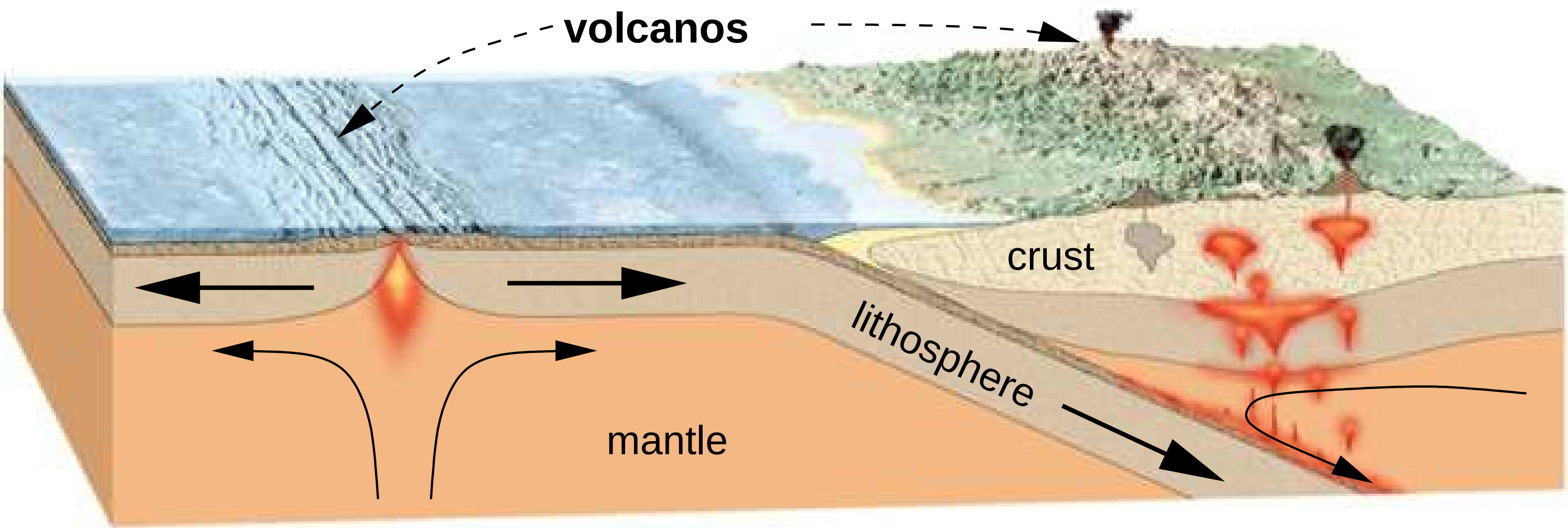
# The deep roots of volcanos:
## localization instabilities in a continuum model of magma dynamics

**Richard F. Katz & Marc Spiegelman**
LDEO/Columbia University

With Ben Holtzman, Peter Kelemen (LDEO/CU), Barry Smith, Matt Knepley (ANL) and Craig Manning (UCLA)

# Plate tectonics, volcanos and magma genesis



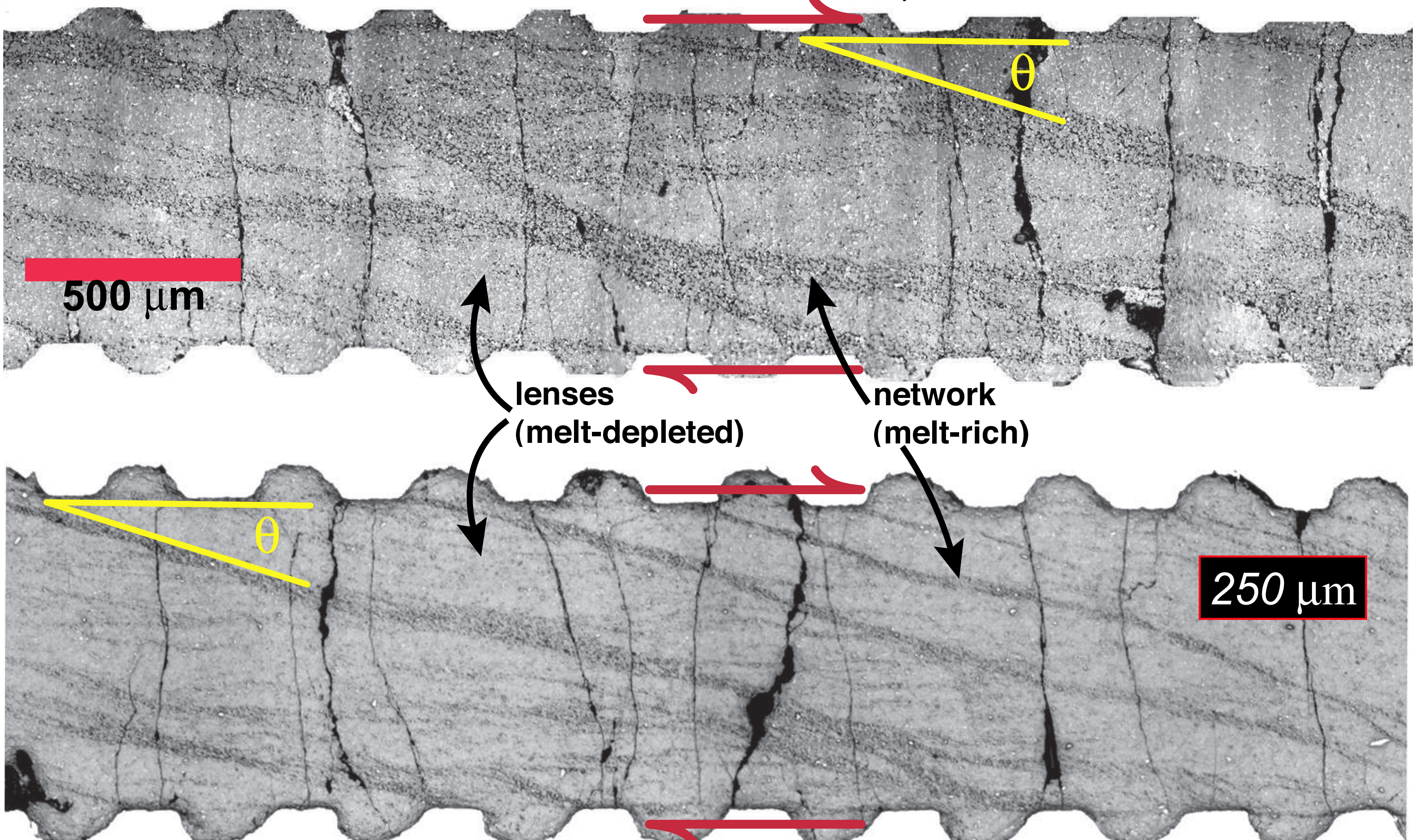
# Chemical localization of magma

**observations by Kelemen et al.**

# Mechanical localization of magma

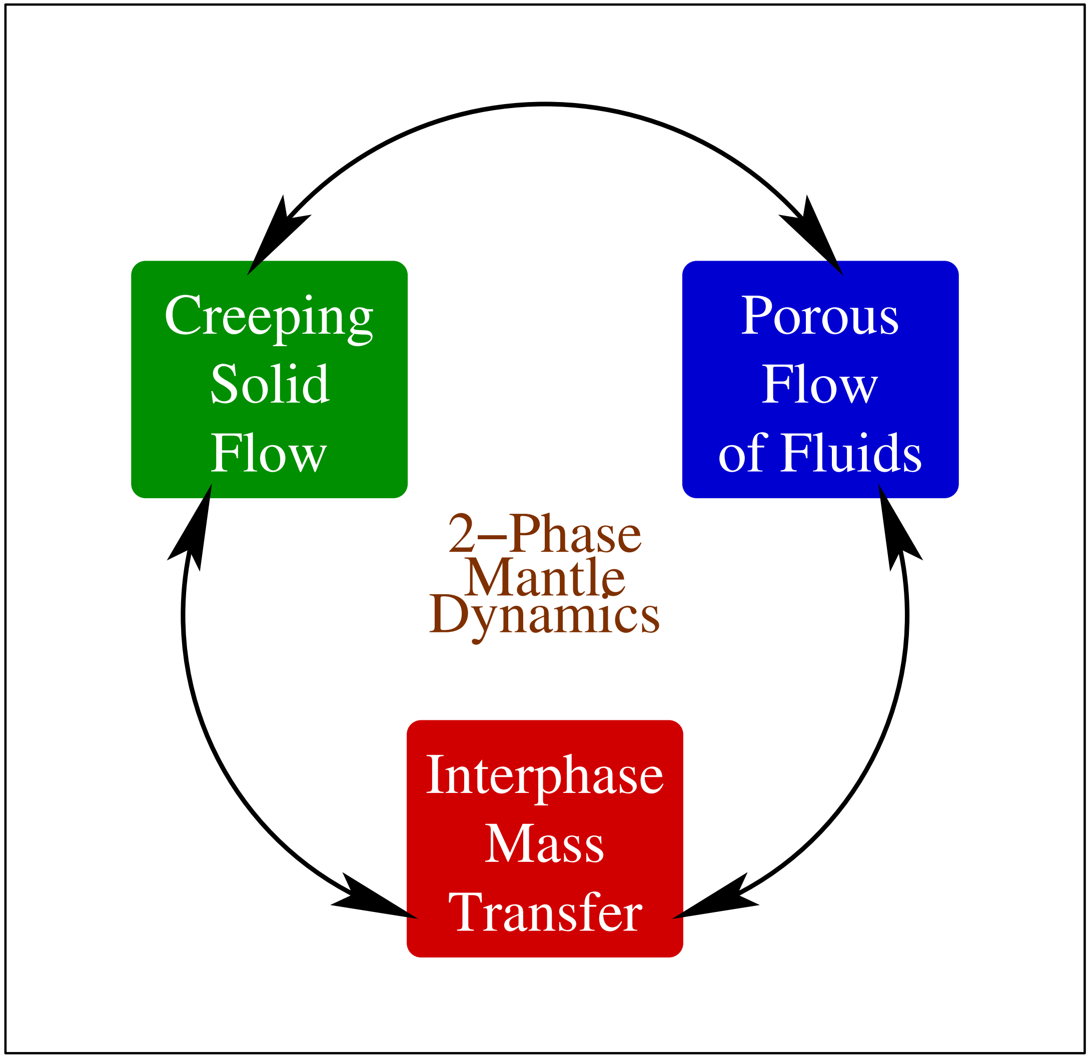
**experiments by Holtzman et al.**

PI-1020, olivine + chromite + 4% MORB, $\gamma = 3.5$, P = 30-60 MPa




Olivine + chromite (4:1) + 4 vol% MORB, const. strain rate, $\gamma = 3.4$

# Magma dynamics theory: key components




**4-7 primary variables**

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**with permeability $k_\phi \propto \phi^n$, shear viscosity $\eta$ and bulk viscosity $\zeta$.**

