

# Simulating PEO melts using connectivity-altering Monte Carlo

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Acknowledgements:

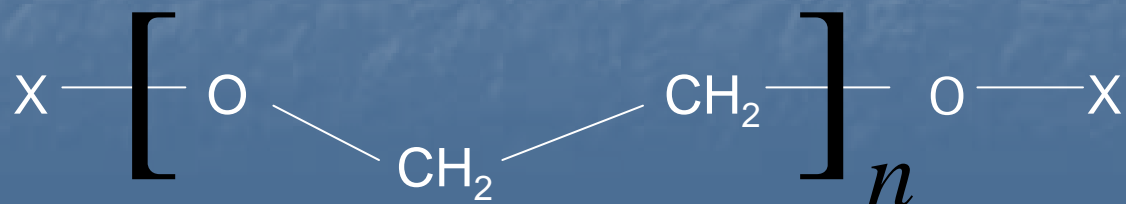
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# Introduction

Poly(ethylene glycol) ( $X=H$ ) and poly(ethylene oxide) dimethyl ether ( $X=CH_3$ ) have the same interior segments, but different endpoints.

The volumetric properties of the two systems as a function of chain length behave quite differently.



# Molecular Simulation Techniques

- Two predominate simulation techniques are used.
- Molecular Dynamics: the system follows Newton's equations of motion, following their real dynamical trajectory.
- Monte Carlo: the system randomly moves from one state to another with no set trajectory.
- The technique we use is Monte Carlo.

# Designing a Monte Carlo Move

$$\rho_{\alpha} T_{\alpha \rightarrow \beta} P_{\alpha \rightarrow \beta} = \rho_{\beta} T_{\beta \rightarrow \alpha} P_{\beta \rightarrow \alpha}$$

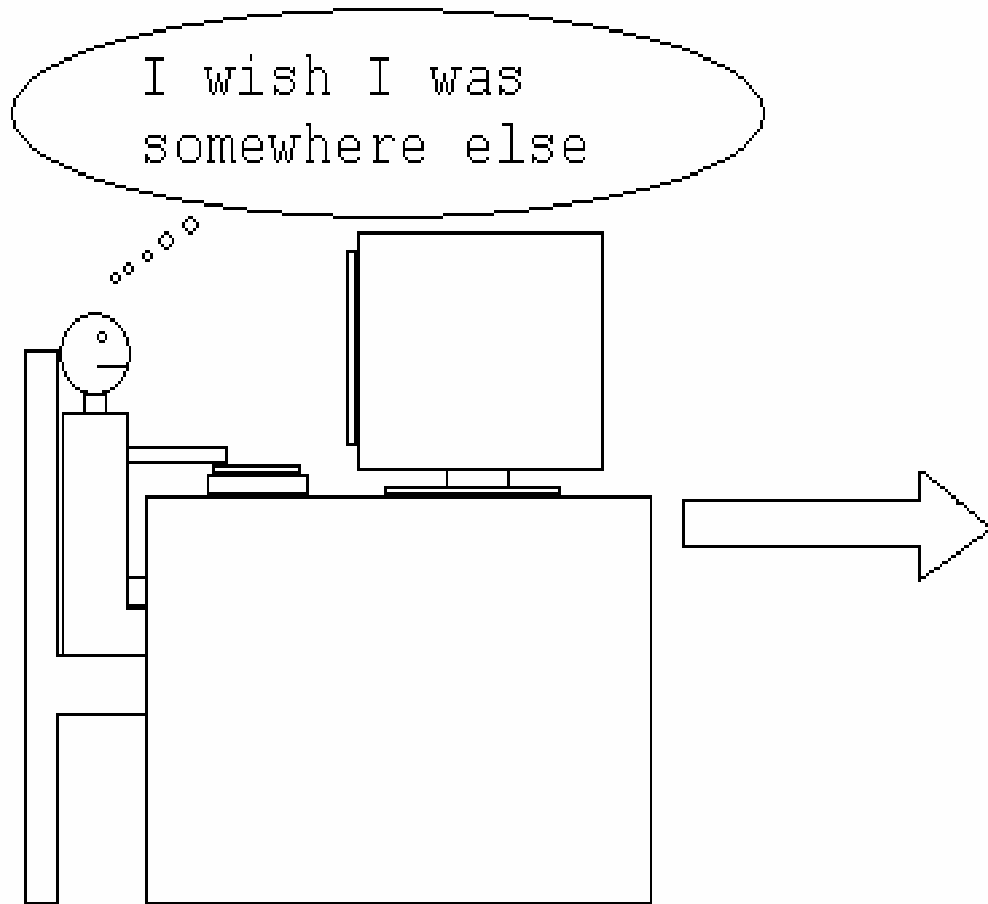
$\rho_{\alpha}$  = prob. density to be in state  $\alpha$

$T_{\alpha \rightarrow \beta}$  = transition prob. for the transfer  
from state  $\alpha$  to  $\beta$ .

$P_{\alpha \rightarrow \beta}$  = acceptance prob. for the transfer  
from state  $\alpha$  to  $\beta$ .

$$P_{\alpha \rightarrow \beta} = \min[1, (T_{\beta \rightarrow \alpha} / T_{\alpha \rightarrow \beta})(\rho_{\beta} / \rho_{\alpha})]$$

# The Benefit of Monte Carlo



# Metropolis Acceptance

$$T_{\alpha \rightarrow \beta} = T_{\beta \rightarrow \alpha}$$

$$P_{\alpha \rightarrow \beta} = \min[1, \rho_{\beta} / \rho_{\alpha}]$$

Translation and rotation moves :

$$\rho_{\beta} / \rho_{\alpha} = \Delta B = \exp[-(u_{\beta} - u_{\alpha}) / k_{\text{B}}T]$$

Volume Fluctuations with outside pressure bath.

