Computational Modeling of Hall Thrusters

Justin W. Koo
Department of Aerospace Engineering
University of Michigan
Ann Arbor, Michigan 48109

Overview

- Motivation
- Hall Thruster Background
- Computational Modeling
- Results
- Acknowledgements

Rocket Equation

- Force balance on a spacecraft

$$M\frac{dv}{dt}+U_{Exit}\frac{dm}{dt}=F_{gravity}+F_{drag}$$

- Neglect gravity and drag forces and integrate

$$M_{final}=M_{initial}\exp\left(\frac{-\Delta V}{U_{Exit}}\right)$$

Mission	**ΔV Requirement**
Earth to LEO	7600 m/s
LEO to GEO	4200 m/s
LEO Escape	3200 m/s
LEO to Moon	3900 m/s
LEO to Mars	5700 m/s

Table from Mechanics and Thermodynamics of Propulsion, 2nd Edition, Peterson and Hill, 1992

Specific Impulse

- Thrust per unit mass propellant (measured in s)

$$I_{sp} = \frac{\int T dt}{\dot{m}_{propellant} g_e} = \frac{U_{exit}}{g_e}$$

- Put this into the rocket equation

$$M_{final} = M_{initial} \exp\left(\frac{-\Delta V}{I_{sp} g_e}\right)$$

Chemical vs EP

- Chemical Propulsion
  - Limits on propellant exit velocity are based on thermodynamic properties of propellant and material properties of combustion systems
  - Typical I_{sp} between 150 s and 450 s
  - High thrust applications (especially Earth to LEO)
- Electric Propulsion (EP)
  - Limits on propellant exit velocity are based on power supply mass and lifetime issues
  - For Hall thrusters typically between 1500 s and 2500 s
  - Low thrust applications (LEO to GEO and beyond)
  - Tradeoff between thrust and I_{sp}

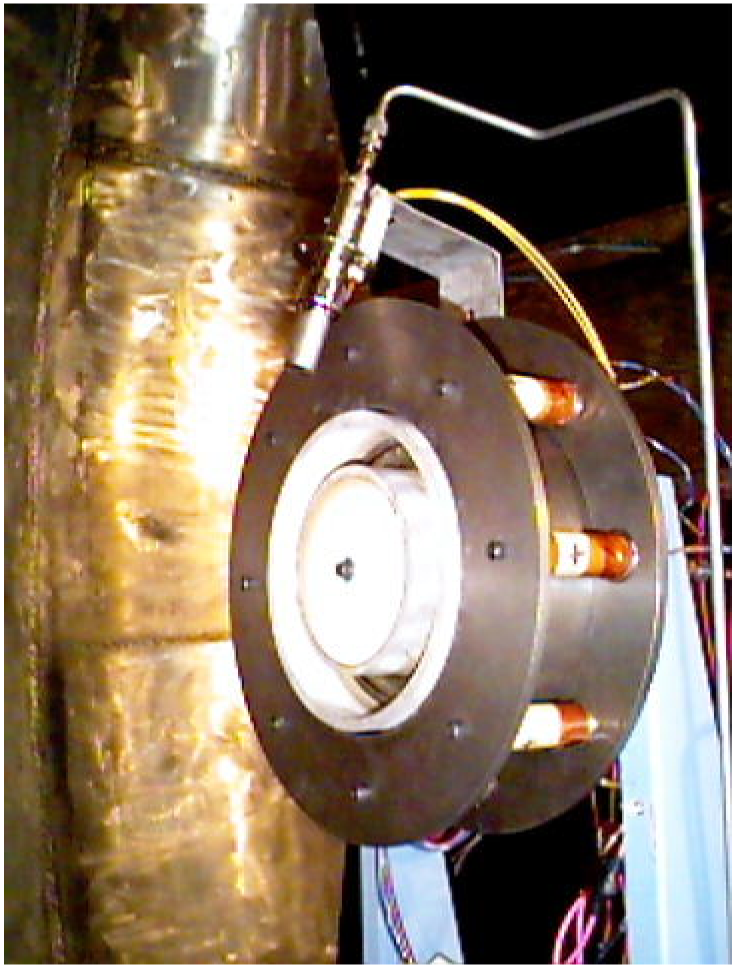
LEO to Mars

- Suppose Δv = 5700 m/s is required
- With typical bipropellant chemical propulsion
  - Isp = 250 s
  - Ideal Payload Fraction = 9.8%
- With EP (Hall Thruster)
  - Isp = 1600 s
  - Ideal Payload Fraction = 69.5%

