Numerical Approaches and Computational Results for

Fluid Dynamics Problems with Immersed Elastic Structures

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Overview

- **A Numerical Approach**
  - The Immersed Boundary Method
  - A Simple Model Problem

- **Some Implementation Issues**
  - SAMRAI and stationary Cartesian grids
  - PETSc and moving curvilinear meshes

- **Very Preliminary Results**

The Immersed Boundary Method

- **General framework for modeling flows with immersed elastic structures or complex geometry**

- **Introduced by Peskin to study fluid dynamics of heart valves (2D model)**
  - 2D model extended by Peskin and McQueen to 3D coupled fluid-mechanical heart model

- **Other application areas have included:**
  - wave propagation in inner ear
  - swimming, fish, bacteria, etc.
  - insect flight
  - flow around sails, flags, and parachutes
  - fluids with suspended elastic particles

A Simple Model Problem

- **viscous incompressible fluid**
- **immersed elastic boundary (“2D water balloon”)**

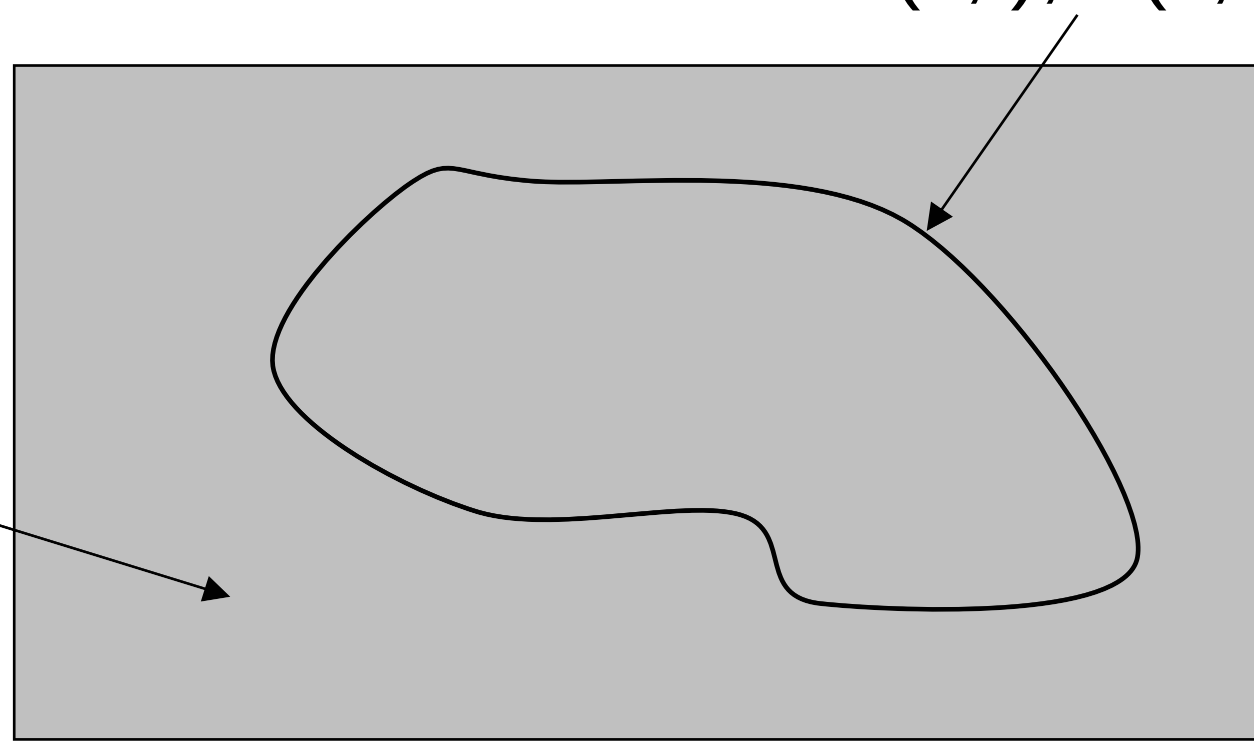
Structure domain: $\mathbf{X}(s,t)$, $\mathbf{F}(s,t)$

Fluid domain:

$\mathbf{u}(\mathbf{x},t)$

$p(\mathbf{x},t)$

$\mathbf{f}(\mathbf{x},t)$



Typical IB Spatial Discretization

- ***Eulerian variables*** **described on a Cartesian grid**
  - fluid velocity: $\mathbf{u}(\mathbf{x},t)$
  - pressure: $p(\mathbf{x},t)$

- ***Lagrangian variables*** **described on <u>moving</u> curvilinear mesh (parameterized by *s*)**
  - structure position: $\mathbf{X}(s,t)$
  - elastic force: $\mathbf{F}(s,t) = \mathbf{F}(\mathbf{X}(s,t))$

- **More generally: Lagrangian variables are parameterized by** $(q,r,s,\ldots)$
  - structure not restricted to “lower dimensional” objects
  - in particular: structure can occupy nonzero volume in the fluid domain

Simple Model Problem *Redux*

- **viscous incompressible fluid**
- **immersed elastic boundary ("2D water balloon")**

Structure domain: $\mathbf{X}(s,t)$, $\mathbf{F}(s,t)$

Fluid domain:

$\mathbf{u}(\mathbf{x},t)$

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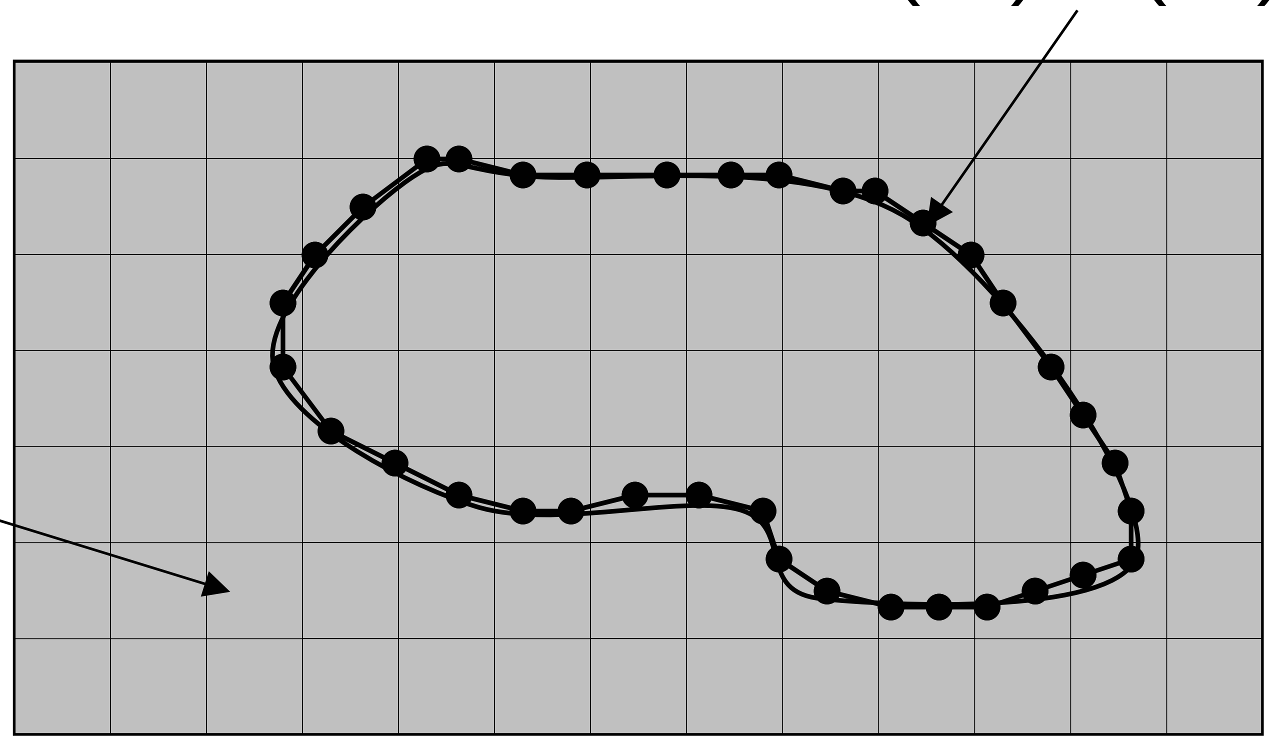
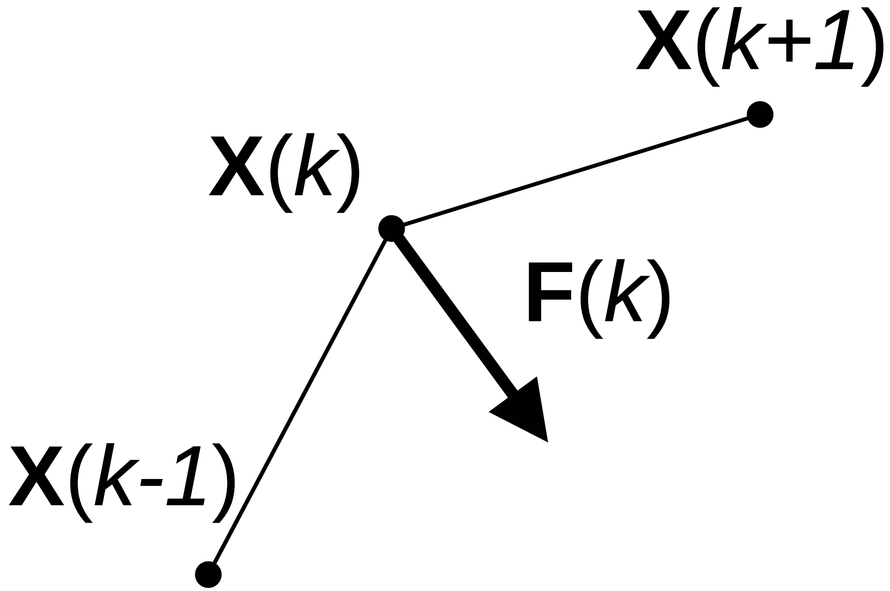


Fluid-to-Structure Interactions

- **Fluid velocity**
  - governed by incompressible Navier-Stokes equations (i.e. viscous incompressible flow)
- **Structure moves at local fluid velocity**
  - structure velocity: $\mathbf{u}(\mathbf{X}(s,t),t)$
- **How does the fluid "feel" the influence of the structure…?**

Spread the Force to the Grid!

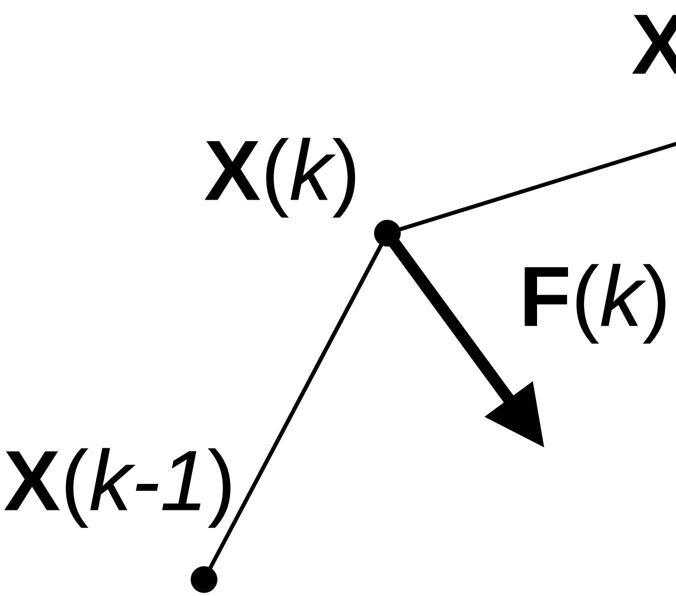
- **<u>Main Idea</u>: boundaries can be represented by the *forces* which they exert on the fluid**
- **How do we define the force on the Cartesian grid?**