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1. **Conservation of mass:** pore fluid

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$$\boldsymbol{\nabla} P = \boldsymbol{\nabla} \cdot \eta \left[(\boldsymbol{\nabla} \mathbf{V}) + (\boldsymbol{\nabla} \mathbf{V})^T\right] + \boldsymbol{\nabla}\left(\zeta - \tfrac{2\eta}{3}\right) \boldsymbol{\nabla} \cdot \mathbf{V} + \bar{\rho} \mathbf{g}$$

4. **Conservation of momentum:** matrix solid (Stokes eqn)

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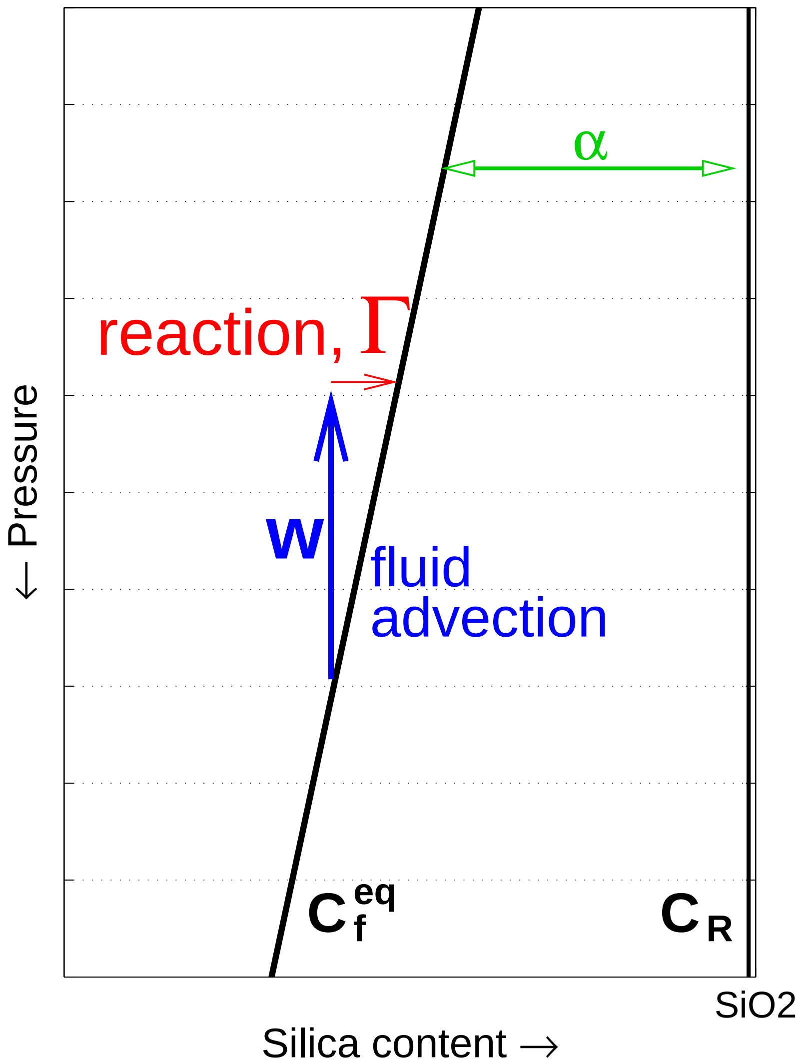
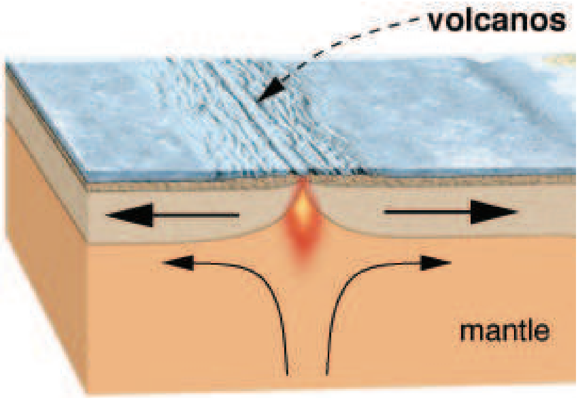
- Parallel semi-Lagrangian advection in development for PETSc.

# Part 1: Chemical localization

**Past work by Aharonov, Spiegelman, Kelemen, Fang & others**

# Reactive open-system melting beneath a ridge



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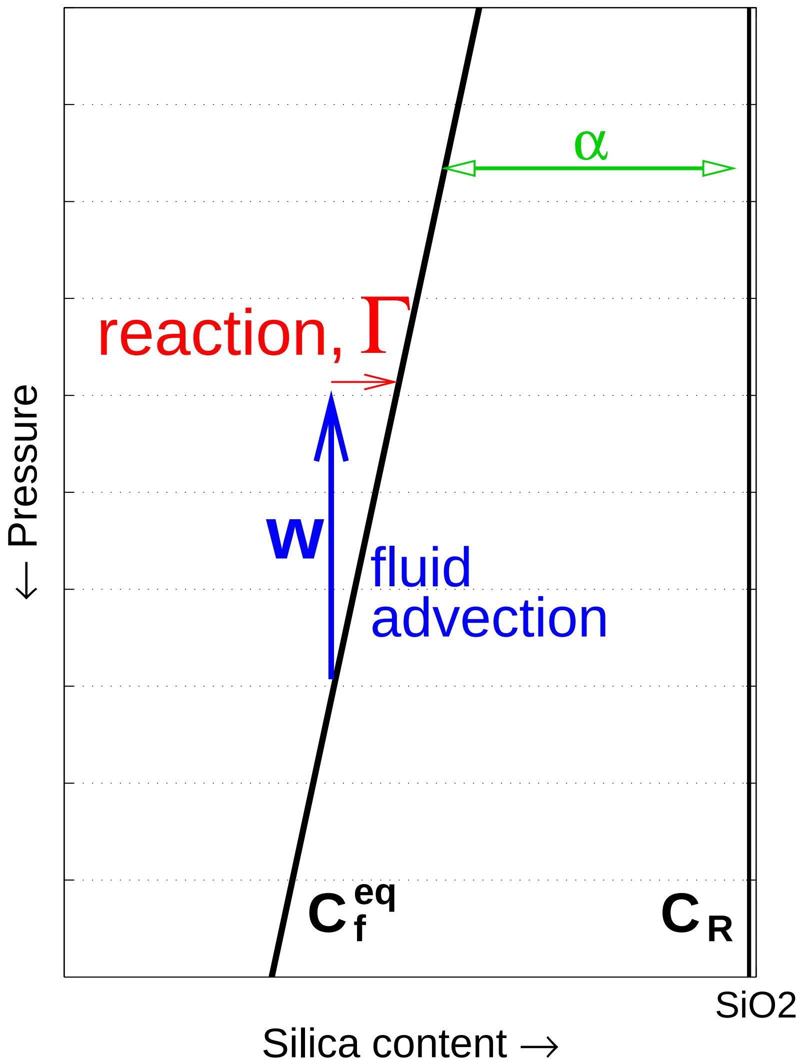


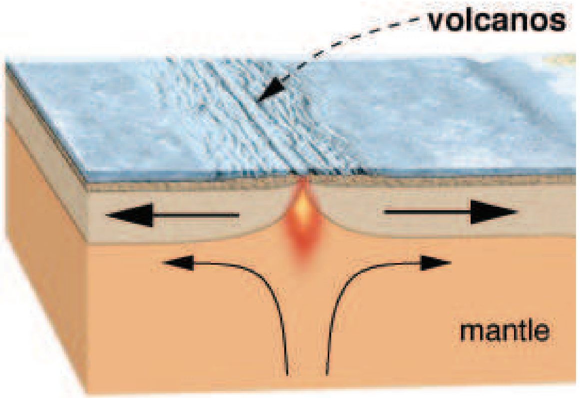


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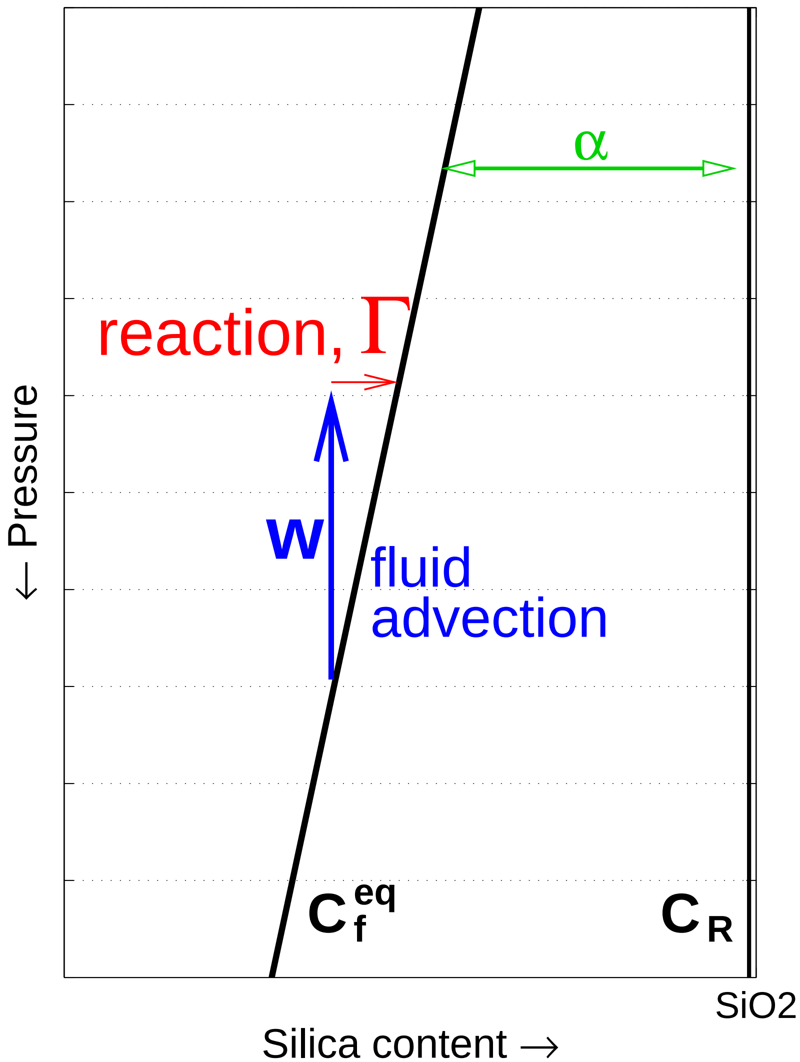


