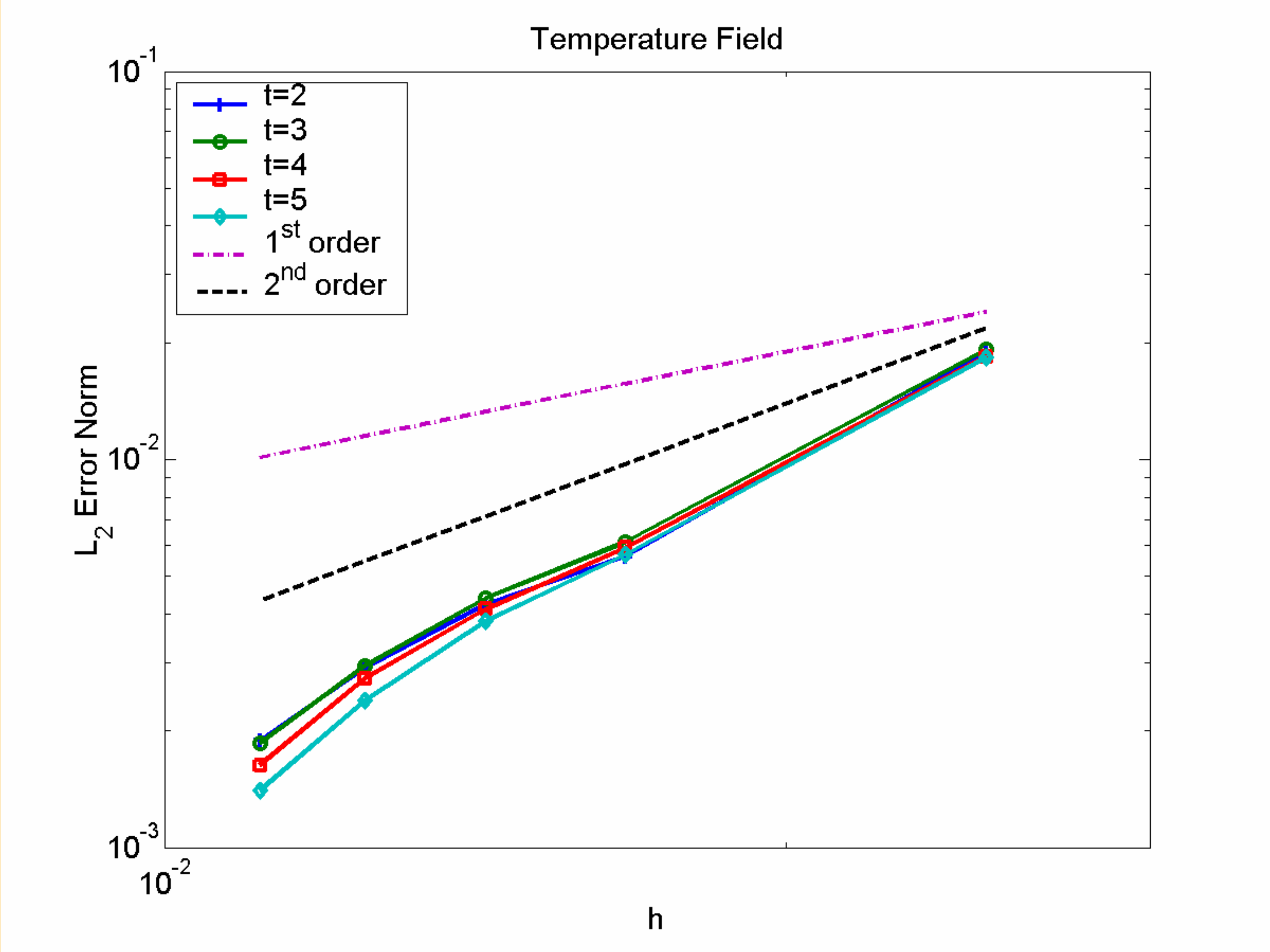
$\alpha_v = 10^{-3}$



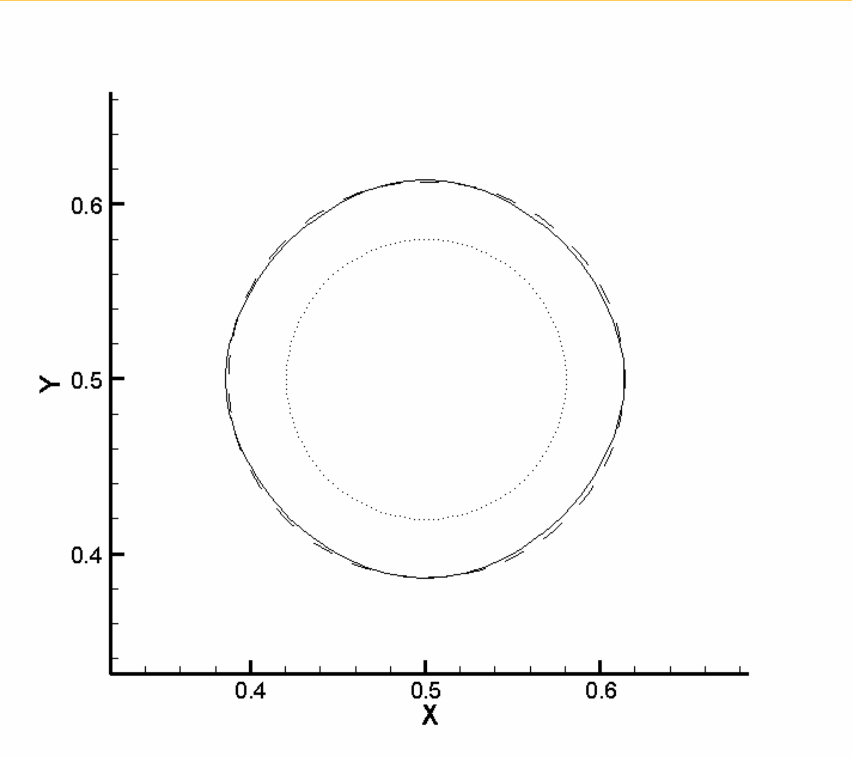