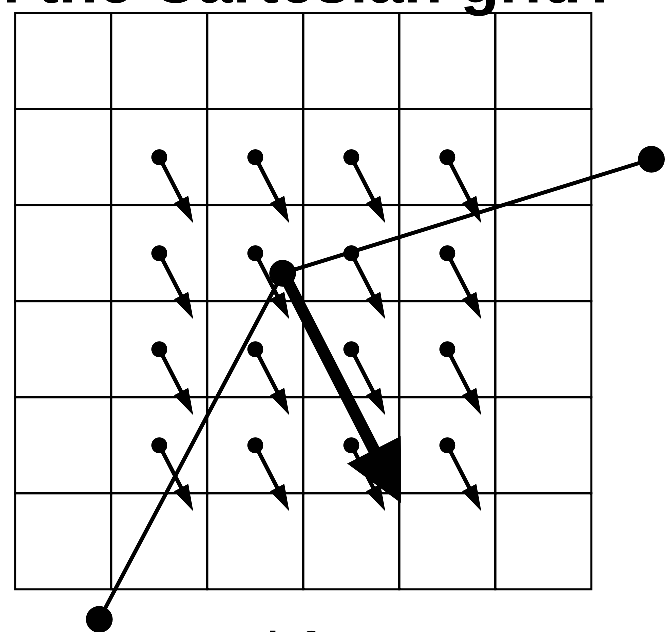
Compute force on
curvilinear mesh

Spread the Force to the Grid!

- <u>Main Idea</u>: boundaries can be represented by the *forces* which they exert on the fluid
- How do we define the force on the Cartesian grid?



