Maximizing Kidney Paired Donation

CSGF Fellows Conference 2005

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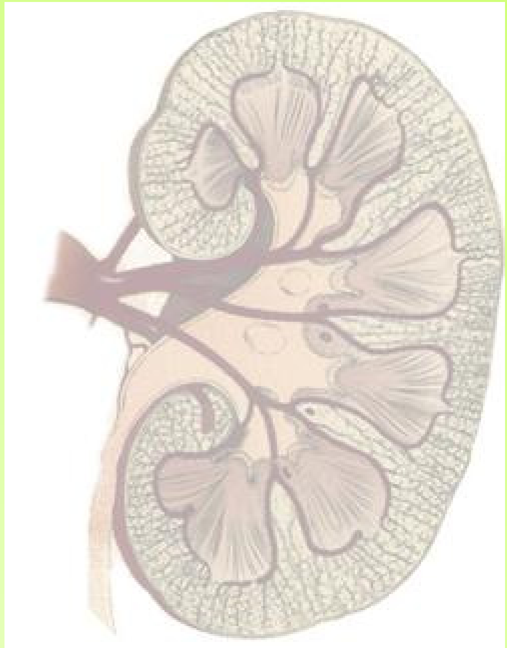


ESRD and kidney donation

- End-stage-renal-disease (kidney failure) strikes tens of thousands of people
  - Kidneys can no longer purify the blood
- Treatments include
  - Dialysis, artificial kidney support, is extremely expensive ($60,000 per year) and debilitating
  - Kidney transplantation generally restores a patient to full health

Deceased and live donors

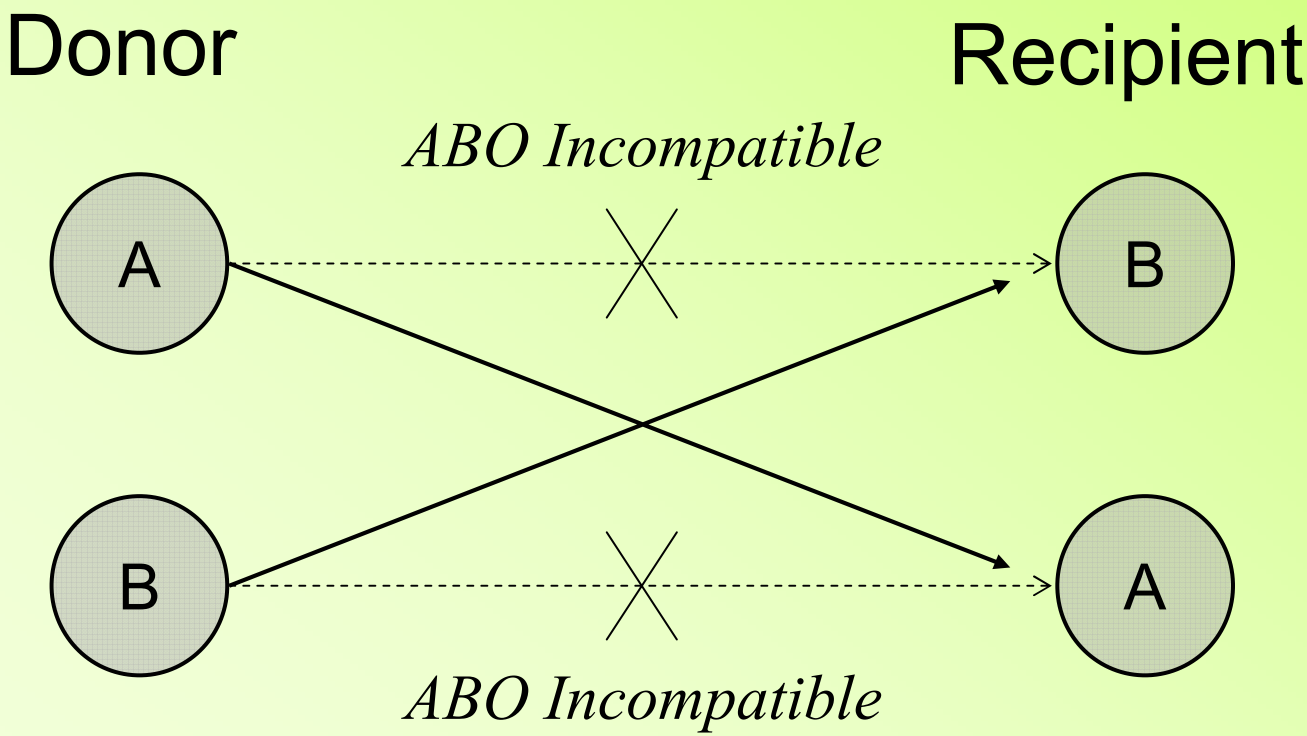
- The demand for deceased donor kidneys far outstrips the supply (UNOS)
  - About 10,000 deceased donor kidneys per year are transplanted
  - Over 60,000 patients are on the waiting list
  - Deceased donation is level
- Live donation is an attractive option
  - Operation is very safe, can be laparoscopic
  - About 6,000 live donations in 2003
  - Spouses, friends, siblings, parents, children

Donor Incompatibilities

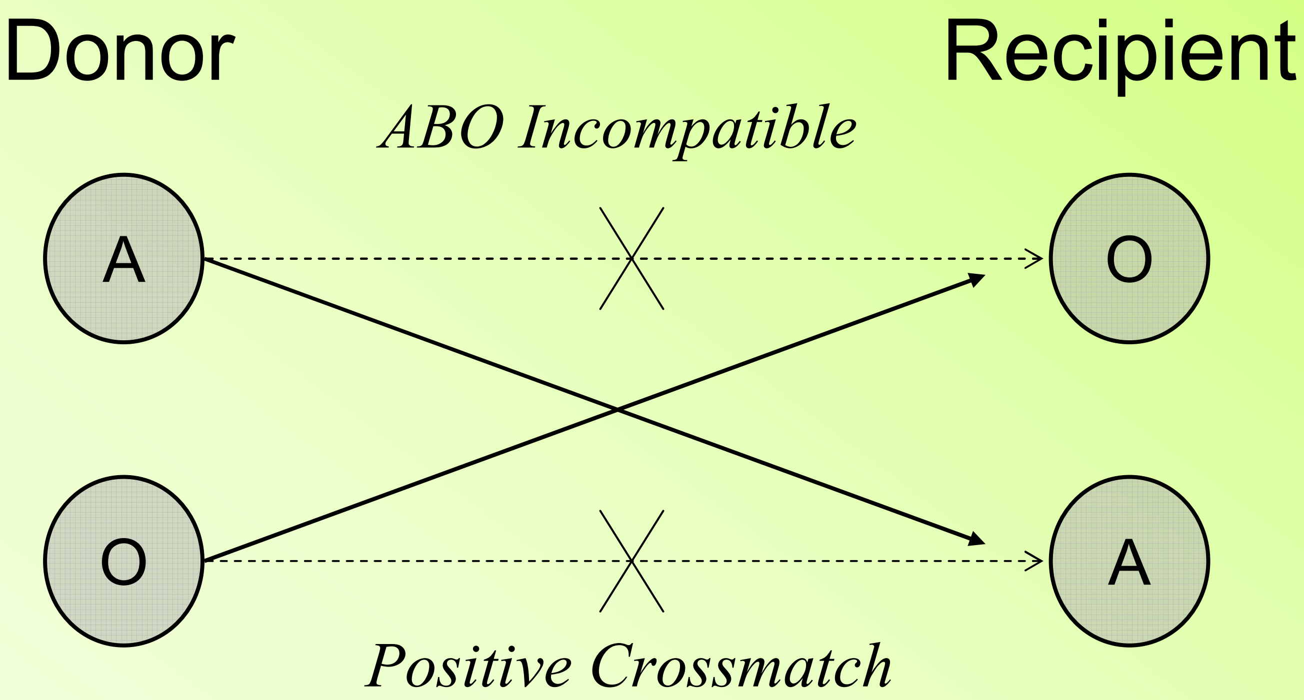
- Many willing live donors are disqualified
  - Bloodtypes: O(46%), A(34%), B(16%), AB(4%)
    - O recipients can accept only an O kidney
    - A recipients can accept only O, A kidneys
    - B recipients can accept only O, B kidneys
    - AB recipients can accept any kidney
  - Positive crossmatch (XM+) predicts rejection
    - Even blood compatible donors might have XM+
    - Crossmatch is difficult to predict
    - **Highly sensitized** patients almost always XM+