1D Test Problem Results (II)



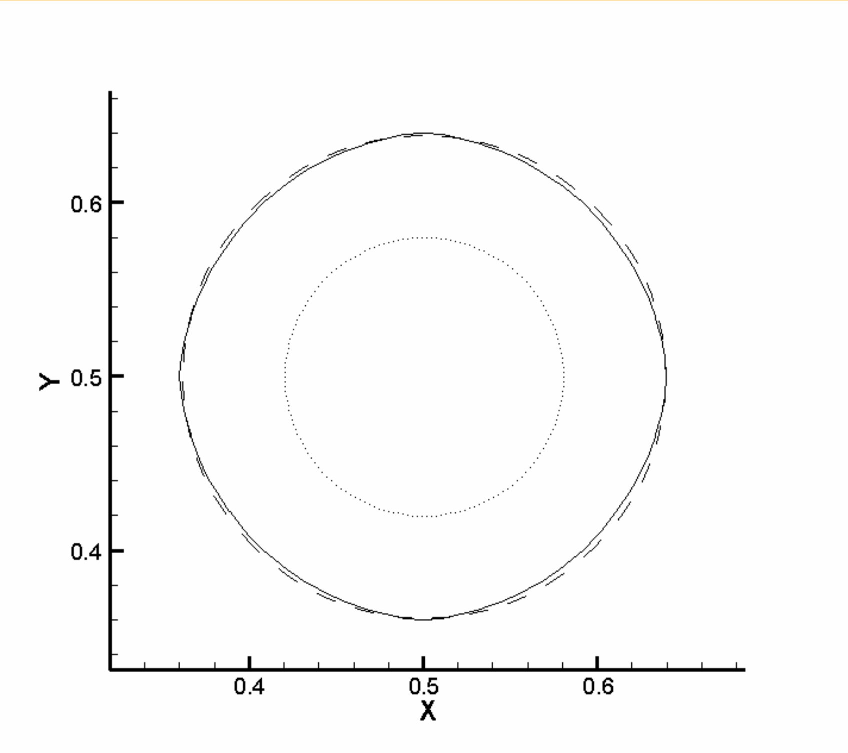
1D Test Problem Results (III)



