Computational Kinematic Design of Robot Manipulators

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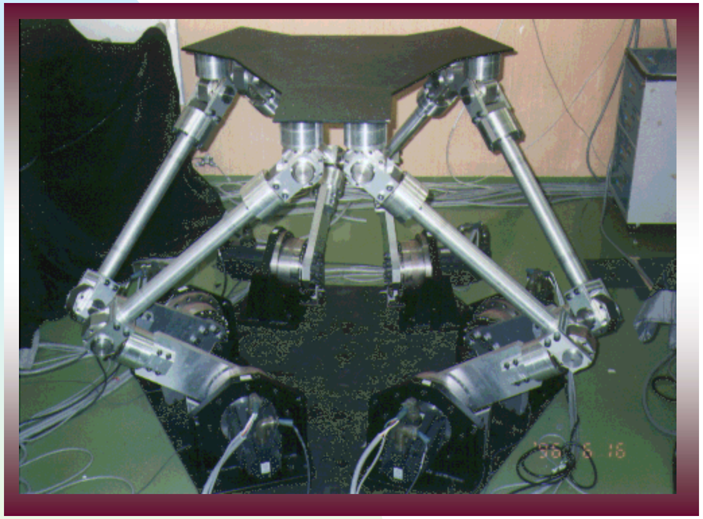
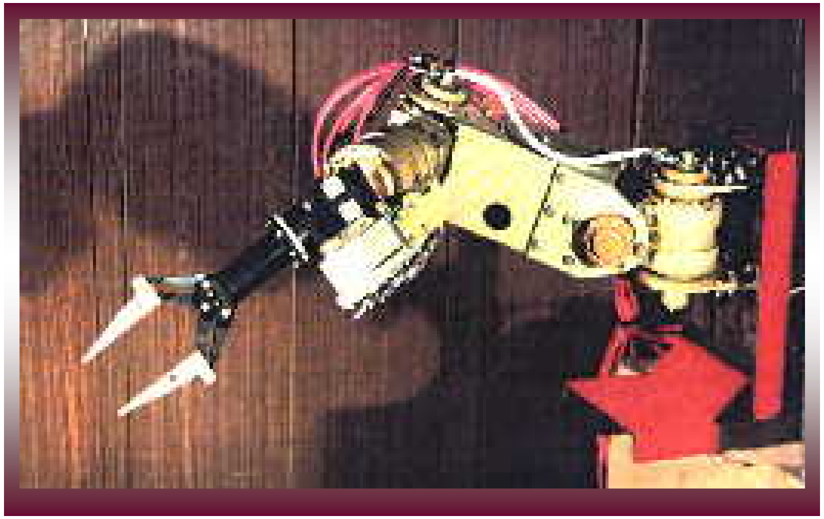
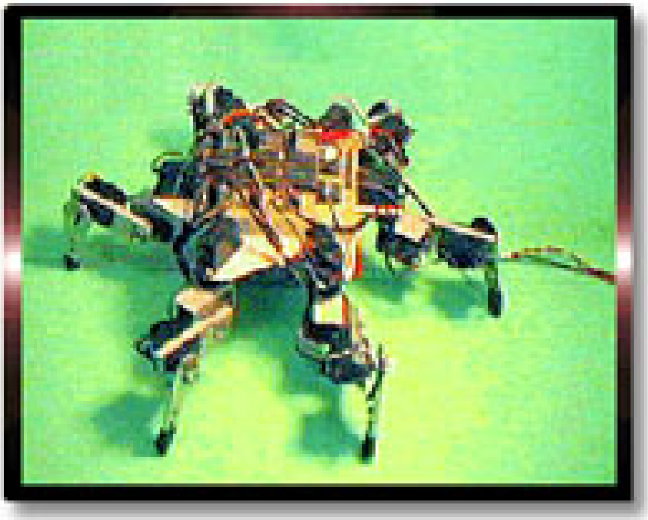
Robot Manipulators

- Manipulators:
  - Mechanical devices composed of Linkages interconnected by actuated and passive Joints
  - To produce desired Motion Properties, generate Force/Torque for Manipulation of objects