- Primary present application is to satellite stationkeeping which requires small ΔV corrections over a number of years with savings of hundreds of pounds on satellites that weigh thousands of pounds

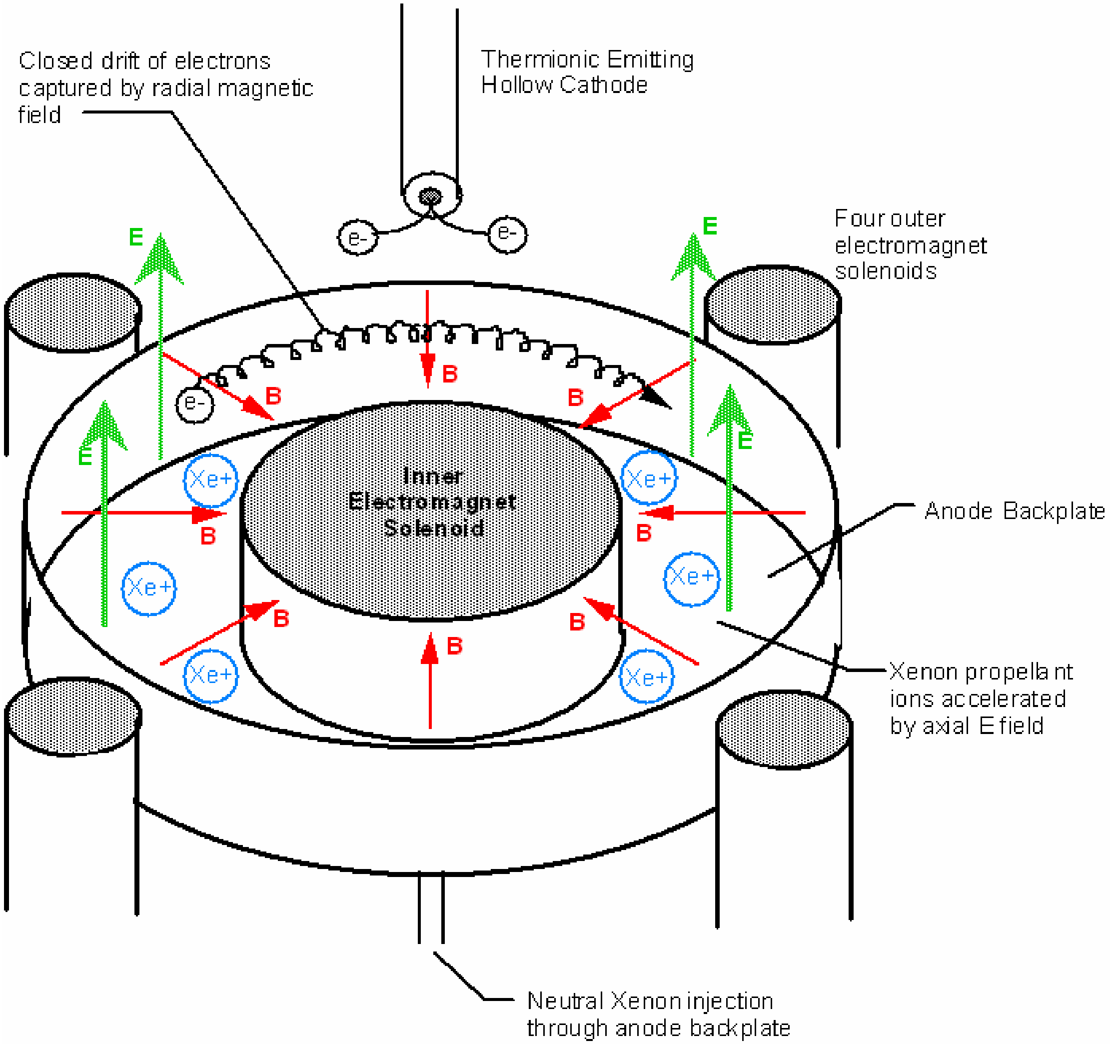
Hall Thruster Performance

- **UM/AFRL P5**: 3 kW, 300 V, 10 A operating condition with a thrust of 180 mN, I_{sp} of 1650 s, and efficiency of 51%
- **NASA-457 M**: >50 kW operating condition with thrust of nearly 3 N, I_{sp} of 1750s - 3250 s, and efficiencies of 46% - 65%