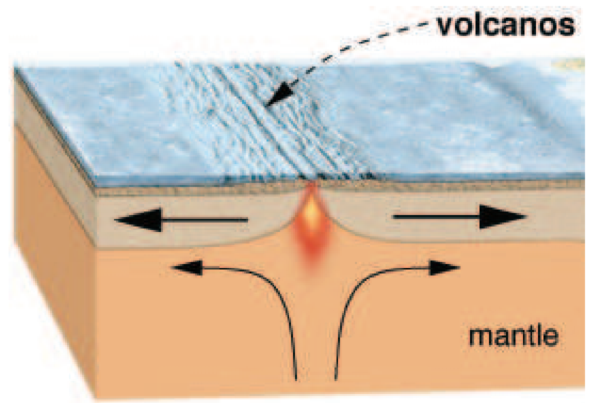


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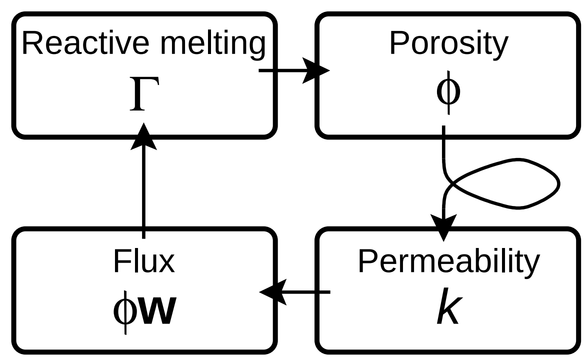
Chemical Instability



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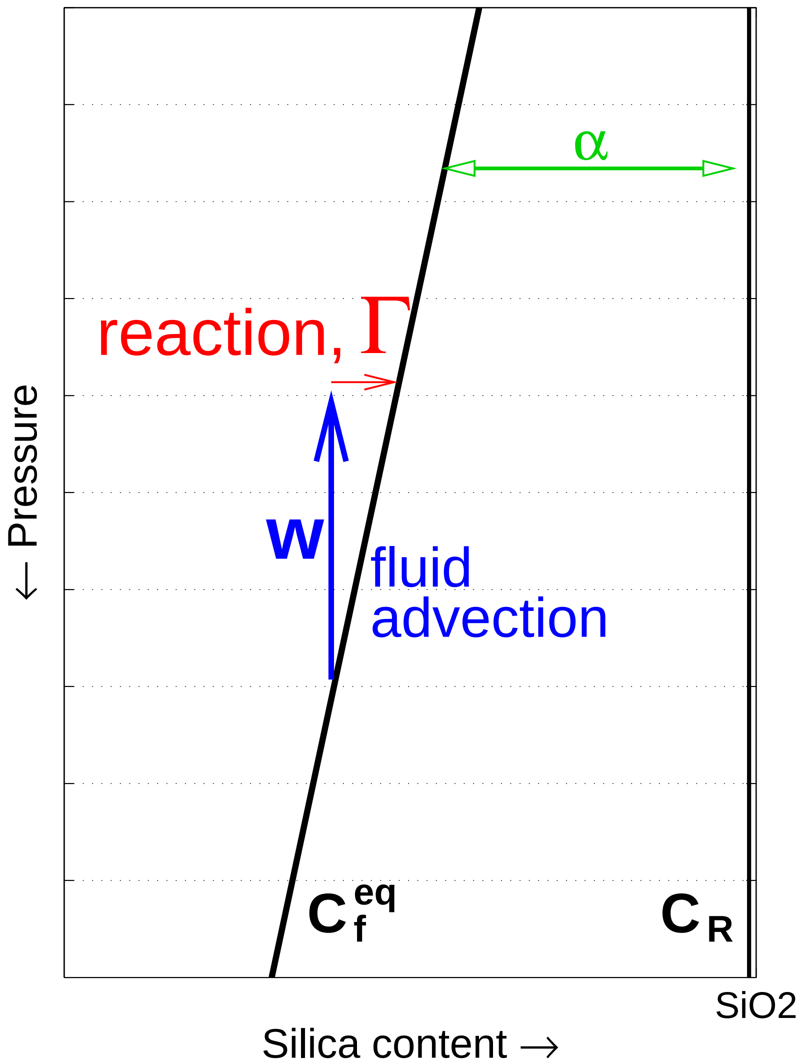


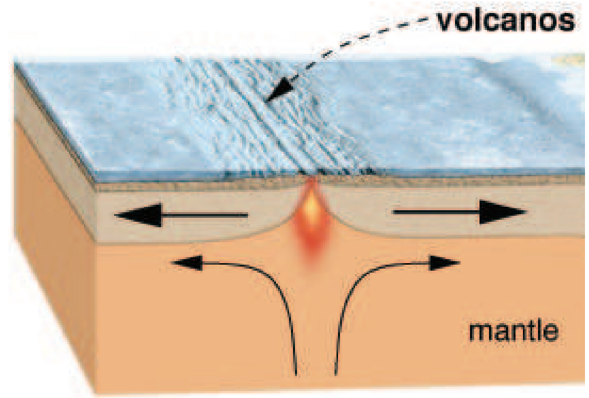


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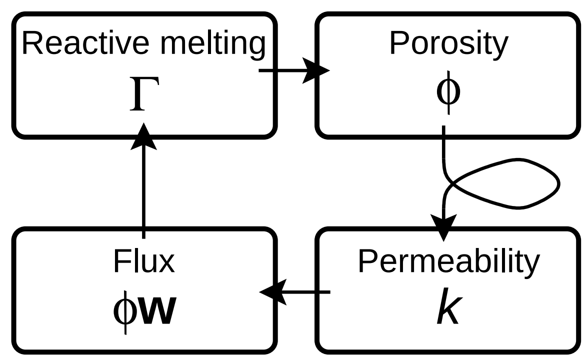
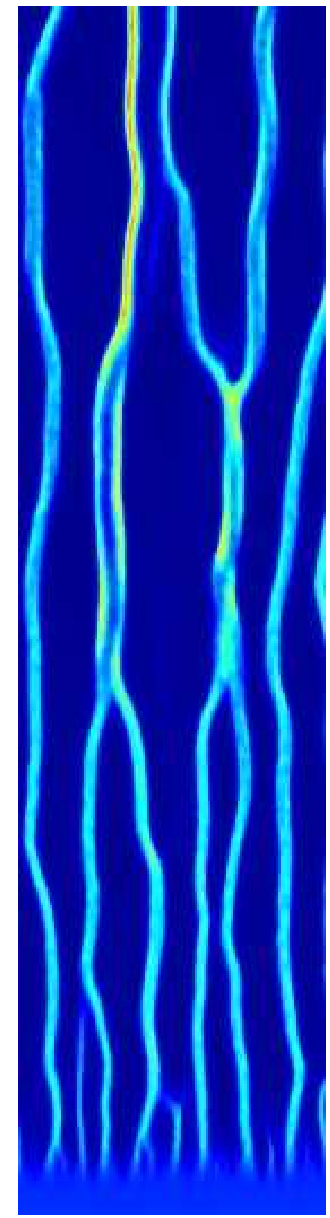
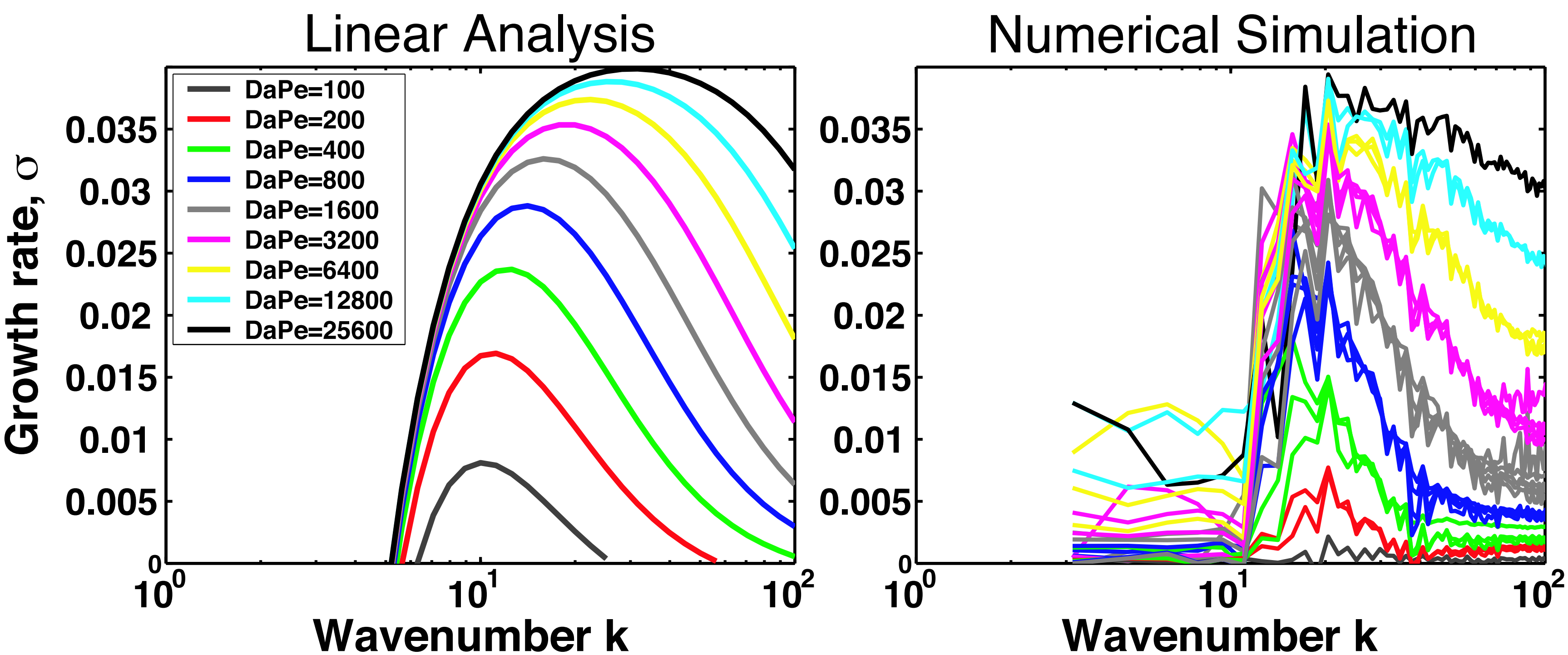