# Configurational-bias Monte Carlo

$$T_{\alpha \rightarrow \beta} \neq T_{\beta \rightarrow \alpha}$$

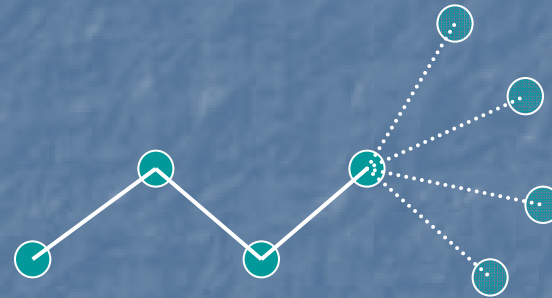
$$T_{\alpha \rightarrow \beta} = \prod_{n=1}^{nstep} (B_i / W)_n$$

$$B_i = \exp(-u_i / k_B T)$$

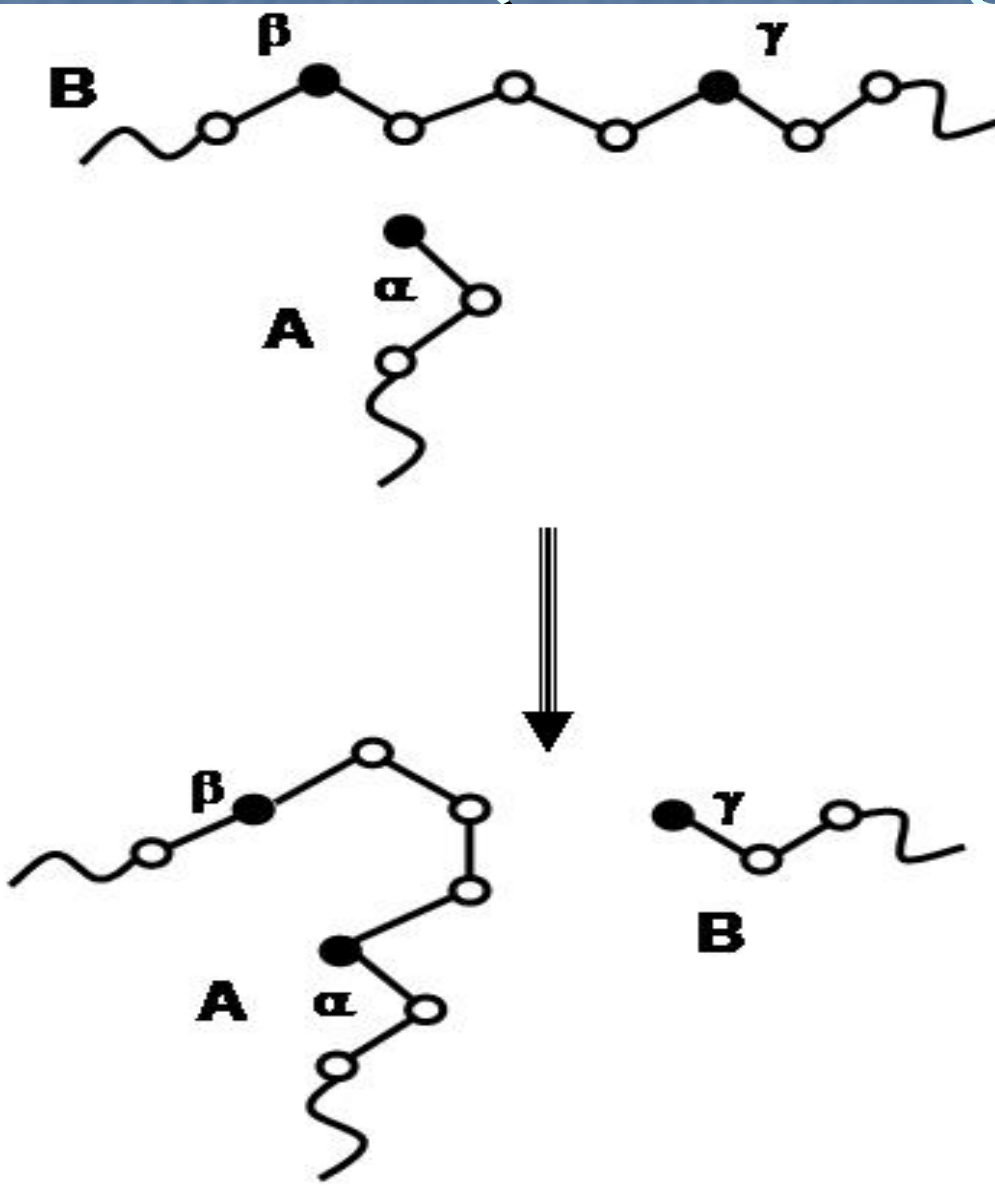
$$W = \sum_{j=1}^{nchoi} B_j$$

$$\rho_\alpha \propto \prod_{n=1}^{nstep} (B_i)_n$$

$$P_{\alpha \rightarrow \beta} = \left[ \prod_{n=1}^{nstep} (W)_n \right]_{\alpha \rightarrow \beta} / \left[ \prod_{n=1}^{nstep} (W)_n \right]_{\beta \rightarrow \alpha}$$