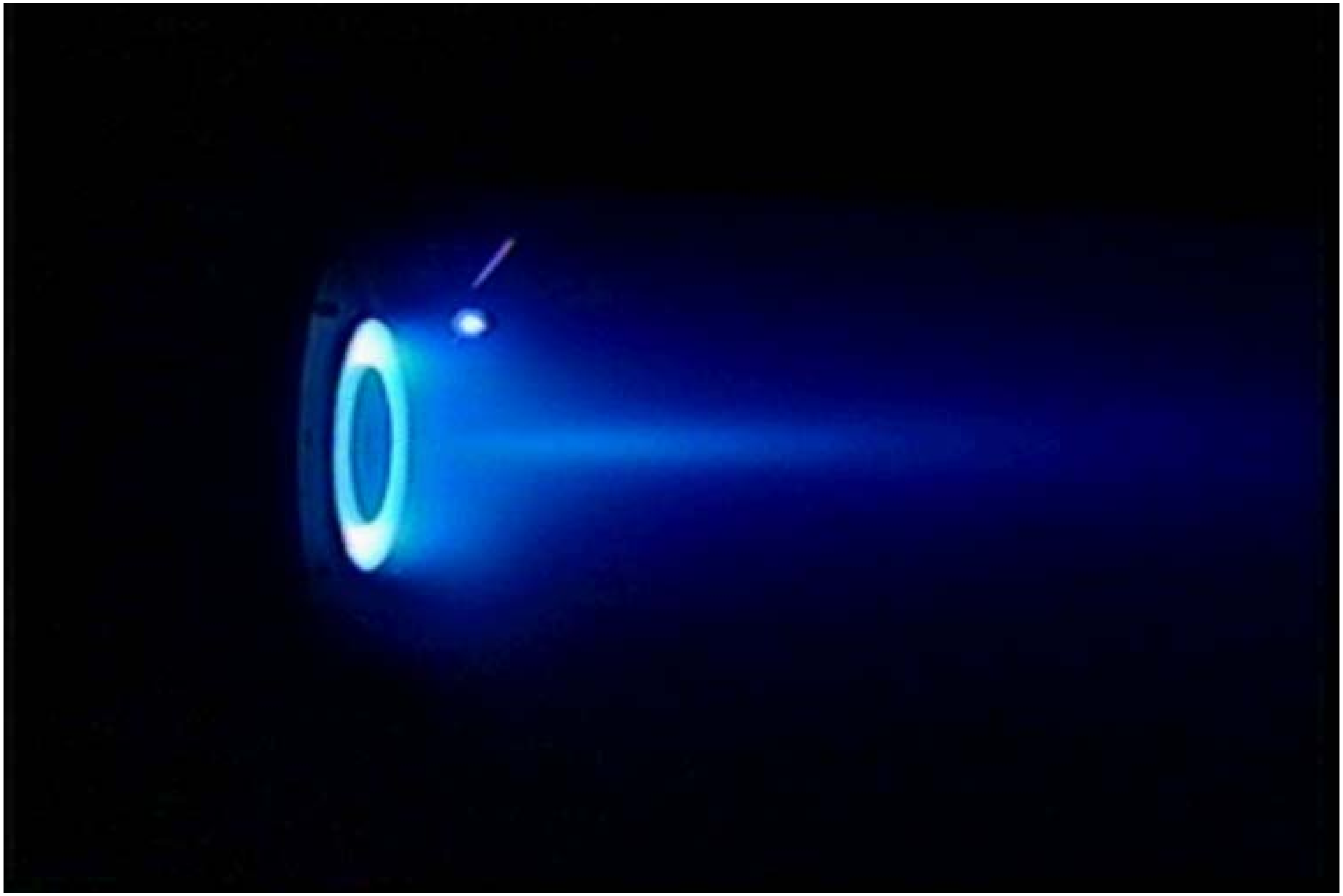
UM/AFRL P5 Photo courtesy of PEPL

Hall Thruster Schematic



Schematic courtesy of PEPL

UM/AFRL P5



UM/AFRL P5 Video courtesy of PEPL

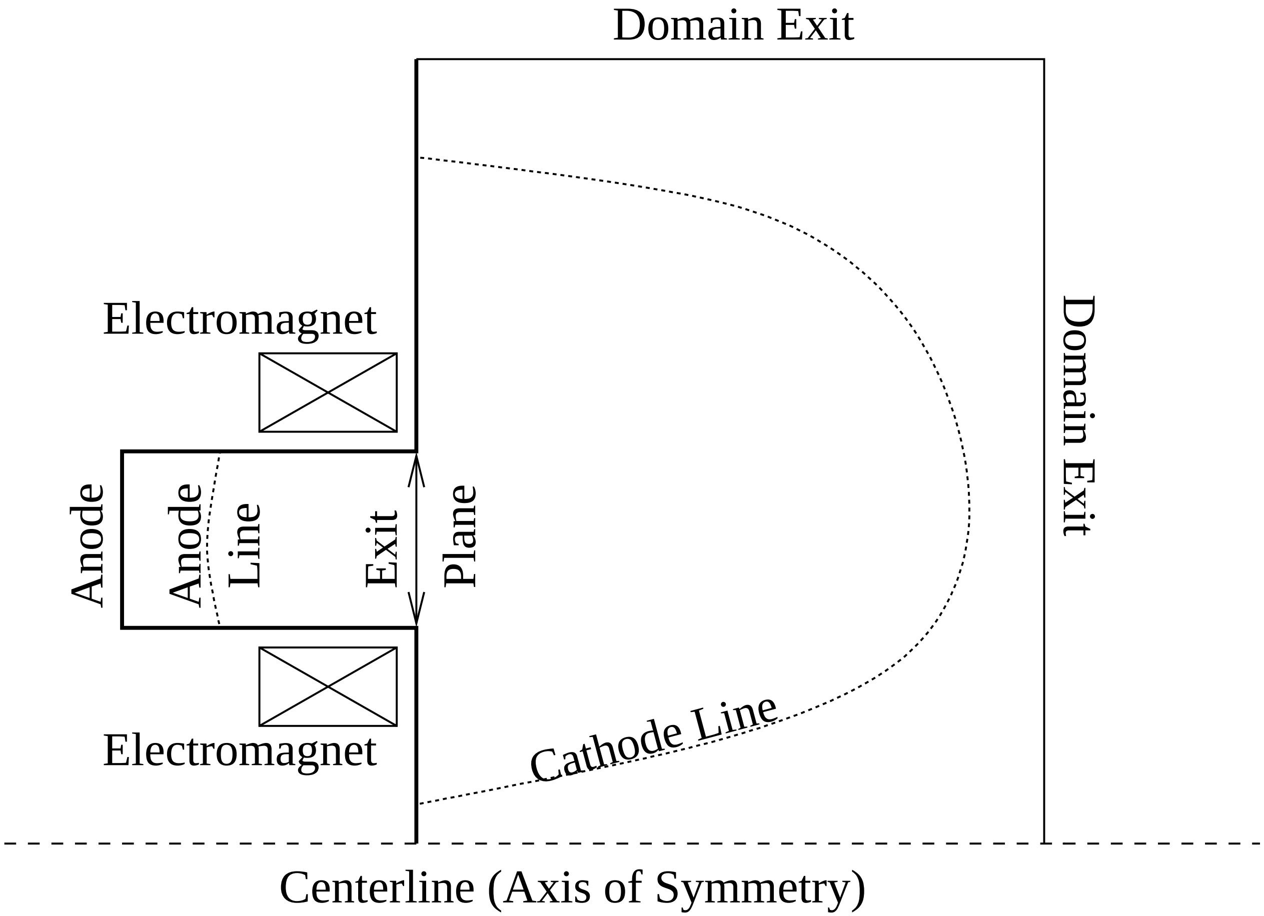
Modeling Benefits

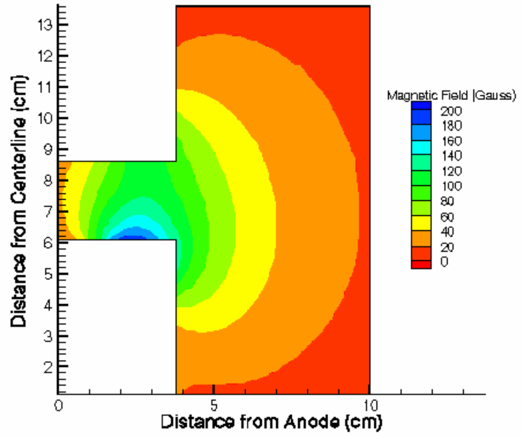
- Many good reasons to develop computational Hall thruster models
  - Spacecraft Integration
    - Existing plume models need better boundary conditions at the device exit
    - Contamination of solar panels and sensitive instruments
  - Quantify chamber effects
    - Vacuum chamber performance of Hall thrusters is affected by finite neutral background density
  - Virtual life tests
    - Thruster lifetimes (>8,000 hours) require erosion modeling to determine lifetime limiting design characteristics
  - Understand physics relevant to thruster operation
    - Experimental measurements inside device are limited by probe dimensions, probe lifetime and other (optical, RF) access issues