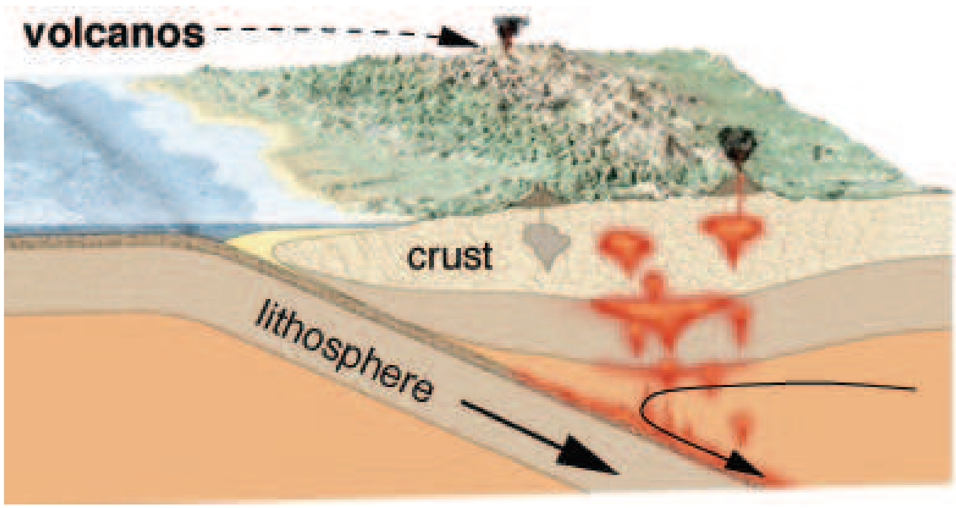



# Verification of simulations

**From Spiegelman, Kelemen & Aharonov, JGR 2001**



# What about subduction zones?



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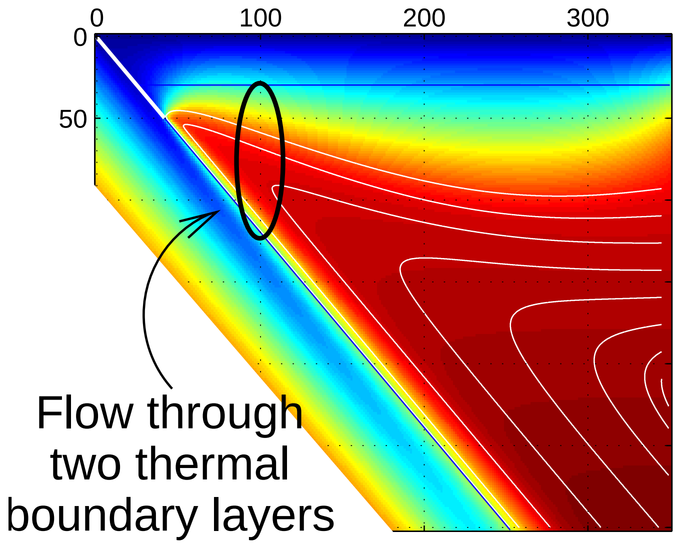


