The deep roots of volcanos: localization instabilities in a continuum model of magma dynamics

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With Ben Holtzman, Peter Kelemen (LDEO/CU), Barry Smith, Matt Knepley (ANL) and Craig Manning (UCLA)
Plate tectonics, volcanos and magma genesis
Chemical localization of magma
observations by Kelemen et al.
Mechanical localization of magma experiments by Holtzman et al.

PI-1020, olivine + chromite + 4% MORB, $\gamma = 3.5$, $P = 30-60$ MPa

Olivine + chromite (4:1) + 4 vol% MORB, const. strain rate, $\gamma = 3.4$
Magma dynamics theory: key components

Creeping Solid Flow
Porous Flow of Fluids
2-Phase Mantle Dynamics
Interphase Mass Transfer

4-7 primary variables
Theory: two-phase magma dynamics (McKenzie ’84)

with permeability $k_\phi \propto \phi^n$, shear viscosity $\eta$ and bulk viscosity $\zeta$. 
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\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}) = 0 \]

1. **Conservation of mass**: pore fluid

\[ \frac{\partial}{\partial t}(1 - \phi) + \nabla \cdot [(1 - \phi) \mathbf{v}] = 0 \]

2. **Conservation of mass**: matrix solid

with permeability \( k_\phi \propto \phi^n \), shear viscosity \( \eta \) and bulk viscosity \( \zeta \).
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\[ \phi (\mathbf{v} - \mathbf{V}) = -\frac{k_\phi}{\mu} [\nabla P - \rho_f \mathbf{g}] \]

1. **Conservation of mass:** pore fluid

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3. **Conservation of momentum:** pore fluid (Darcy’s law)

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\]

\[
\nabla P = \nabla \cdot \eta \left[ (\nabla \mathbf{V}) + (\nabla \mathbf{V})^T \right]
\]
\[
+ \nabla \left( \zeta - \frac{2\eta}{3} \right) \nabla \cdot \mathbf{V} + \bar{\rho} \mathbf{g}
\]

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4. **Conservation of momentum:** matrix solid (Stokes eqn)

with permeability $k_\phi \propto \phi^n$, shear viscosity $\eta$ and bulk viscosity $\zeta$. 
Computational Method

- 2D finite volume discretization on a Cartesian staggered mesh.
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- Parallel semi-Lagrangian advection in development for PETSc.
Part 1: Chemical localization

Past work by Aharonov, Spiegelman, Kelemen, Fang & others
Reactive open-system melting beneath a ridge
Reactive open-system melting beneath a ridge

\[ \Gamma \propto \phi_w \frac{\partial C_f^{eq}}{\alpha \frac{\partial P}{\partial \alpha}} \]
Reactive open-system melting beneath a ridge

\[ \Gamma \propto \phi W \frac{\partial C_{eq}^f}{\partial P} \]

Chemical Instability

- Reactive melting
- Porosity
- Flux
- Permeability
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Chemical Instability

- Reactive melting
- Porosity \( \phi \)
- Flux \( \phi_w \)
- Permeability \( k \)
Verification of simulations

From Spiegelman, Kelemen & Aharonov, JGR 2001

Linear Analysis

Numerical Simulation

Growth rate, $\sigma$

Wavenumber k

DaPe=100
DaPe=200
DaPe=400
DaPe=800
DaPe=1600
DaPe=3200
DaPe=6400
DaPe=12800
DaPe=25600
What about subduction zones?
What about subduction zones?

Flow through two thermal boundary layers
What about subduction zones?

Computational domain

Flow through two thermal boundary layers

volcanos

lithosphere

crust
An unexpected result...
Part 2: Mechanical localization

Paintings by Ben Holtzman
Basic mechanics of shear bands, \( \eta = \eta(\phi) \)
Basic mechanics of shear bands, $\eta = \eta(\phi)$
Basic mechanics of shear bands, $\eta = \eta(\phi)$

What is the role of rheology?

$$\eta(\phi, \mathbf{V}) = \eta_0 e^{-\alpha \phi} f(\dot{\epsilon}_{ij}) \frac{1-n}{n}$$
Experiment and Computation

Olivine + chromite (4:1) + 4 vol% MORB, const. strain rate, $\gamma = 3.4$

- Lenses (melt-depleted)
- Network (melt-rich)

Porosity (Simulation), $\gamma = 1.51$

Perturbation Vorticity (Simulation), $\gamma = 1.51$
Linear Analysis

Angle, degrees

Growth rate of porosity, $\frac{ds}{dt}$

- $n=1$
- $n=2$
- $n=3$
- $n=4$
Verifying Simulation with Linear Analysis

Linear Analysis

Numerical Simulation

Angle, degrees

Strain, $\gamma$
Comparing simulations with experimental data

![Graph comparing simulations with experimental data. The x-axis represents the angle in degrees, and the y-axis represents the shear strain. The graph shows data points for different simulations labeled as n = 1, n = 2, n = 3, and n = 4, as well as experimental data labeled as φ₀ = 0.02 and φ₀ = 0.06.]
An emergent picture of magma dynamics

Painting by Ben Holtzman

reaction-driven

stress-driven

asthenosphere

lithosphere

low strain rate

high strain rate
The future: multi-scale subduction dynamics

Need robust, scalable multi-scale solvers. Multigrid? Adaptive grid refinement?
Conclusions

- Quantitative understanding of magma genesis requires computational models capable of resolving magma dynamics.

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• Strong interaction of scales $\rightarrow$ separation of length-scales probably not valid.
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