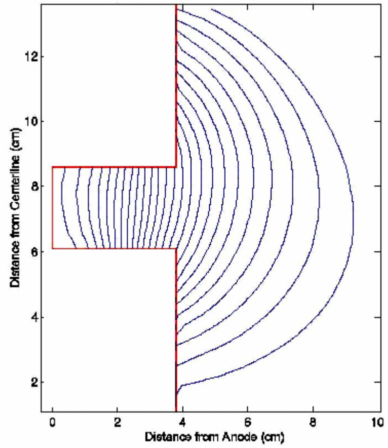
Computational Model

- 2-D axisymmetric hybrid PIC-MCC
- Domain includes acceleration channel and near-field of dielectric wall-type Hall thruster
- Based on a quasi-neutral plasma description
- Heavy particles (Xe, Xe^{+}, Xe^{++}) are treated with a PIC-MCC model
- 1-D energy model assumes isothermal maxwellian electron distribution along magnetic field lines
- Plasma potential based on Ohm's Law formulation
- Anode region potential model based on generalized analytic Bohm criterion

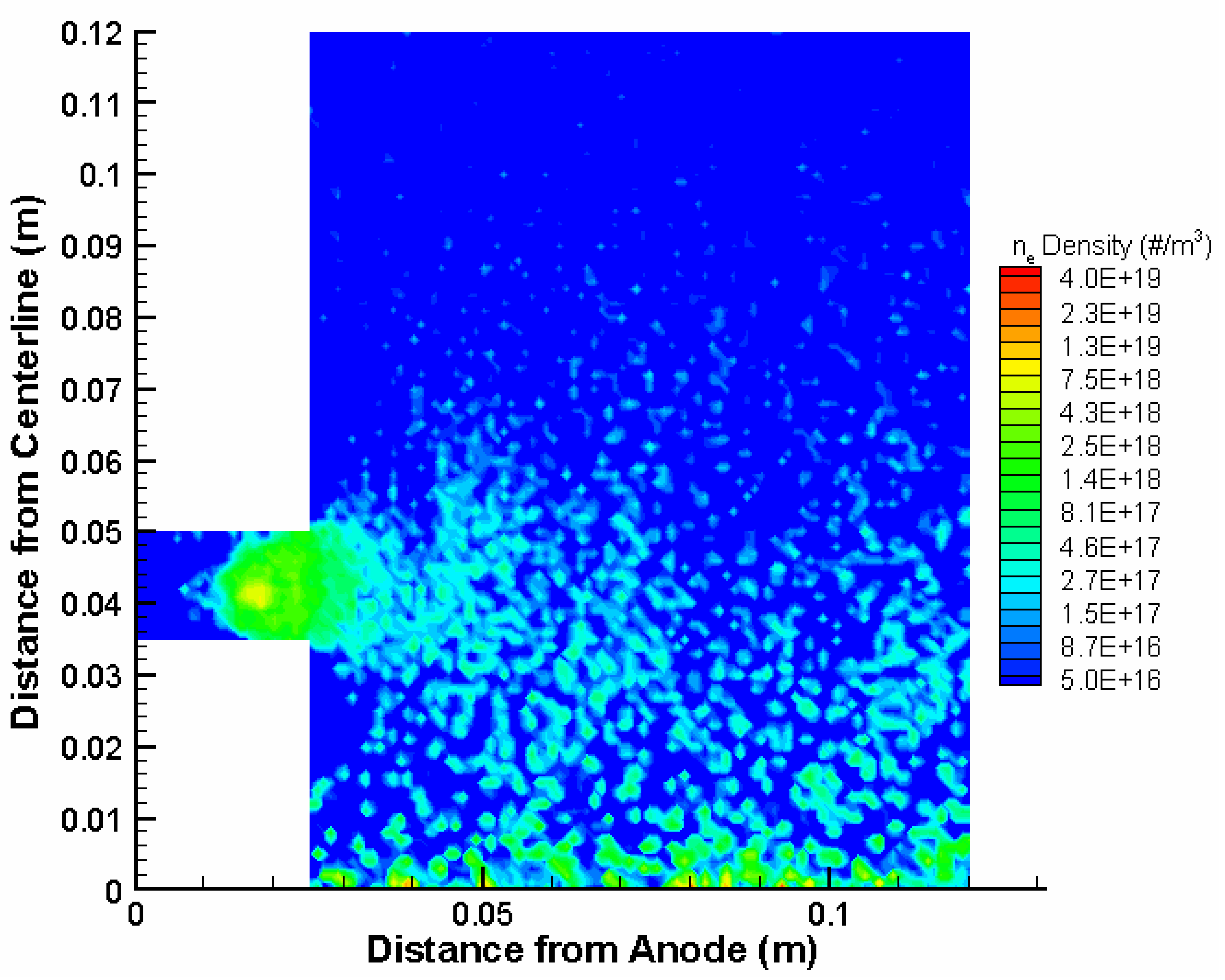
Computational Schematic



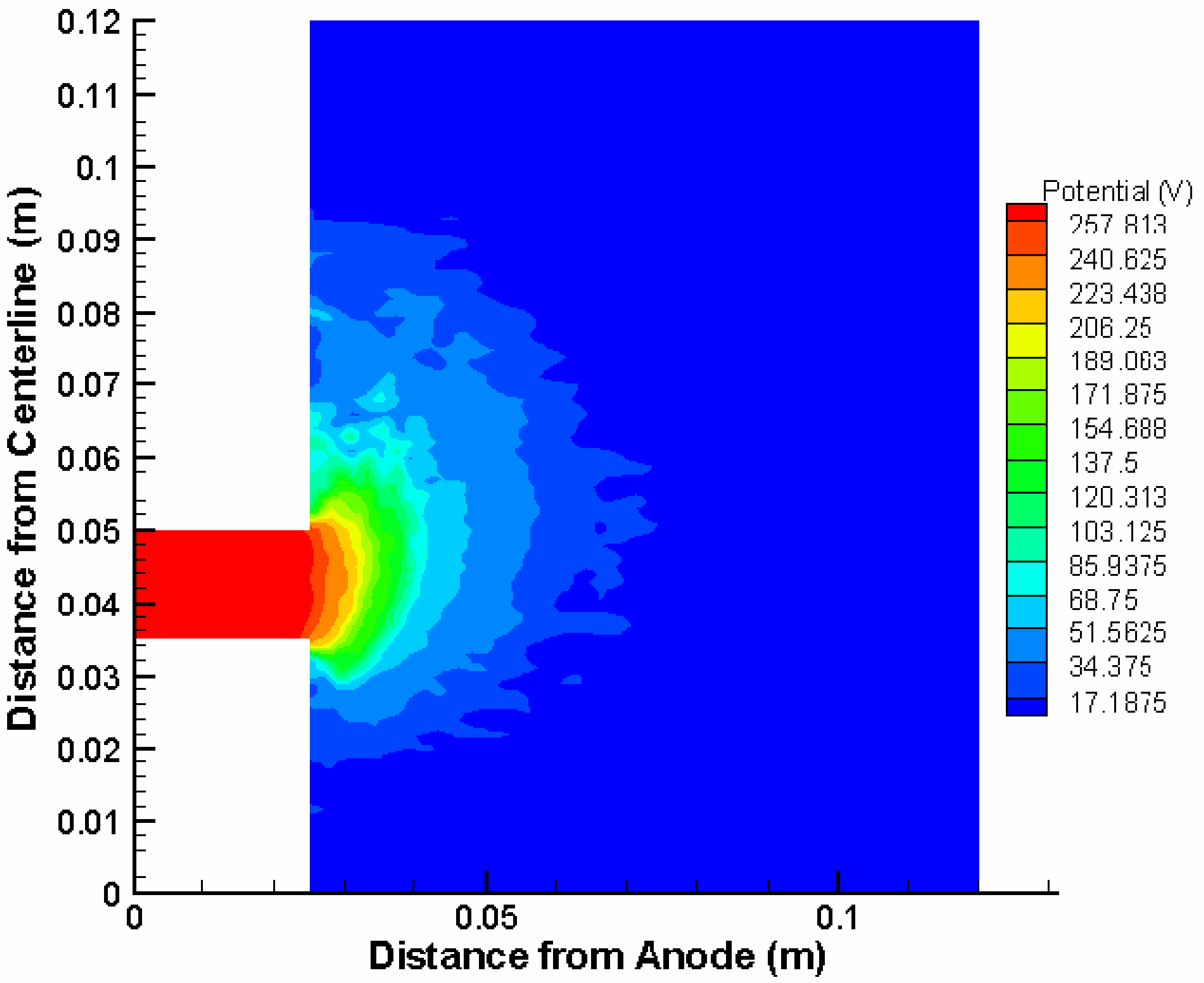




SPT-100 Plasma Density



SPT-100 Potential