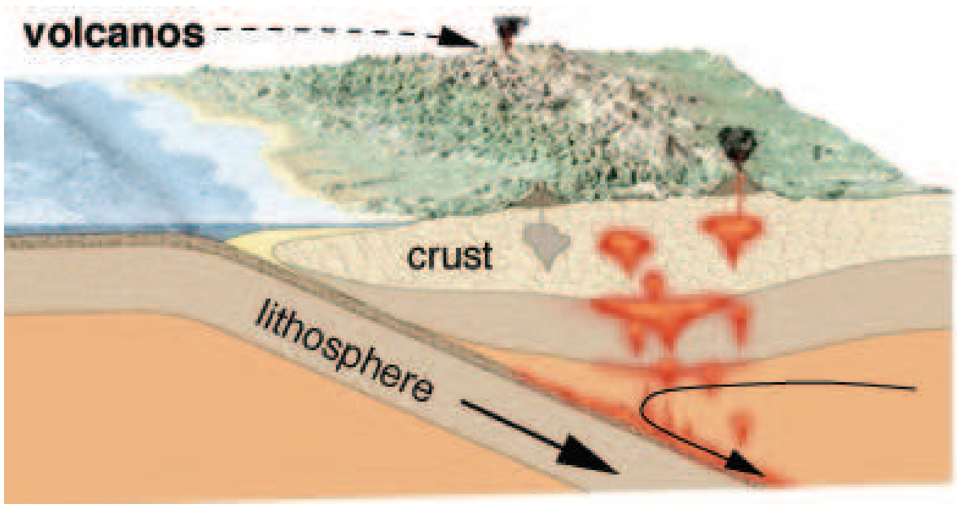

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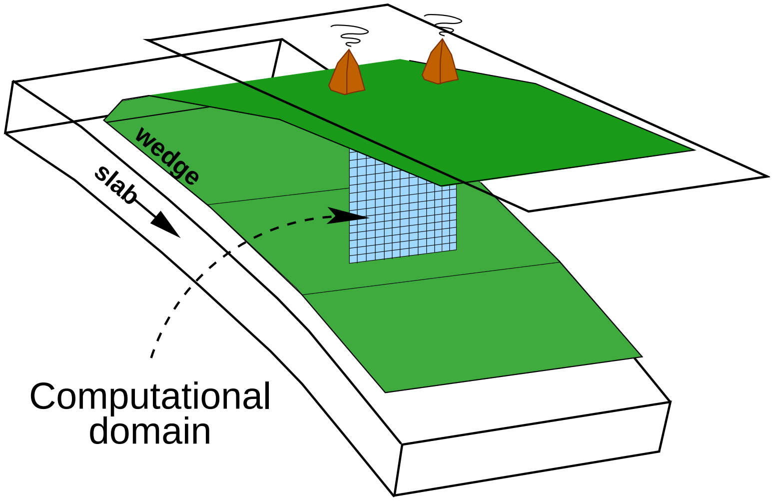
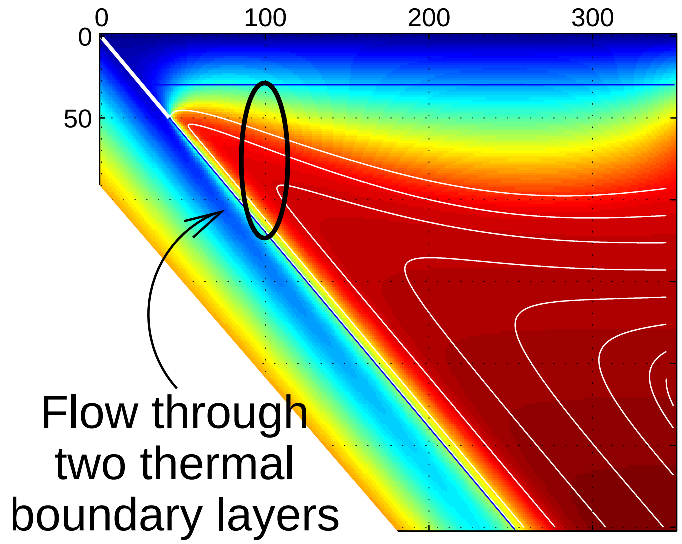
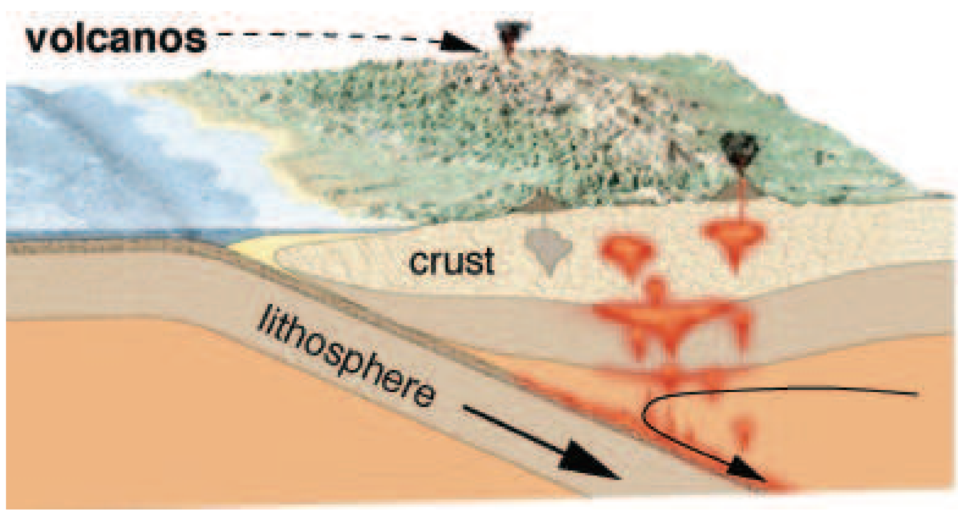
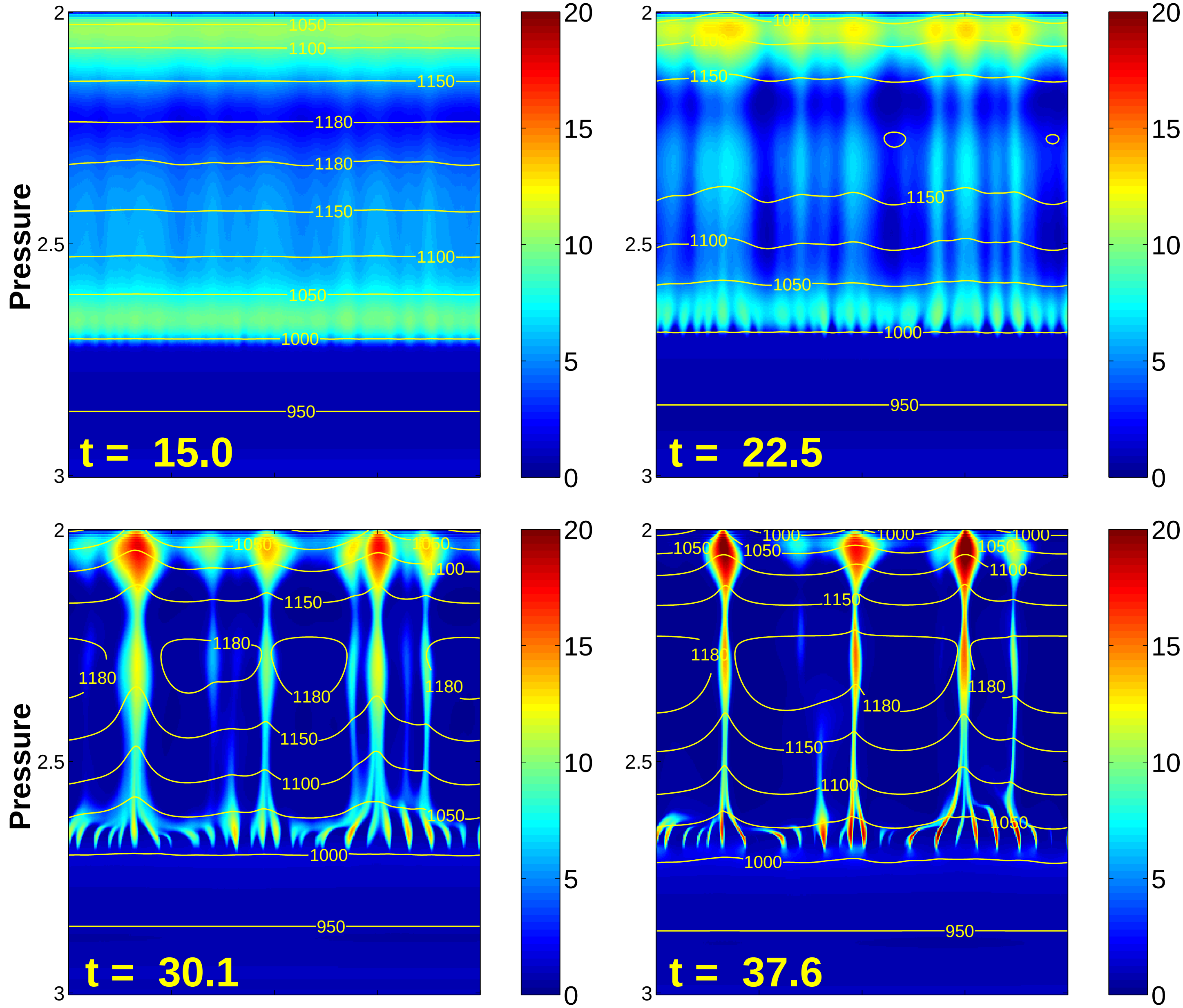


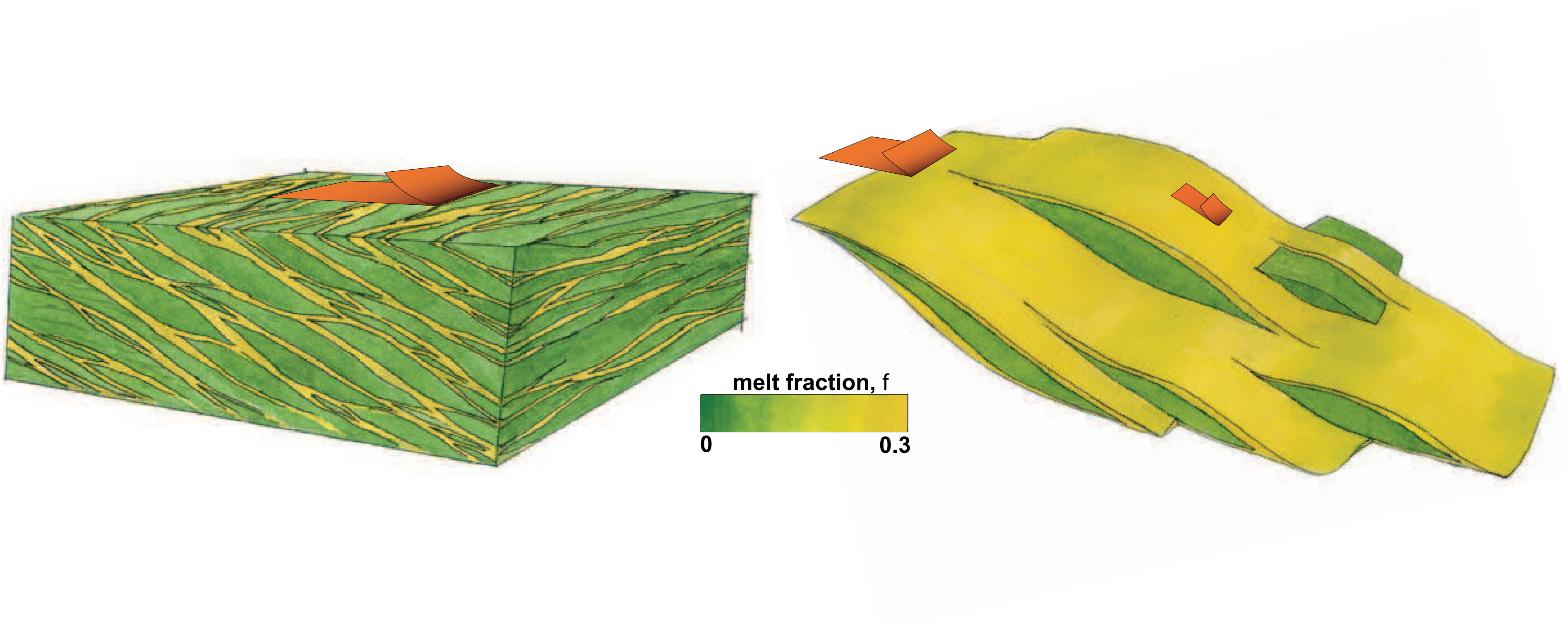



# An unexpected result...

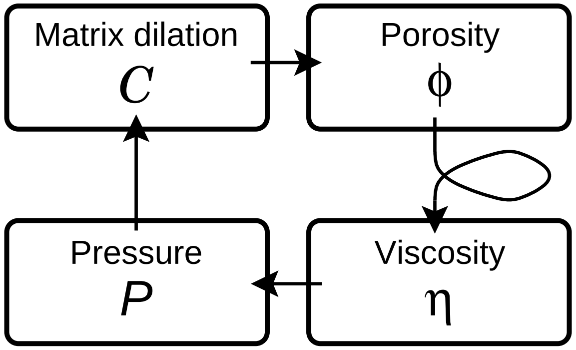


# Part 2: Mechanical localization

**Paintings by Ben Holtzman**



# Basic mechanics of shear bands, $\eta = \eta(\phi)$



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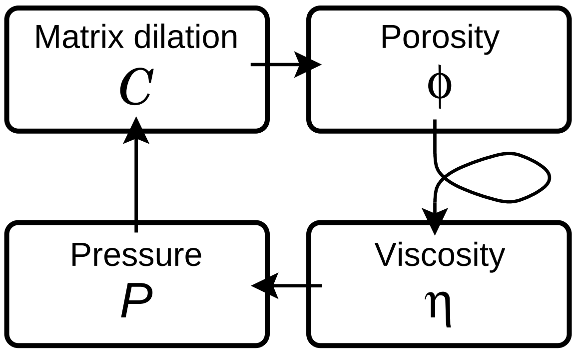


