

**Numerical Approaches and Computational Results for  
Fluid Dynamics Problems with  
Immersed Elastic Structures**

**Boyce Griffith**

Courant Institute of Mathematical Sciences  
New York University

with

**Charles Peskin** (Courant Institute)

and

**Richard Hornung** (Lawrence Livermore National Lab)

# Overview

---

---

- **A Numerical Approach**
  - The Immersed Boundary Method
  - A Simple Model Problem
  
- **Some Implementation Issues**
  - SAMRAI and stationary Cartesian grids
  - PETSc and moving curvilinear meshes
  
- **Very Preliminary Results**

# The Immersed Boundary Method

---

---

- **General framework for modeling flows with immersed elastic structures or complex geometry**
- **Introduced by Peskin to study fluid dynamics of heart valves (2D model)**
  - 2D model extended by Peskin and McQueen to 3D coupled fluid-mechanical heart model
- **Other application areas have included:**
  - wave propagation in inner ear
  - swimming, fish, bacteria, etc.
  - insect flight
  - flow around sails, flags, and parachutes
  - fluids with suspended elastic particles

# A Simple Model Problem

---

---

- viscous incompressible fluid
- immersed elastic boundary (“2D water balloon”)

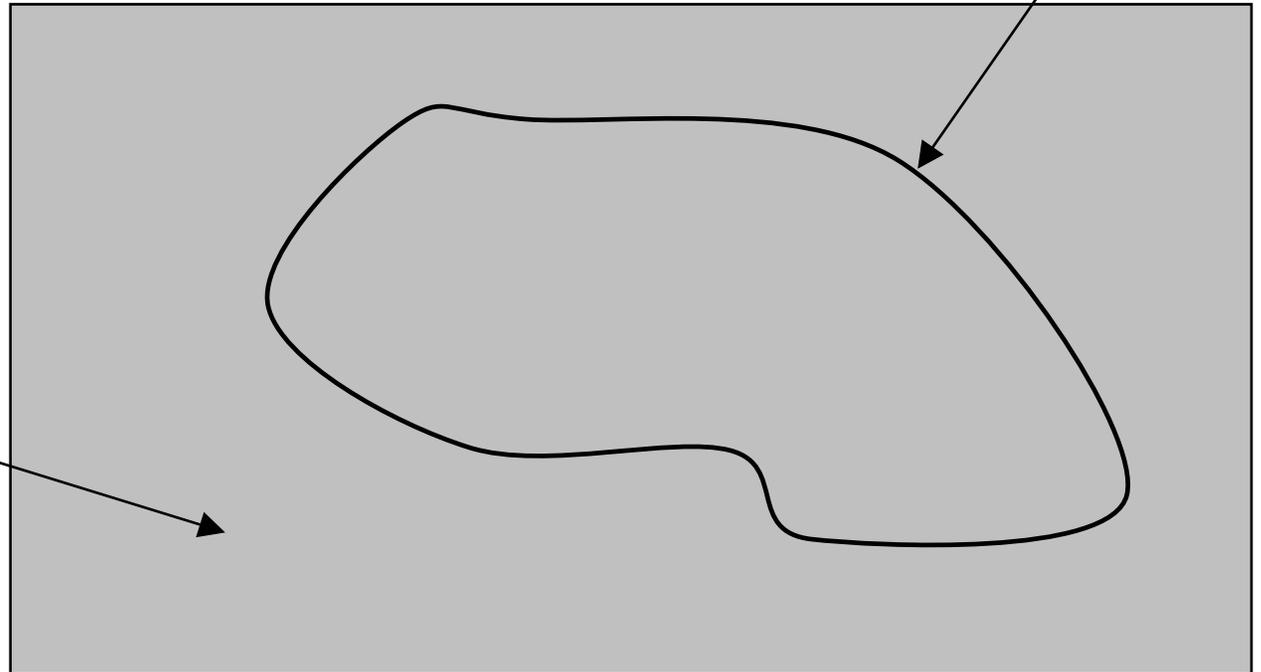
Structure domain:  $\mathbf{X}(s, t)$ ,  $\mathbf{F}(s, t)$

Fluid domain:

$\mathbf{u}(\mathbf{x}, t)$

$p(\mathbf{x}, t)$

$\mathbf{f}(\mathbf{x}, t)$



# Typical IB Spatial Discretization

---

- ***Eulerian variables*** described on a Cartesian grid
  - fluid velocity:  $\mathbf{u}(\mathbf{x}, t)$
  - pressure:  $p(\mathbf{x}, t)$
  
- ***Lagrangian variables*** described on moving curvilinear mesh (parameterized by  $s$ )
  - structure position:  $\mathbf{X}(s, t)$
  - elastic force:  $\mathbf{F}(s, t) = \mathbf{F}(\mathbf{X}(s, t))$
  
- **More generally: Lagrangian variables are parameterized by  $(q, r, s, \dots)$** 
  - structure not restricted to “lower dimensional” objects
  - in particular: structure can occupy nonzero volume in the fluid domain

# Simple Model Problem *Redux*

- viscous incompressible fluid
- immersed elastic boundary (“2D water balloon”)

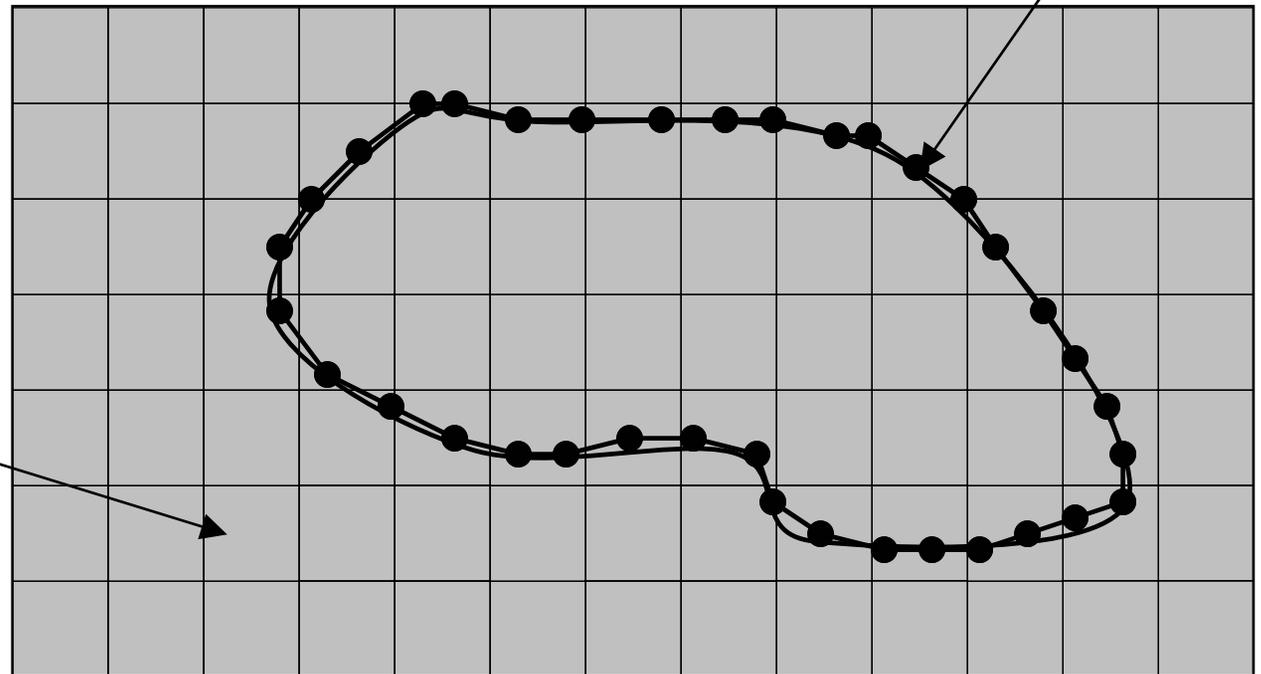
Structure domain:  $\mathbf{X}(s, t)$ ,  $\mathbf{F}(s, t)$

