Dynamics of a Gas Bubble in an Inclined Channel at Finite Reynolds Number

Catherine Norman

Michael J. Miksis

Northwestern University

An investigation of the dynamics of bubbles in vertical and inclined parallel walled channels

Applications:

- Multiphase flow
- Micro fluidics
- In the bloodstream (Decompression sickness)

Outline

- Formulation
- Numerical method
- Numerical results & comparion to experiments
  - Bond number
  - Reynolds number
  - Inclination angle

Formulation

Full Navier - Stokes equations:

- Continuity of Mass

$$\nabla \cdot \mathbf{U} = 0$$

- Momentum Equation

$$\frac{\partial \mathbf{U}}{\partial t} = -(\mathbf{U}\cdot\nabla)\mathbf{U} - \frac{\nabla p}{\rho} + \frac{\mu\nabla^2\mathbf{U}}{\rho} - \frac{\sigma\kappa(\phi)\nabla H(\phi))}{\rho} - \mathbf{g}$$

Boundary Conditions:

- Inflow and outflow
- No-slip along walls

Numerical Method

- Navier-Stokes solved with projection method
- Finite differences, 2nd order in space
- Multigrid conjugate gradient method to solve Possion equation for pressure.

Level Set Equation

Signed Distance Function

$$\phi(\mathbf{x},t)\begin{cases} >0 & \text{in the liquid,} \\ =0 & \text{on the interface} \\ <0 & \text{in the gas.}\end{cases}$$

Time Derivative

$$\phi\left(\mathbf{x}(t),t\right)=0$$

$$\frac{\partial\phi}{\partial t}+\mathbf{x}'(t)\cdot\nabla\phi=0$$

$$\boxed{\frac{\partial\phi}{\partial t}+\mathbf{U}\cdot\nabla\phi=0}$$

Use velocity on interface to maintain a signed distance function

$$\mathrm{F}=\mathbf{x}'(t)\;\hat{n}=\mathbf{U}\cdot\left(\frac{\nabla\phi}{\mid\nabla\phi\mid}\right)$$

$$\boxed{\frac{\partial\phi}{\partial t}+\mathrm{F}\mid\nabla\phi\mid=0}$$

Level Set Method

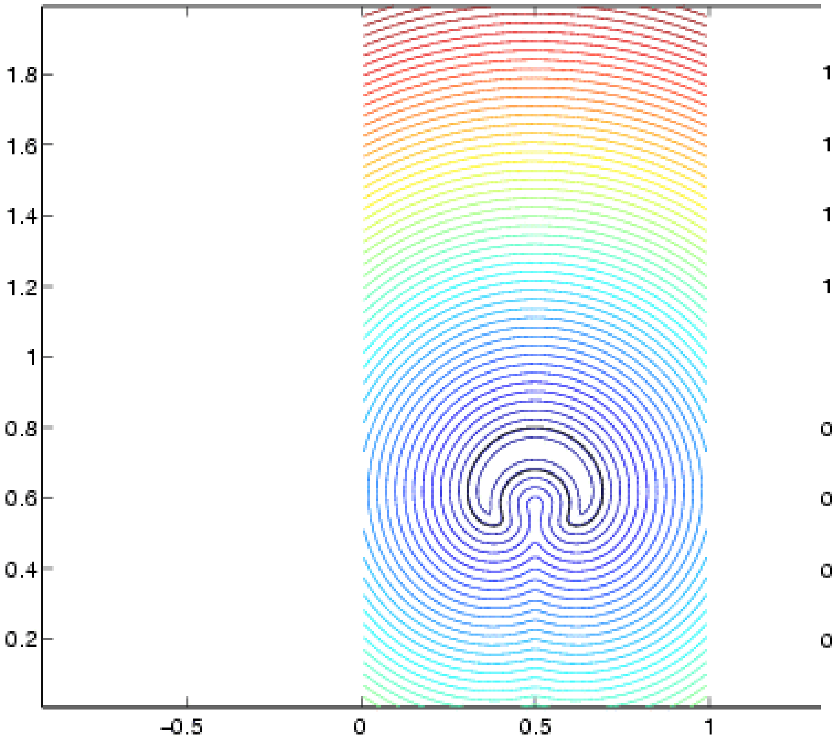
- Velocity Extension
  Use bicubic interpolation to find velocity near interface (Chopp 2001)