# Connectivity-Altering MC (End-Bridging Move)



End-bridging (EB)  
move from  $\alpha$  to  $\beta$

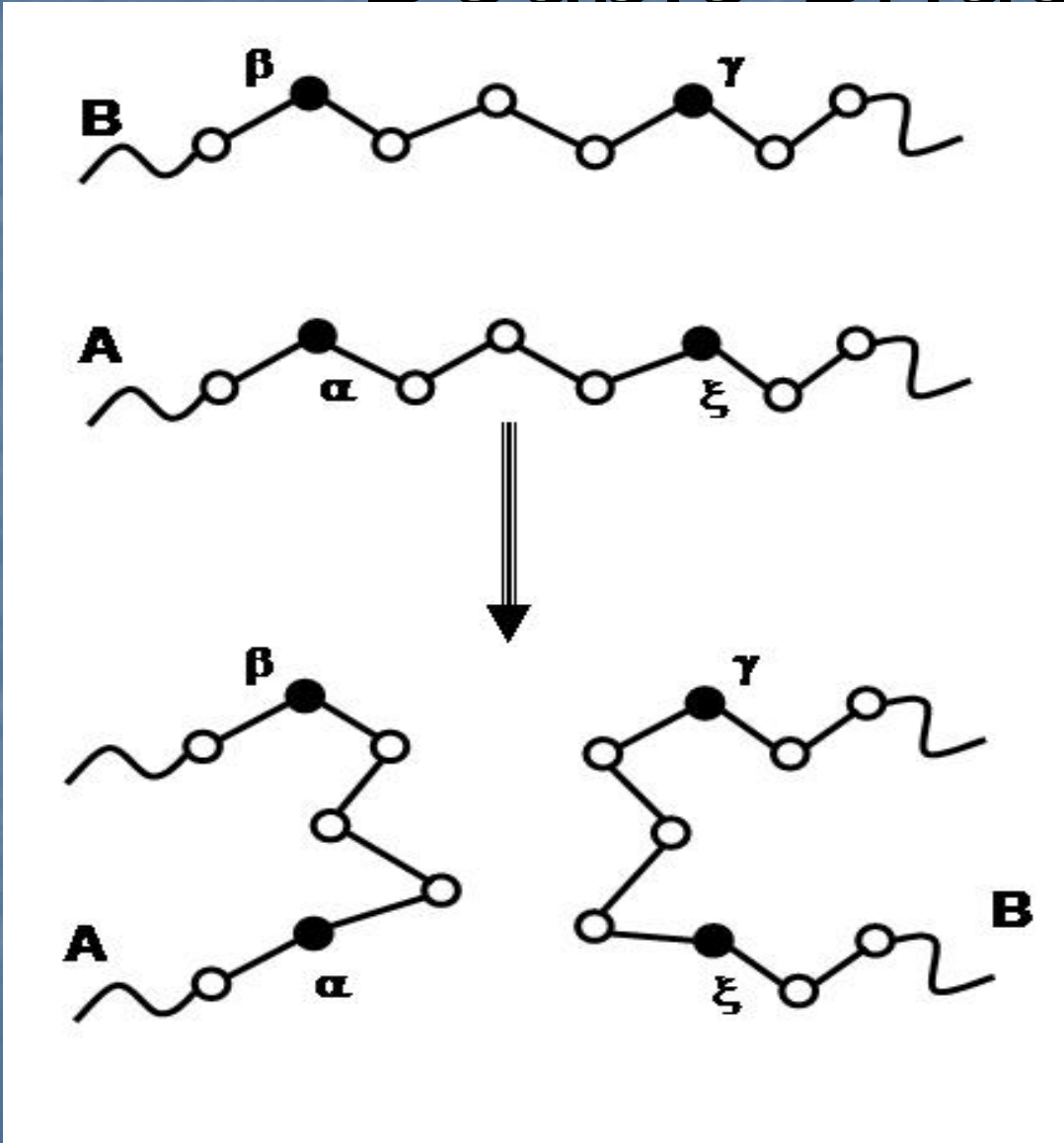
Use SAFE-CBMC to  
regrow the  
segments

For PEG, regrow OH  
group from  $\gamma$ .

Repeat for reverse  
move



# Double Bridging Move



Double-bridging (DB) move from  $\alpha$  to  $\beta$ , and from  $\gamma$  to  $\xi$ . Use SAFE-CBMC to regrow the segments. Repeat for reverse move.

# TraPPE-UA Force Field

- Transferable potentials for phase equilibria-united atom form.
- Enforces transferability in the fitting of new groups.
- Utilizes pseudo-atoms for alkyl groups located at carbon centers, and treats all non-alkyl atoms explicitly.
- Fixed bond lengths

# Potential Form

Harmonic bond bending potential

$$u_{\text{bend}}(\theta) = \frac{k_0}{2}(\theta - \theta_0)^2$$

Cosine series for dihedral interactions

$$u_{\text{torsion}}(\phi) = c_0 + c_1(1 + \cos(\phi)) + c_2(1 - \cos^2(\phi)) \\ + c_3(1 + \cos^3(\phi))$$

LJ + coulombic terms for non - bonded interactions

$$u_{\text{NB}}(r_{ij}) = 4\epsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left( \frac{\sigma_{ij}}{r_{ij}} \right)^6 \right] + \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

# Parameterizing TraPPE-UA

- Bonded interactions are taken from existing force fields.
- Lennard-Jones and Coulombic interactions are fit to reproduce vapor-liquid coexistence curves.
- For PEODME, all nonbonded parameters were fit to small ether molecules.
- For PEG, all nonbonded parameters were fit to small alkanol and glycol molecules.
- Oxygens have negative charges, and methyl, methylene, and hydrogens have positive charges.

# Simulation Details

- Polydisperse melts of PEG and PEODME were simulated with chain length evenly distributed over a selected range.
- Simulation runs consisted of four runs of 200,000 MC cycles (one MC cycle is  $N$  MC moves).
- Periodic boundary conditions were used.
- Pressure and temperature were set for each simulation run.