Fluid domain:

$\mathbf{u}(\mathbf{x}, t)$

$p(\mathbf{x}, t)$

$\mathbf{f}(\mathbf{x}, t)$



# Fluid-to-Structure Interactions

---

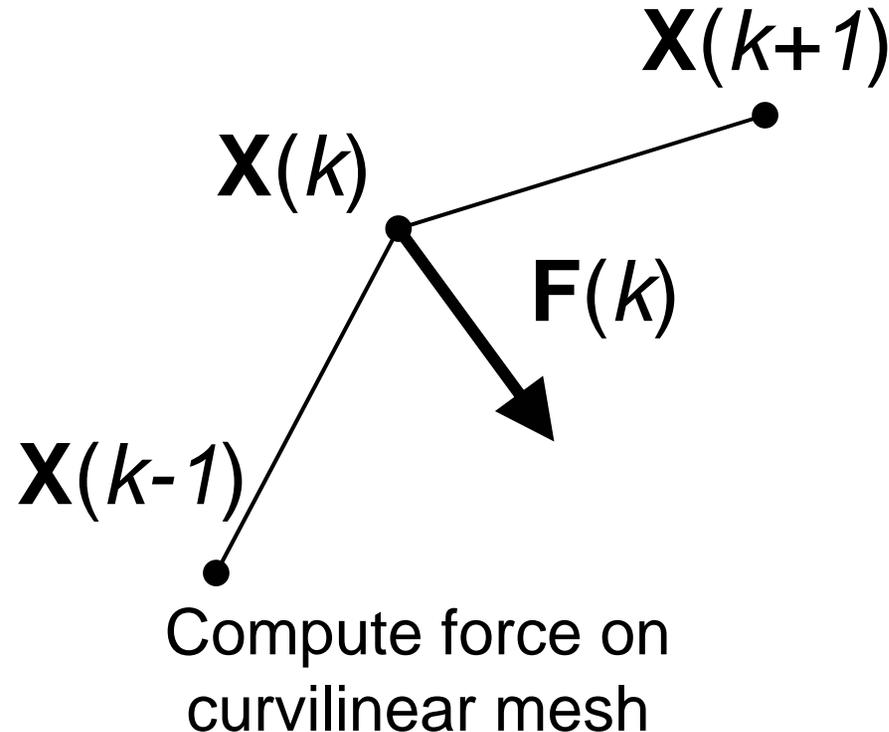
- **Fluid velocity**
  - governed by incompressible Navier-Stokes equations (i.e. viscous incompressible flow)
- **Structure moves at local fluid velocity**
  - structure velocity:  $\mathbf{u}(\mathbf{X}(s,t), t)$
- **How does the fluid “feel” the influence of the structure...?**

# Spread the Force to the Grid!

---

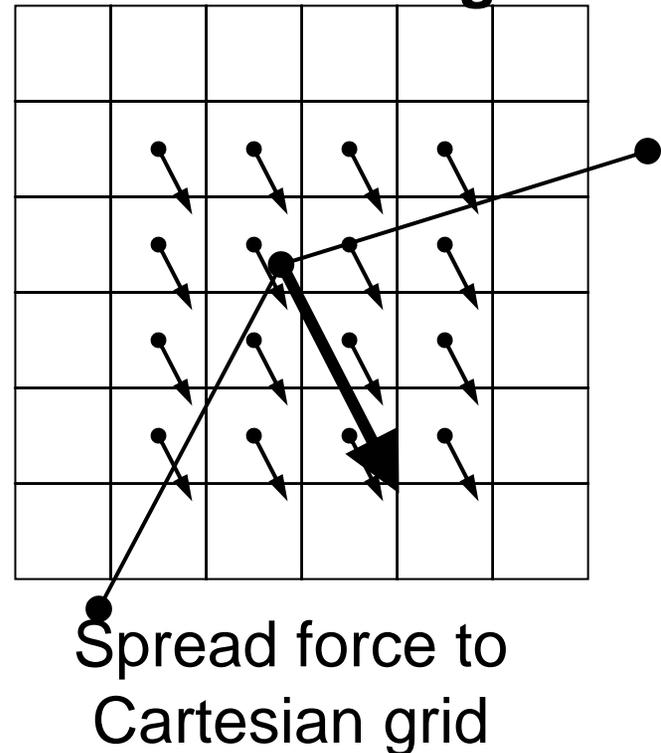
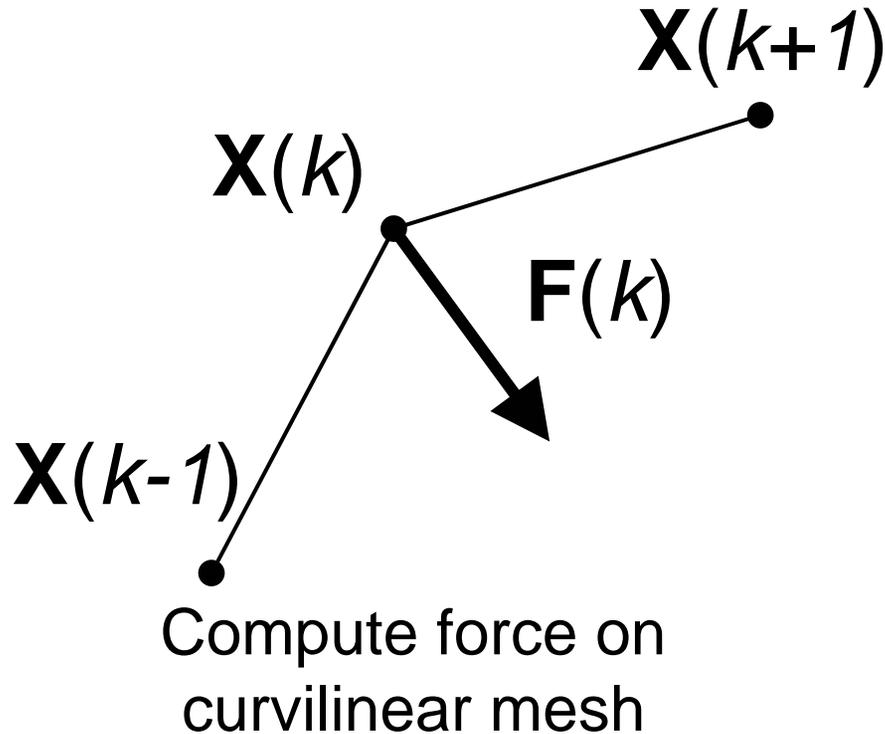
---