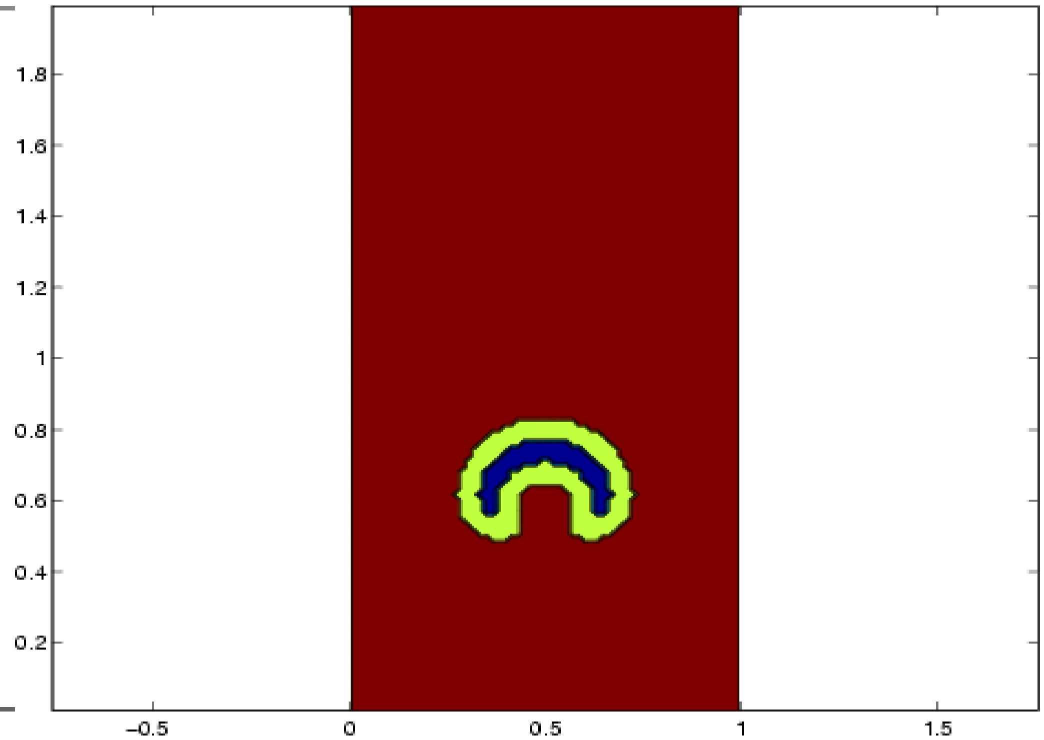
- Fast Marching Method to extend **F** further

- 2^{nd} order upwinding method
  to advance level set (Sethian 1999)