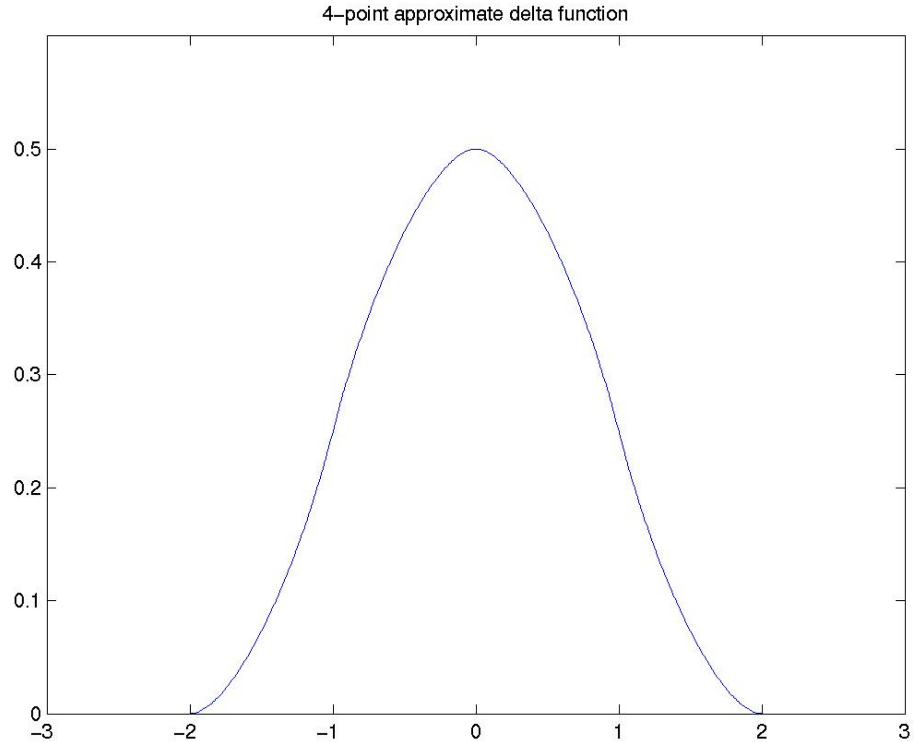
Compute force on curvilinear mesh

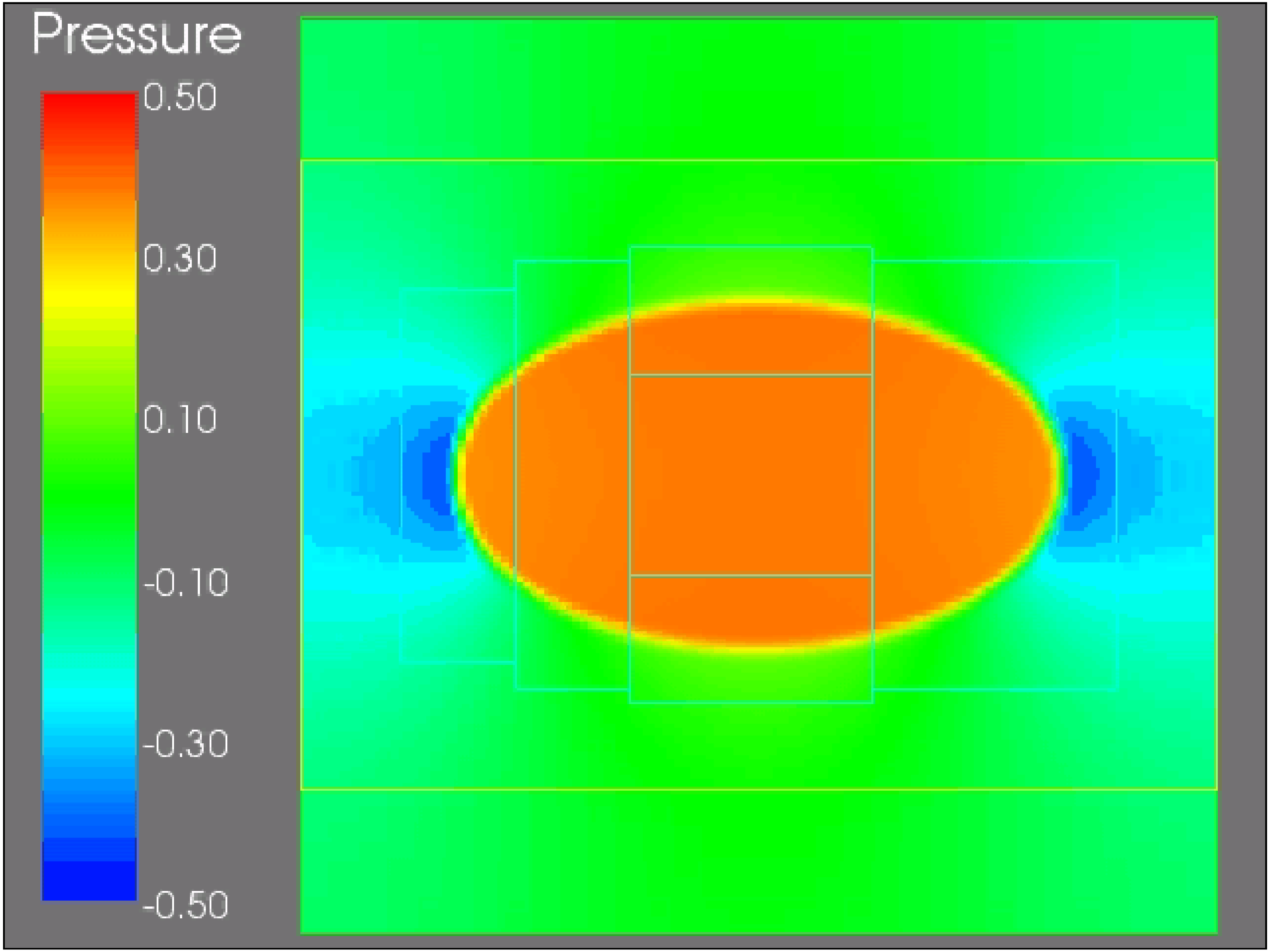


Spread force to Cartesian grid

Smoothing Out Force Spreading

- Force spreading weights determined by smoothed approximation to the Dirac delta function.
- Use same smoothed delta function for interpolation.

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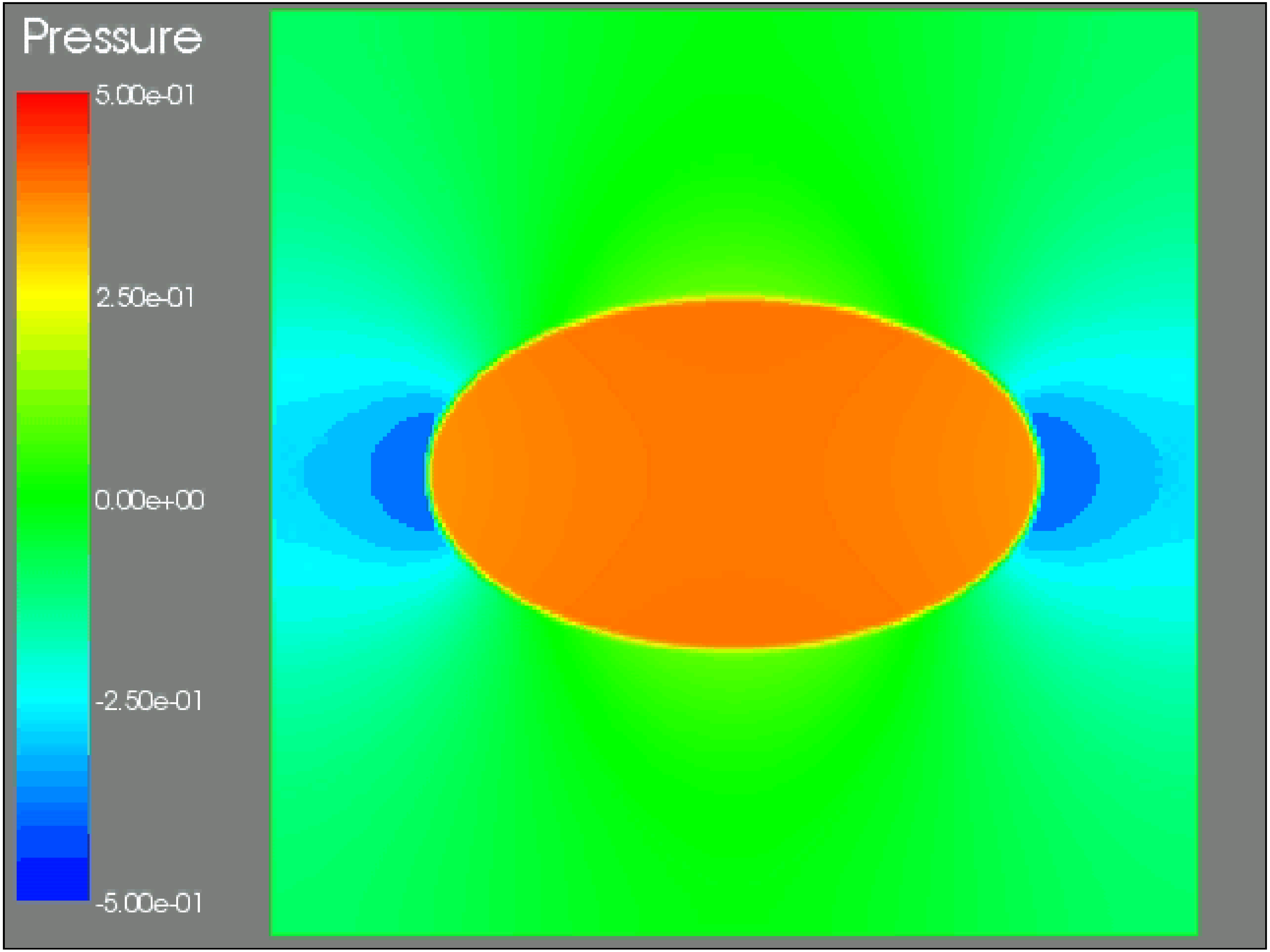
0.10
-0.10
-0.30
-0.50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-5.00e-01

