An Adaptive 3D Cartesian Approach for the Parallel Computation of Inviscid Flow About Static and Dynamic Configurations

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Motivation & Background

- **Computational Fluid Dynamics (CFD)**
  - Becoming a mature field
  - Problems involving moving or deforming objects remain a challenge
  - Unstructured grid approach
    - Grid shearing in regions of large relative motion
    - Grid generation is sensitive to body definition
  - Chimera approach
    - Data interpolation is difficult in regions between close objects
    - Body-fitted grids are sensitive to body definition
Approach Foundation

- Cartesian grid approach
  - Independent to body definition
  - Very little user interaction
- Approach requirements
  - Inviscid compressible flow
  - Three-dimensional Cartesian framework
  - Arbitrary geometric configurations
  - Adaptive mesh refinement
  - Parallel computation
- Inviscid compressible flow
  - Euler Equations
  \[
  \frac{\partial}{\partial t} \int_{\Omega} U \, d\Omega + \int_{S} \mathbf{F}_r \cdot \mathbf{n} \, dS = 0
  \]
  \[
  U = (\rho, \rho u, \rho v, \rho w, \rho E)^T
  \]
  \[
  \mathbf{F}_r = \begin{pmatrix}
  \rho(v - v_s) \\
  \rho u(v - v_s) + p \hat{i} \\
  \rho v(v - v_s) + p \hat{j} \\
  \rho w(v - v_s) + p \hat{k} \\
  (\rho E + p)(v - v_s) + pv_s
  \end{pmatrix}
  \]
  - Equation of state
  \[
  p = \rho e(\gamma - 1)
  \]
Three-dimensional Cartesian framework
- A spatial region is represented by a block of $N \times N \times N$ cells
- The cells constitute a structured Cartesian grid
- A finite-volume flow solver is applied to each cell
  - The MUSCL approach is used to achieve higher order spatial accuracy
  - Two-stage Runge-Kutta time integration utilized
  - Fluxes obtained via Roe’s approximate Riemann solver
• Arbitrary Geometric Configurations
  – Configuration components are defined by closed triangulated surfaces
  – Cells must be identified as one of three types: flow; solid; or intersected
  – Intersected cells must be cut
    • Cart3D – Aftosmis, NASA Ames
  – Introduced issues
    • Prohibitively small time steps may result from cut-cells with a very small volume
    • Split cells can also be produced
• Adaptive Mesh Refinement
  – Utilizes a block-octtree data structure
    • A block of cells is stored in each node of an octtree
    • Blocks deeper in the tree represent smaller sub-regions within the domain
  – Flow-based adaptation
    • Adapts to flow features such as shock and expansion waves
  – Geometry-based adaptation
    • Adapts to geometric features such as local surface curvature
Parallel Computation
- PARAMESH – MacNeice, NASA Goddard
- Blocks are distributed across processors to balance work
- Each processor maintains a copy of the geometric configuration
- Layers of ghost cells are used to facilitate processing each structured block independently
Dynamic Configurations

- Component motion
  - Components can be moved independently
  - Motion restricted to prescribed rigid-body motion
- Necessary considerations
  - Runge-Kutta formulation permits a varying control volume
  - Cell geometry needed at three instances during an update
  - Time step calculation and flux computation must now include facial velocities

\[
\text{Res}(\bar{U}V) \equiv - \sum_{i=1}^{n\text{Faces}} \Phi_i A_i
\]

\[
(\bar{U}V)^{(1)} = (\bar{U}V)^{(0)} + \frac{\Delta t}{2} \text{Res} \left( (\bar{U}V)^{(0)} \right)
\]

\[
(\bar{U}V)^{(2)} = (\bar{U}V)^{(0)} + \Delta t \text{Res} \left( (\bar{U}V)^{(1)} \right)
\]
Dynamic Configurations

- Topologic Transformations
  - Cell volumes can appear or disappear during a solution update
  - Results from a cell transforming from solid to cut or vice versa
  - The formulation of the time step calculation does not admit transformations between flow and solid cells
  - Runge-Kutta time integration can not tolerate this
Cell Merging