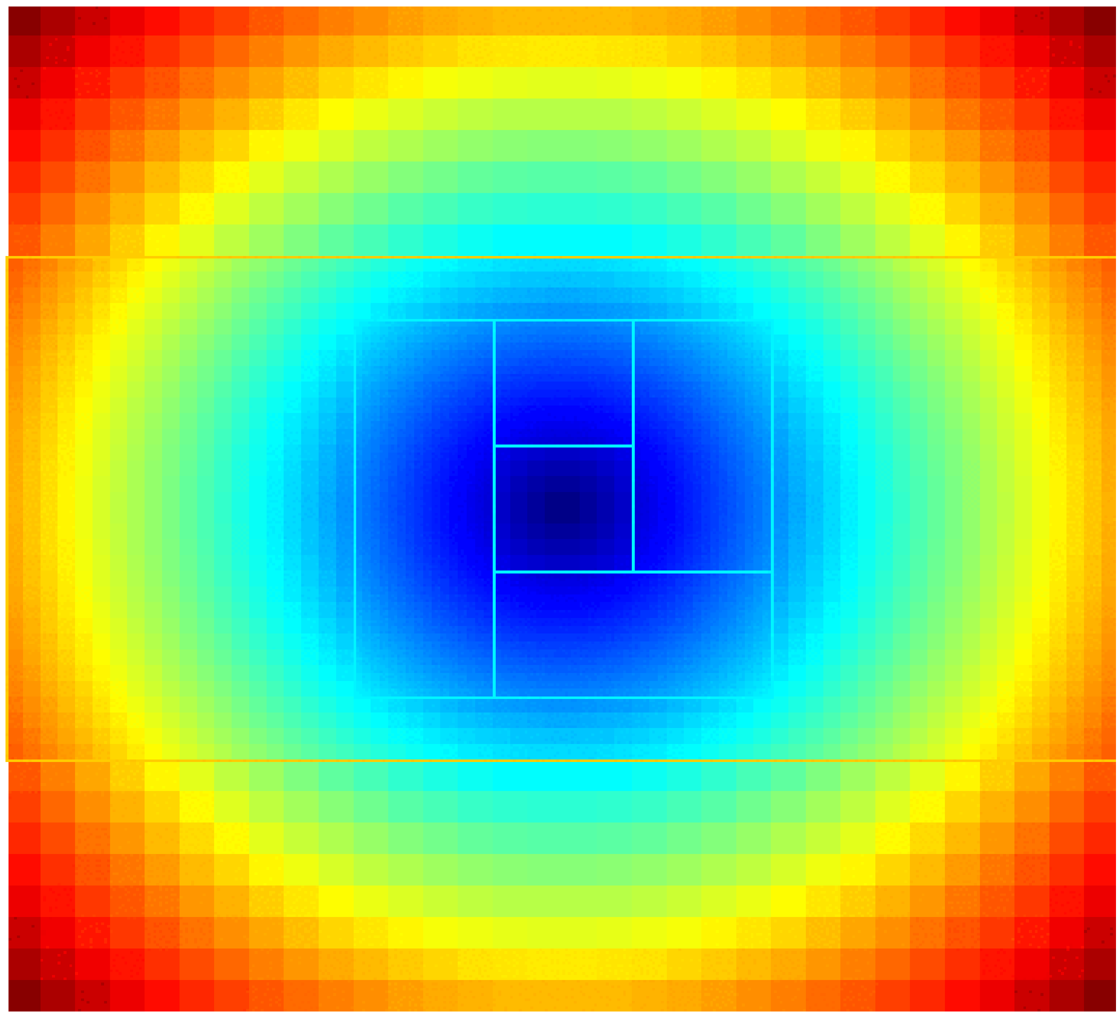
Narrow Banding

Only solve level set equation near interface





Adaptive Mesh Refinement



Parameters

- Initial value problem: Unit of velocity $U = \sigma / \mu$

- Reynolds number = Re = $\rho U r / \mu = \rho \sigma r / \mu^2$
  Re = 250, 1000, ..., 8000
  Re based on rise velocity 1 - 70.

- Bond number = $B = \rho g r^2 / \sigma$

- Channel width = 1
- Initial radius = r = 0.1425

- Density ratio, bubble / liquid = 0.01
- Viscosity ratio = 1/1

Results

- Vertical Channel
  - Steady
  - Periodic oscillations
  - Path instability, zigzag, spiral motion
  - Rupture
- Inclination angle study
- Contact line motion, start bubble on wall