Examples of Manipulators

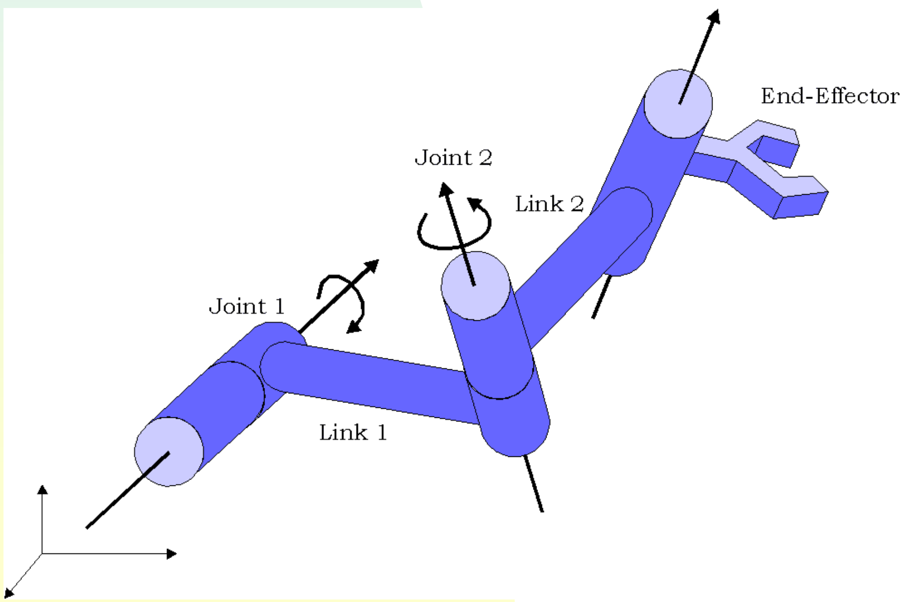




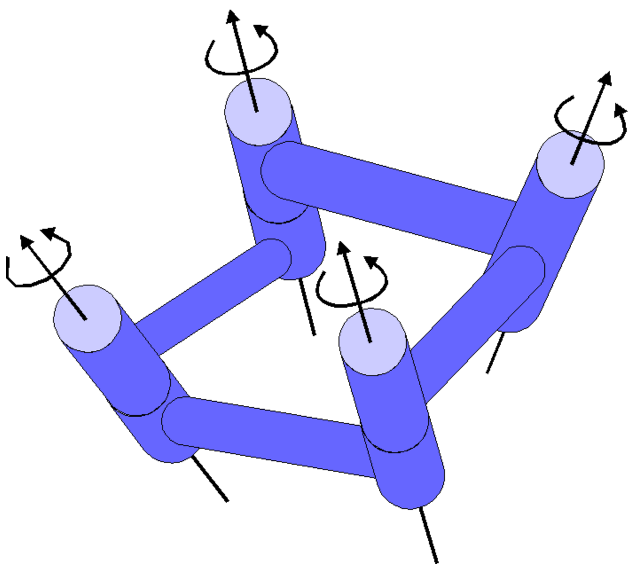


Robot Manipulators Classification

- Classification according to Topology:
- Open Loop Manipulator:



- Closed-Loop Manipulator:



Robot Manipulators Classification

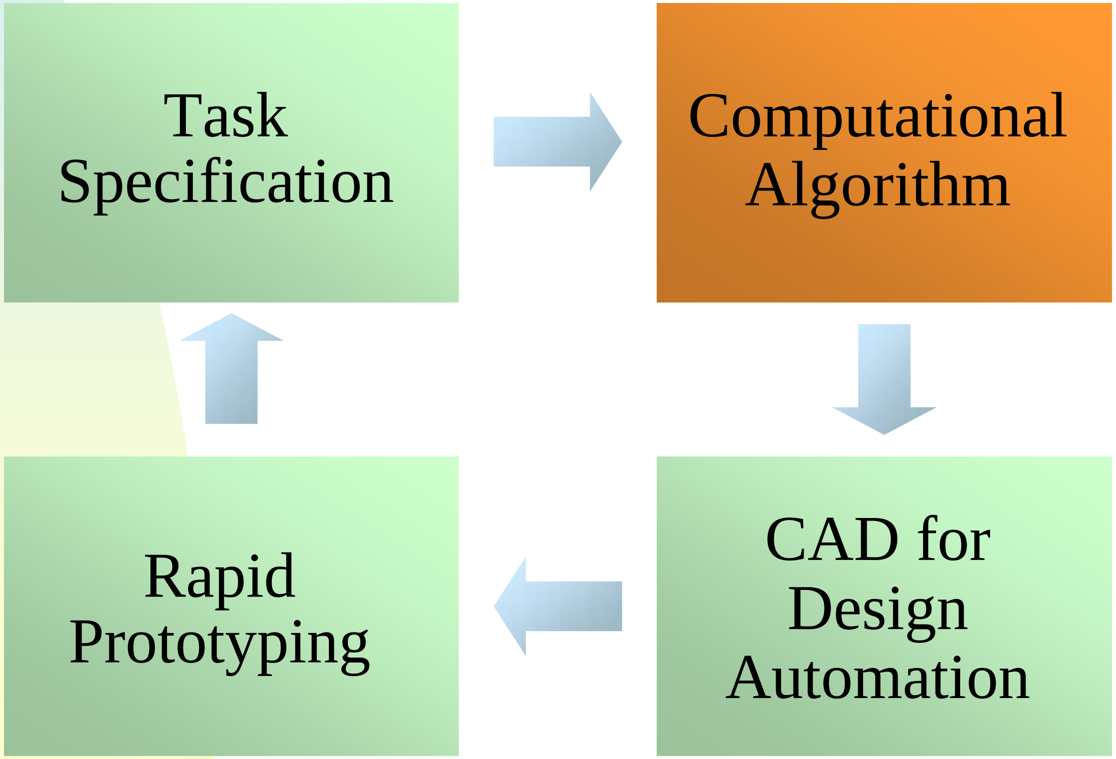
- Classification according to Type of Joints:
  - R=Revolute Joint (Rotation Motion)
  - P=Prismatic Joint (Translation Motion)
  - C=Cylindrical Joint (Both Rotation and Translation)
  - S=Spherical Joint (Ball and Socket Joint, 3 DOF)

Kinematic Design of Manipulators

- Kinematic Design of Manipulators:
  - Design of Robot Manipulators that satisfy certain geometric constrains or properties
  - Examples:
    - Design for Trajectory Following
    - Design for Rigid Body Guidance

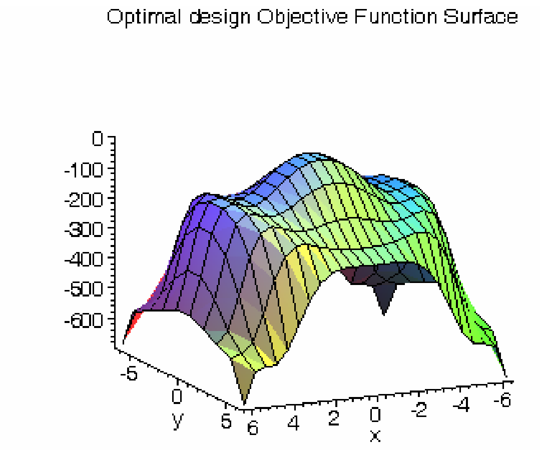
Computational Kinematic Design Procedures