Kidney Paired Donation (KPD)

- Donations must be simultaneous



Kidney Paired Donation (KPD)



Simulation and Algorithms

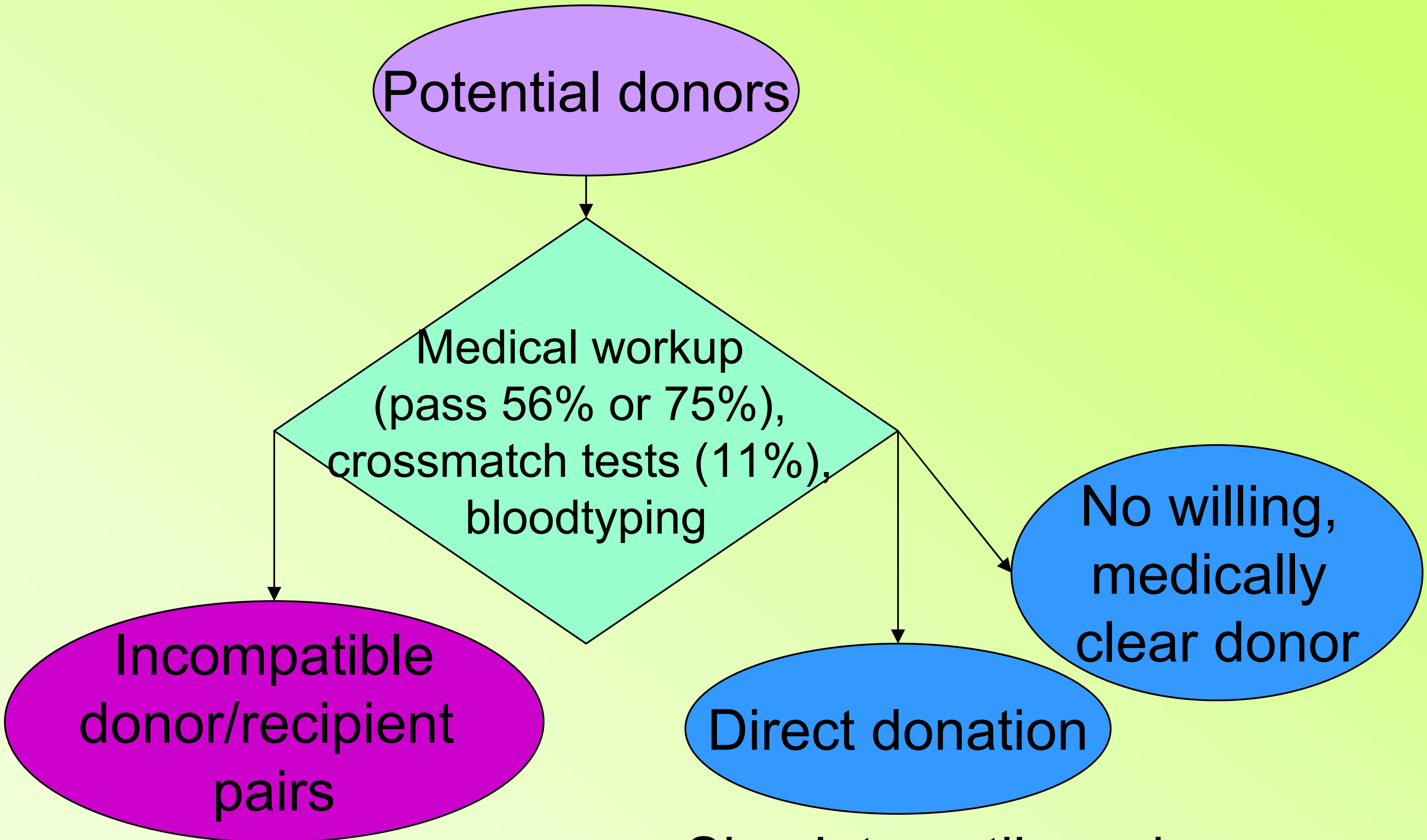
- Because it is critically important not to waste offers of live donor kidneys, the matching algorithm must be analyzed in advance of any national implementation
- Simulated patients and virtual exchanges can answer questions about the outcomes of a matching algorithm

Donor / Recipient bloodtypes

- ABO distribution
  (Zenios, Woodle, Ross, Transplantation 72:4, 2001)
  (Zenios, Management Science 48:3, 2002)
- Each recipient has up to four potential donors
  - Mother, father, spouse, sibling
- Bloodtypes distributed as in population
  - Parent has two alleles
    - AA, AO, BB, BO, OO, or AB
  - A child receives one allele from each parent

Decision tree model of family

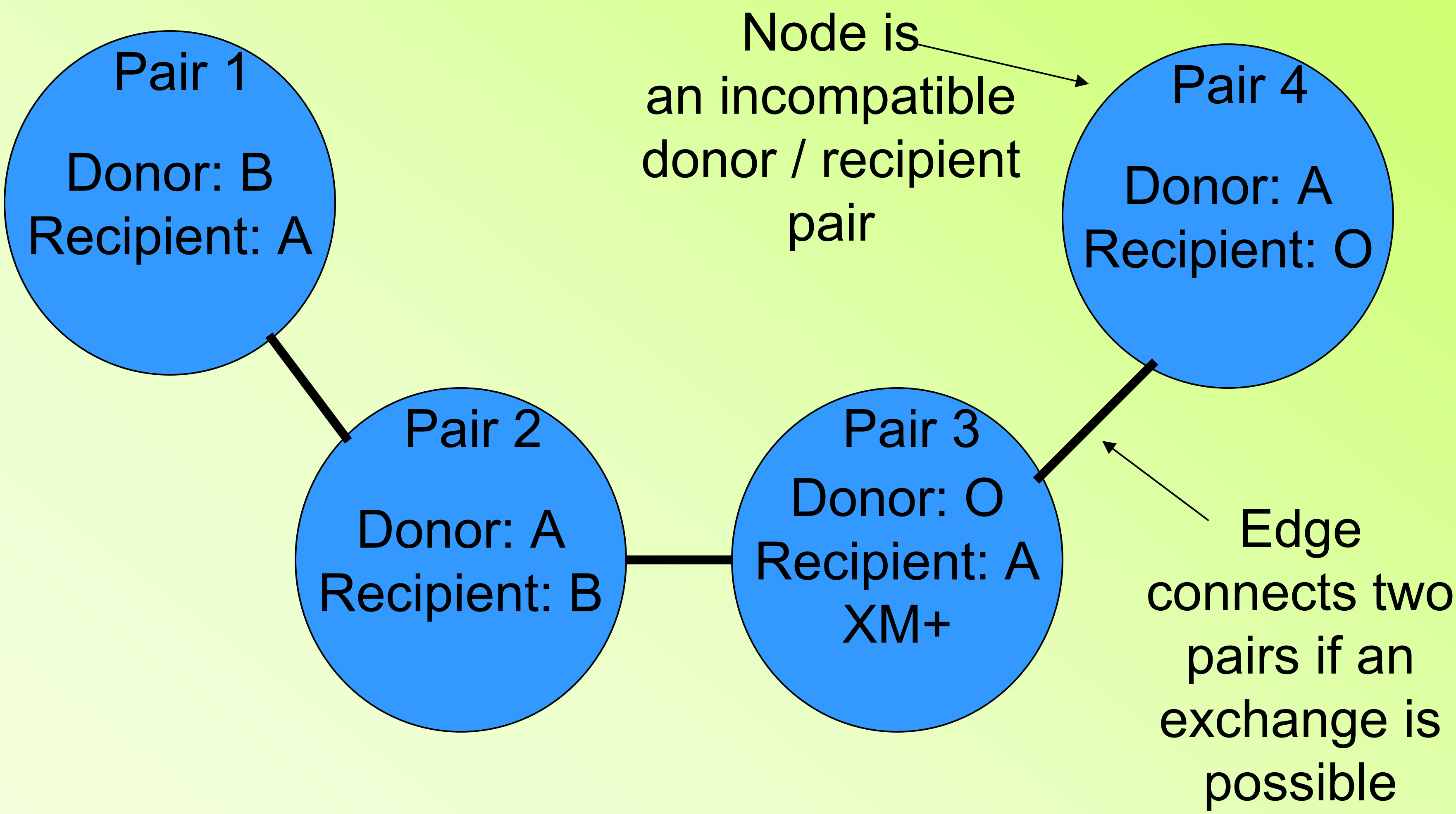
(Zenios, Woodle, Ross, Transplantation 72:4, 2001)