Project Goals

- **Structured AMR fluid solver**
  - approximate projection method
  - using **SAMRAI** (LLNL)
- **Implicit timestepping**
  - equations are **very** stiff
  - analytic Jacobian is **dense** and **not** available – use Newton-Krylov methods
  - using **PETSc** (ANL)
- **Use this with Peskin and McQueen's 3D heart model!**

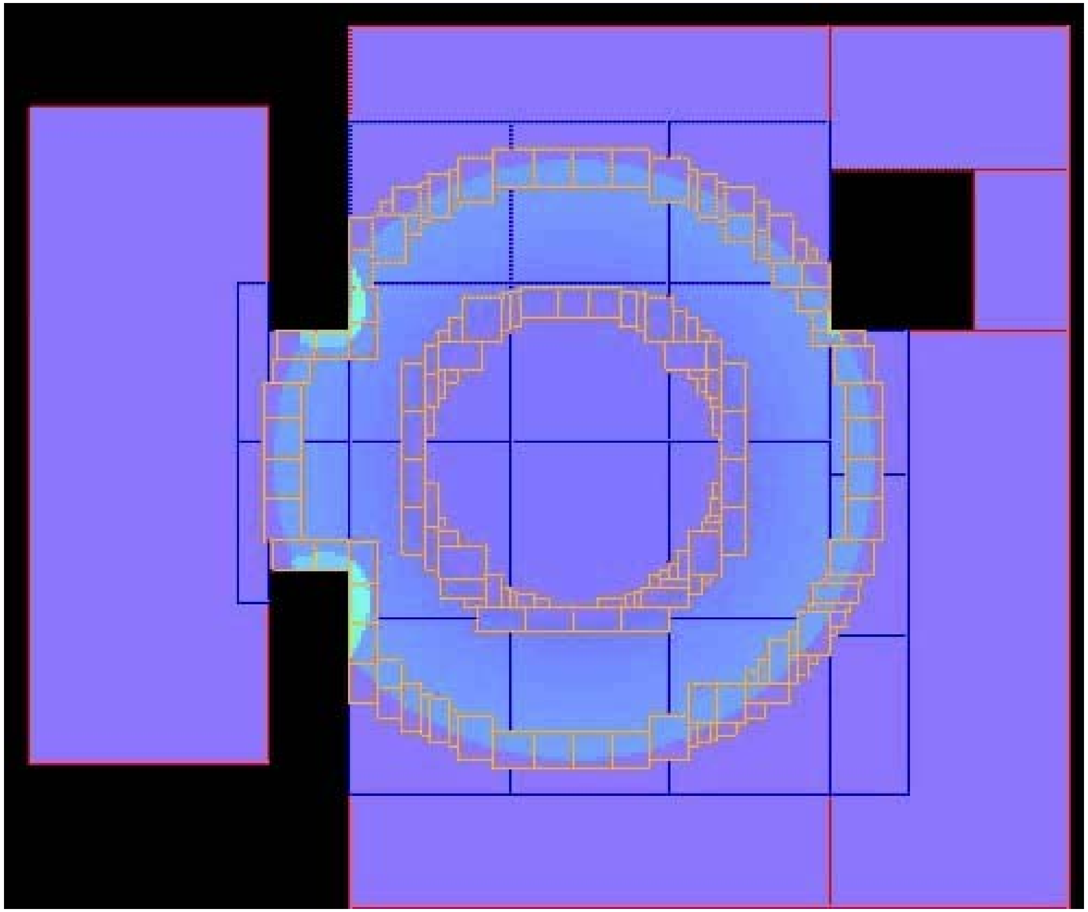
SAMR employs a dynamic structured "patch hierarchy"

Mesh and data:

- data stored on "logically-rectangular" patches (e.g., arrays)
- any "orthogonal" coordinate system (e.g., Cartesian, cylindrical, etc.)

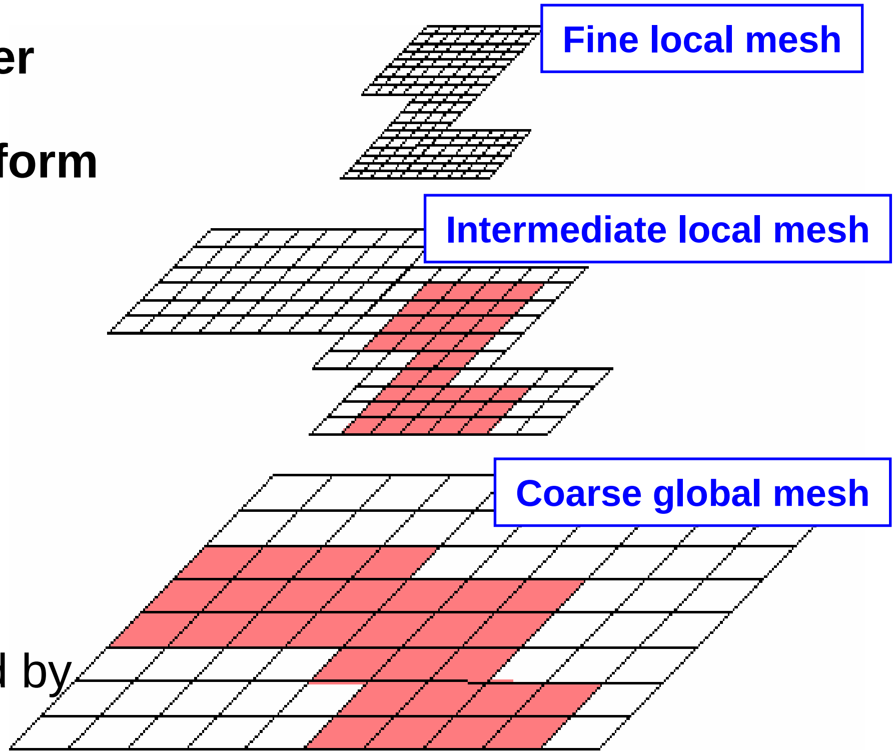
Basic SAMR ingredients:

- problem formulation for locally-refined meshes
- (serial) numerical routines for individual patches
- inter-patch data transfer operations (copying, coarsening, refining, ...)