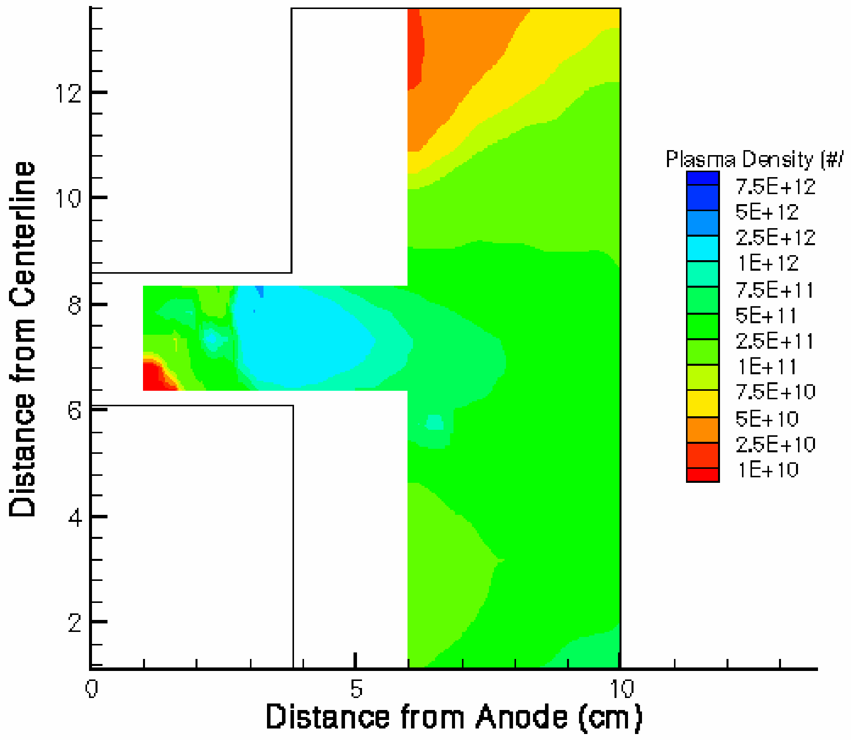
Mobility Modeling

- Calculated from classical transverse electron mobility form:

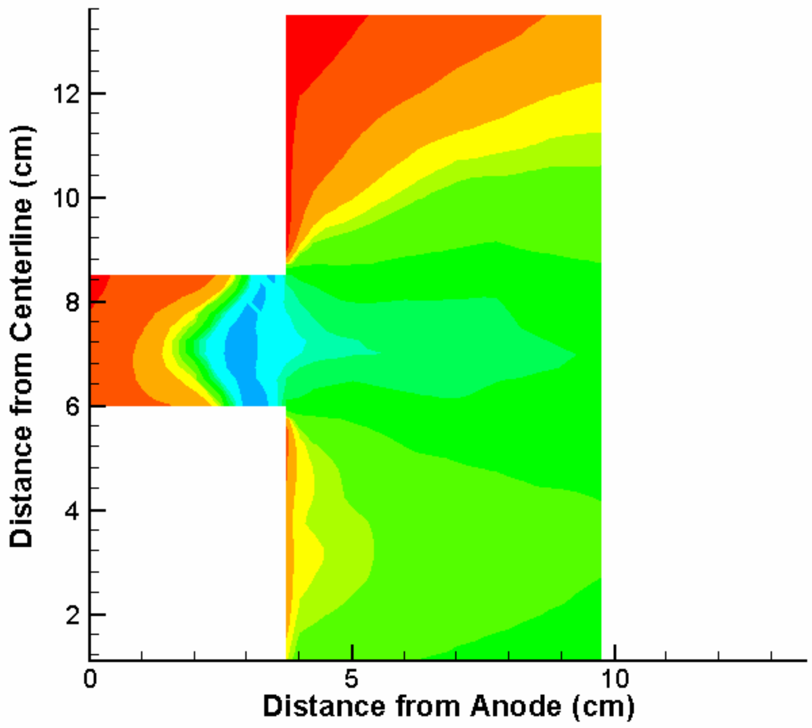
$$\mu_e = \frac{e}{m\nu_{mom}} \frac{1}{1+\left(\frac{\omega_{B,e}}{\nu_{mom}}\right)^2}$$

- Electron momentum transfer frequency is supplemented by a modeled bohm mobility term or wall-collision term

