Numerical Approaches and Computational Results for Fluid Dynamics Problems with Immersed Elastic Structures

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Overview

- **A Numerical Approach**
  - The Immersed Boundary Method
  - A Simple Model Problem

- **Some Implementation Issues**
  - SAMRAI and stationary Cartesian grids
  - PETSc and moving curvilinear meshes

- **Very Preliminary Results**
The Immersed Boundary Method

- General framework for modeling flows with immersed elastic structures or complex geometry

- Introduced by Peskin to study fluid dynamics of heart valves (2D model)
  - 2D model extended by Peskin and McQueen to 3D coupled fluid-mechanical heart model

- **Other application areas have included:**
  - wave propagation in inner ear
  - swimming, fish, bacteria, etc.
  - insect flight
  - flow around sails, flags, and parachutes
  - fluids with suspended elastic particles
A Simple Model Problem

- viscous incompressible fluid
- immersed elastic boundary ("2D water balloon")

Structure domain: \( \mathbf{X}(s,t), \mathbf{F}(s,t) \)

Fluid domain:
- \( u(x,t) \)
- \( p(x,t) \)
- \( f(x,t) \)
Typical IB Spatial Discretization

- **Eulerian variables** described on a Cartesian grid
  - fluid velocity: \( u(x,t) \)
  - pressure: \( p(x,t) \)

- **Lagrangian variables** described on moving curvilinear mesh (parameterized by \( s \))
  - structure position: \( X(s,t) \)
  - elastic force: \( F(s,t) = F(X(s,t)) \)

More generally: Lagrangian variables are parameterized by \( (q,r,s,\ldots) \)
- structure not restricted to “lower dimensional” objects
- in particular: structure can occupy nonzero volume in the fluid domain
Simple Model Problem *Redux*

- viscous incompressible fluid
- immersed elastic boundary ("2D water balloon")

Structure domain: $X(s,t), F(s,t)$

Fluid domain:
- $u(x,t)$
- $p(x,t)$
- $f(x,t)$
Fluid-to-Structure Interactions

- Fluid velocity
  - governed by incompressible Navier-Stokes equations (i.e. viscous incompressible flow)

- Structure moves at local fluid velocity
  - structure velocity: \( u(X(s,t),t) \)

- How does the fluid “feel” the influence of the structure...?
Spread the Force to the Grid!

- **Main Idea**: boundaries can be represented by the forces which they exert on the fluid
- How do we define the force on the Cartesian grid?

\[
\begin{align*}
X(k) &\rightarrow X(k+1) \\
F(k) &\rightarrow X(k-1)
\end{align*}
\]

Compute force on curvilinear mesh
Spread the Force to the Grid!

- **Main Idea**: boundaries can be represented by the **forces** which they exert on the fluid

- How do we define the force on the Cartesian grid?

\[ \mathbf{X}(k+1) \]

Compute force on curvilinear mesh

Spread force to Cartesian grid
Smoothing Out Force Spreading

- Force spreading weights determined by smoothed approximation to the Dirac delta function.
- Use same smoothed delta function for interpolation.
Project Goals

- Structured AMR fluid solver
  - approximate projection method
  - using SAMRAI (LLNL)

- Implicit timestepping
  - equations are very stiff
  - analytic Jacobian is dense and not available – use Newton-Krylov methods
  - using PETSc (ANL)

- Use this with Peskin and McQueen’s 3D heart model!
SAMR employs a dynamic structured “patch hierarchy”

Mesh and data:
- data stored on “logically-rectangular” patches (e.g., arrays)
- any “orthogonal” coordinate system (e.g., Cartesian, cylindrical, etc.)

Basic SAMR ingredients:
- problem formulation for locally-refined meshes
- (serial) numerical routines for individual patches
- inter-patch data transfer operations (copying, coarsening, refining, ...)

Structure of SAMR computational mesh

- Hierarchy of levels of mesh resolution
- Finer levels are nested within coarser
- Cells on each level are clustered to form logically-rectangular patches

Motivation:
- low overhead mesh description
- bookkeeping for computation and communication is simple (boxes)
- simple model of data locality
- amortize communication overhead by computing over a patch
- well-suited to structured solvers, hierarchical methods, local time refinement, etc.
How is the Lagrangian Grid Distributed?

- **Option 1:** Each processor gets roughly equal number of nodes from the Lagrangian mesh

  - **Advantages**
    - Essentially no duplicated computations
    - Ignoring communications, load balancing is nearly automatic

  - **Disadvantages**
    - Complicated mapping from points to fluid grid
    - Huge amounts of unstructured communication
How are Fibers and Points Distributed?

- **Option 2**: All nodes live on the same patch as their corresponding fluid grid cell

**Advantages**
- Lower communications requirements and overhead

**Disadvantages**
- Moderate amount of duplicated computational work
- Requires non-uniform load balancing
- Still need to maintain mappings from fluid cells to Lagrangian indices
Sample Explicit Timestep

- Fill ghost data on patch
- Move structure to predicted half-timestep position
Sample Explicit Timestep

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- Move structure to predicted half-timestep position
- Spread half-timestep force
- Compute end-timestep flow
- Move structure to end-timestep position
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