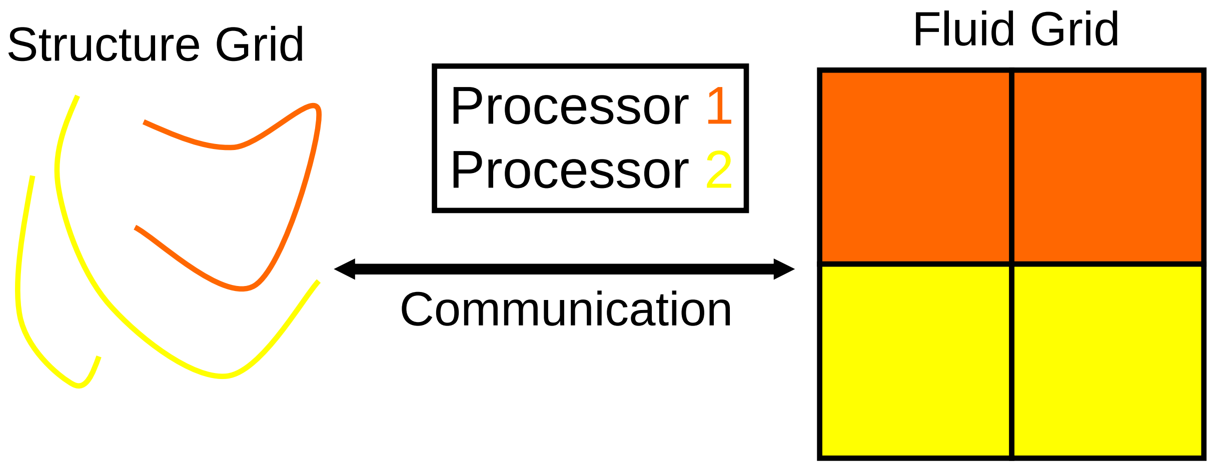
Structure of SAMR computational mesh

- **Hierarchy of levels of mesh resolution**
- **Finer levels are nested within coarser**
- **Cells on each level are clustered to form logically-rectangular patches**
- **Motivation:**
  - low overhead mesh description
  - bookkeeping for computation and communication is simple (boxes)
  - simple model of data locality
  - amortize communication overhead by computing over a patch
  - well-suited to structured solvers, hierarchical methods, local time refinement, etc.



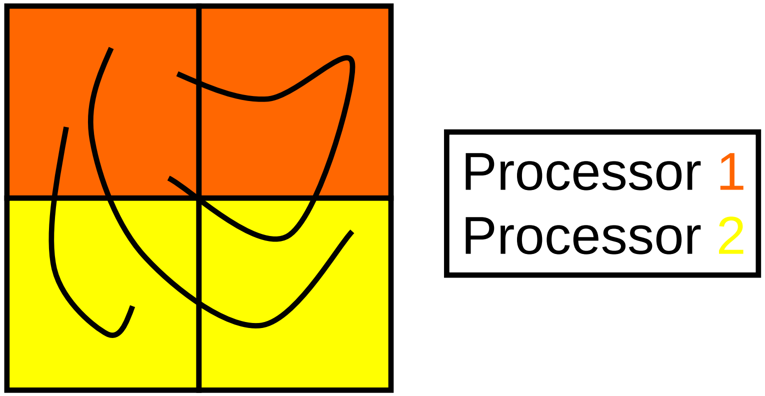
How is the Lagrangian Grid Distributed?

- **<u>Option 1:</u> Each processor gets roughly equal number of nodes from the Lagrangian mesh**
- **Advantages**
  - Essentially no duplicated computations
  - Ignoring communications, load balancing is nearly automatic
- **Disadvantages**
  - Complicated mapping from points to fluid grid
  - Huge amounts of unstructured communication



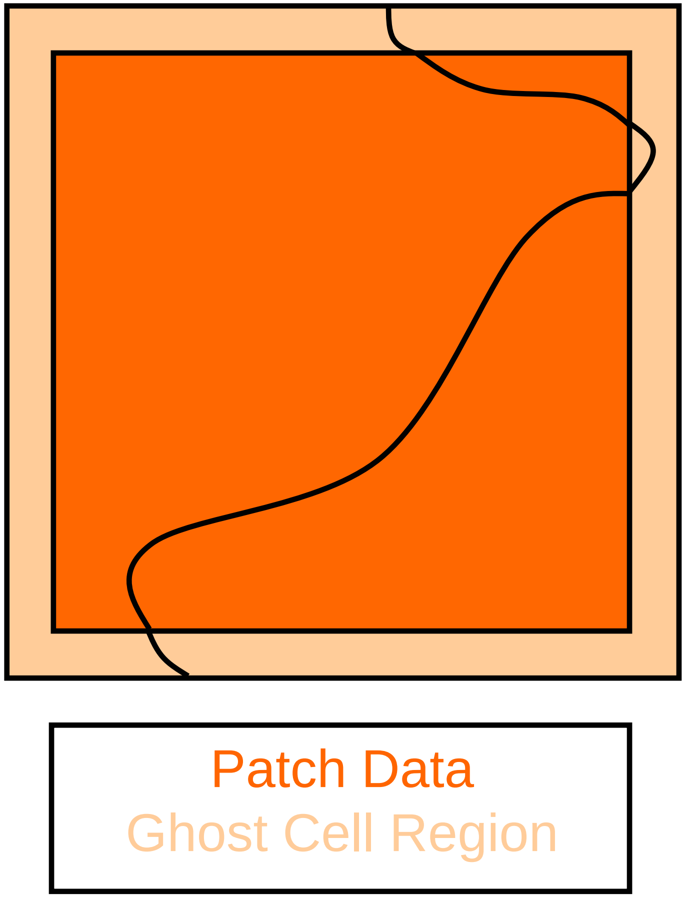
How are Fibers and Points Distributed?