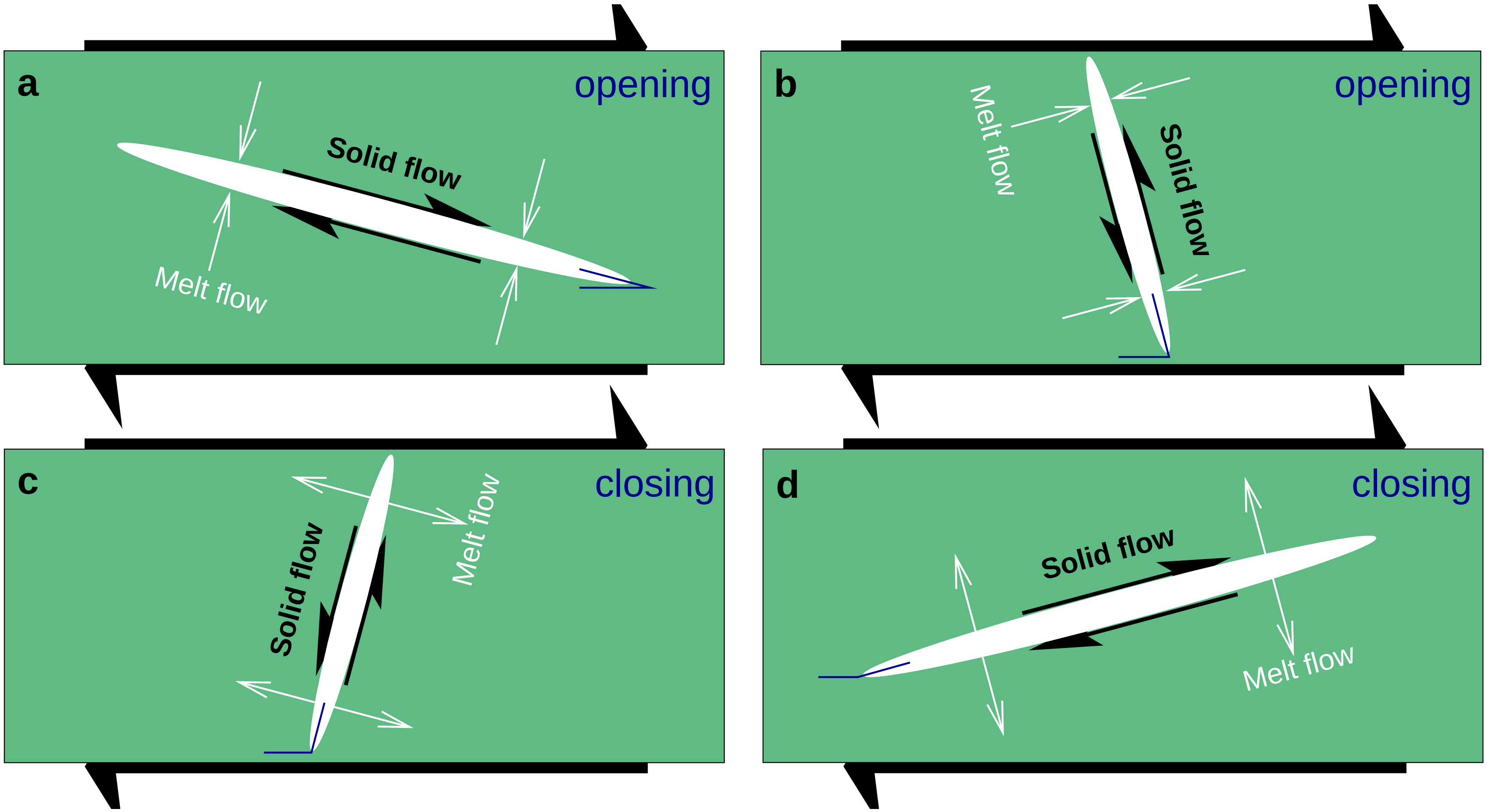

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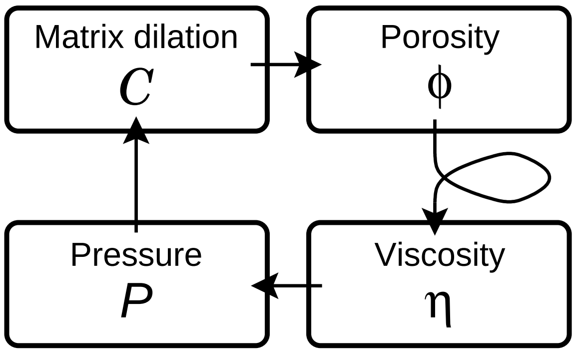
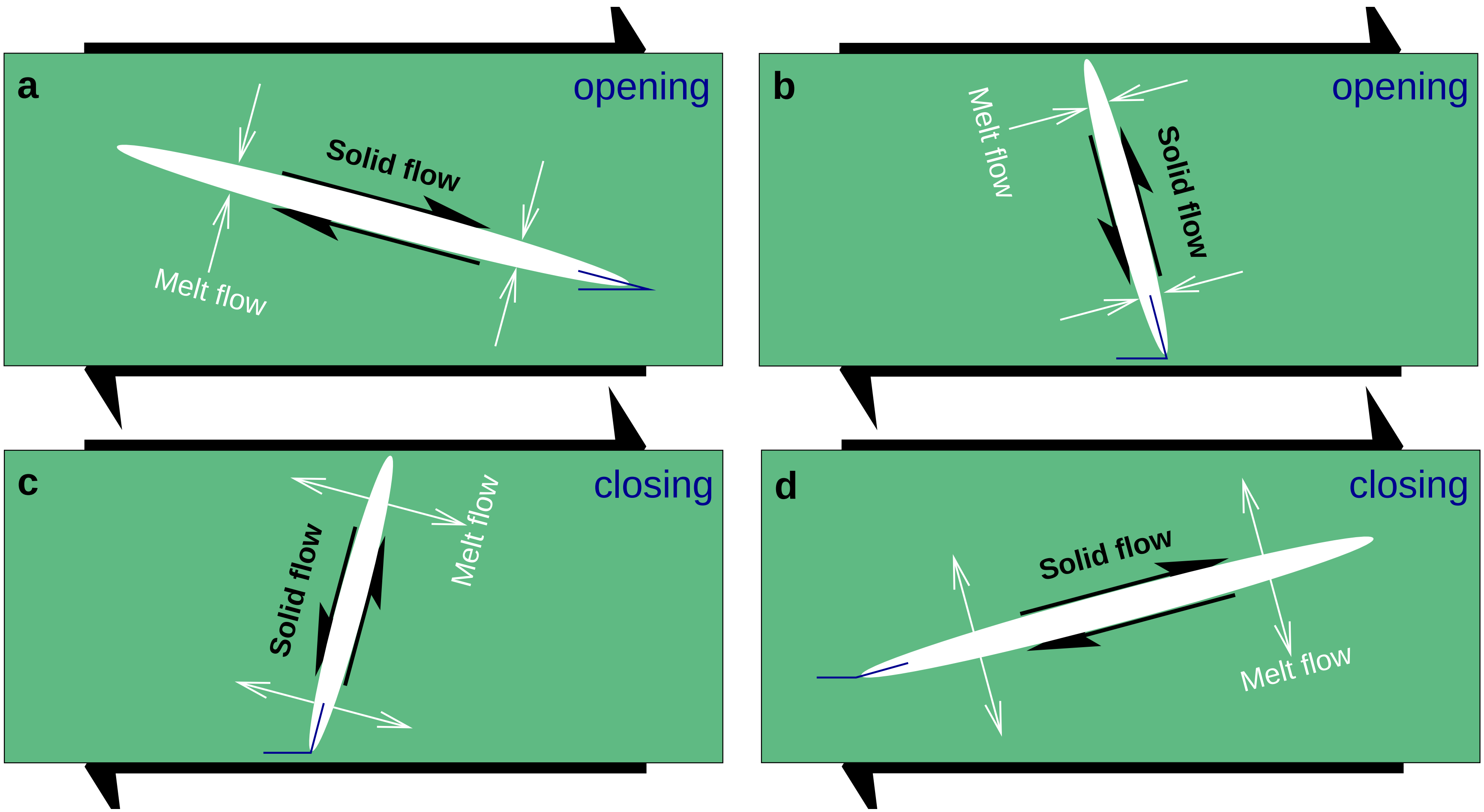
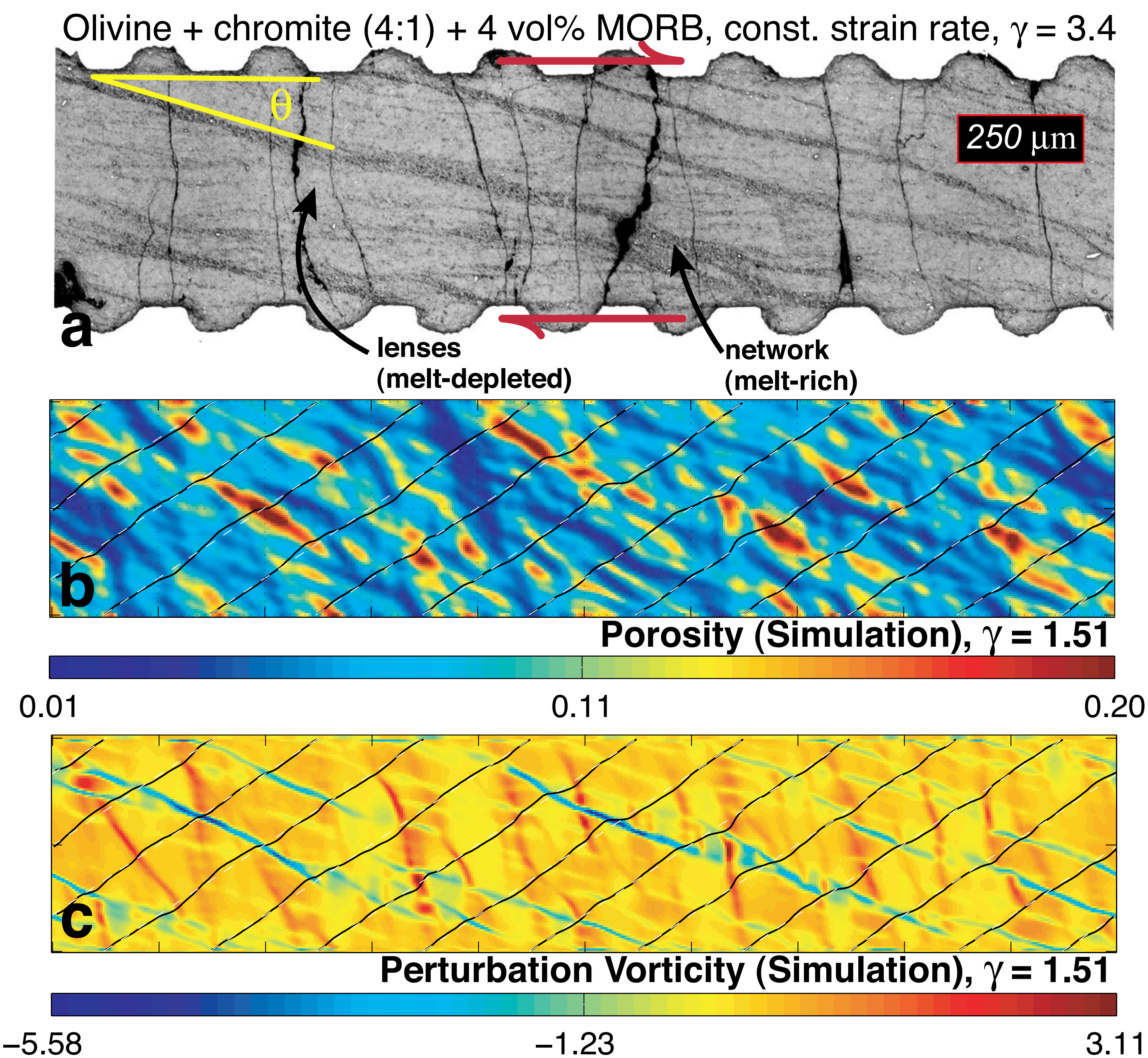



What is the role of rheology?