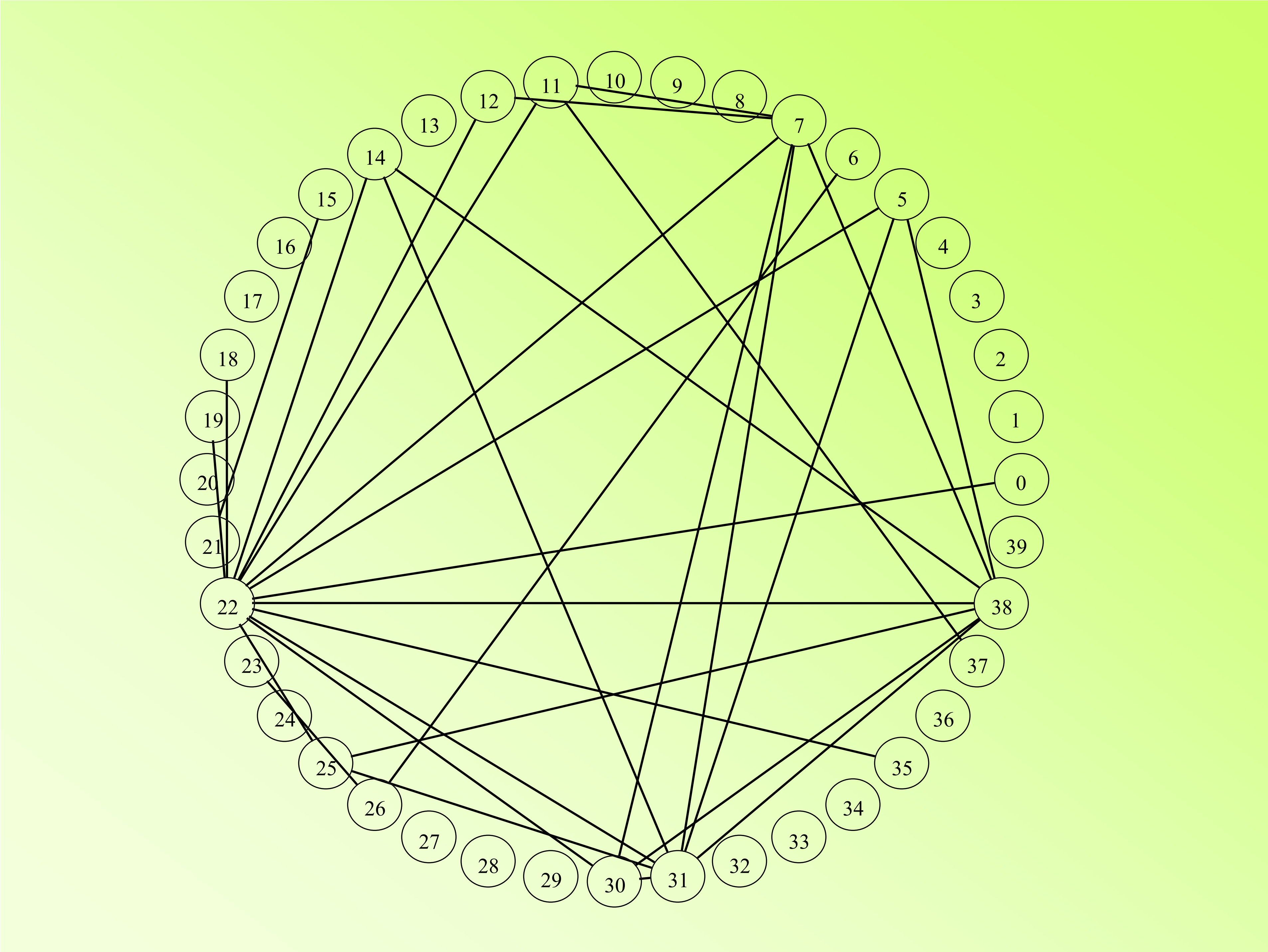


3584 pairs annually

Simulate until reach
# of real live donors

Graph showing recipient/donor pairs

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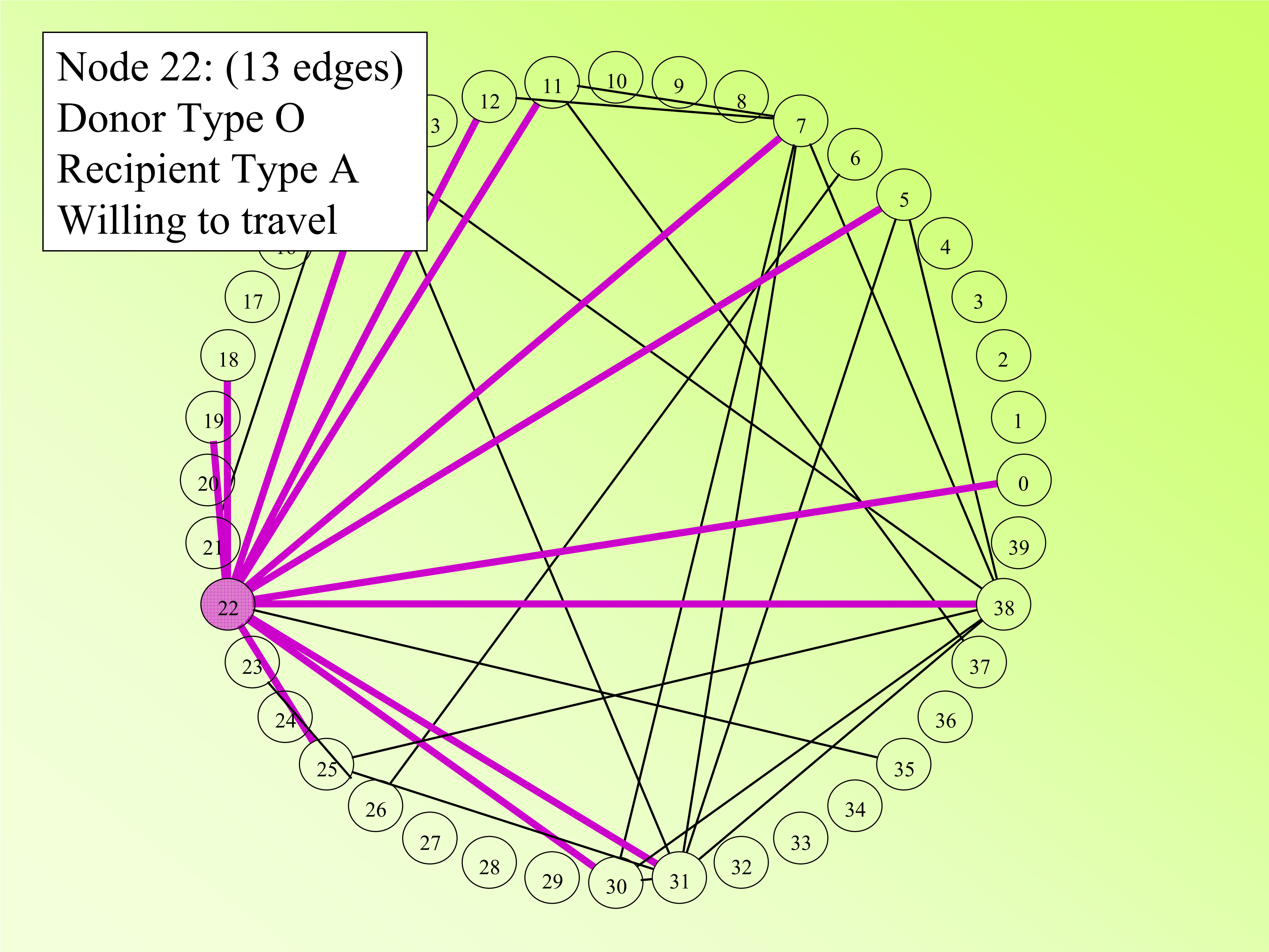
Node 22: (13 edges)
Donor Type O