- Main Idea: boundaries can be represented by the *forces* which they exert on the fluid
- How do we define the force on the Cartesian grid?



# Spread the Force to the Grid!

- Main Idea: boundaries can be represented by the *forces* which they exert on the fluid
- How do we define the force on the Cartesian grid?

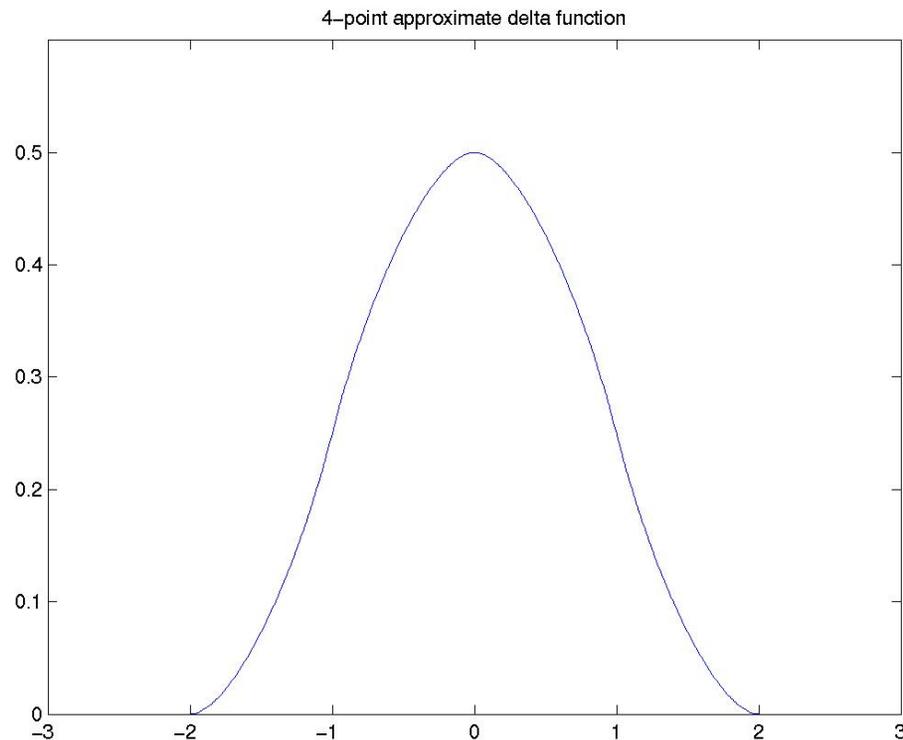


# Smoothing Out Force Spreading

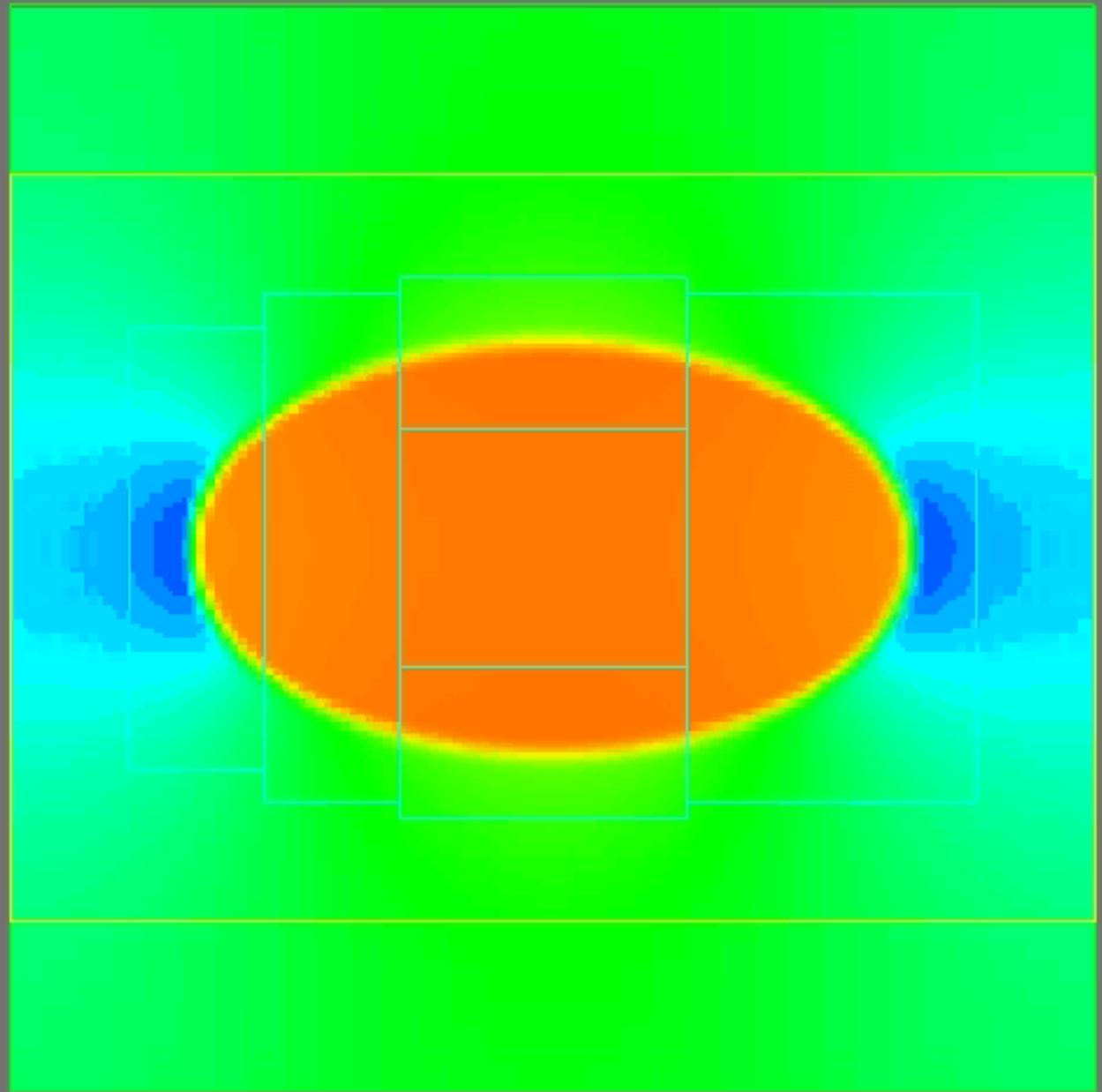
---

---

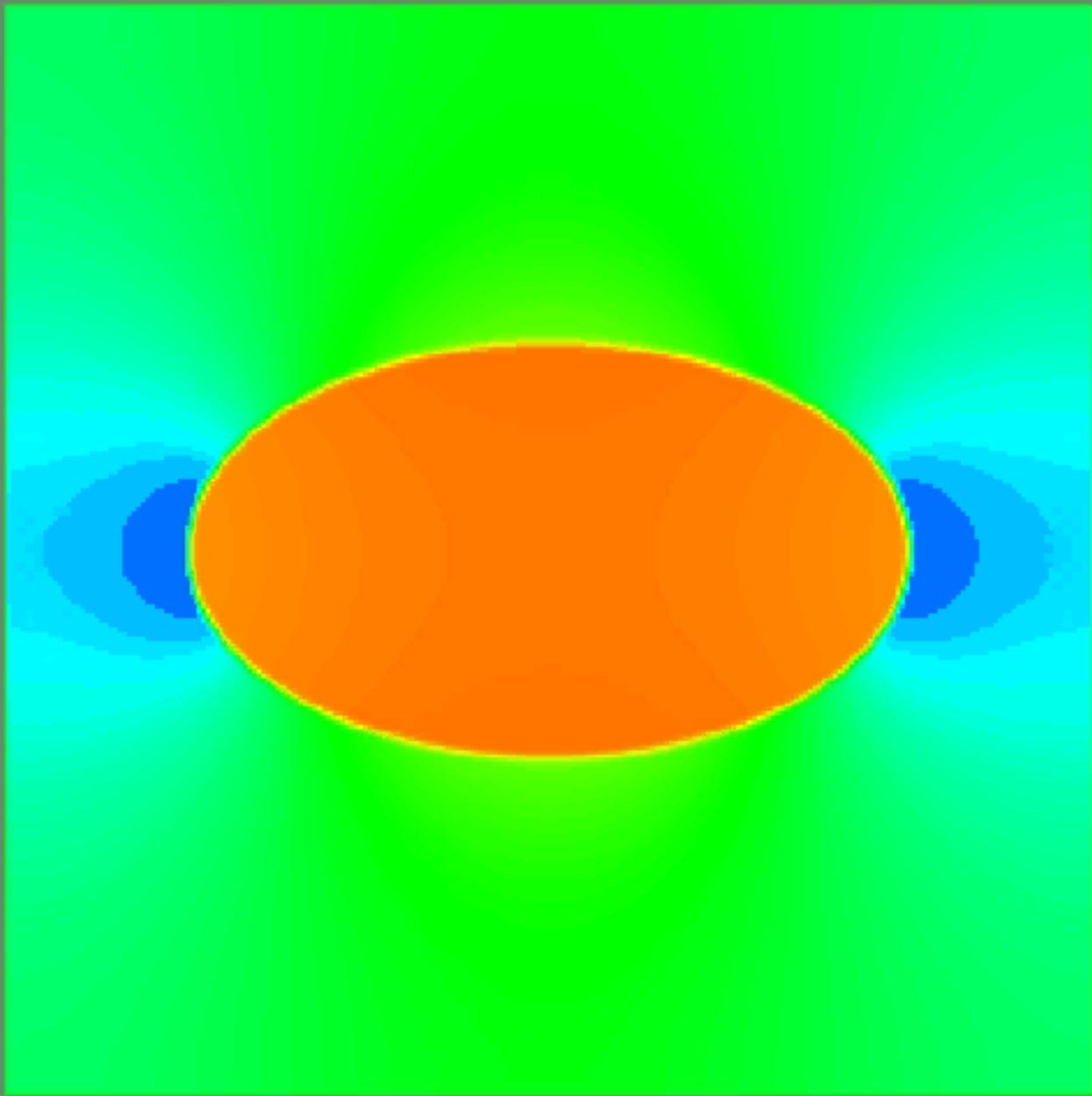
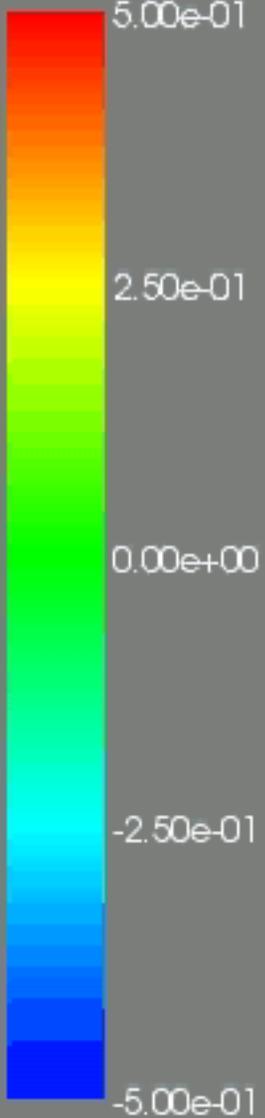
- Force spreading weights determined by smoothed approximation to the Dirac delta function.
- Use same smoothed delta function for interpolation.