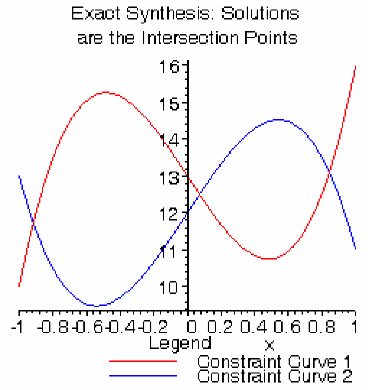
- Given: Specific Task, Performance Indexes
- Find: Manipulator Type, Dimensions



Classification of Kinematic Design Methods

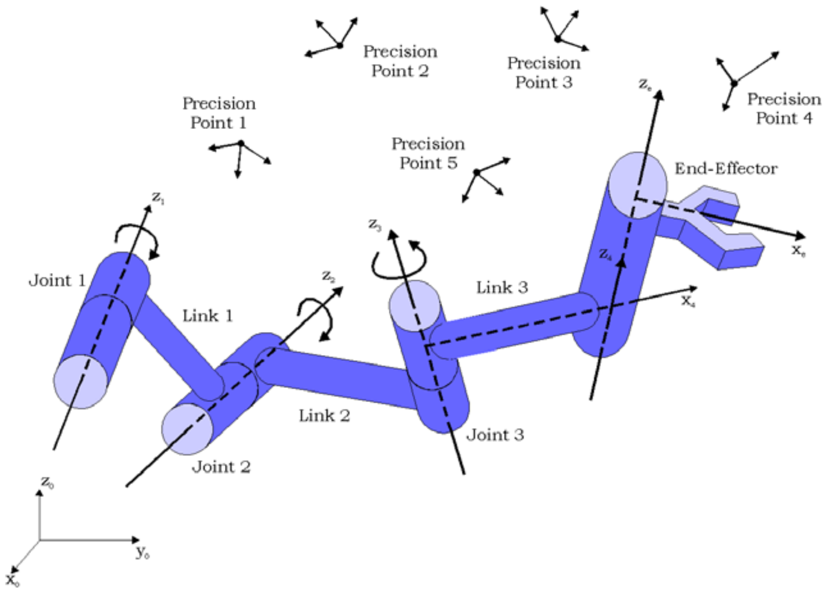
- Optimal (Approximate) Design
  - When Many Solutions Exist
  - When Design Constraints can only be Satisfied Approximately
  - e.g. Workspace Optimization
- Exact (Analytical) Synthesis
  - Constraints are Satisfied Exactly
  - e.g. Rigid Body Guidance





Rigid Body Guidance of Spatial Manipulators

- Given:
  - n Precision Points (Position & Orientation)
  - Type of Manipulator
- Find:
  - All Manipulators which End-Effector could reach all n Precision Points Exactly
  - Compute Geometric Parameters



Rigid Body Guidance

- Result in Algebraic (Polynomial) Kinematic Constraints Equations:
  - Each Equation represent a Hypersurface
  - Solutions are the Intersecting Points of Hypersurfaces
  - Multiple Solutions Exist
- Advantages:
  - High Precision
  - High Repeatability

Possible Applications

- Special Purpose Industrial Manipulators:
  - For Repetitive Tasks
- Vehicle Components
  - Landing Gear Mechanisms
  - Transmission Mechanisms
- Deployable Structures
  - Civil Engineering / Space Applications

Computational Methods in Rigid Body Guidance

- Algebraic Methods
  - Technique from Computational Algebraic Geometry and Commutative Algebra
  - Reduce n by n Polynomial System to 1 Polynomial in 1 Unknown
  - e.g. Resultants, Grobner Basis
- Direct Numerical Methods
  - Compute Numerical Solutions Directly from Polynomial System
  - e.g. Polynomial (Homotopy) Continuation Method, Interval Arithmetic

Previous Work in Spatial Design

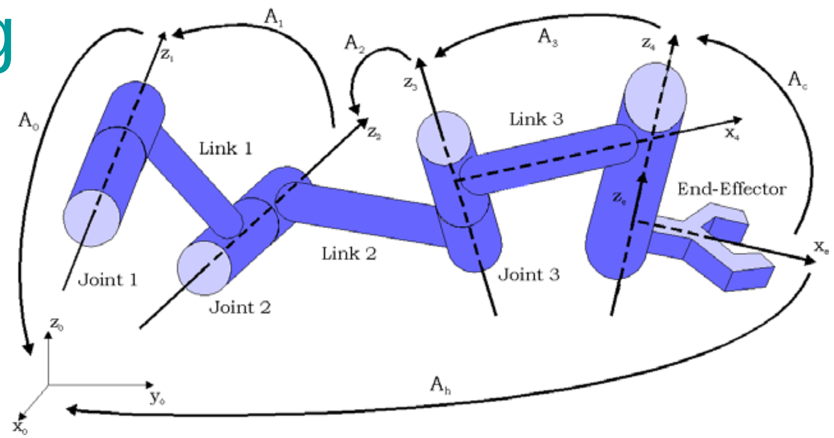
- Algebraic Method:
  - 2R: Tsai & Roth (1972); Mavroidis, Alam & Lee (1999); McCarthy (1999)
  - 2C: Roth (1967); Murray & McCarthy (1999); Huang & Chang (2000)
  - S-S Binary Link: Innocenti (1994)
  - Slider-Slider Sphere Dyad, Cylinder Cylinder Binary Link: Neilsen and Roth (1995)
- Continuation Method:
  - 3R 3 Precision Points: Lee & Mavroidis (2002)

Design Problems Solved in Rutgers Robotic Lab

- 2R 3 Positions (1999): Resultant, 2 Real Solutions
- 3R 3 Positions (2001): Continuation Method, 8 Solutions
- 3R 4 Positions (2002): Continuation Method, 36 Solutions
- 3R 5 Positions (2002): Interval Arithmetic, Partially solved
- PRR 4 Positions (2002): Resultant, 12 Solutions