Recipient Type A
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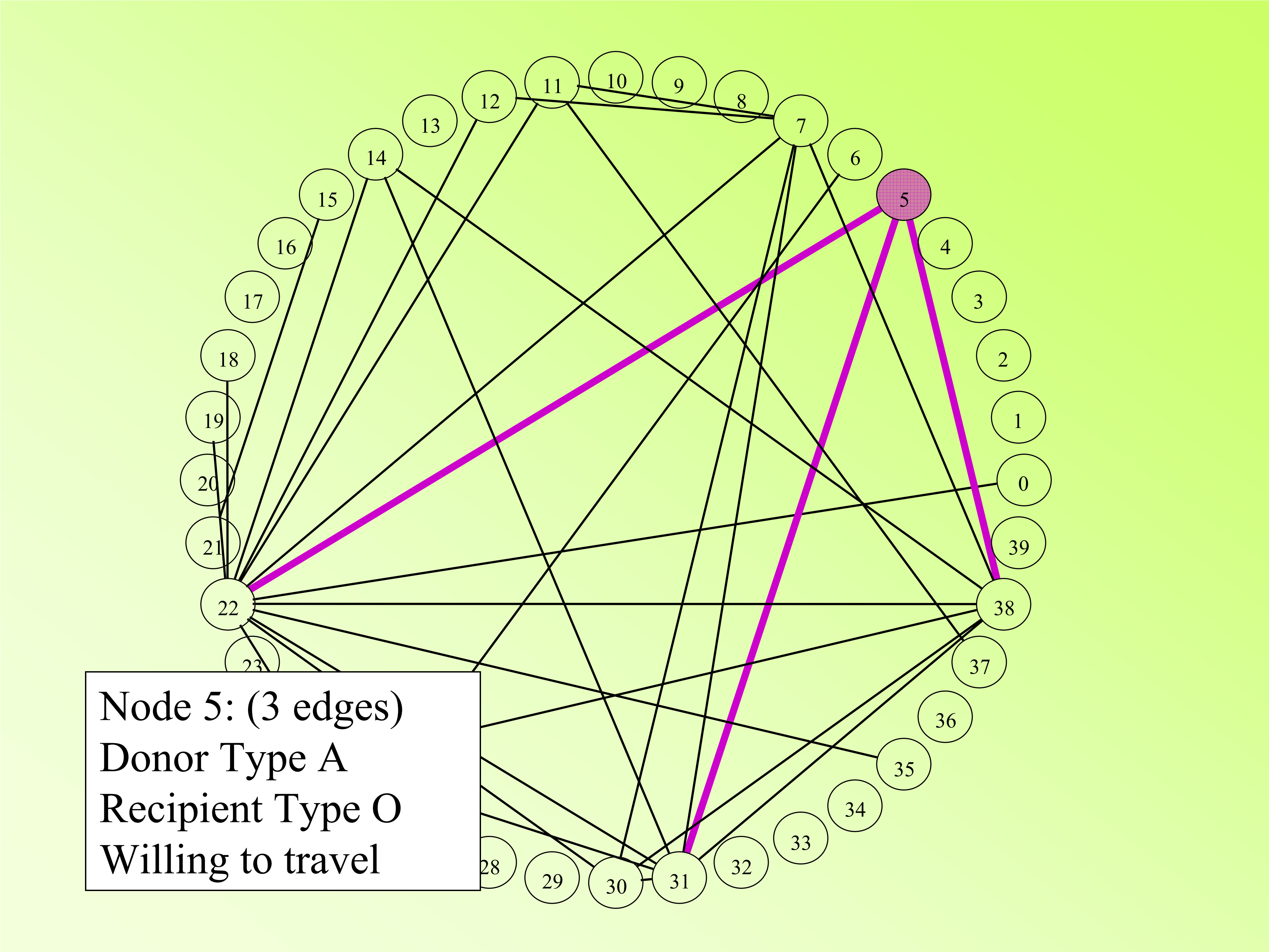
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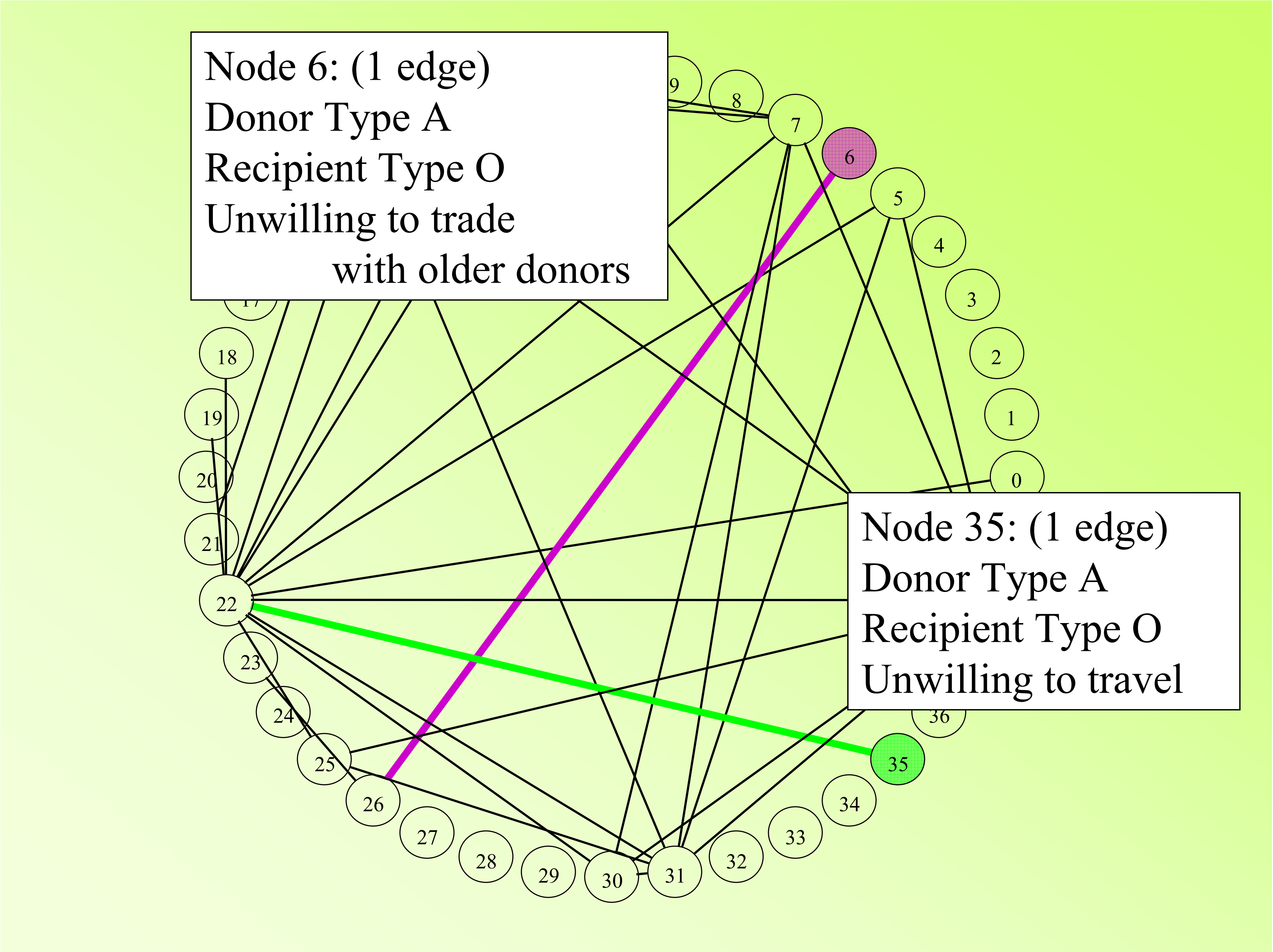
Node 5: (3 edges)
Donor Type A
Recipient Type O
Willing to travel