- Motivation for cell merging
  - Prohibitively small time steps
  - Cell-type transformations during a time step
- Concept of cell merging
  - Multiple simply connected cells are grouped together to avoid flow solving issues
  - Each group of cells is treated as an individual composite cell during a solution update
  - At the end of the update each member cell receives its appropriate portion of the updated solution
Cell Merging

- Implementation requirements
  - Time complexity similar to that of a solution update
  - Accommodate adaptive mesh refinement
  - Parallelizable

- Cell-merging algorithm core
  - Identify problematic cells
  - Generate and score valid merging choices for each problem cell
  - Choose a merged-cell cover
Cell Merging

- Choosing a merged-cell cover
  - A merged-cell cover is a set of merged-cells that satisfies the following conditions:
    - Every problem cell is part of a merged cell or is covered
    - No merged cells overlap
  - For each problem cell within the considered region, choose the best choice that does not introduce an overlap
  - If all the problem cells were not covered, make a second pass and try to choose the best choice that covers an uncovered cell while not introducing an overlap

- Parallelization
  - Cell merge each block independently
  - Cell merge within larger regions, as necessary, by traversing back through the octtree
  - As a last resort merge the grid as a whole
Cell-Merging Usage

• With a static configuration:
  – Cell merge once at the beginning of the simulation
  – Cell merge again only if flow adaptation alters the grid

• With a dynamic configuration:
  – Cell merge every time step
  – Circular dependency exists between the global time step and the computed merged-cell cover
    • Sometimes a viable merged-cell cover can be computed by assuming a stationary configuration
    • Starting from this assumption, the dependency is resolved by iterating to find a viable merged-cell cover for the associated time step prior to performing solution update
Computational Results

• Shock-Wave Interaction with Two Cylinders
  – Recreated from an example given by Berger & LeVeque in AIAA 89-1930-CP
  – Two cylinders are positioned such that one is slightly ahead of the other
  – A shock wave moving at Mach 2.81 interacts with the cylinders
  – An animation of the normalized density contours is presented on the next slide through a simulation time of 0.06 seconds.
Computational Results

$\rho = 0.00$

- $x$-axis range: $-1$ to $1$
- $z$-axis range: $-2$ to $2$

- Color scale:
  - Red: $11.20$
  - Orange: $10.08$
  - Yellow: $8.87$
  - Green: $8.75$
  - Cyan: $6.74$
  - Blue: $5.62$
  - Magenta: $4.50$
  - Gray: $3.39$
  - White: $0.04$

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Computational Results

- Moving Symmetric Diamond Airfoil
  - Diamond profile has a 5° half-angle
  - Airfoil is impulsively given a velocity
  - Equivalent flow relative to the airfoil: Mach 2 at a 5° angle-of-attack
    - Airfoil moves horizontally
    - Ambient air has a non-zero vertical component
  - An animation of the Mach contours is presented on the next slide through a simulation time of 4.0 sec.
Computational Results

$t = 0.0$

Mach
0.300
0.270
0.240
0.210
0.180
0.150
0.121
0.091
0.061
0.031
0.001

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Computational Results

- Comparison to an equivalent steady-state simulation
  - Relative Mach numbers are plotted
  - Results for the fully developed regions above and below the wake are in good agreement
  - The wake region results also show similarities, but the wake region has not become fully developed yet
  - Much more refinement is evident in the wake region of the moving case because the flow is not fully developed

Steady-State Results

Moving Airfoil, t = 4.0
Computational Results

• Ordinance Firing
  – Transonic flow over an Onera M6 wing with three under-the-wing ordinances
  – Two ordinances fired at separate times
    • The outermost ordinance is fired at $t = 0.0$
    • The innermost ordinance is fired at $t = 3.0$
  – An animation of the Mach contours is presented on the next slide through a simulation time of 6.0 sec.
Conclusions & Future Work

• Conclusions
  – Developed a parallel block-adaptive Cartesian code to compute compressible flow about static configurations
  – Implemented dynamic configurations
  – Developed a cell-merging algorithm within the parallel block-adaptive Cartesian framework
  – Demonstrated the capabilities of the approach

• Future Work
  – Incorporate split cells
  – Sophisticate permissible component motion
  – Implement the use of hybrid prismatic-Cartesian grids to solve viscous flow problems