Computational Modeling of Hall Thrusters

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Overview

- Motivation
- Hall Thruster Background
- Computational Modeling
- Results
- Acknowledgements
Rocket Equation

- Force balance on a spacecraft

\[ M \frac{dv}{dt} + U_{Exit} \frac{dm}{dt} = F_{\text{gravity}} + F_{\text{drag}} \]

- Neglect gravity and drag forces and integrate

\[ M_{\text{final}} = M_{\text{initial}} \exp \left( \frac{-\Delta V}{U_{Exit}} \right) \]

<table>
<thead>
<tr>
<th>Mission</th>
<th>( \Delta V ) Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth to LEO</td>
<td>7600 m/s</td>
</tr>
<tr>
<td>LEO to GEO</td>
<td>4200 m/s</td>
</tr>
<tr>
<td>LEO Escape</td>
<td>3200 m/s</td>
</tr>
<tr>
<td>LEO to Moon</td>
<td>3900 m/s</td>
</tr>
<tr>
<td>LEO to Mars</td>
<td>5700 m/s</td>
</tr>
</tbody>
</table>

Table from Mechanics and Thermodynamics of Propulsion, 2nd Edition, Peterson and Hill, 1992
Specific Impulse

- Thrust per unit mass propellant (measured in s)

\[ I_{sp} = \frac{\int Tdt}{m_{propellant} g_e} = \frac{U_{exit}}{g_e} \]

- Put this into the rocket equation

\[ M_{final} = M_{initial} \exp\left(\frac{-\Delta V}{I_{sp} g_e}\right) \]
Chemical vs EP

- **Chemical Propulsion**
  - Limits on propellant exit velocity are based on thermodynamic properties of propellant and material properties of combustion systems
  - Typical $I_{sp}$ between 150 s and 450 s
  - High thrust applications (especially Earth to LEO)

- **Electric Propulsion (EP)**
  - Limits on propellant exit velocity are based on power supply mass and lifetime issues
  - For Hall thrusters typically between 1500 s and 2500 s
  - Low thrust applications (LEO to GEO and beyond)
  - Tradeoff between thrust and $I_{sp}$
LEO to Mars

• Suppose $\Delta v = 5700$ m/s is required
• With typical bipropellant chemical propulsion
  – $I_{sp} = 250$ s
  – Ideal Payload Fraction = 9.8%
• With EP (Hall Thruster)
  – $I_{sp} = 1600$ s
  – Ideal Payload Fraction = 69.5%

• Primary present application is to satellite stationkeeping which requires small $\Delta V$ corrections over a number of years with savings of hundreds of pounds on satellites that weigh thousands of pounds
Hall Thruster Performance

- **UM/AFRL P5**: 3 kW, 300 V, 10 A operating condition with a thrust of 180 mN, $I_{sp}$ of 1650 s, and efficiency of 51%
- **NASA-457 M**: >50 kW operating condition with thrust of nearly 3 N, $I_{sp}$ of 1750s - 3250 s, and efficiencies of 46% - 65%
Hall Thruster Schematic

Schematic courtesy of PEPL
Modeling Benefits

• Many good reasons to develop computational Hall thruster models
  – Spacecraft Integration
    • Existing plume models need better boundary conditions at the device exit
    • Contamination of solar panels and sensitive instruments
  – Quantify chamber effects
    • Vacuum chamber performance of Hall thrusters is affected by finite neutral background density
  – Virtual life tests
    • Thruster lifetimes (>8,000 hours) require erosion modeling to determine lifetime limiting design characteristics
  – Understand physics relevant to thruster operation
    • Experimental measurements inside device are limited by probe dimensions, probe lifetime and other (optical, RF) access issues
Computational Model

- 2-D axisymmetric hybrid PIC-MCC
- Domain includes acceleration channel and near-field of dielectric wall-type Hall thruster
- Based on a quasi-neutral plasma description
- Heavy particles (Xe, Xe\(^+\), Xe\(^{++}\) ) are treated with a PIC-MCC model
- 1-D energy model assumes isothermal maxwellian electron distribution along magnetic field lines
- Plasma potential based on Ohm’s Law formulation
- Anode region potential model based on generalized analytic Bohm criterion
Computational Schematic

Domain Exit

Electromagnet

Anode Line

Exit

Plane

Cathode Line

Centerline (Axis of Symmetry)
SPT-100 Plasma Density
Mobility Modeling

- Calculated from classical transverse electron mobility form:

\[ \mu_e = \frac{e}{m v_{mom}} \frac{1}{1 + \left( \frac{\omega_{B,e}}{v_{mom}} \right)^2} \]

- Electron momentum transfer frequency is supplemented by a modeled bohm mobility term or wall-collision term
P5 Mean Plasma Density

Experimental

Computational
P5 Mean Potential

Experimental

Computational
Acknowledgements

- Department of Energy Computational Science Graduate Fellowship