Node 6: (1 edge)
Donor Type A
Recipient Type O
Unwilling to trade
with older donors
Node 35: (1 edge)
Donor Type A
Recipient Type O
Unwilling to travel
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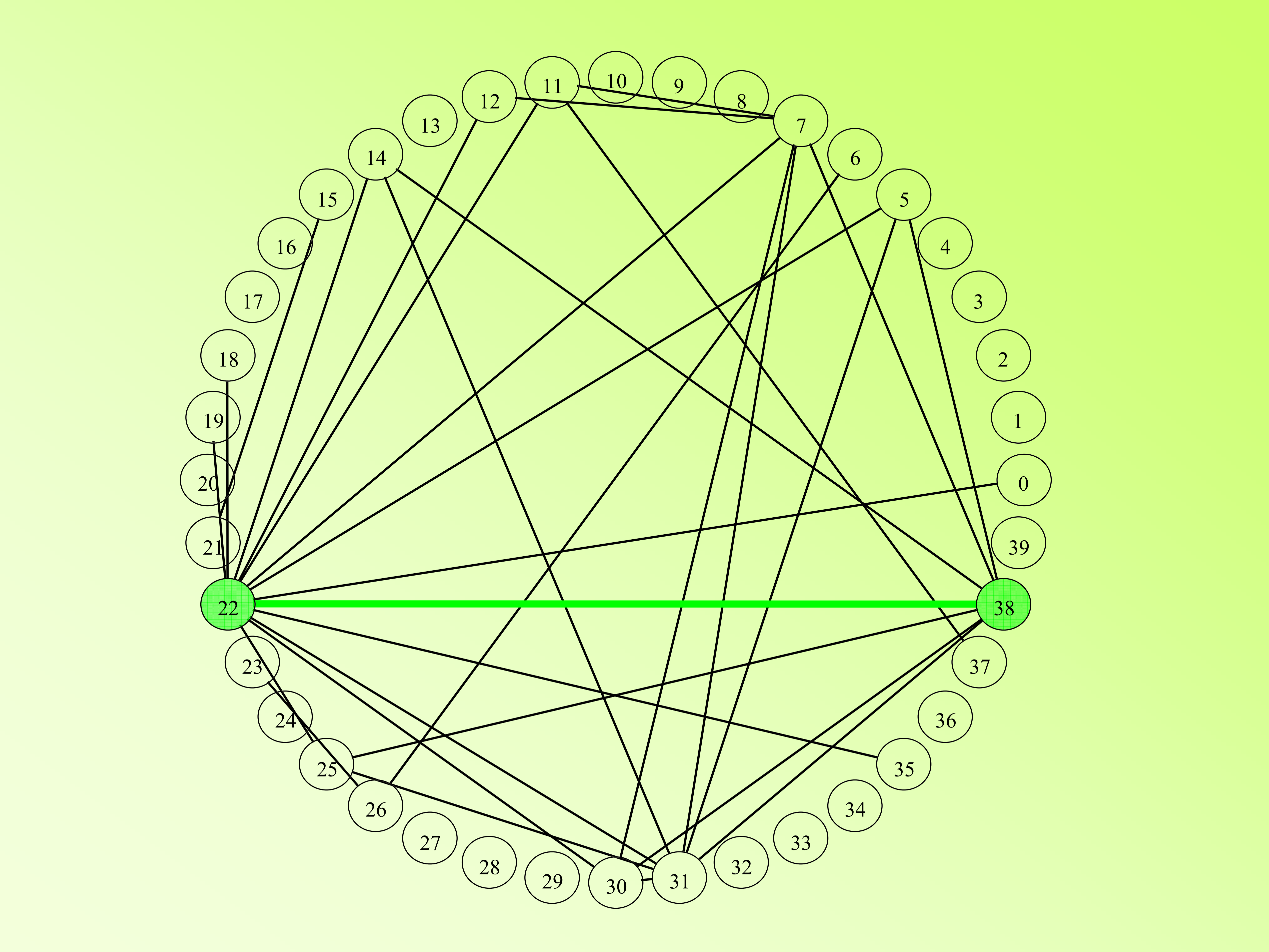