Pressure



Pressure

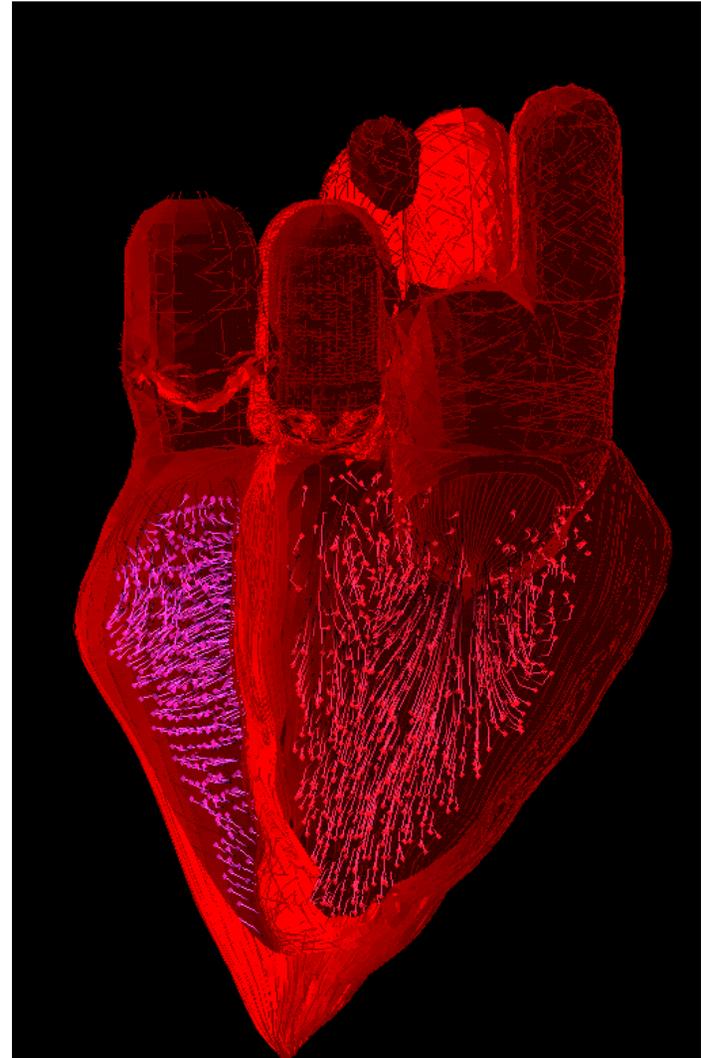


# Project Goals

---

---

- **Structured AMR fluid solver**
  - approximate projection method
  - using **SAMRAI** (LLNL)
- **Implicit timestepping**
  - equations are **very** stiff
  - analytic Jacobian is **dense** and **not** available – use Newton-Krylov methods
  - using **PETSc** (ANL)
- **Use this with Peskin and McQueen's 3D heart model!**



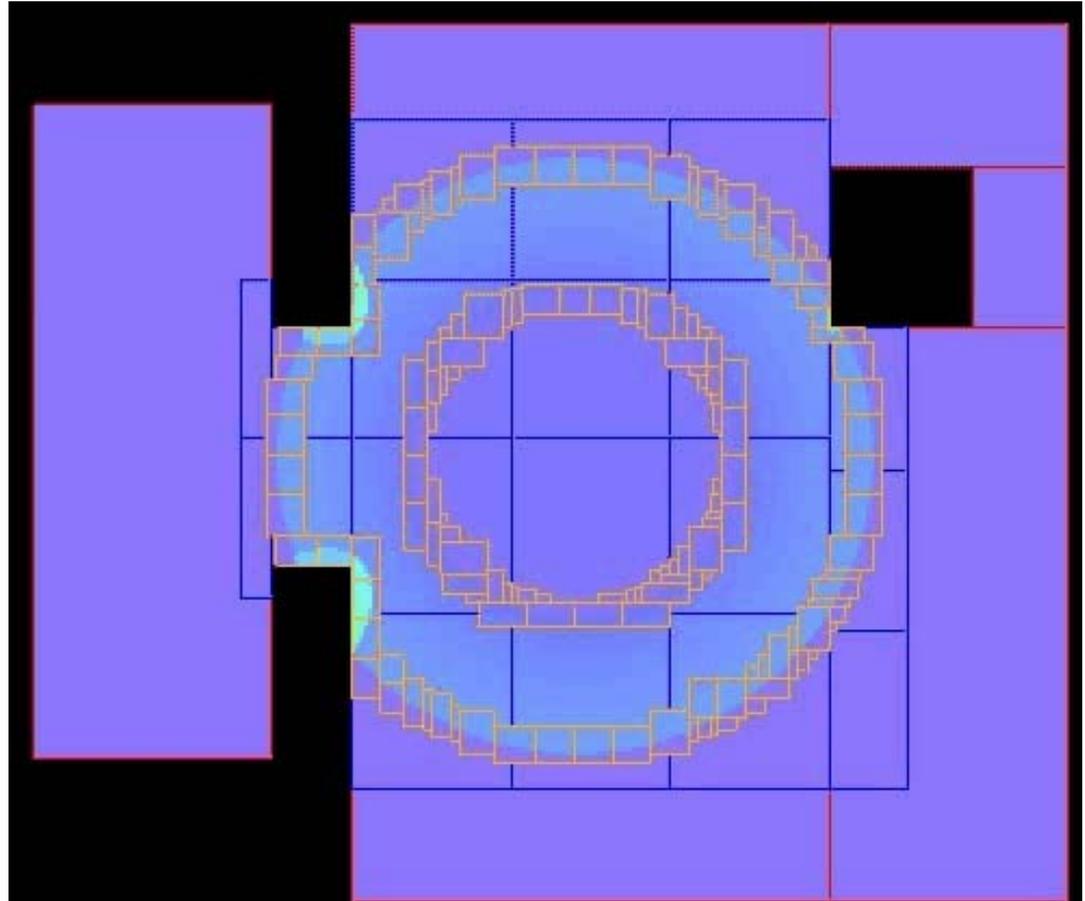
# SAMR employs a dynamic structured “patch hierarchy”

## Mesh and data:

- data stored on “logically-rectangular” patches (e.g., arrays)
- any “orthogonal” coordinate system (e.g., Cartesian, cylindrical, etc.)