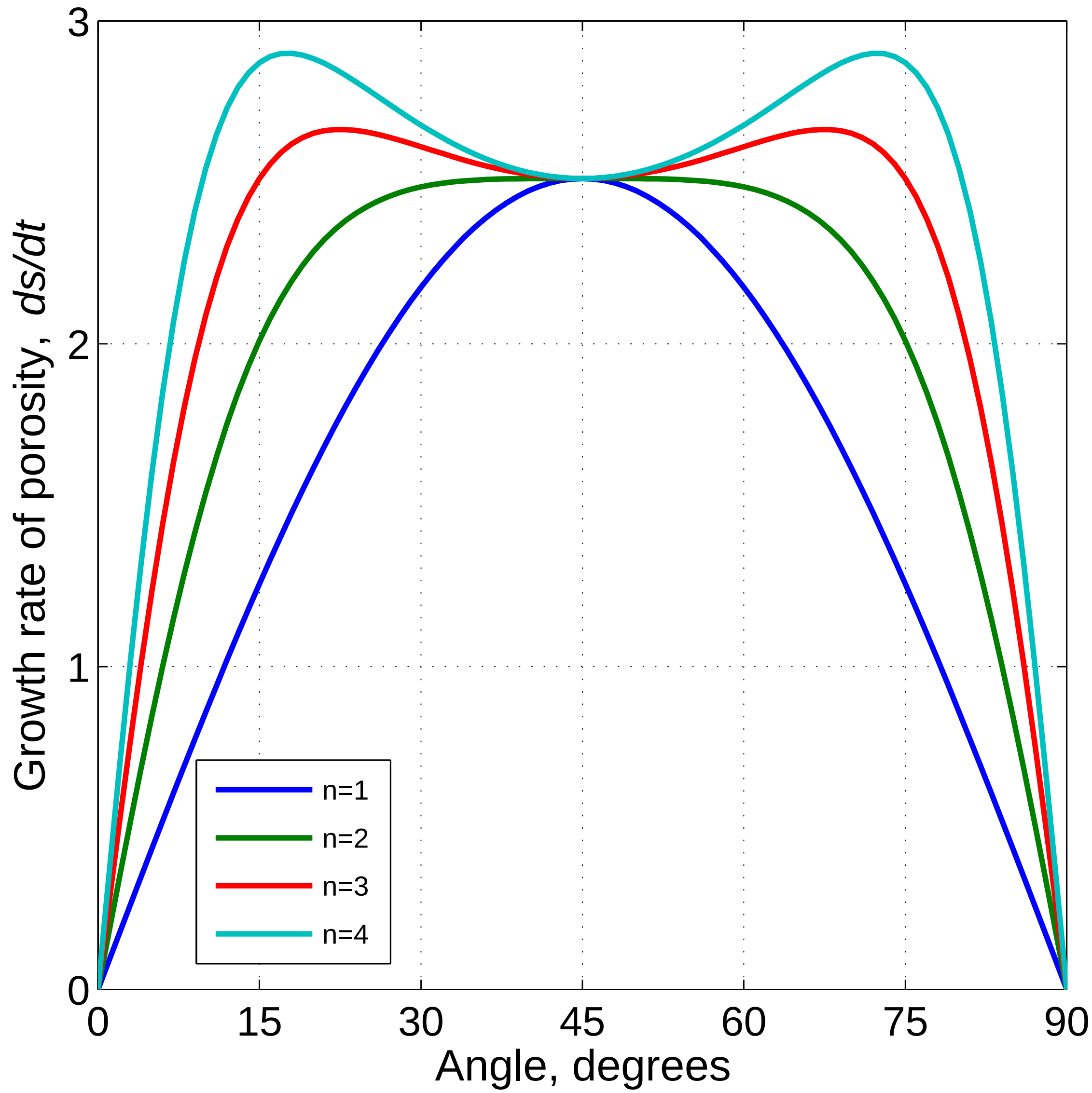
$$\eta(\phi, \mathbf{V}) = \eta_0 \, e^{-\alpha\phi} \, f(\dot{\epsilon}_{ij})^{\frac{1-n}{n}}$$