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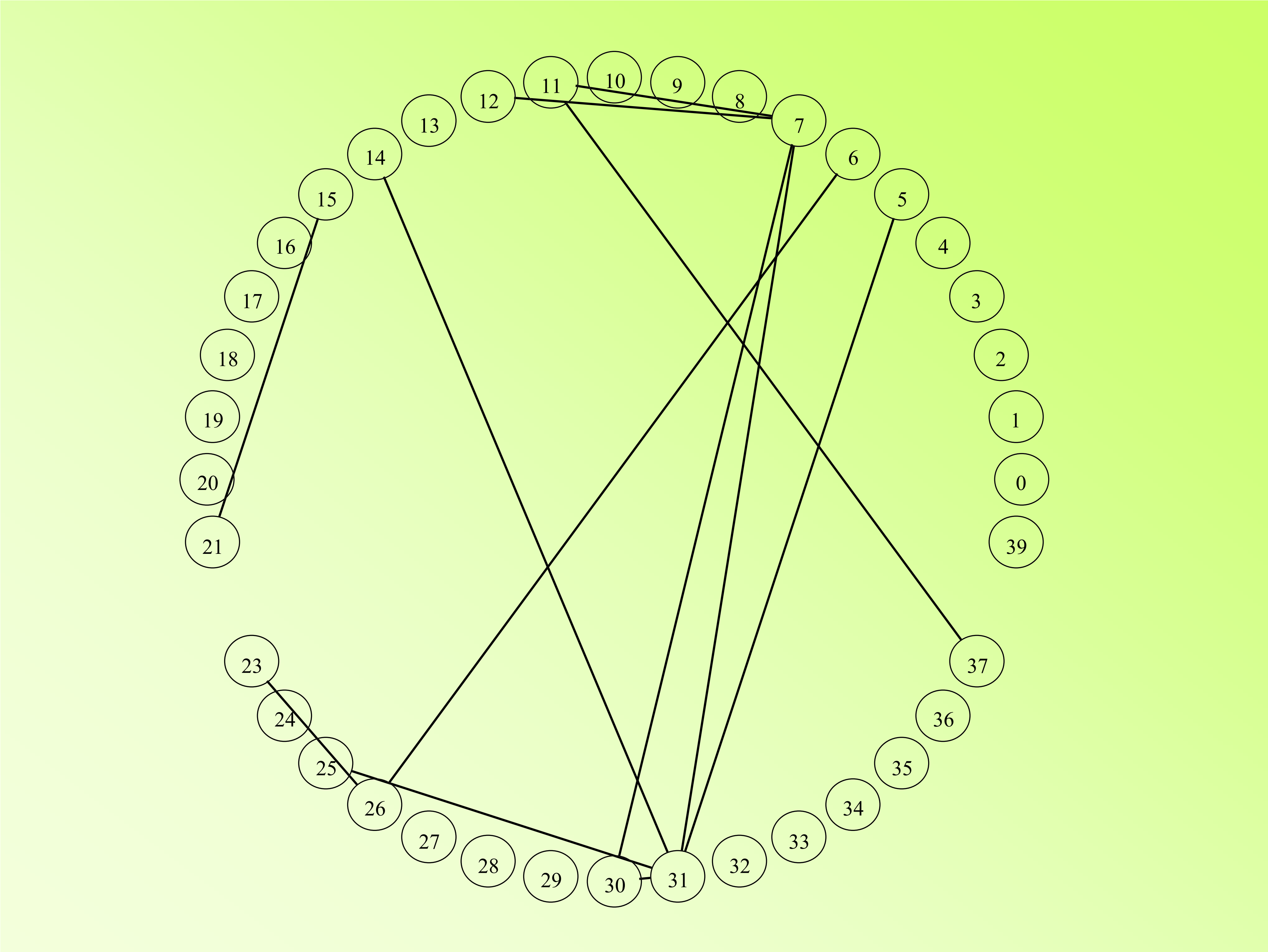

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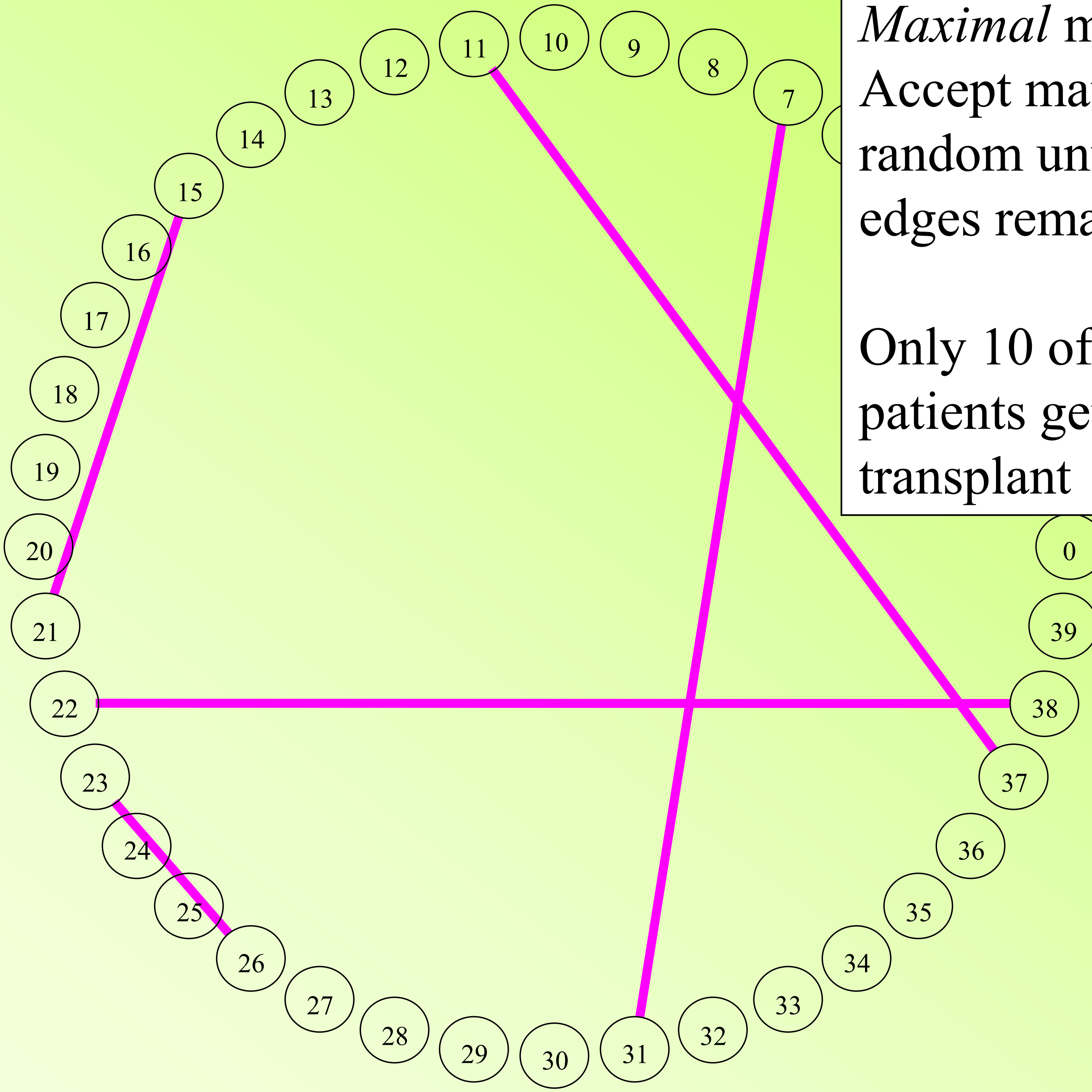
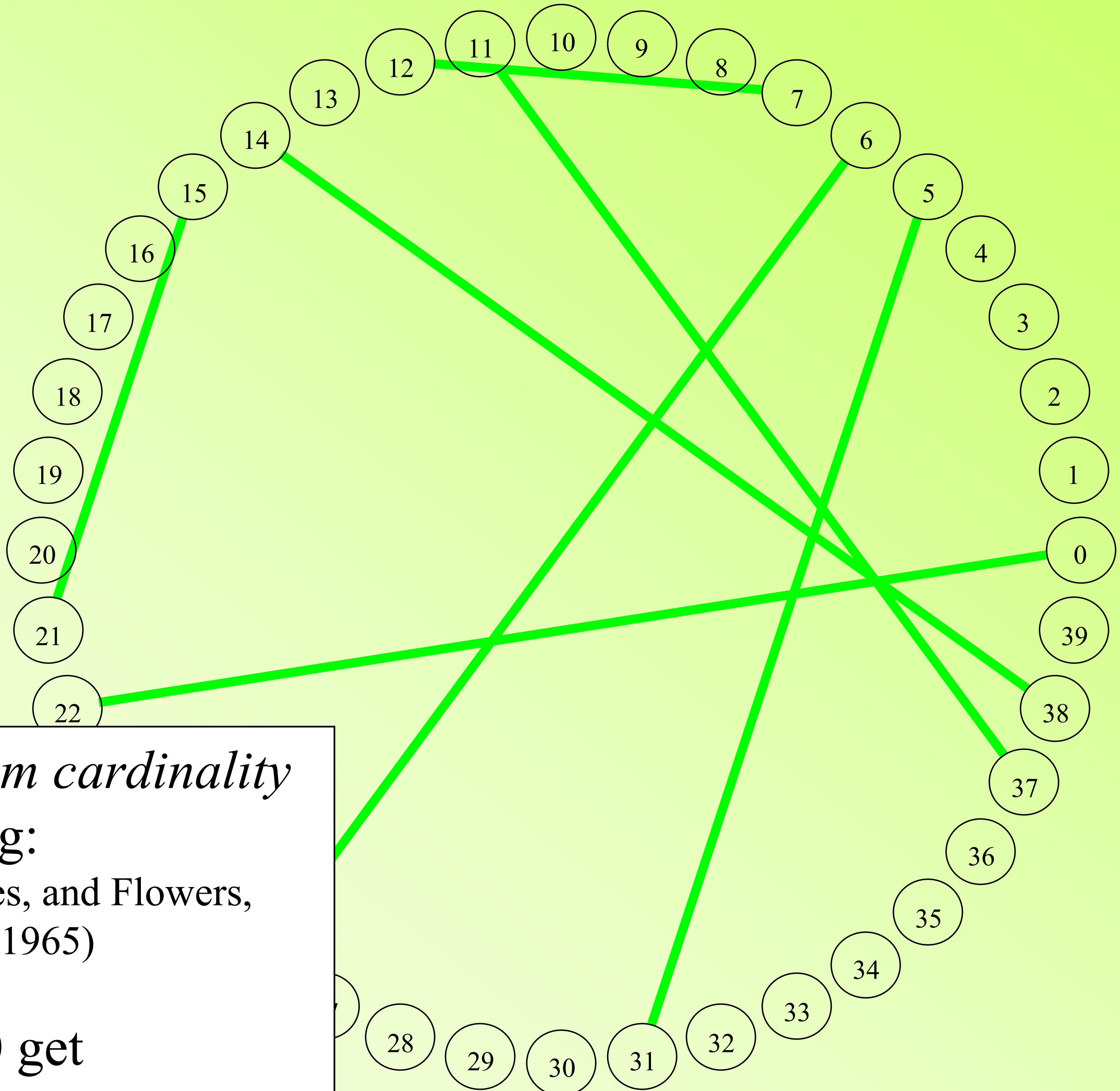