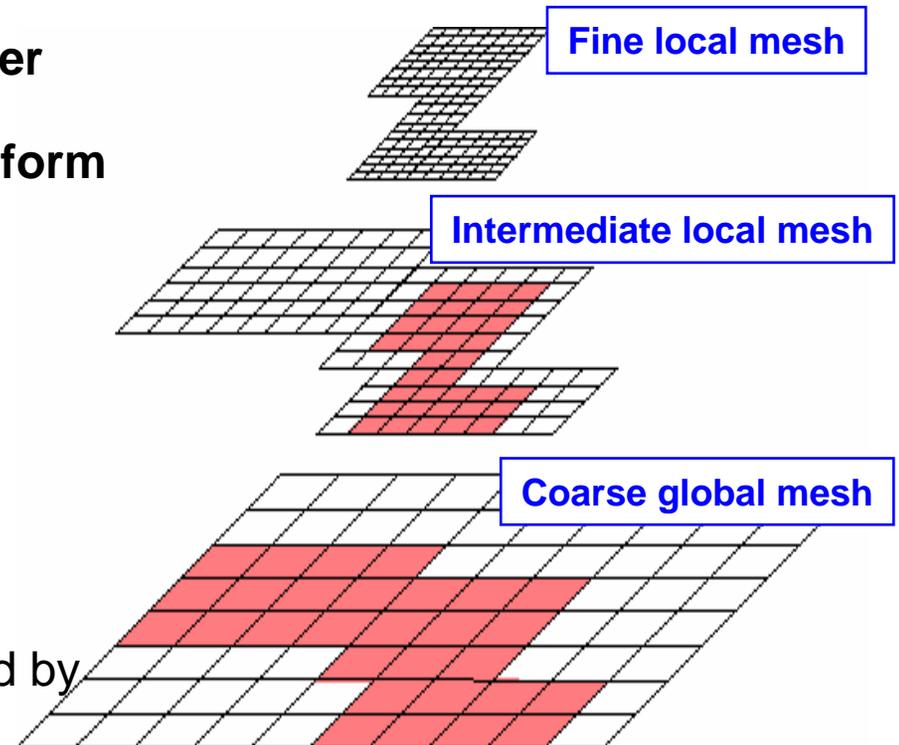
## **Basic SAMR ingredients:**

- problem formulation for locally-refined meshes
- (serial) numerical routines for individual patches
- inter-patch data transfer operations (copying, coarsening, refining, ...)



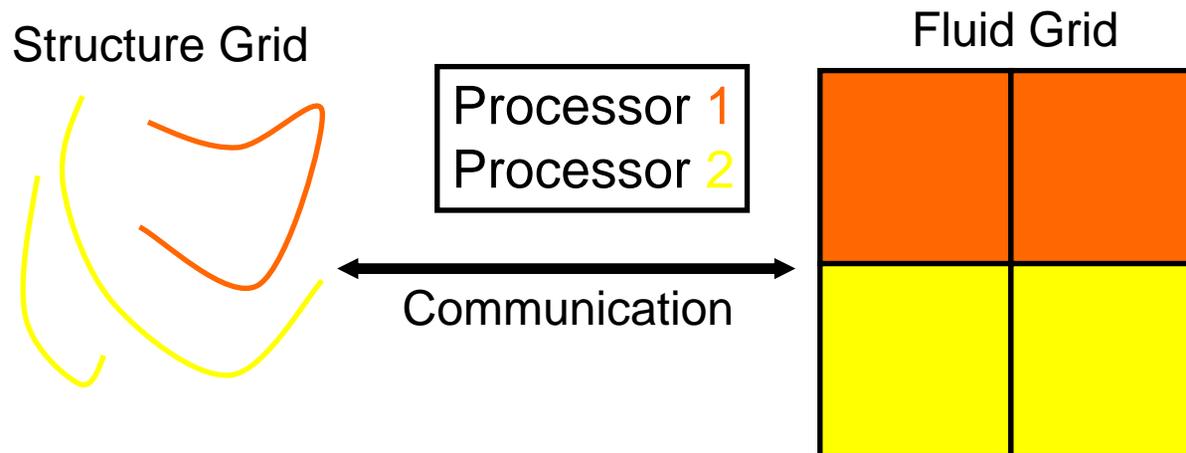
# Structure of SAMR computational mesh

- Hierarchy of levels of mesh resolution
- Finer levels are nested within coarser
- Cells on each level are clustered to form logically-rectangular patches
- Motivation:
  - low overhead mesh description
  - bookkeeping for computation and communication is simple (boxes)
  - simple model of data locality
  - amortize communication overhead by computing over a patch
  - well-suited to structured solvers, hierarchical methods, local time refinement, etc.



# How is the Lagrangian Grid Distributed?

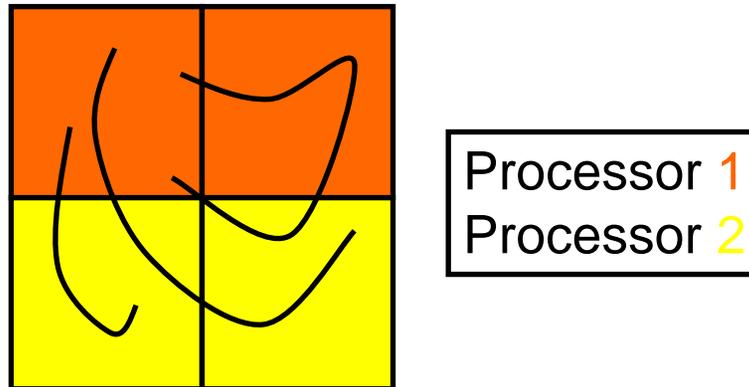
- **Option 1**: Each processor gets roughly equal number of nodes from the Lagrangian mesh
- **Advantages**
  - Essentially no duplicated computations
  - Ignoring communications, load balancing is nearly automatic
- **Disadvantages**
  - Complicated mapping from points to fluid grid
  - Huge amounts of unstructured communication



# How are Fibers and Points Distributed?

---

- **Option 2:** All nodes live on same patch as their corresponding fluid grid cell
- **Advantages**
  - Lower communications requirements and overhead
- **Disadvantages**
  - Moderate amount of duplicated computational work
  - Requires non-uniform load balancing
  - Still need to maintain mappings from fluid cells to Lagrangian indices