- **<u>Option 2:</u> All nodes live on same patch as their corresponding fluid grid cell**
- **Advantages**
  - Lower communications requirements and overhead
- **Disadvantages**
  - Moderate amount of duplicated computational work
  - Requires non-uniform load balancing
  - Still need to maintain mappings from fluid cells to Lagrangian indices



Sample Explicit Timestep

- **Fill ghost data on patch**
- **Move structure to predicted half-timestep position**



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- **Spread half-timestep force**
- **Compute end-timestep flow**
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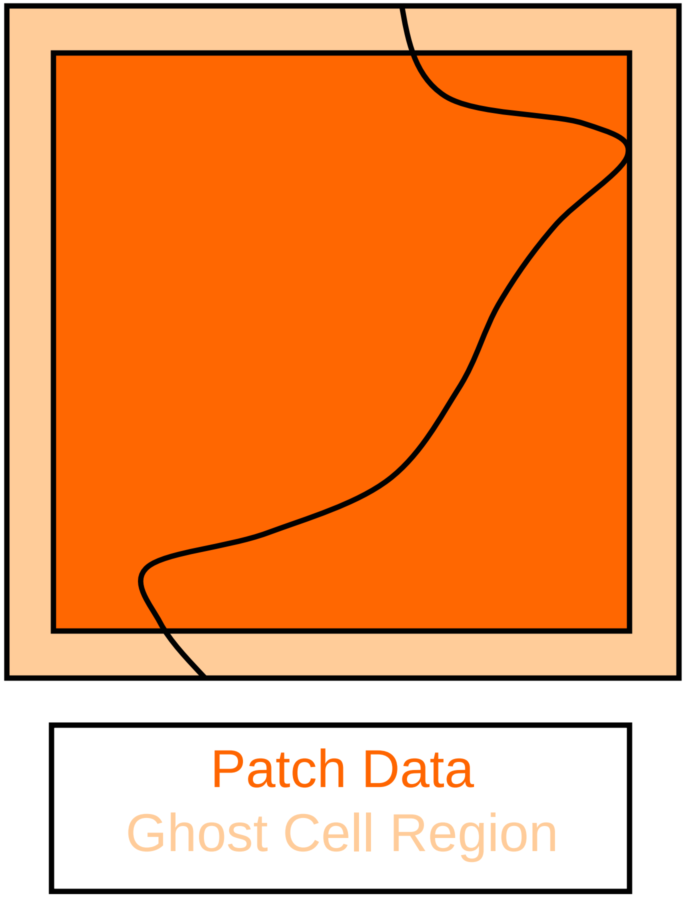

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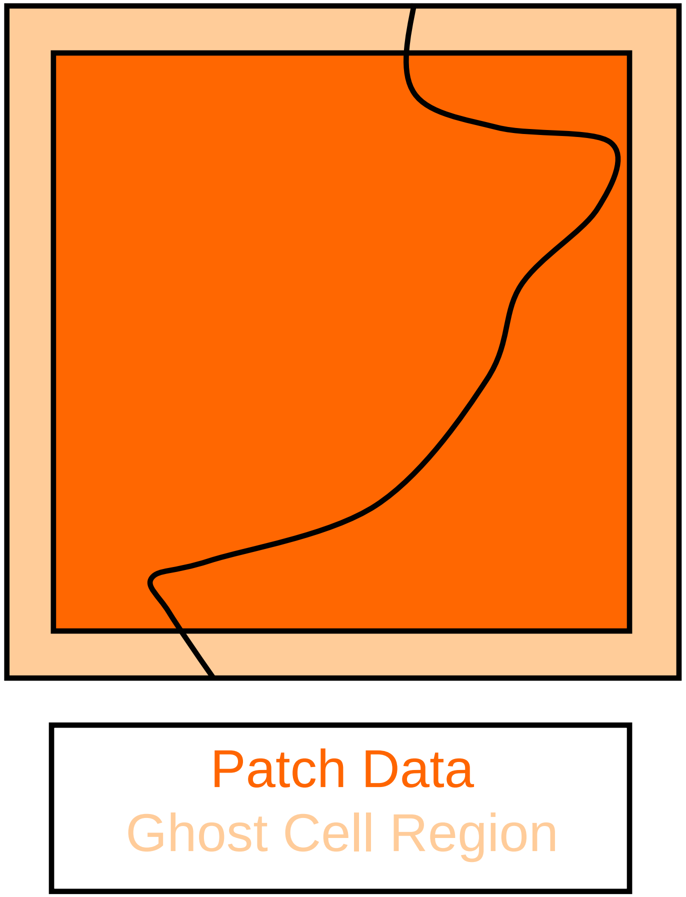