Design Equations (General Formulation)

- Use Denavit & Hartenberg Parameters and Transformation Matrices:
  - A_i & A_c : Link Transformation



$$\mathbf{A_i} = \begin{pmatrix} c_i & -s_i c_{\alpha_i} & s_i s_{\alpha_i} & a_i c_i \\ s_i & c_i c_{\alpha_i} & -c_i s_{\alpha_i} & a_i s_i \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{A_c} = \begin{pmatrix} c_\psi & -s_\psi & 0 & 0 \\ s_\psi & c_\psi & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{A_h} = \begin{pmatrix} l_1 & m_1 & n_1 & x_d \\ l_2 & m_2 & n_2 & y_d \\ l_3 & m_3 & n_3 & z_d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

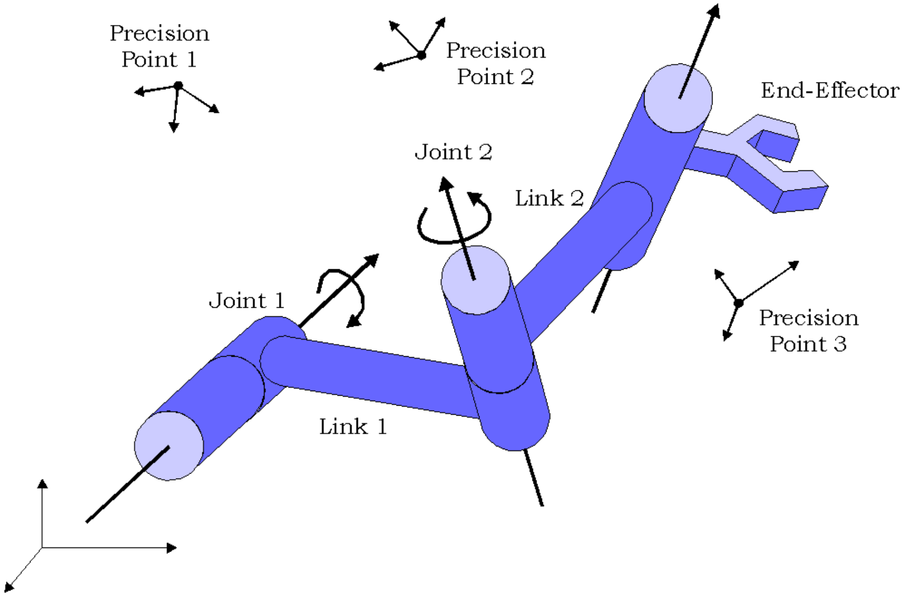
- Loop Closure (Matrix) Equation with n Links

$$\mathbf{A}_0 \mathbf{A}_1 \cdots \mathbf{A}_n \mathbf{A_c} = \mathbf{A_h}$$

Example One: 2R Design with Algebraic Method (Resultant)…

2R Design with Algebraic Method

- Robot Geometry:
  - 2 Revolute Joints
  - 3 Precision Points



- Loop Closure (4x4 Matrix) Equation

$$\mathbf{A}_0\mathbf{A}_1 = \mathbf{A}_\mathbf{h}\mathbf{A}_\mathbf{c}^{-1}\mathbf{A}_2^{-1}$$

Design Equations

- Scalar Design Equations

$$
\begin{aligned}
c_0 s_1 s_{\alpha_1} + s_0 c_{\alpha_0} c_1 s_{\alpha_1} + s_0 s_{\alpha_0} c_{\alpha_1} &= s_{\alpha_2} p_1 + c_{\alpha_2} n_1 && (1)\\
s_0 s_1 s_{\alpha_1} - c_0 c_{\alpha_0} c_1 s_{\alpha_1} - c_0 s_{\alpha_0} c_{\alpha_1} &= s_{\alpha_2} p_2 + c_{\alpha_2} n_2 && (2)\\
- s_{\alpha_0} c_1 s_{\alpha_1} + c_{\alpha_0} c_{\alpha_1} &= s_{\alpha_2} p_3 + c_{\alpha_2} n_3 && (3)\\
a_1 c_1 c_0 - a_1 s_1 s_0 c_{\alpha_0} + d_1 s_0 s_{\alpha_0} + a_0 c_0 &= -a_2 q_1 - d_2 s_{\alpha_2} p_1 - d_2 c_{\alpha_2} n_1 - n_1 d + x_d && (4)\\
a_1 c_1 s_0 + a_1 s_1 c_0 c_{\alpha_0} - d_1 c_0 s_{\alpha_0} + a_0 s_0 &= -a_2 q_2 - d_2 s_{\alpha_2} p_2 - d_2 c_{\alpha_2} n_2 - n_2 d + y_d && (5)\\
a_1 s_1 s_{\alpha_0} + d_1 c_{\alpha_0} + d_0 &= -a_2 q_3 - d_2 s_{\alpha_2} p_3 - d_2 c_{\alpha_2} n_3 - n_3 d + z_d && (6)
\end{aligned}
$$

$$p_i = l_i s_\psi + m_i c_\psi \qquad q_i = l_i c_\psi - m_i s_\psi \qquad i = 1, 2, 3$$

- 18 Nonlinear Equations in 18 Unknowns
- Elimination using Resultant:
  - Final Polynomial in 1 Unknown

$$k_6 t_0^6 + k_5 t_0^5 + k_4 t_0^4 + k_3 t_0^3 + k_2 t_0^2 + k_1 t_0 + k_0 = 0$$