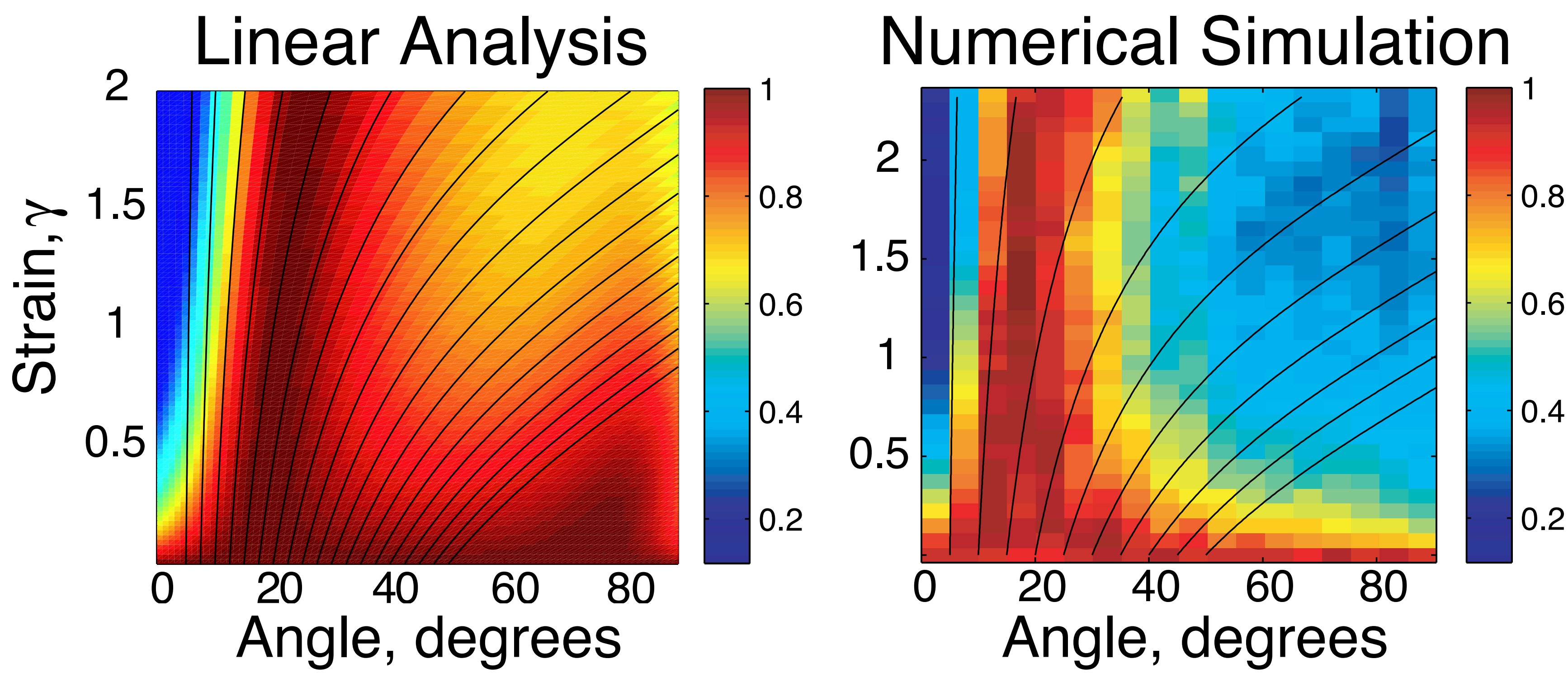
# Experiment and Computation



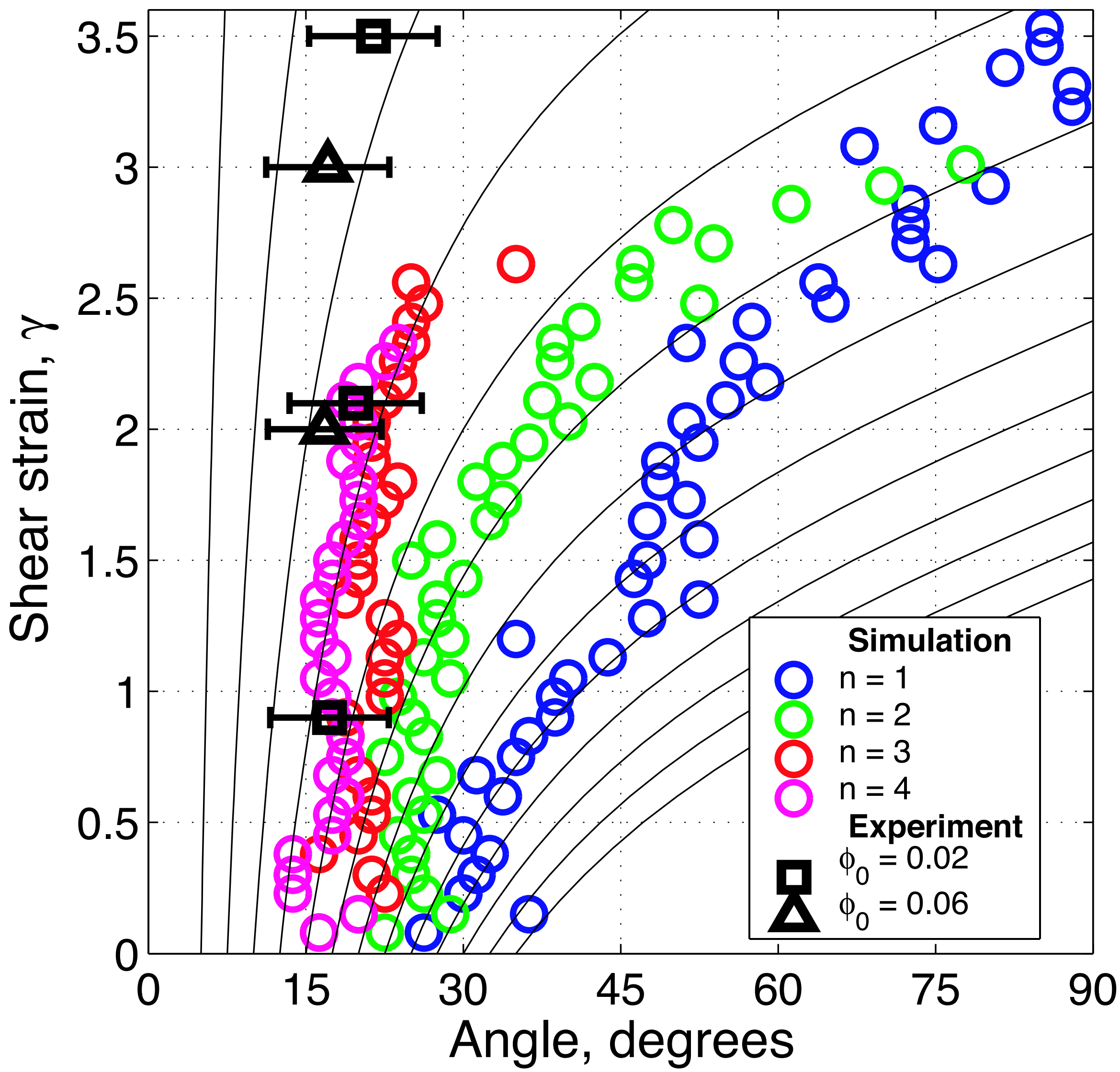
# Linear Analysis



# Verifying Simulation with Linear Analysis



# Comparing simulations with experimental data



# An emergent picture of magma dynamics

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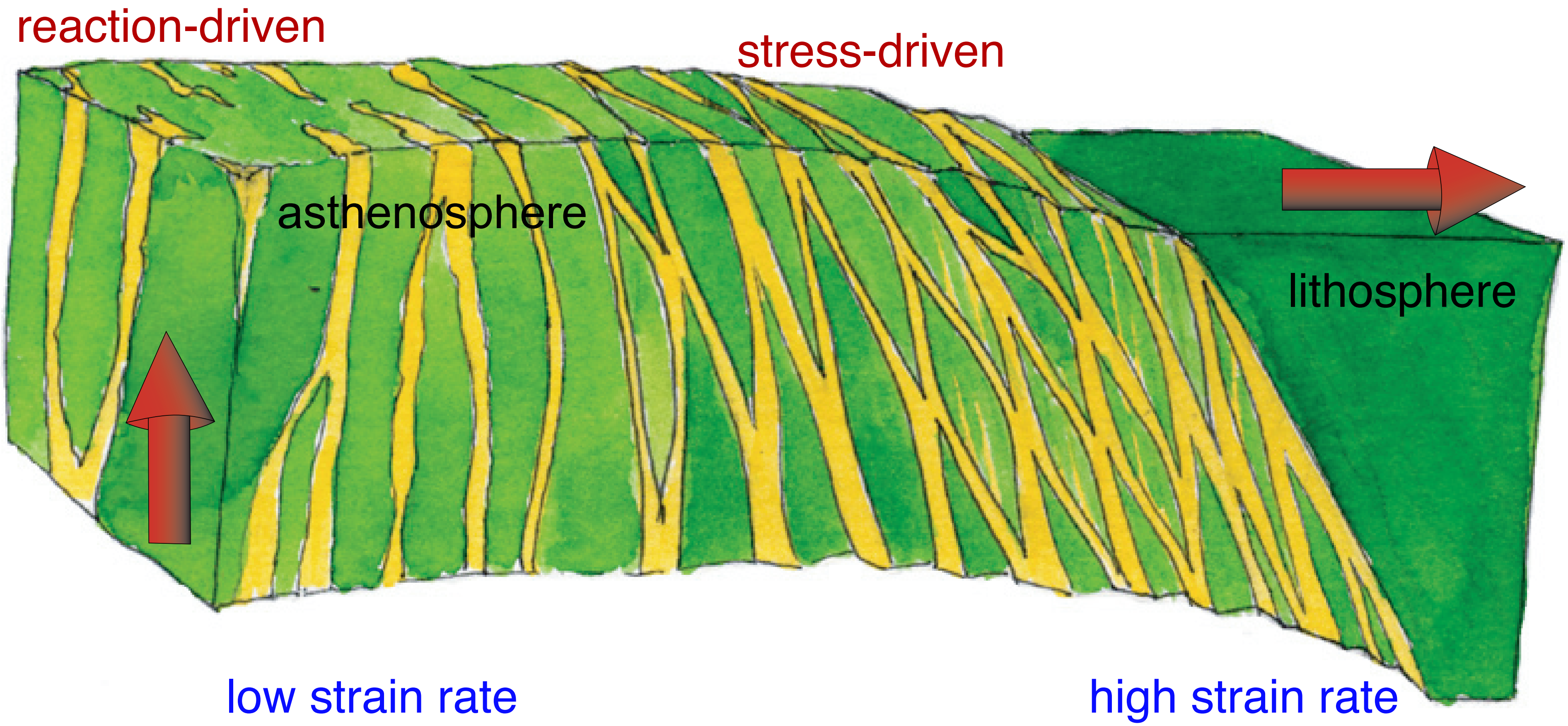
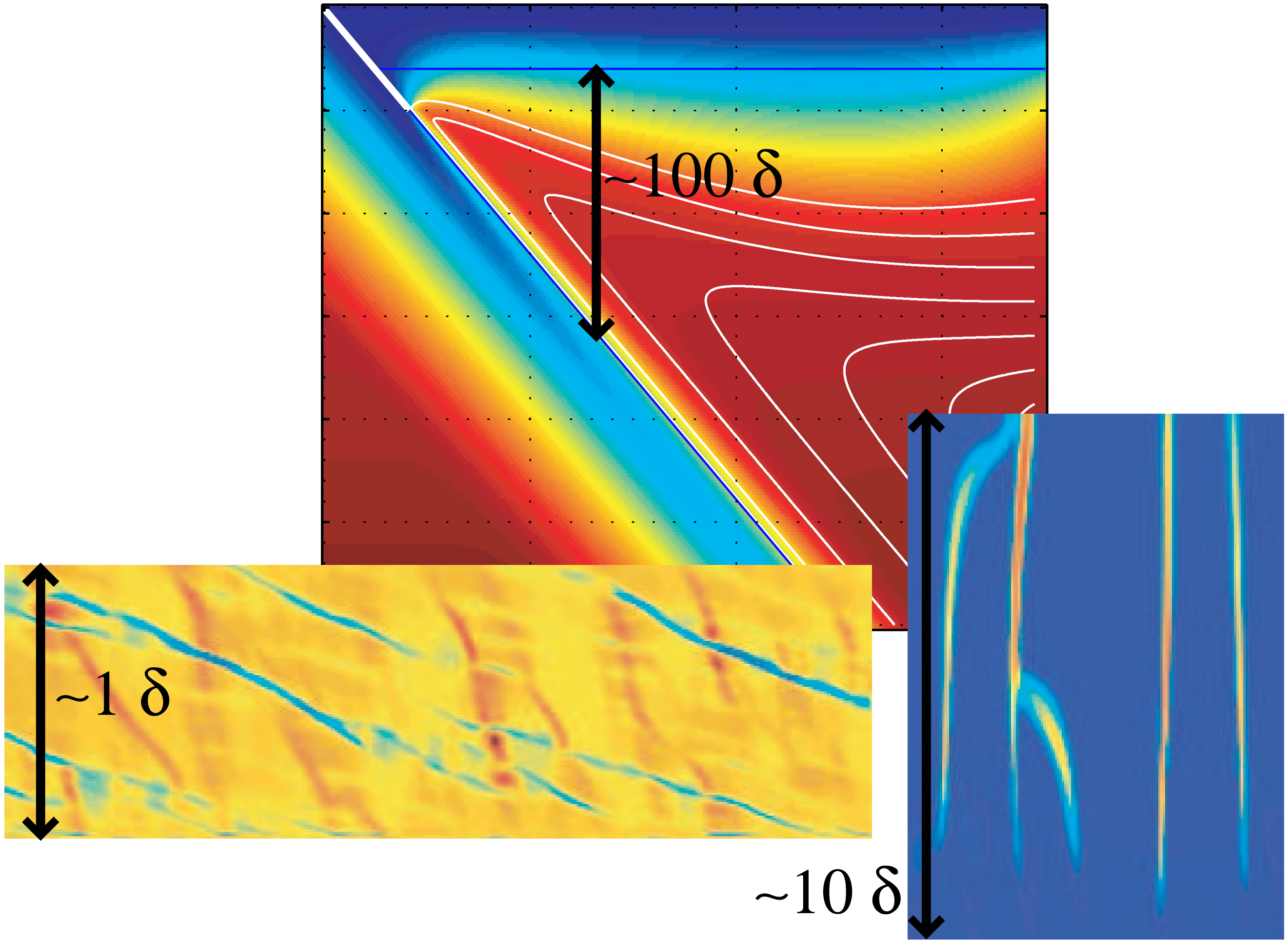


# The future: multi-scale subduction dynamics




**Need robust, scalable multi-scale solvers. Multigrid?**

**Adaptive grid refinement?**

## Conclusions

- Quantitative understanding of magma genesis requires computational models capable of resolving magma dynamics.

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# Computational challenges

- Localization problems are computationally demanding.
- Strong interaction of scales $\longrightarrow$ separation of length-scales probably not valid.

## Thanks to:

DOE CSGF Department of Energy Computational Science Graduate Fellowship