Maximal matching: Accept matches at random until no edges remain.

Only 10 of 40 patients get a transplant



Maximum cardinality matching:

Paths, Trees, and Flowers, Edmonds (1965)

14 of 40 get transplants

Maximizing # of transplants

- First-Accept method finds maximal matching
  - When an incompatible pair arrives, search the entire current list for a feasible trade, and if one is found, perform that exchange
- Maximum cardinality matching method
  - Over the entire pool, periodically choose the exchanges that result in the greatest benefit
  - Combinatorial explosion: for 100 donor/recipient pairs, there are millions of possible matchings, for 1000, there are 10^{250}
  - Edmonds' algorithm finds, efficiently, the exchanges that yield the maximum number of transplants

Flexibility: edge weights

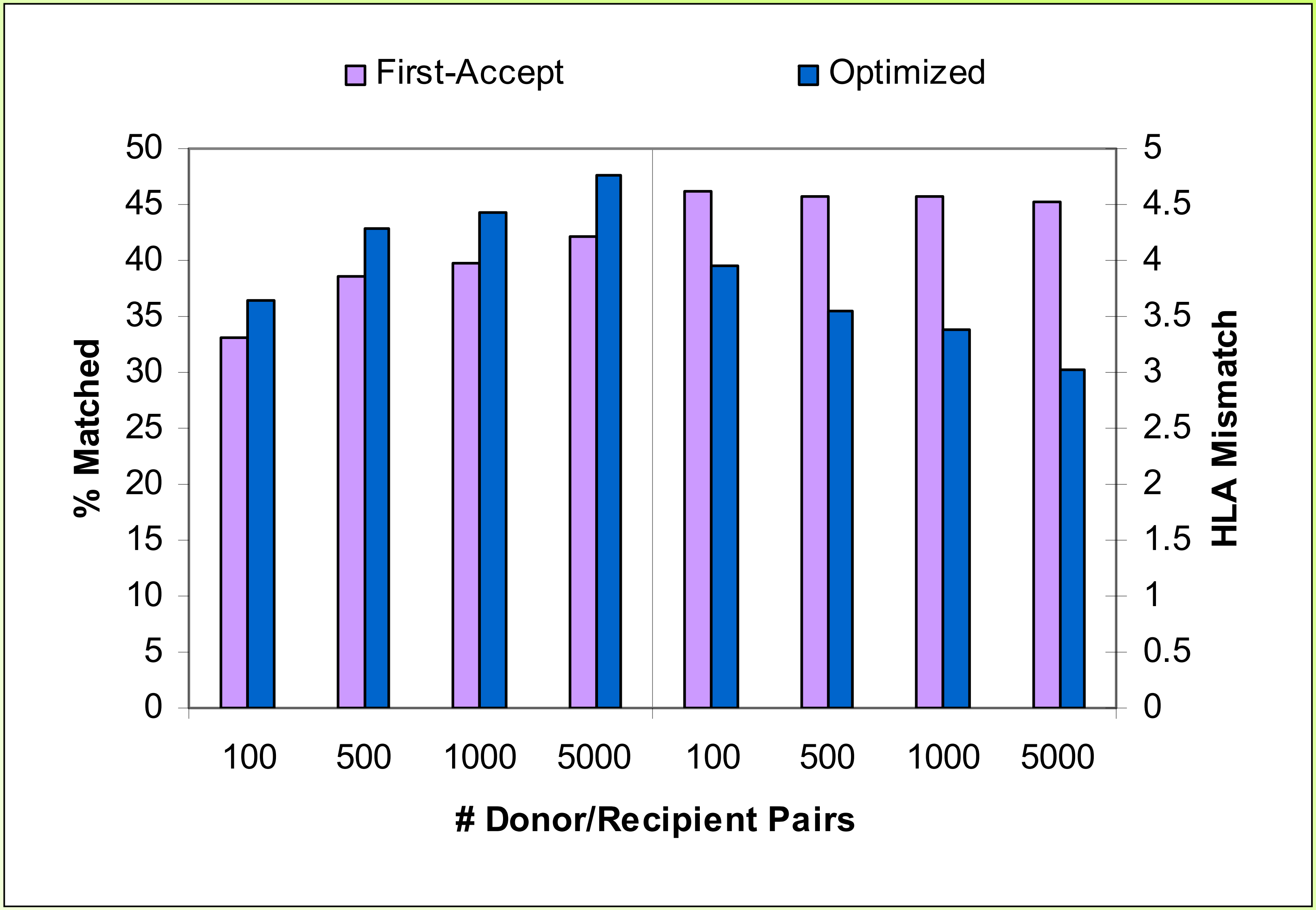
- All exchanges are equal, but some exchanges are more equal than others
  - Bonus points for disadvantaged groups, like O patients, the highly sensitized, or those who have been waiting a long time
  - Points to reward clinical predictors of good outcomes: HLA antigen mismatch
  - Patient priorities: donor age versus different transplant center
- Maximum edge weight matching problem: find the matching that maximizes the sum of the included edge weights (Edmonds, 1965)