# Number, molecular weight, and polydispersity of polymers

Name	$N$	$M_n$ (g/mol)	$M_w/M_n$
PEG-600	30	723	1.05
PEG-1540	20	1560	1.08
PEG-3000	12	3014	1.08
PEG-4500	9	4467	1.08
PEG-6000	6	5921	1.08
PEG-18500	2	18520	1.08
PEODME-600	40	619	1.02
PEODME-1000	30	1015	1.03
PEODME-3000	12	2998	1.07
PEODME-4300	8	4319	1.08
PEODME-6000	5	5993	1.08

# Percentage and acceptance of select MC moves

	rep	EB	DB
PEODME	(20%)	(20%)	(20%)
	1.5	0.08	0.0035
PEG	(0%)	(30%)	(30%)
	N/A	0.025	0.0031

# Center of Mass Movement

Top: PEODME-  
3000

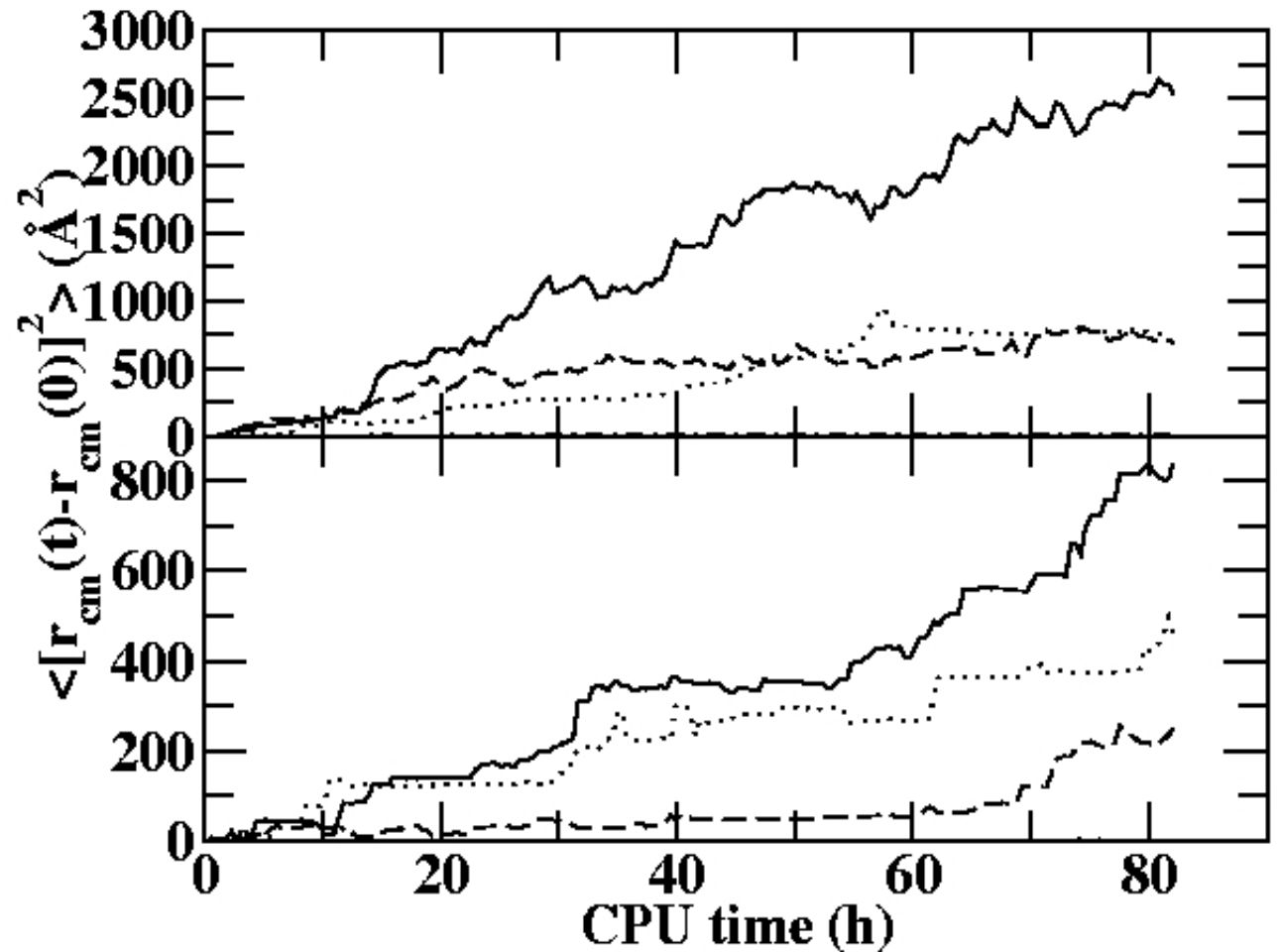
Bottom: PEG-  
3000

Solid line: both EB  