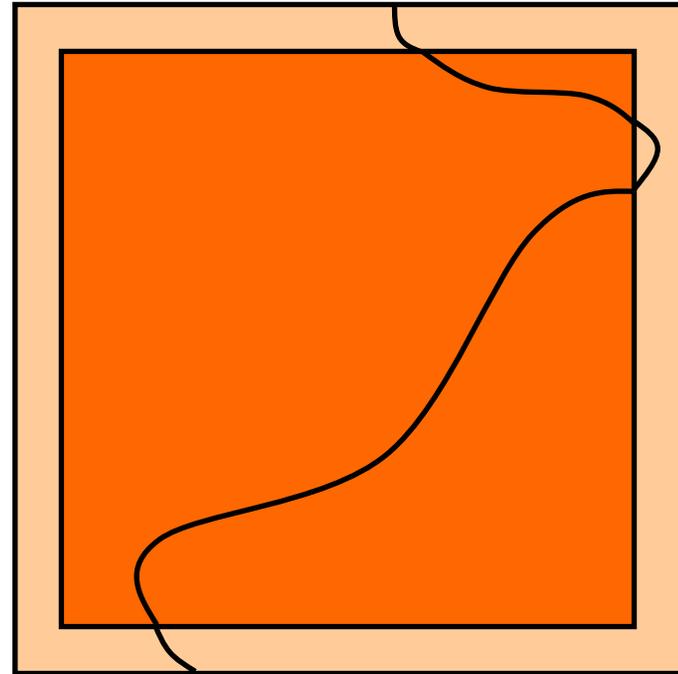


# Sample Explicit Timestep

---

---

- Fill ghost data on patch
- Move structure to predicted half-timestep position



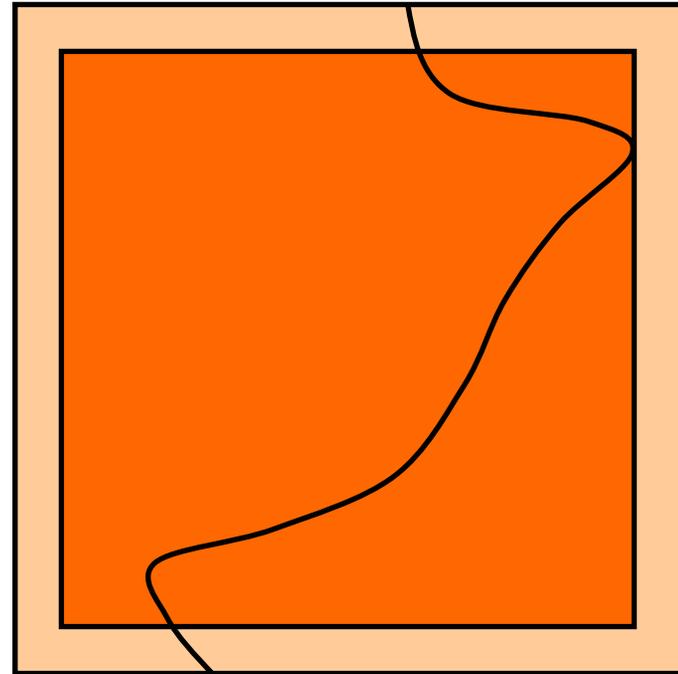
Patch Data  
Ghost Cell Region

# Sample Explicit Timestep

---

---

- Fill ghost data on patch
- Move structure to predicted half-timestep position
- Spread half-timestep force
- Compute end-timestep flow
- Move structure to end-timestep position



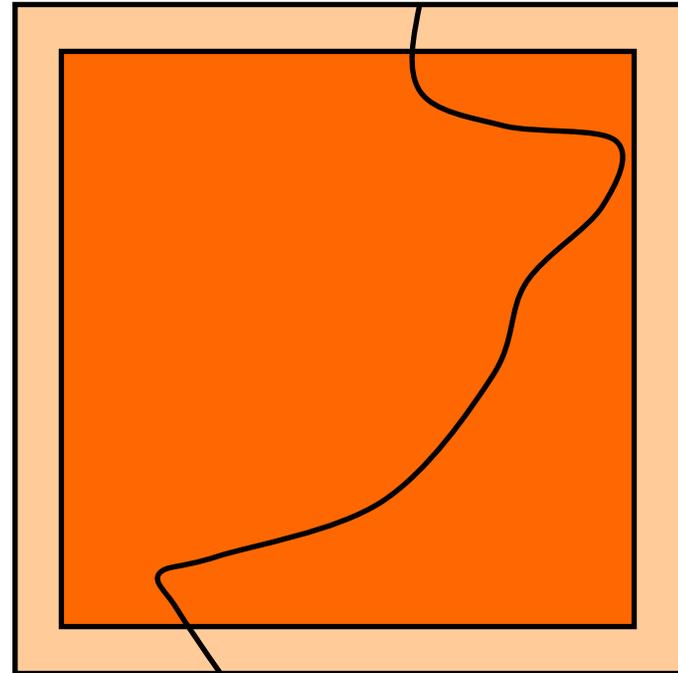
Patch Data  
Ghost Cell Region

# Sample Explicit Timestep

---

---

- Fill ghost data on patch
- Move structure to predicted half-timestep position
- Spread half-timestep force
- Compute end-timestep flow
- Move structure to end-timestep position



Patch Data  
Ghost Cell Region

Bdry Points

10

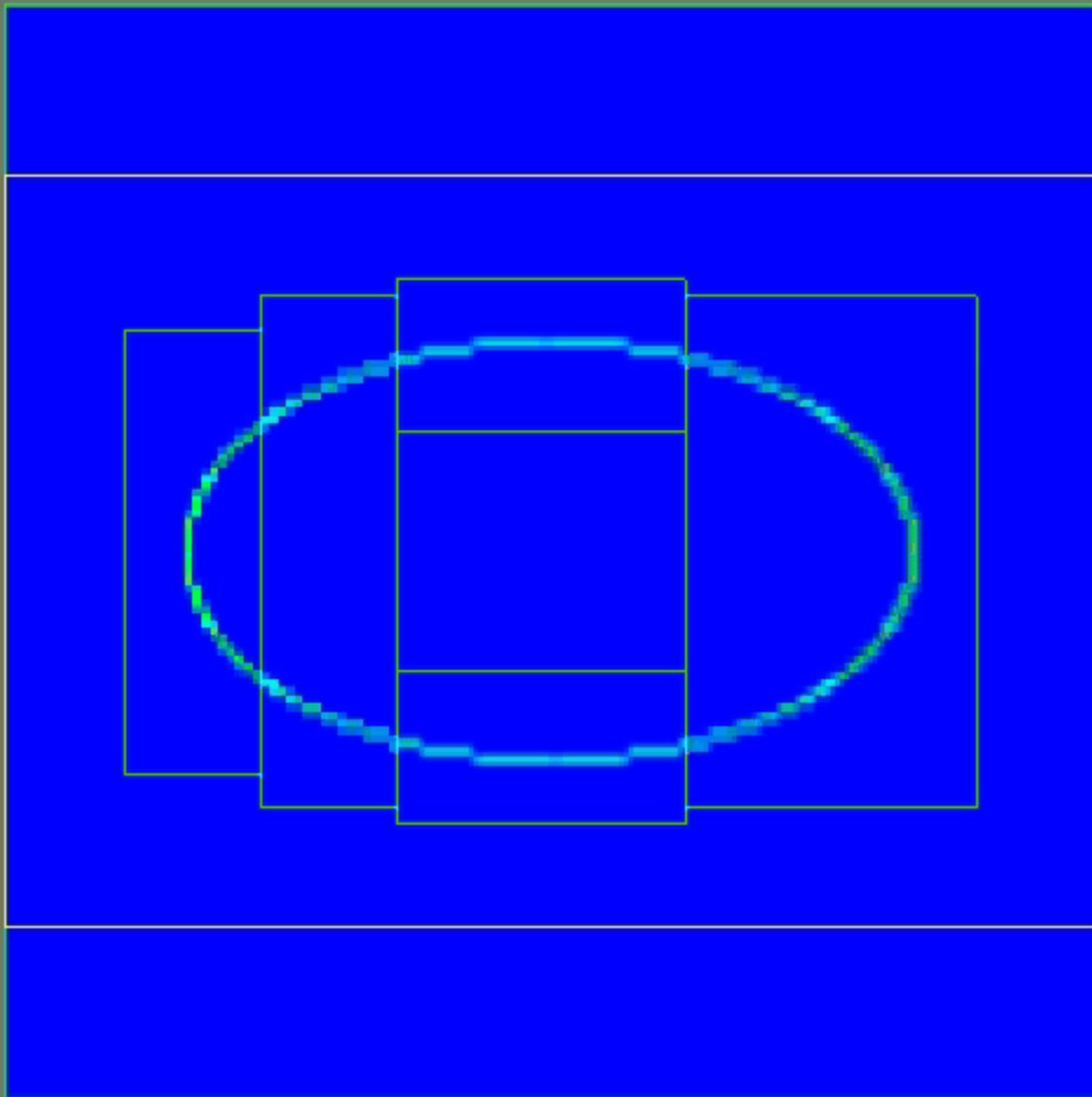
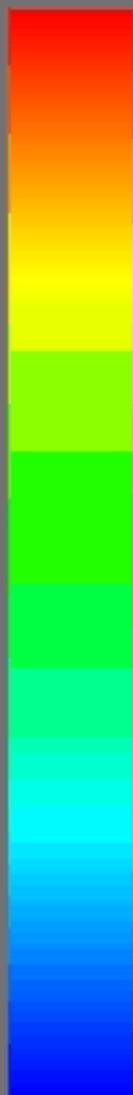
8