Numerical Example

- End-Effector Configurations:

$$A_{h1} = \begin{pmatrix} 0.65623 & -0.11296 & 0.74605 & 162.03673 \\ -0.21885 & -0.97472 & 0.04491 & 82.18408 \\ 0.72212 & -0.19275 & -0.66437 & 48.74290 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A_{h2} = \begin{pmatrix} 0.14391 & 0.92897 & 0.34102 & 72.96453 \\ -0.98949 & 0.13986 & 0.03655 & 42.77492 \\ -0.01373 & -0.34270 & 0.93934 & 113.39977 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{h3} = \begin{pmatrix} 0.76457 & 0.64084 & -0.06892 & 43.27298 \\ -0.47704 & 0.49073 & -0.72911 & 16.14093 \\ -0.43341 & 0.59034 & 0.68091 & 88.68218 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Final Polynomial:

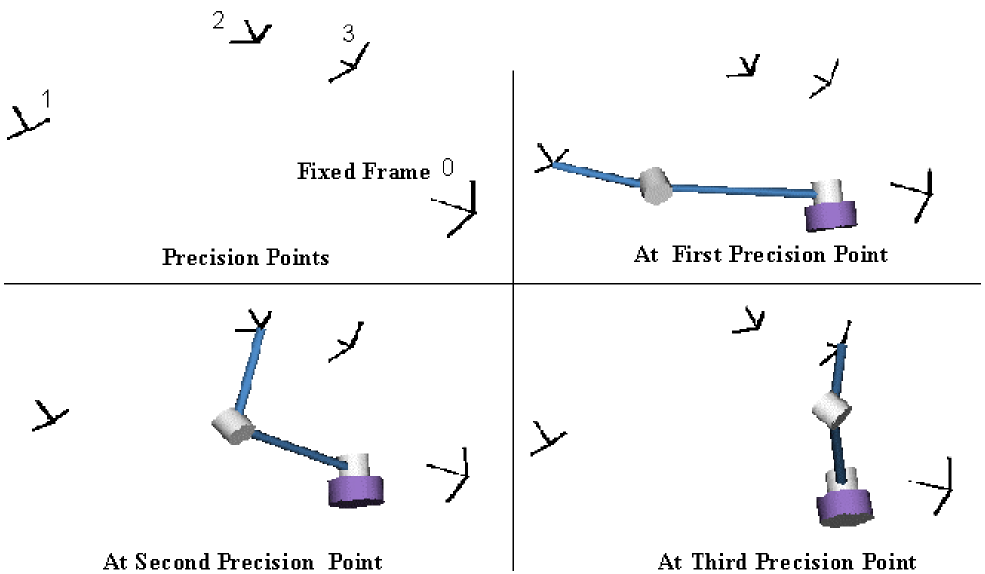
$$-43215.44589t_0^6 + 48238.55854t_0^5 - 2559.15192t_0^4 + 50727.96989t_0^3 - 114681.08559t_0^2 + 75359.73016t_0 - 16254.11701 = 0$$

- Solutions for t_0, 6 Solutions, 2 Real & 4 Complex:

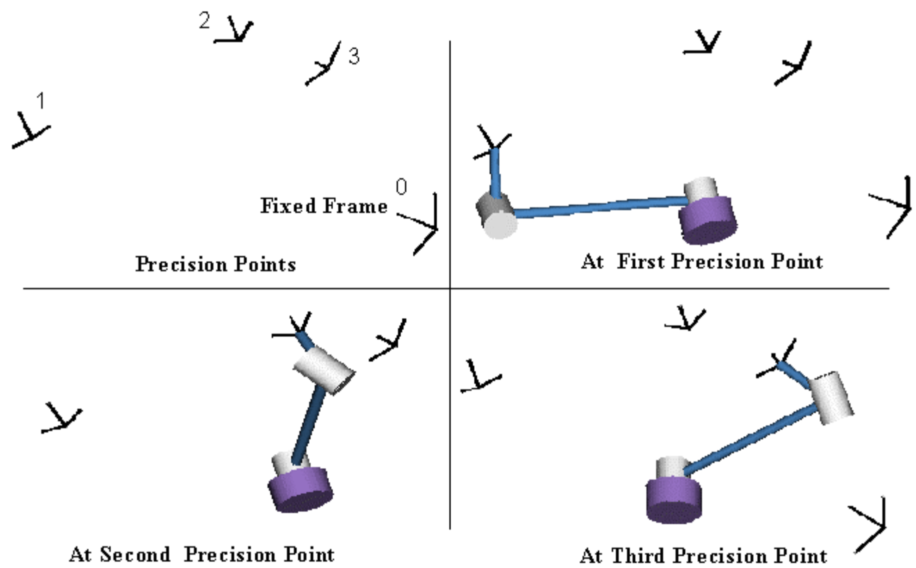
$$\begin{array}{lll} -0.80246 - 0.98242\,\mathrm{I}, & 0.66386 - 0.23534\,\mathrm{I}, & 0.57735, \\ -0.80246 + 0.98242\,\mathrm{I}, & 0.66386 + 0.23534\,\mathrm{I}, & 0.816079 \end{array}$$

Computer Aided Visualization

- The Two Real Solutions:
- Solutions 1:



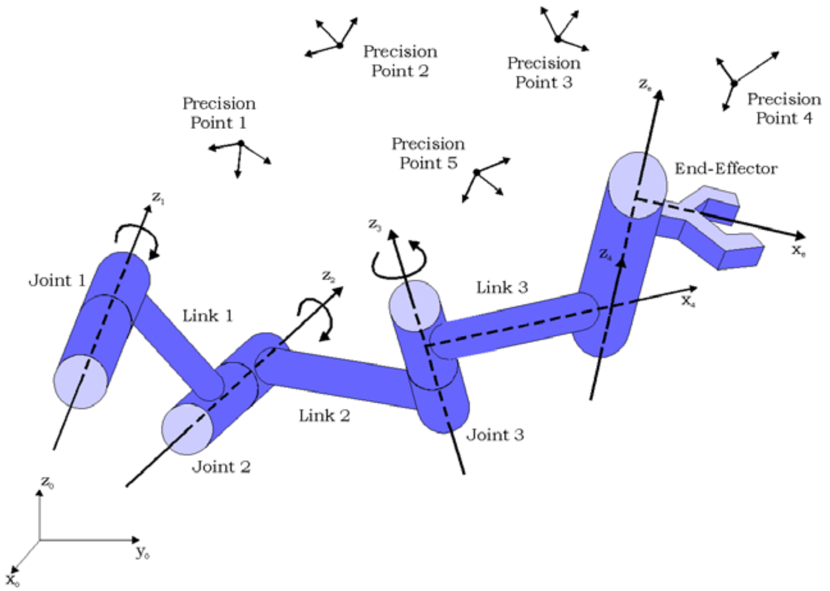
- Solutions 2:



Example 2: 3R 3pp Design with Continuation Method…

3R 3pp Design with Continuation Method

- Given:
  - 3 Precision Points
- Find:
  - All Manipulators which End-Effector could reach all 3 Points
- Loop Closure Matrix Equation:



$$\mathbf{A}_1\mathbf{A}_2 = \mathbf{A}_0^{-1}\mathbf{A}_\mathbf{h}\mathbf{A}_\mathbf{c}^{-1}\mathbf{A}_3^{-1}$$

Design Equations

- Simplification by Algebraic Manipulations → 10 Equations in 10 Unknowns

$$\sum_{X_j, X_k \in W} f_{i,X_j,X_k}(\alpha_1) X_j X_k = 0 \quad i = 1,2,3$$

$$\sum_{X_j, X_k \in W} g_{i,X_j,X_k}(\alpha_1) X_j X_k = 0 \quad i = 1,2,3$$

$$\sum_{X_j, X_k \in W} h_{i,X_j,X_k}(\alpha_1) X_j X_k = 0 \quad i = 1,2,3$$

$$c\alpha_1^2 + s\alpha_1^2 - 1 = 0 \qquad W = \{\mathrm{F, G, H, S, P, Q, R, d_2, 1}\}$$

- Solution Method: Continuation Method
- Software: PHC (Verschelde, 1996)
- 8 Solutions satisfy the Design Constraints

Numerical Results

- End-Effector Configurations:

$$A_{h1} = \begin{pmatrix} 0 & 0 & 1 & 6 \\ 0 & -1 & 0 & 6 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

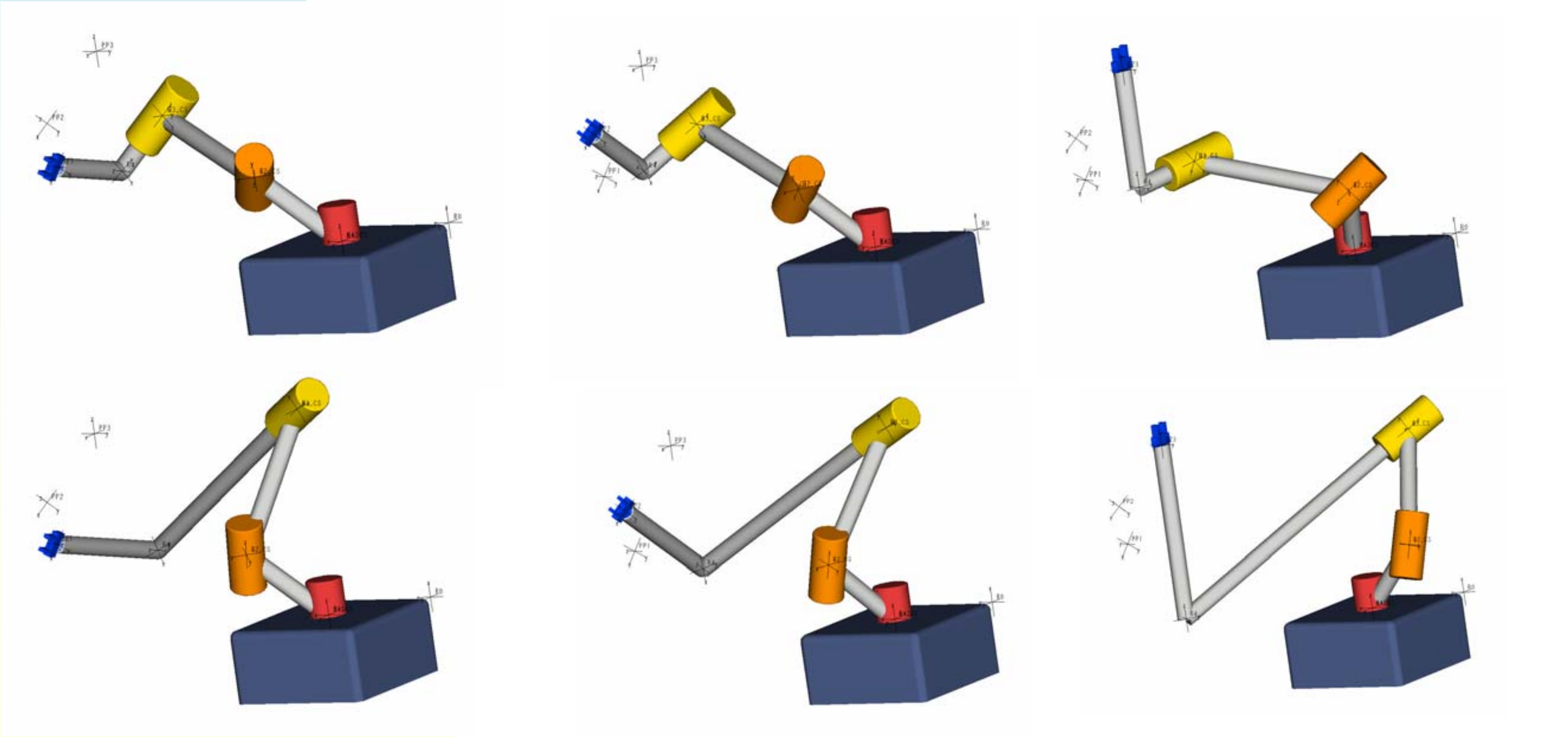
$$A_{h2} = \begin{pmatrix} -0.707107 & -0.5 & 0.5 & 4 \\ 0.5 & -0.853553 & -0.146447 & 8.24264 \\ 0.5 & 0.146447 & 0.853553 & 5.41421 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{h3} = \begin{pmatrix} -0.866025 & -0.433013 & 0.25 & 2.63397 \\ 0.433013 & -0.899519 & -0.0580127 & 8.78109 \\ 0.25 & 0.0580127 & 0.966506 & 4.45096 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- 8 Solutions: 4 Real & 4 Complex