and DB moves

Dotted line: DB  
move only

Dashed line: EB  
move only

Dot-dashed line:  
neither DB or  
EB moves  
used





# End-Group Movement

Top: PEODME-  
3000

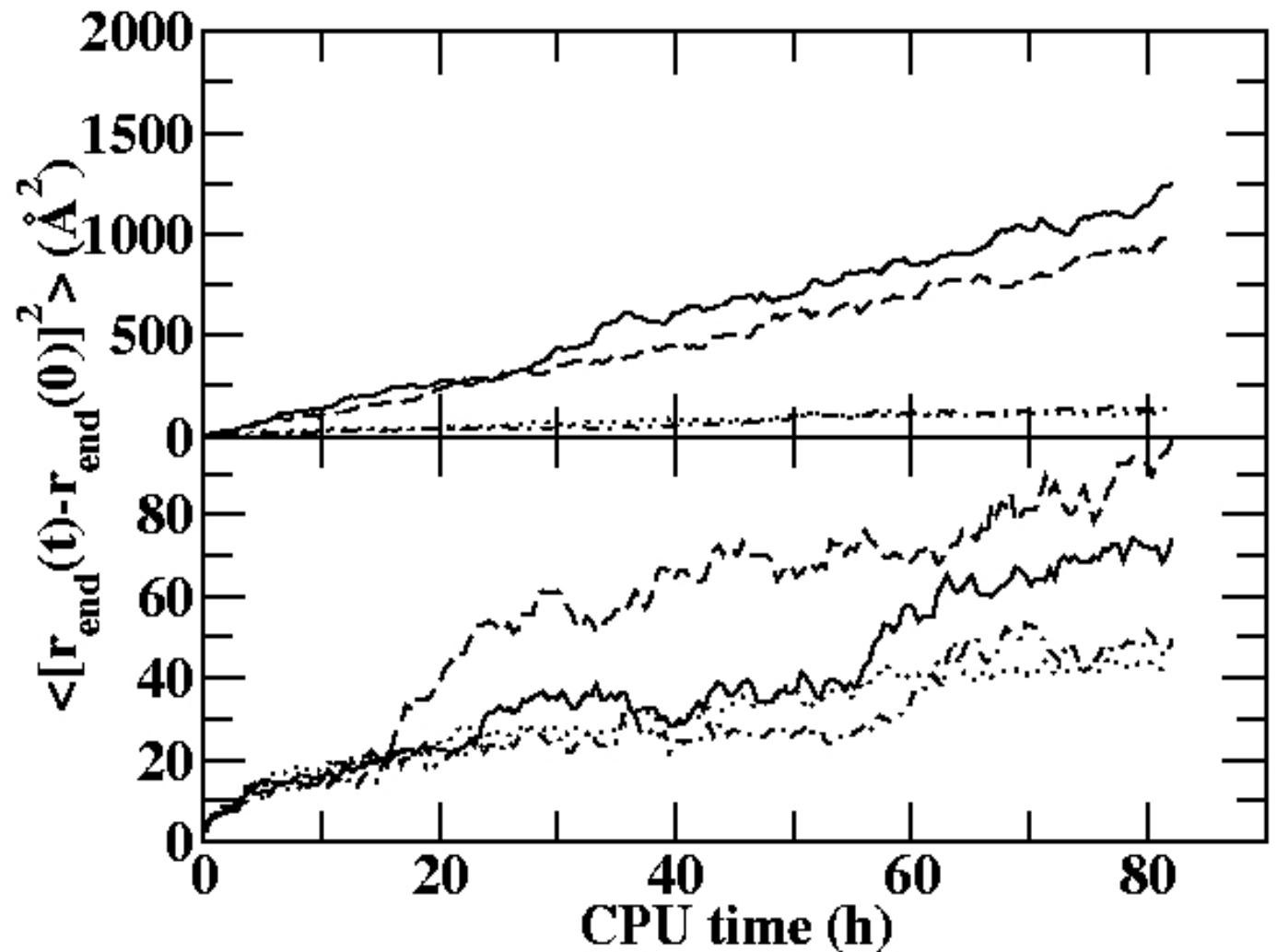
Bottom: PEG-  
3000

Solid line: both EB  
and DB moves

Dotted line: DB  
move only

Dashed line: EB  
move only

Dot-dashed line:  
neither DB or  
EB moves  
used



# Decay of End-to-End Vector

Top: PEODME-3000

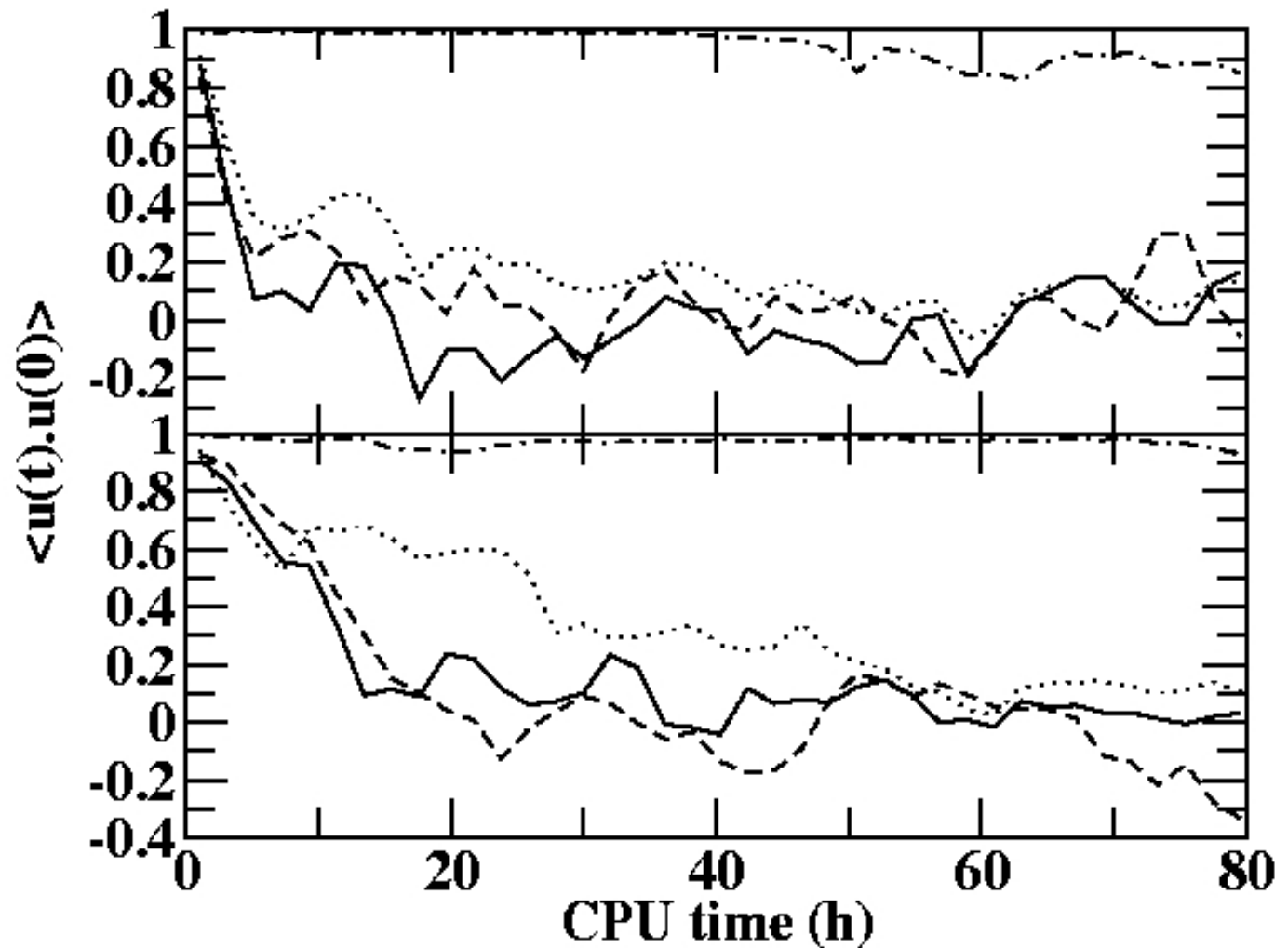
Bottom: PEG-3000

Solid line: both EB and DB moves

Dotted line: DB move only

Dashed line: EB move only

Dot-dashed line: neither DB or EB moves used