6

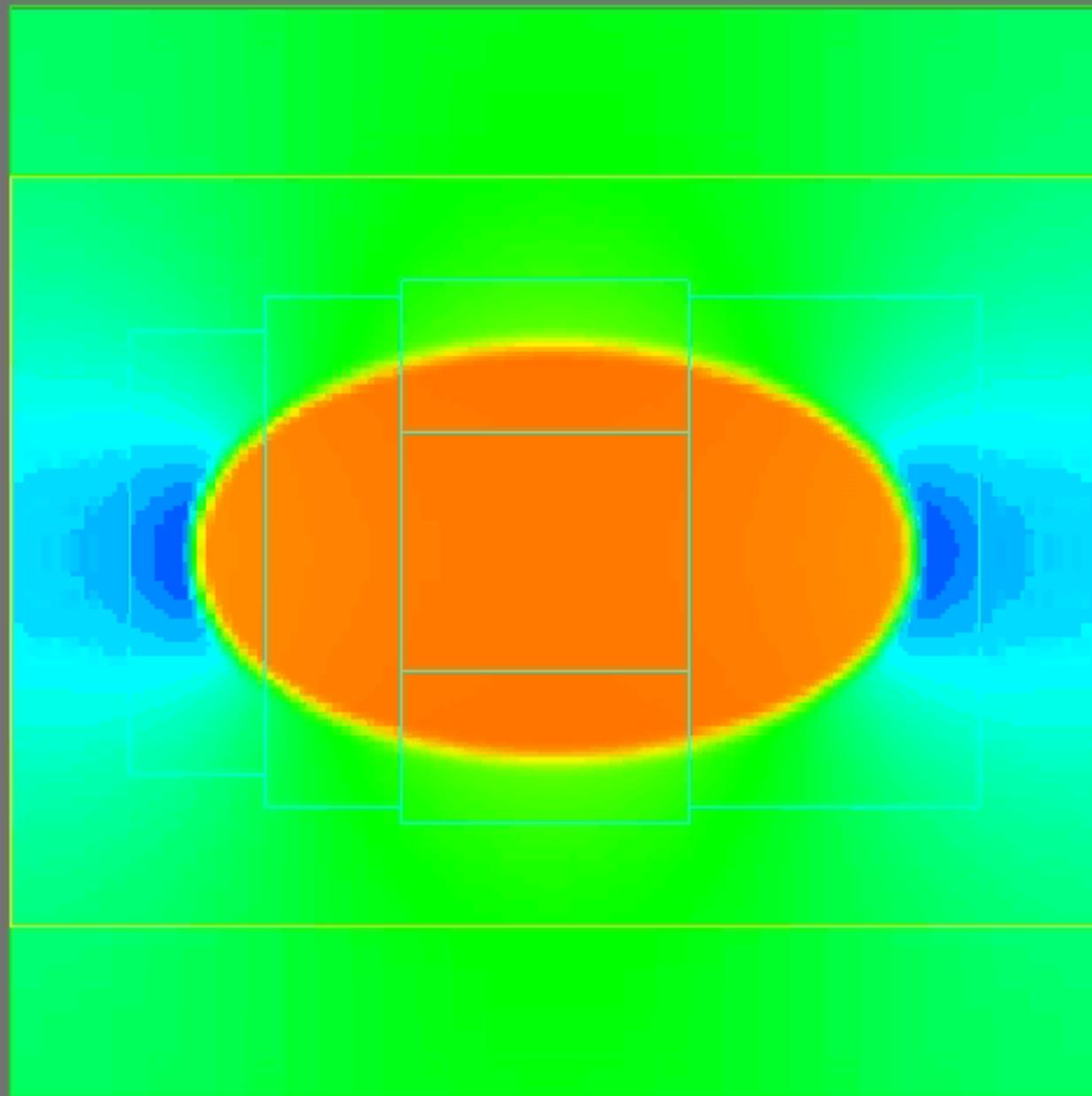
4

2

0



Pressure



# Acknowledgements

---

---

- **Charlie Peskin & Rich Hornung**
- **Dave McQueen (CIMS)**
- **Steve Smith (LLNL)**
- **Brian Gunney (LLNL)**
- **David Keyes (LLNL/Columbia)**
  
- **DOE CSGF Program**
- **The Krell Institute**

# Auspices Statement

---

---

- **Portions of this work were performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.**
- **Some slides are from documents UCRL-PRES-144527 and UCRL-CODE-2002-004**