2D Test Results (I)

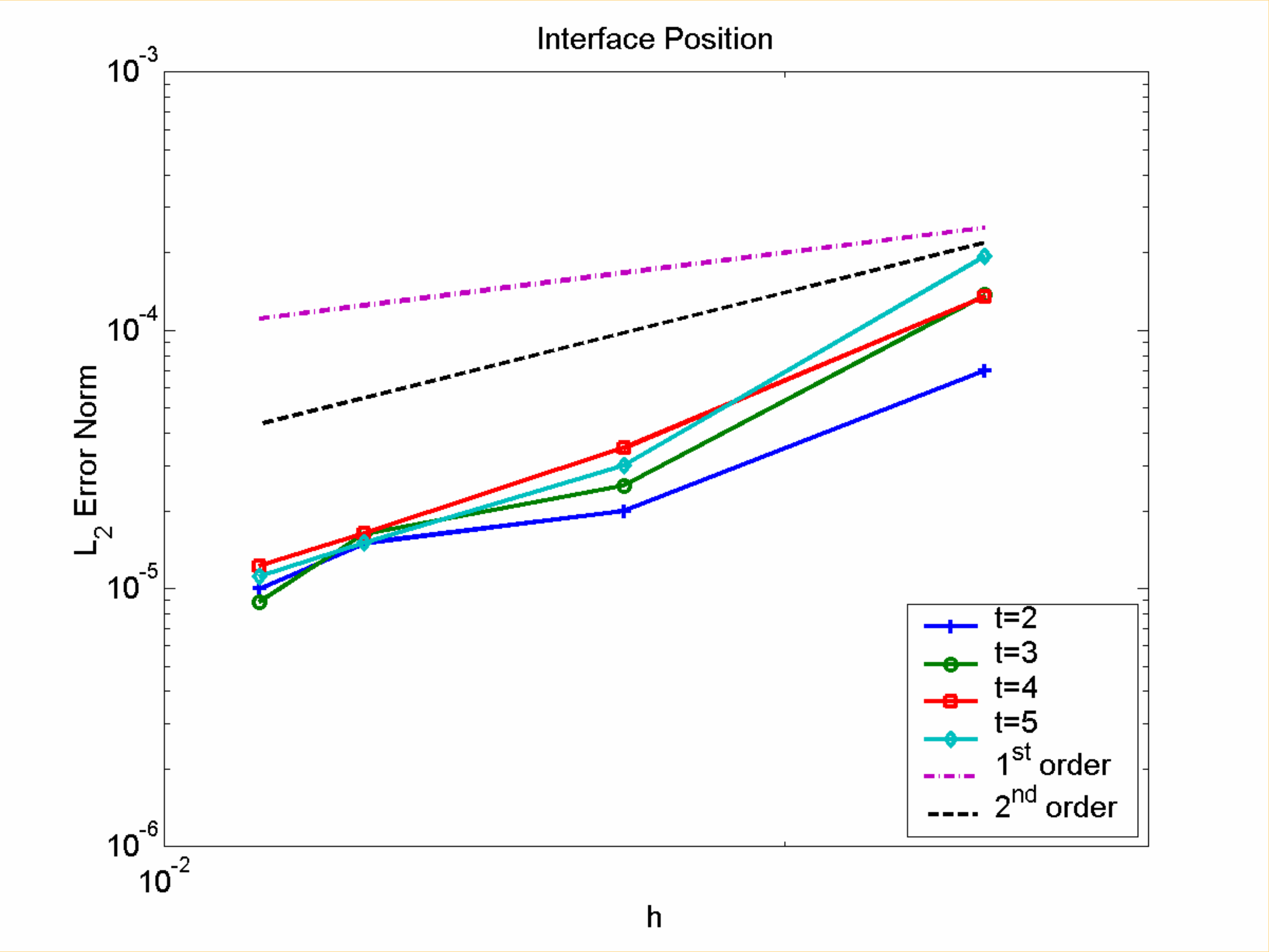


2 sec

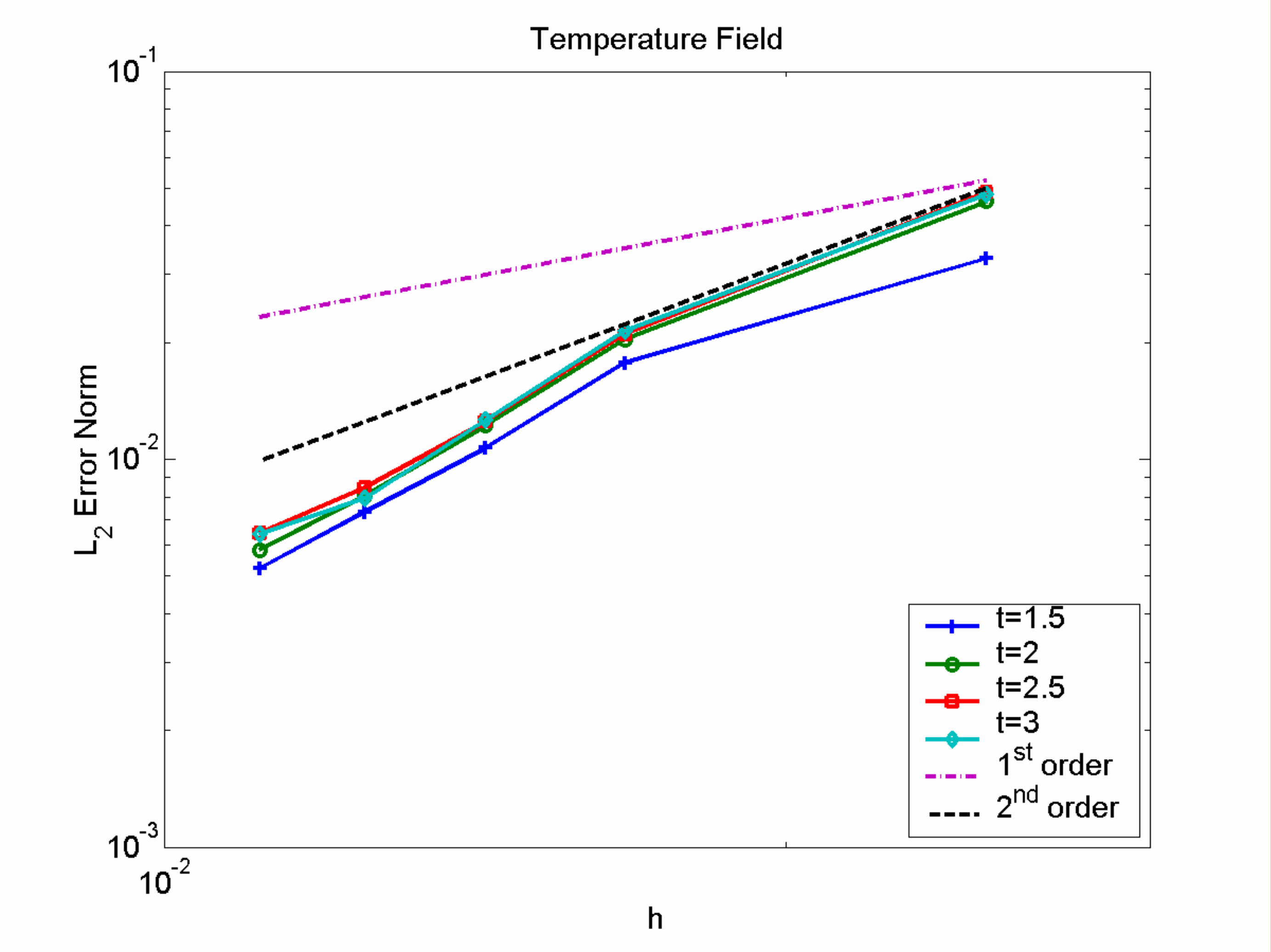


3 sec

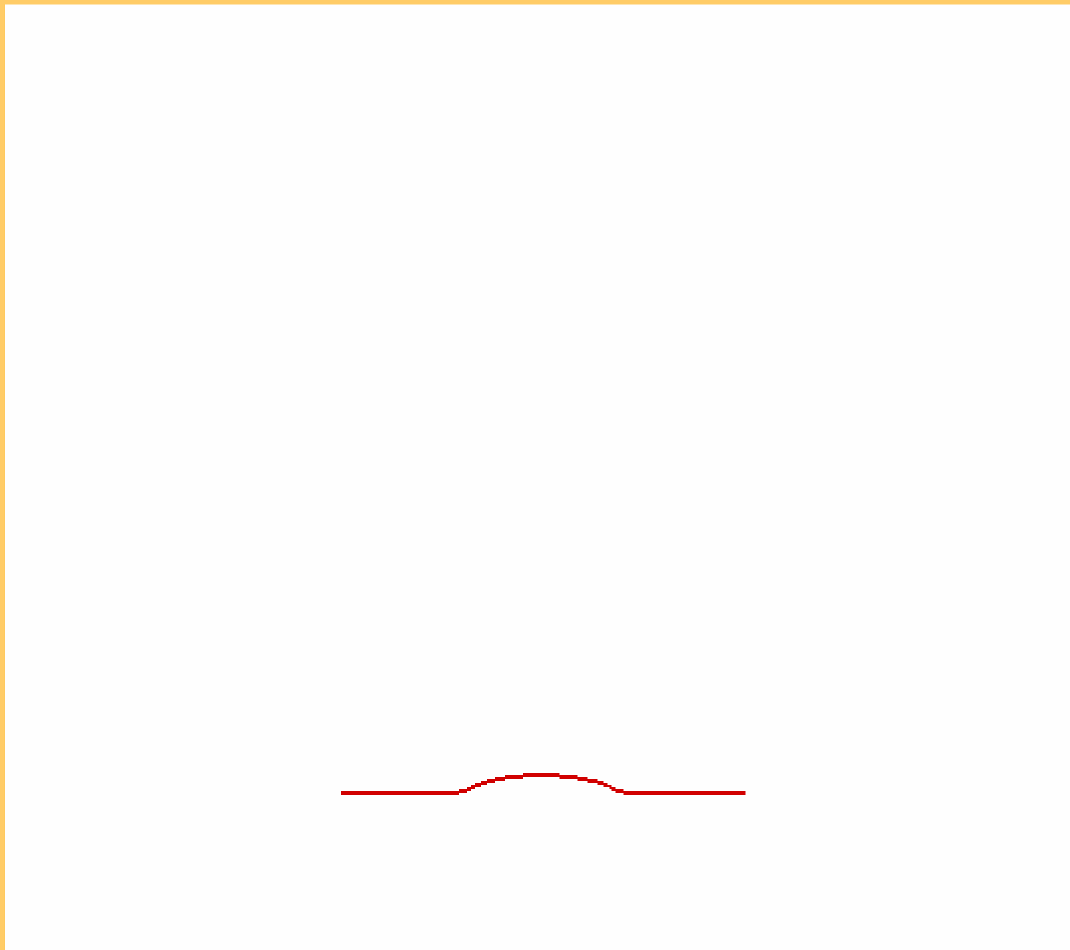
2D Test Results (II)



2-D Test Results (III)



Film Boiling Movie



Conclusions

- The new liquid-vapor phase-transition technique extends the modeling capabilities of the level set method
  - (1) It handles different properties in each respective phase
  - (2) It captures the liberation or absorption of latent heat
  - (3) It captures the discontinuous temperature gradients across the interface
  - (4) It can handle temperature gradients in both phases