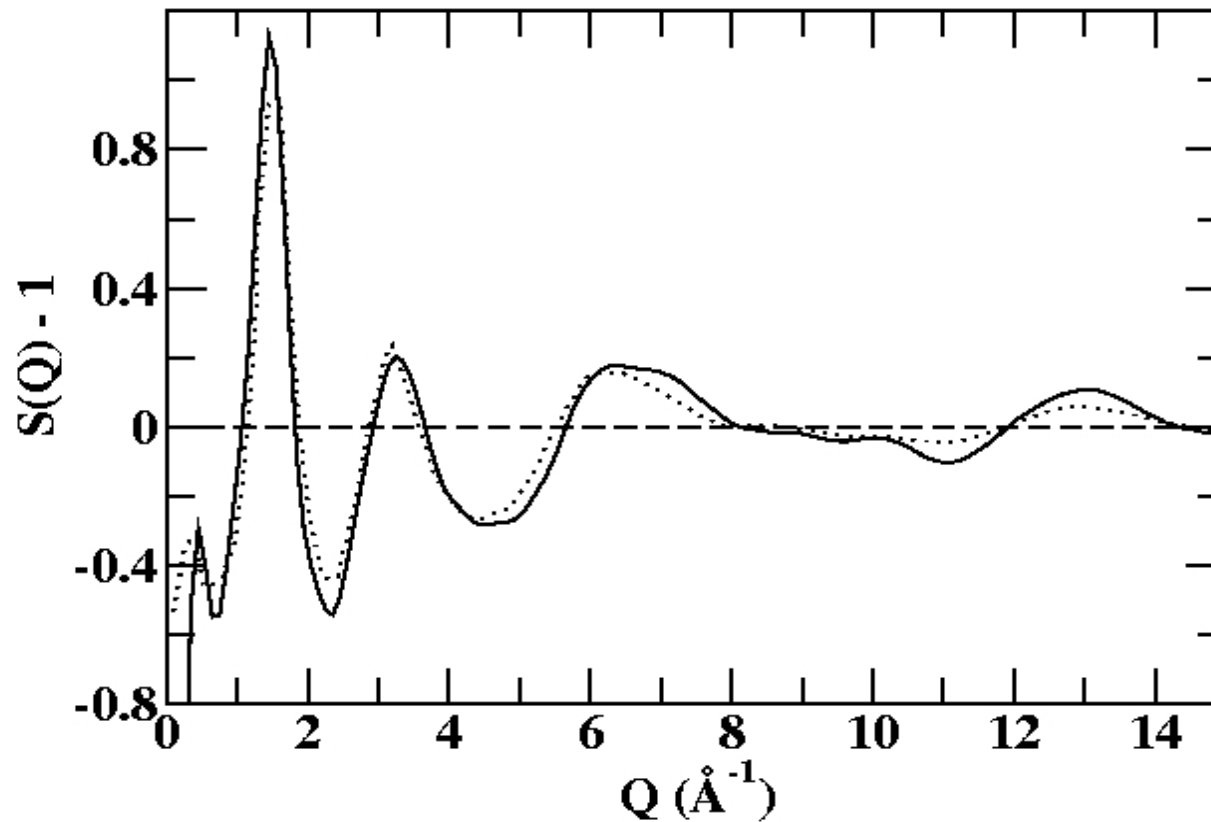
# Specific Densities

	$T$ (K)	$p$ (MPa)	density (g/cm <sup>3</sup> )	
			sim	exp
PEG-600	383.25	0.1	1.061 <sub>2</sub>	1.059
PEG-600	471.75	0.1	0.990 <sub>2</sub>	0.990
PEG-600	471.75	100.0	1.061 <sub>4</sub>	1.055
PEG-1540	474.65	0.1	0.966 <sub>5</sub>	0.977
PEG-18500	471.75	0.1	0.988 <sub>2</sub>	0.990
PEOMME-750	325.25	0.1	1.086 <sub>2</sub>	1.083
PEODME-600	343.35	0.1	1.042 <sub>2</sub>	1.034
PEODME-600	303.05	0.1	1.064 <sub>2</sub>	1.067
PEODME-600	303.05	100.0	1.111 <sub>1</sub>	1.111
PEODME-1000	343.35	0.1	1.063 <sub>g</sub>	1.051