P5 Mean Plasma Density

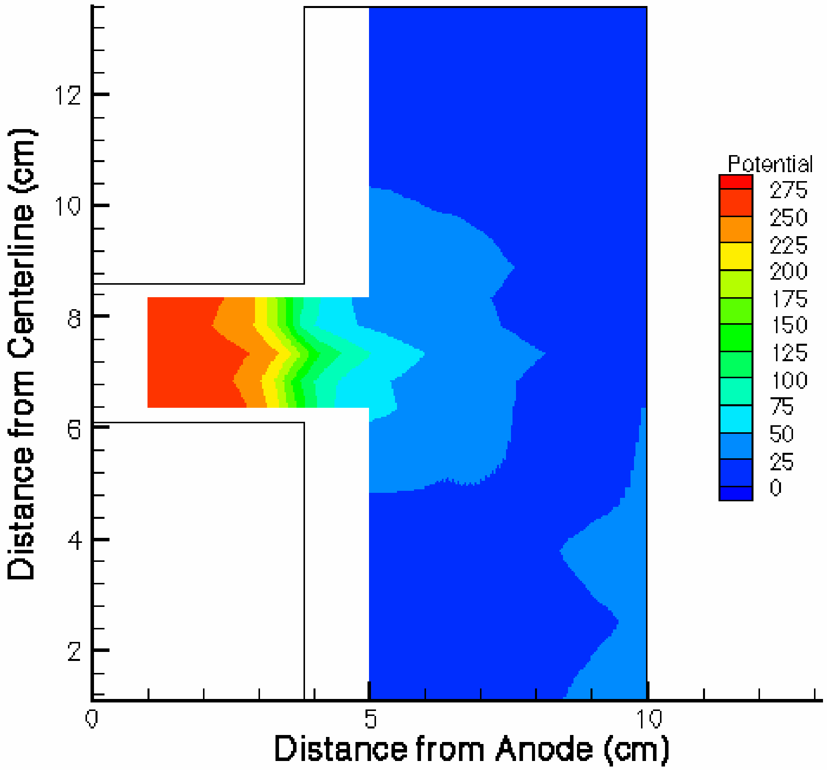


Experimental

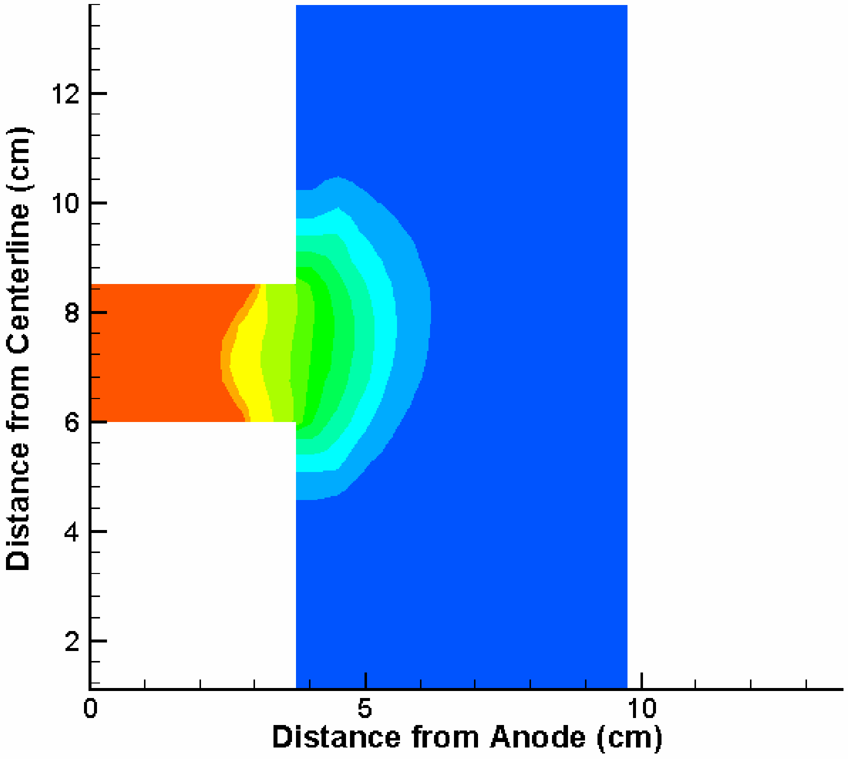


Computational

P5 Mean Potential



Experimental



Computational

Acknowledgements

- Department of Energy Computational Science Graduate Fellowship