Advantage of Optimization

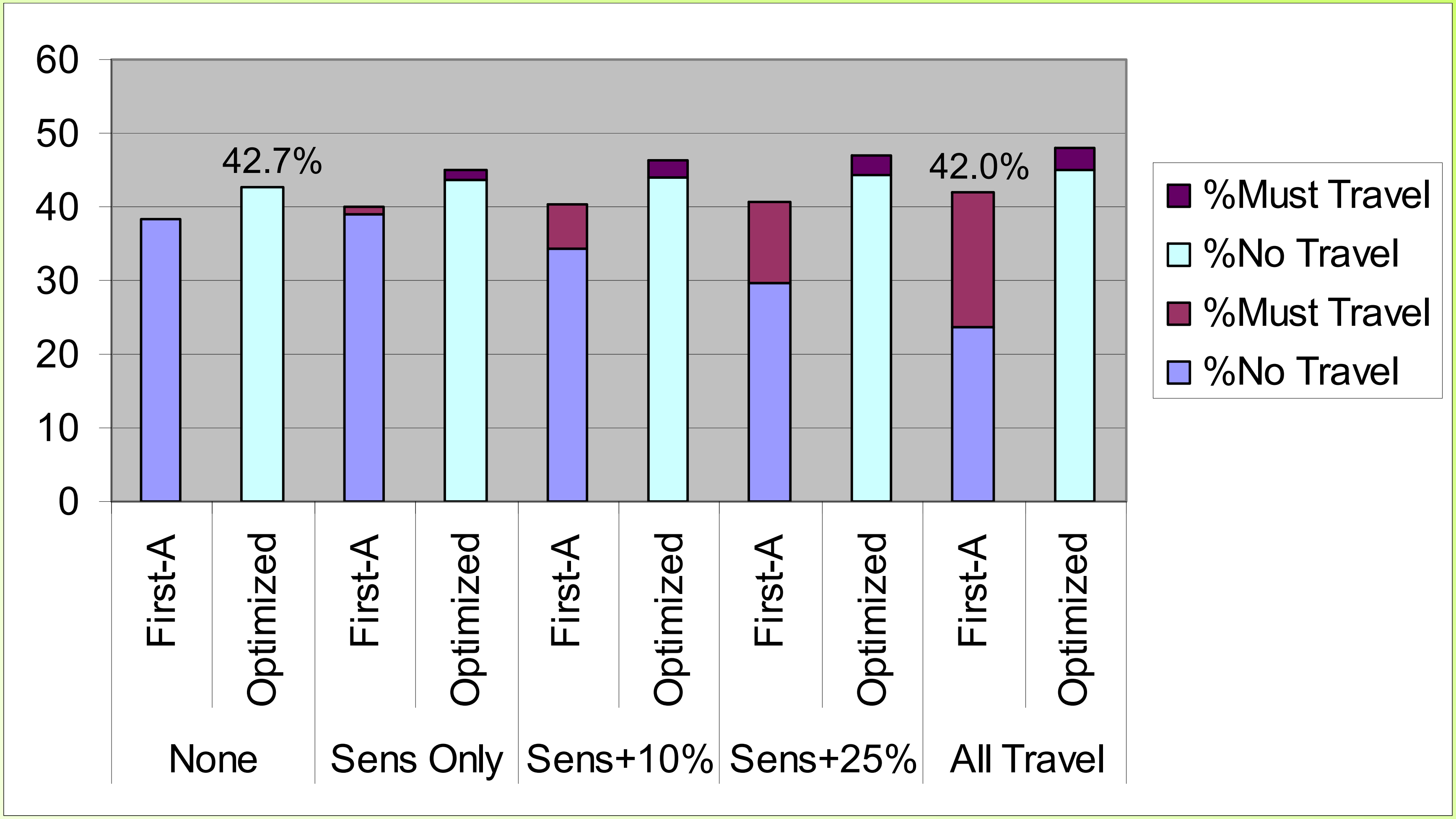
- First-Accept and Optimized matching algorithms tested
  - 30 simulated pools of 4000 patients
  - First-Accept averages
    - 1673 transplants
  - Optimized averages
    - 1891 transplants

(Segev, Gentry, et al., Journal of the American Medical Association, 2005)

PKE pool size advantage



Minimizing travel outside region



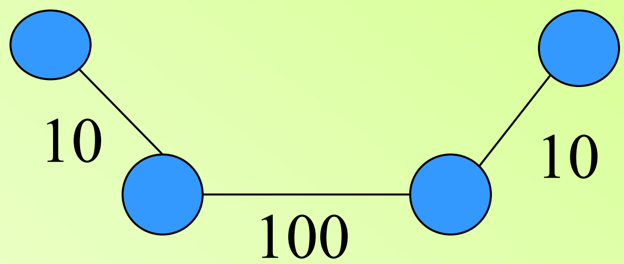
Financial benefit to mathematical literacy

- Long wait times on dialysis are extremely expensive
- $290 million saved over dialysis using any form of paired kidney donation
- $50 million additional saved over dialysis using maximum edge weight matching for paired kidney donation

(Segev, Gentry, et al., Journal of the American Medical Association, 2005)

Edge weight points system

- Maximum edge weight does not necessarily yield a maximum cardinality matching

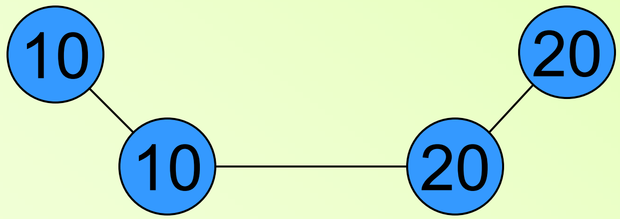


- We use a 100 point bonus for all edges, then differentiate between edges in the range of 0-10 points
  - within .4% of the size of maximum cardinality matching, with vastly reduced travel requirements and HLA antigen mismatch

Alternative optimal matchings

- Roth et al. have suggested using maximum node weight matchings
  - Advantage: all maximum node weight matchings are also maximum cardinality matchings
  - Disadvantage: node weights can not capture priorities we wish to place on particular exchanges, like a very good compatibility match, or an exchange between patients at the same hospital that does not require travel

(Roth, Sonmez, Ünver, Pairwise Kidney Exchange, working paper, 2005)



Waiting times and accumulation

- Pairs that do not match will accumulate
  - Waiting list like that for deceased donors
  - How will the waiting list evolve? How long will patients wait?
- Iterated optimization rather than static
  - As the waiting list grows, so do computation times for the optimal solution
- Currently purchasing a Linux cluster for embarassingly parallel Monte Carlo simulations

All incompatible donor / recipient pairs

# Pairs	dO	dA	dB	dAB
rO	657	1066	354	51
rA	257	429	174	107
rB	83	175	91	72
rAB	12	27	17	11

Unmatched after paired donation: 52%

# Pairs	dO	dA	dB	dAB
rO	214	849	293	50
rA	26	155	23	90
rB	14	22	39	62
rAB	1	7	5	6

(Gentry, Segev, Montgomery, Am. J. Transplantation, 2005)

Deceased donation requires a simpler optimization than KPD