# Static Structure Factor ( $T=383\text{K}$ )

sim, PEODME-3000: solid line

exp, PEG-20000: dotted line



# Characteristic Ratio Plot

Top: PEODME

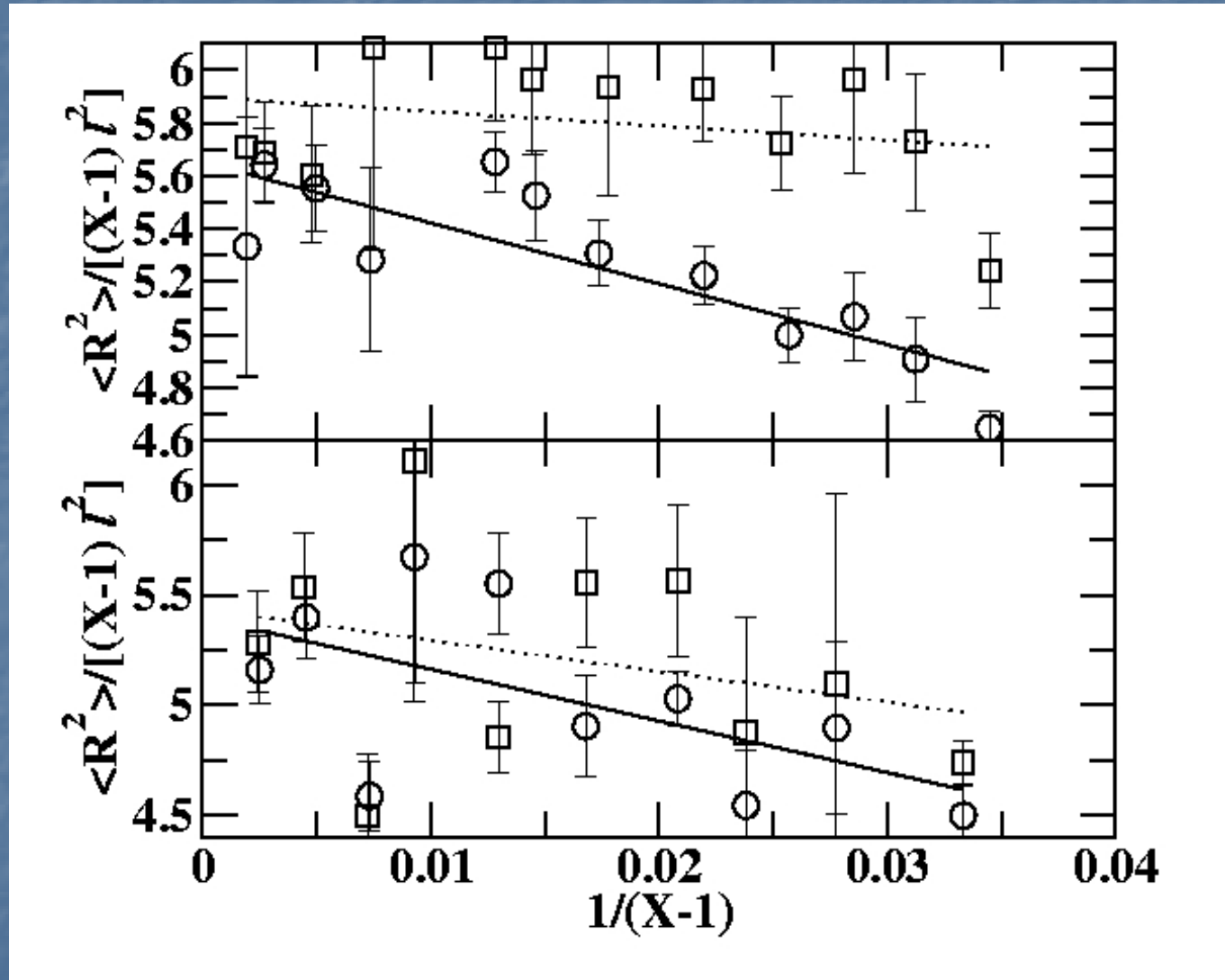
Bottom: PEG

Circles: points  
using end-  
to-end  
distance

Squares: points  
using radii of  
gyration

Solid lines: fit to  
circles

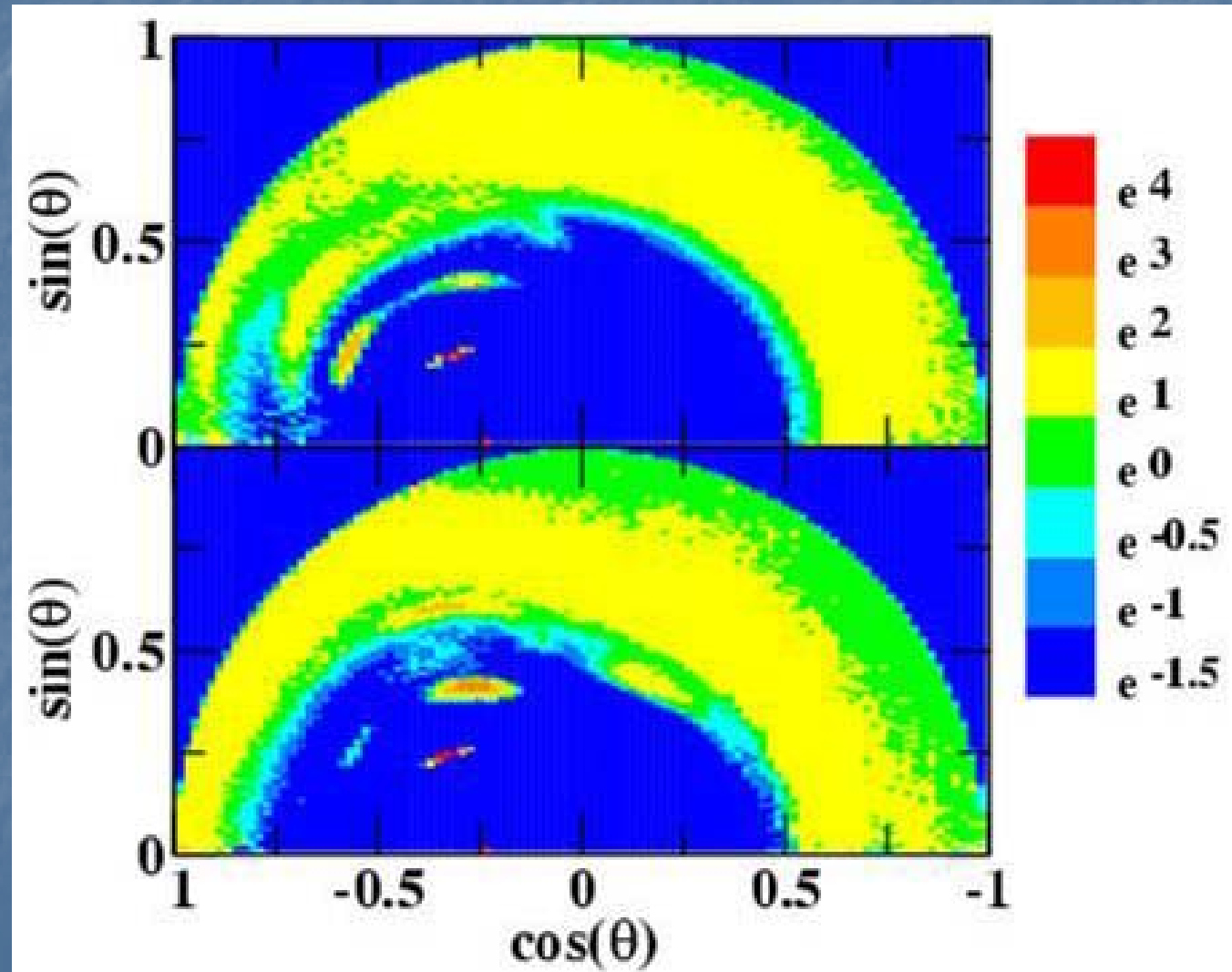
Dotted lines: fit  
to squares



Experiment  $C_\infty = 5.5$

# 3-d Radial Distribution Function

3-d radial-angular distribution function:  $\theta$ =angle for **O-CH<sub>3</sub>**—(any other heavy atom) for PEODME (top) and **CH<sub>2</sub>-O**—(any other heavy atom) for PEG (bottom).



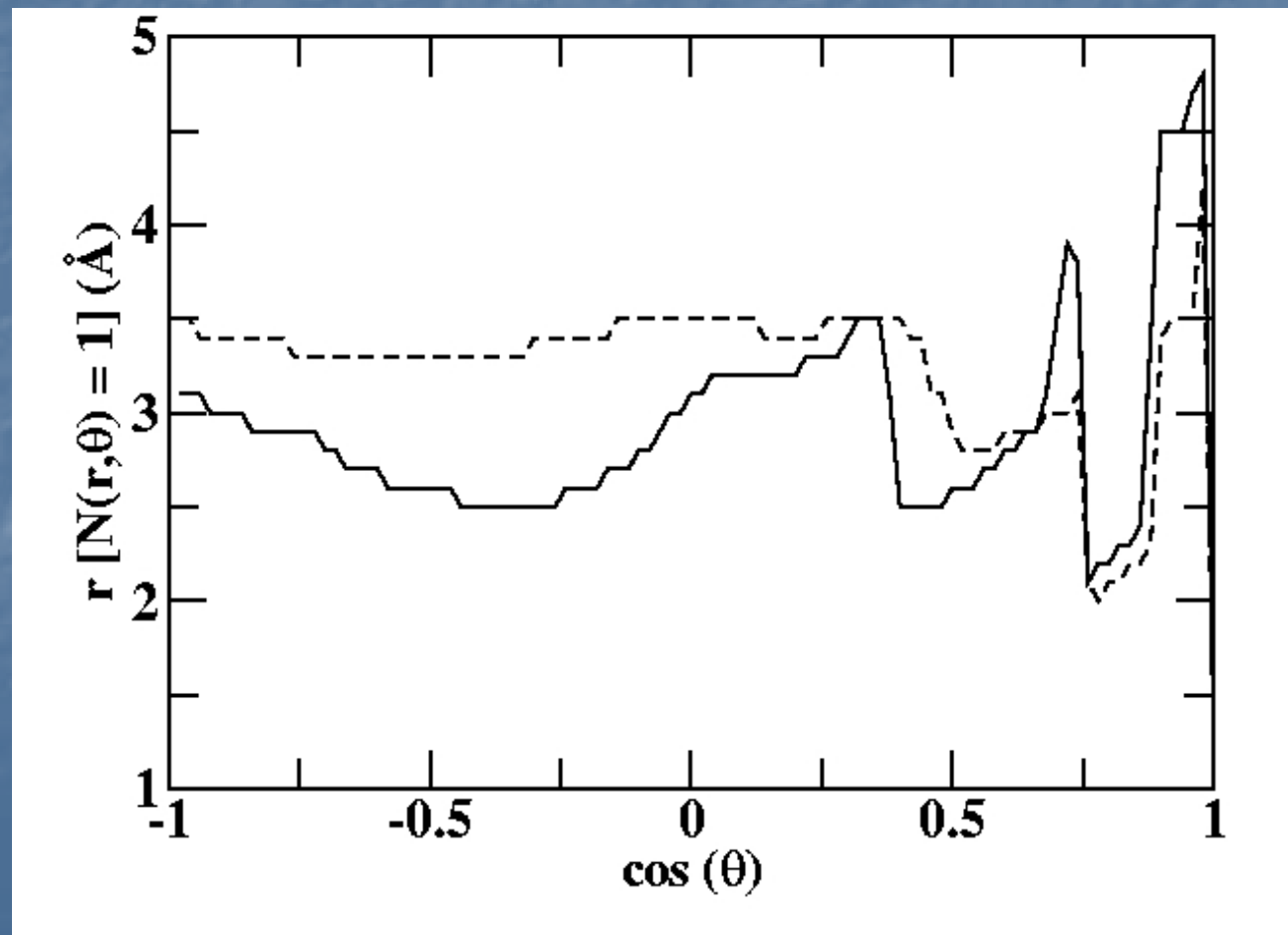
# Integral of 3-d RDF

Distance where the integral of the 3-d radial-angular distribution function is unity at fixed  $\theta$ , or the distance where another heavy atom is most likely present at fixed  $\theta$ .

Integrating out these curves give a total end-group excluded volume: 162 angstroms for PEODME and 127 angstroms for PEG.

solid line:  
PEG

dotted line:  
PEODME



# Conclusions

- Endbridging and double-bridging do reasonably well in equilibrating the PEO melts, but better for PEODME than for PEG.
- The TraPPE-UA force field does an excellent job of reproducing densities, and a good job of reproducing structural properties of PEO.
- The Molecular weight dependence of the volumetric properties can be related to differences in excluded volume of endgroups.