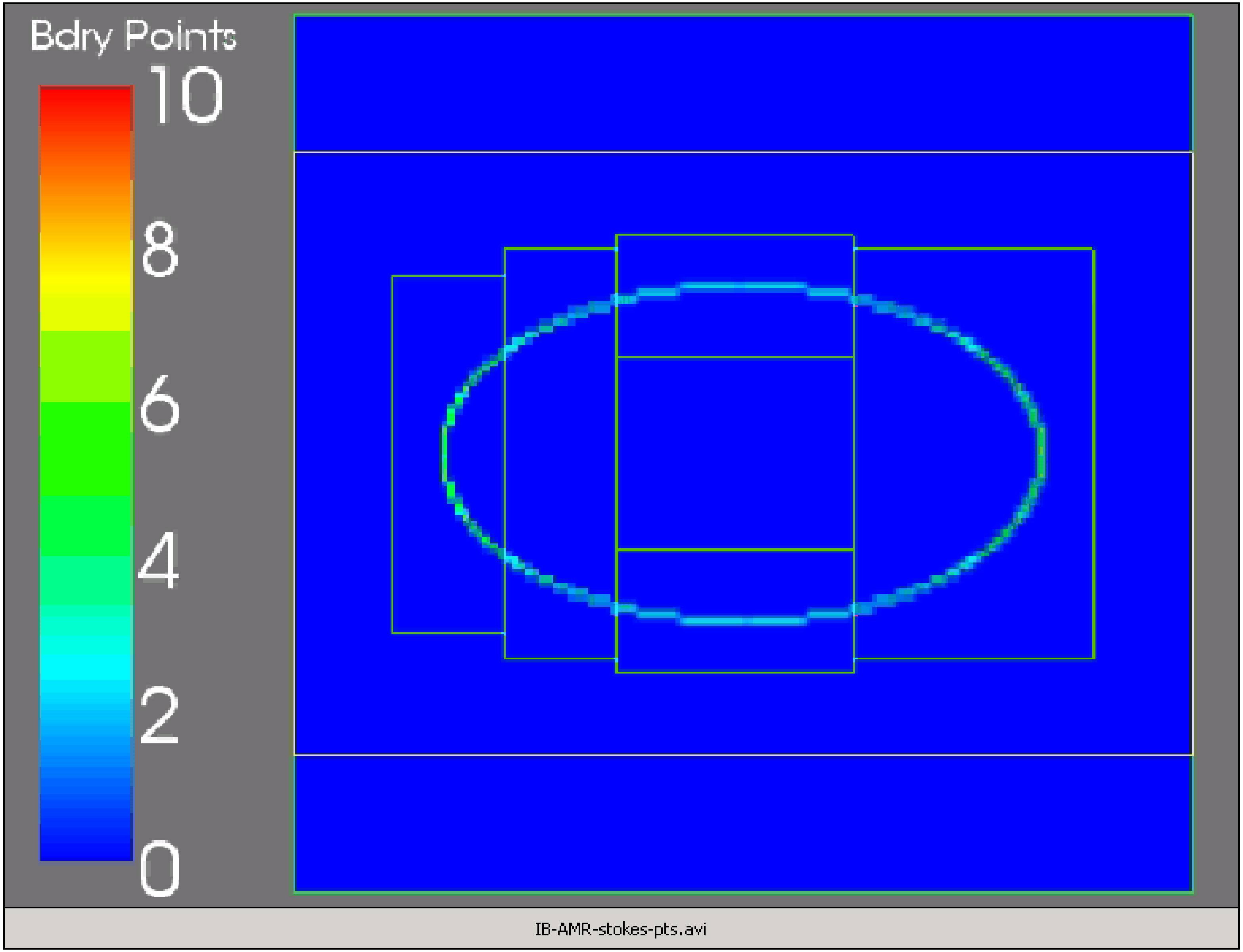
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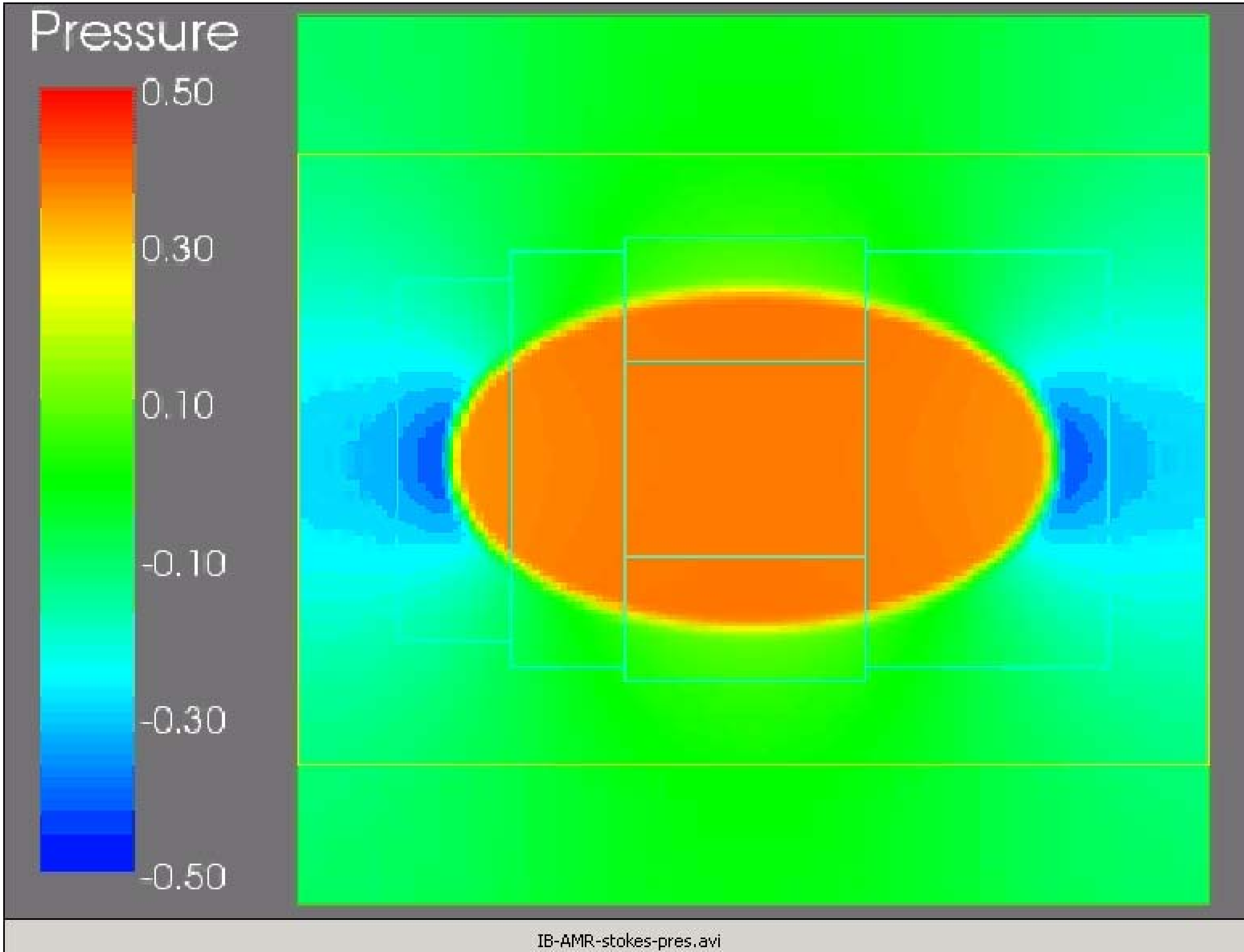
Pressure
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