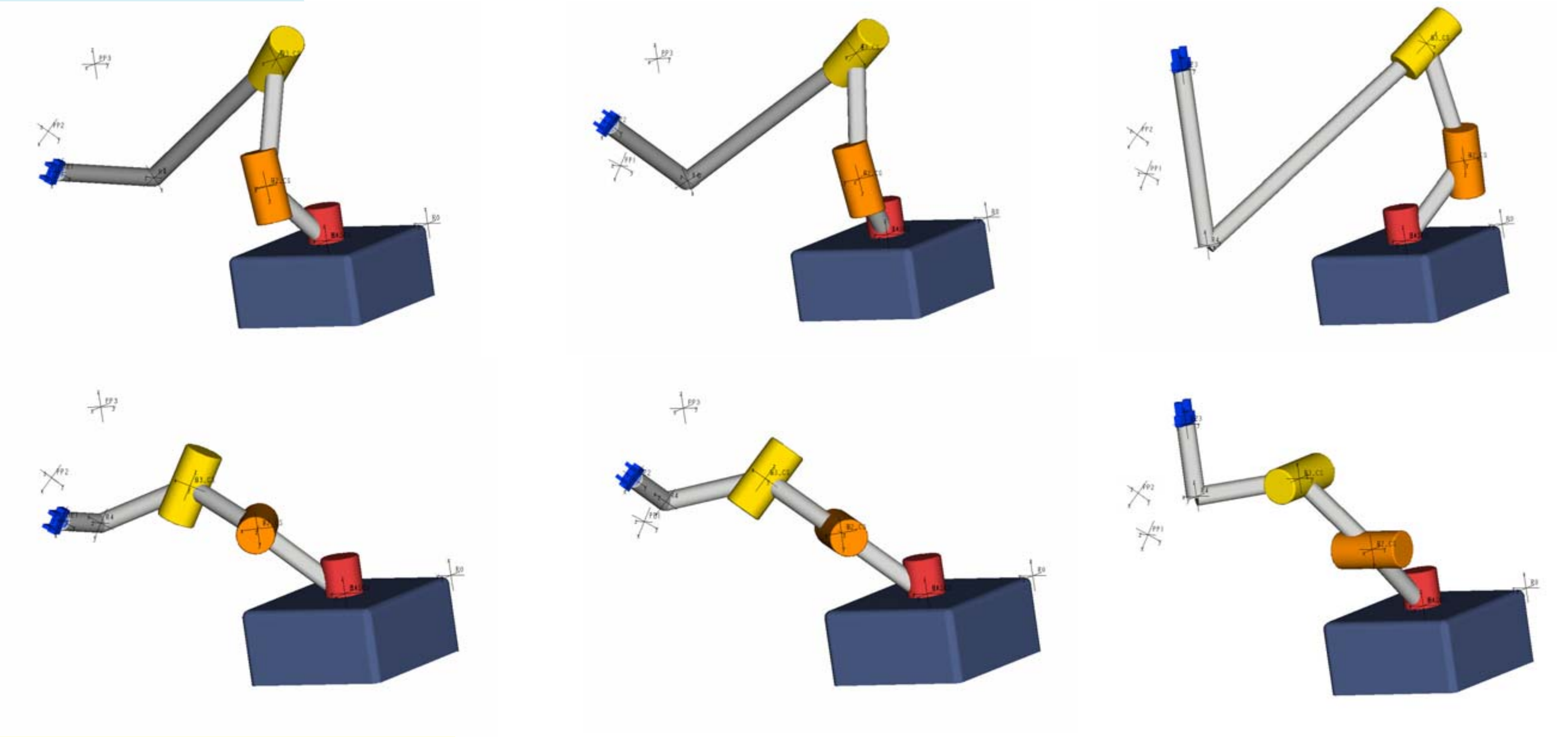
Computer Aided Visualization (I)

- (Real) Solution 1 and 2 at Three Precision Points:



Computer Aided Visualization (II)

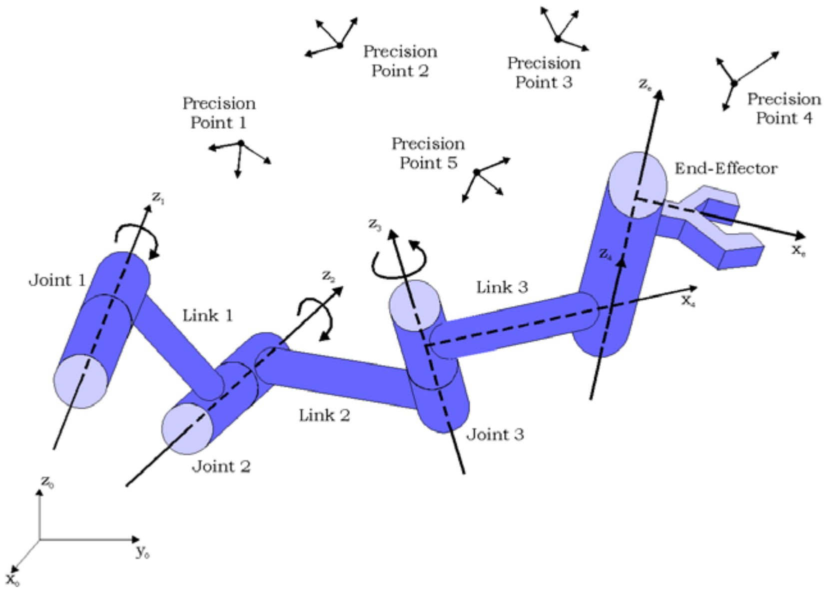
- (Real) Solution 3 and 4 at Three Precision Points:



Example 3: 3R 5pp Design with Interval Analysis…

3R 5pp Design with Interval Analysis

- Given:
  - 5 Precision Points
- Find:
  - All Manipulators which End-Effector could reach all 5 Points



- Solution Method: Interval Analysis
  - Search for all Real Solutions within a Predefined Bounded Region in R^n

Numerical Results

- Implementation:
  - C++ Interval Analysis Library ALIAS
  - PVM on Cluster of PC's (5 days on 26 PC's)
- End-Effector Configurations:

$$\mathbf{A}_{h1} = \begin{pmatrix} -.6396094375 & .1435961208 & 0.755168803 & 8.310644971 \\ -.6265434807 & .4717800207 & -.6203764008 & -1.993959918 \\ -.4453571983 & -.869944691 & -.2117857403 & 4.525646630 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{A}_{h2} = \begin{pmatrix} .4273095207 & -.3048426696 & .8511624523 & 8.462432080 \\ .7180580935 & -.4576191690 & -.5243827518 & 3.909344844 \\ .5493624920 & .8352578302 & .02334971838 & 3.781393231 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_{h3} = \begin{pmatrix} .2085023533 & .2490486651 & .9457809106 & 8.213357066 \\ -.4704189878 & -.8222864878 & .3202357065 & 4.720930002 \\ .8574571385 & -.5116831972 & -.0542914469 & 1.906020548 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{A}_{h4} = \begin{pmatrix} -.2651650429 & .5540136722 & .7891491309 & 6.610088080 \\ -.8775374786 & .2004602816 & -.4355957403 & -.9786178219 \\ -.3995190528 & -.8080127018 & .4330127018 & 7.933012701 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

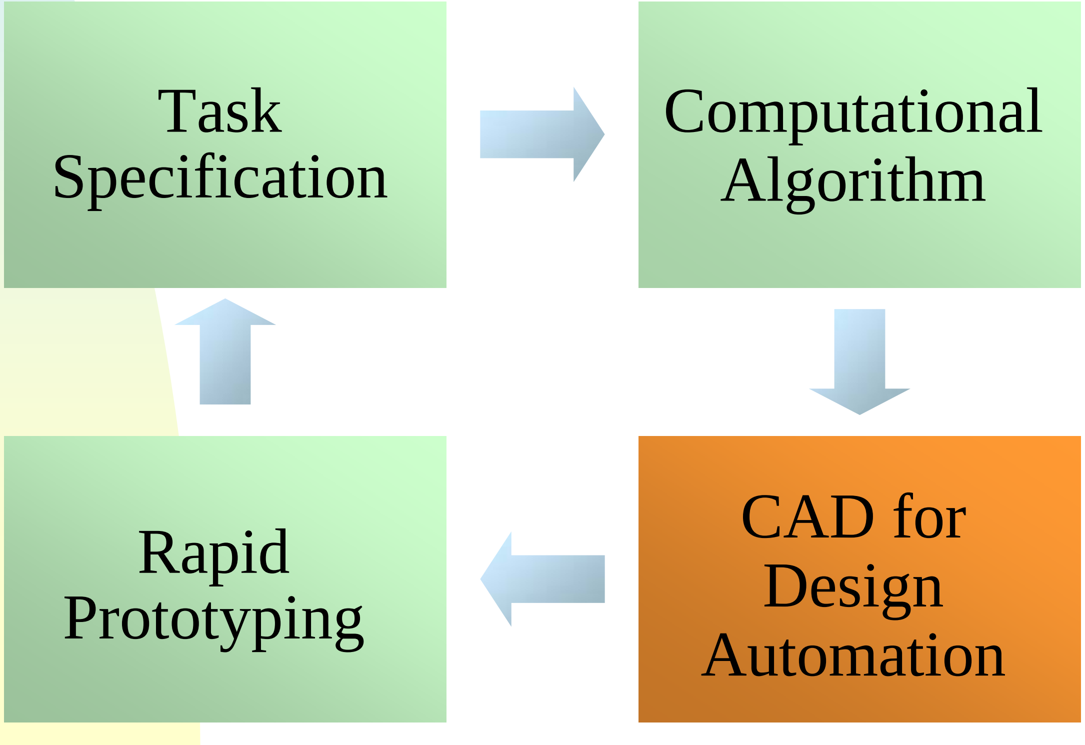
$$\mathbf{A}_{h5} = \begin{pmatrix} -.5451561411 & -.5421432835 & .6394415077 & 7.498628082 \\ -.2838098567 & -.5983617170 & -.7492764648 & -2.362107226 \\ .7888325214 & -.5899524689 & .1723349570 & -.5803329915 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Results: 13 (Real) Solutions

Future Work…

Future Work

- Develop Algorithms for Other Open and Closed Loop Manipulator Design Problems
- Manipulator Design Automation with CAD



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