Observed Rising Bubbles - Steady

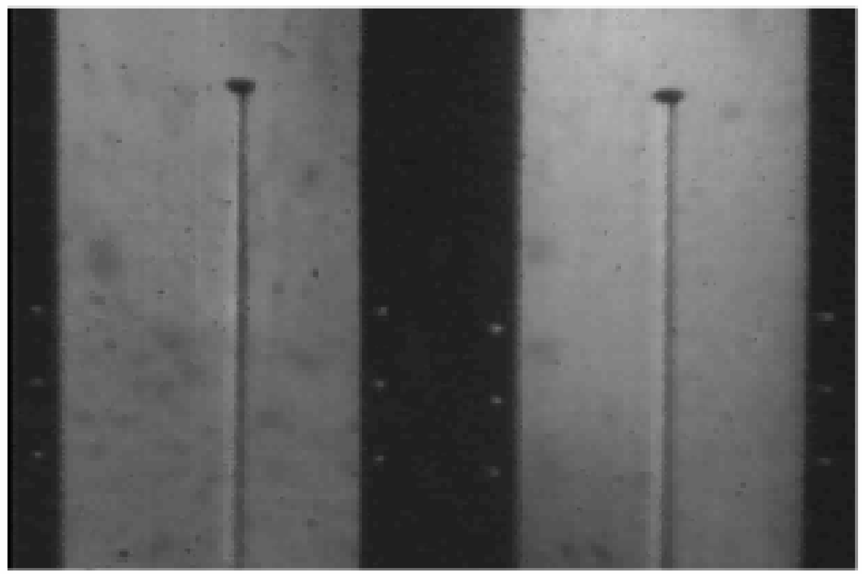


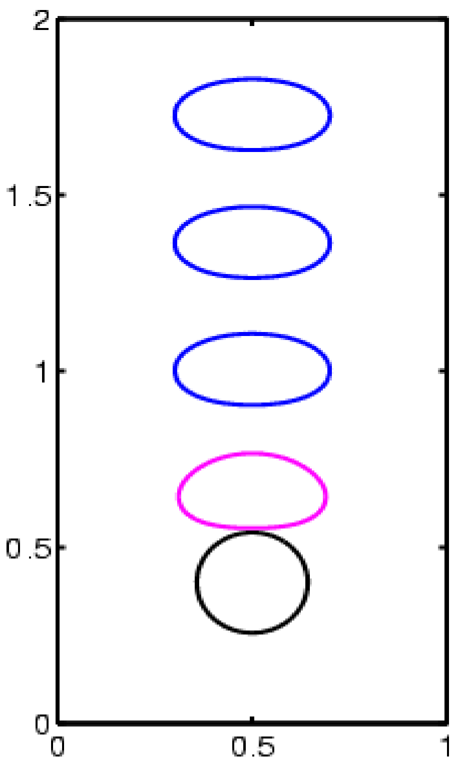
FIGURE 4.1: *The single-threaded wake behind a rectilinear rising bubble (r_{eq} = 0.79 mm). On the left the XZ view and on the right the YZ view. The black areas are part of the reference system outside the water tank. The walls of the tank and the mirror inside the tank are over 20 bubble radii away from the bubble.*

A.W.G. de Vries, Ph.D. Thesis 2001, Univ of Twente

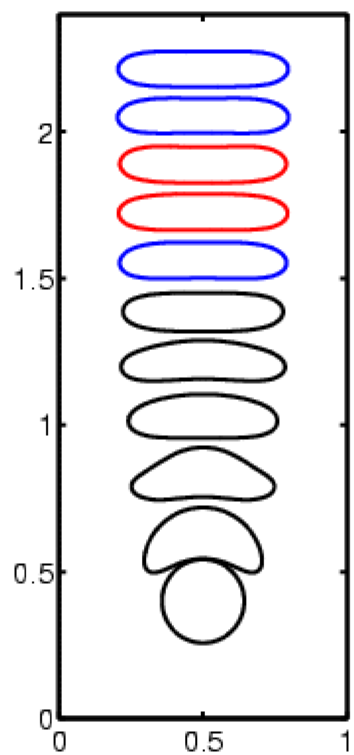
Schlieren visualization (refractive index gradient due to temperature)
View from XZ & YZ plane

Vertical Channel - Steady

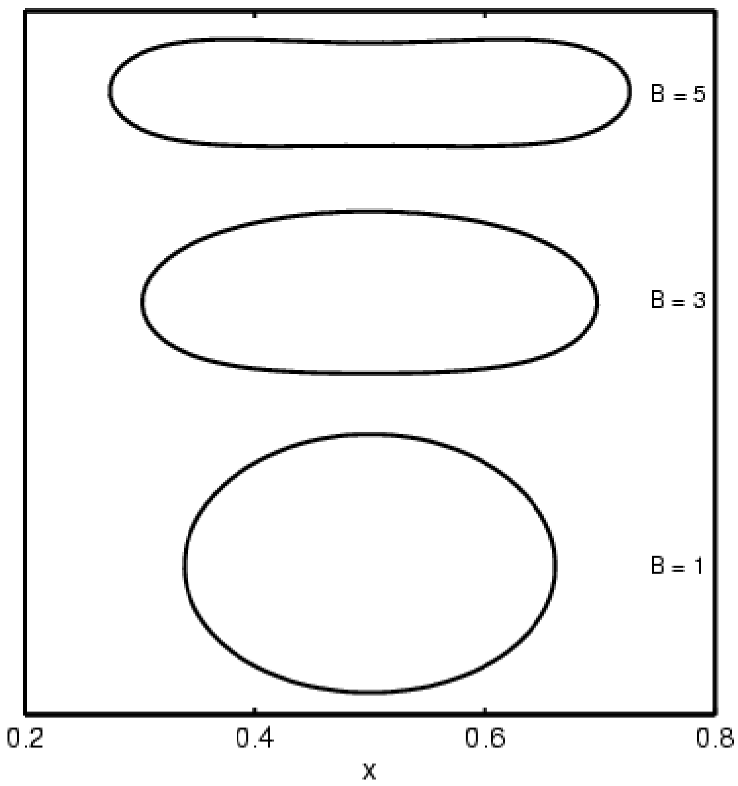
Re = 250



2D, B = 1

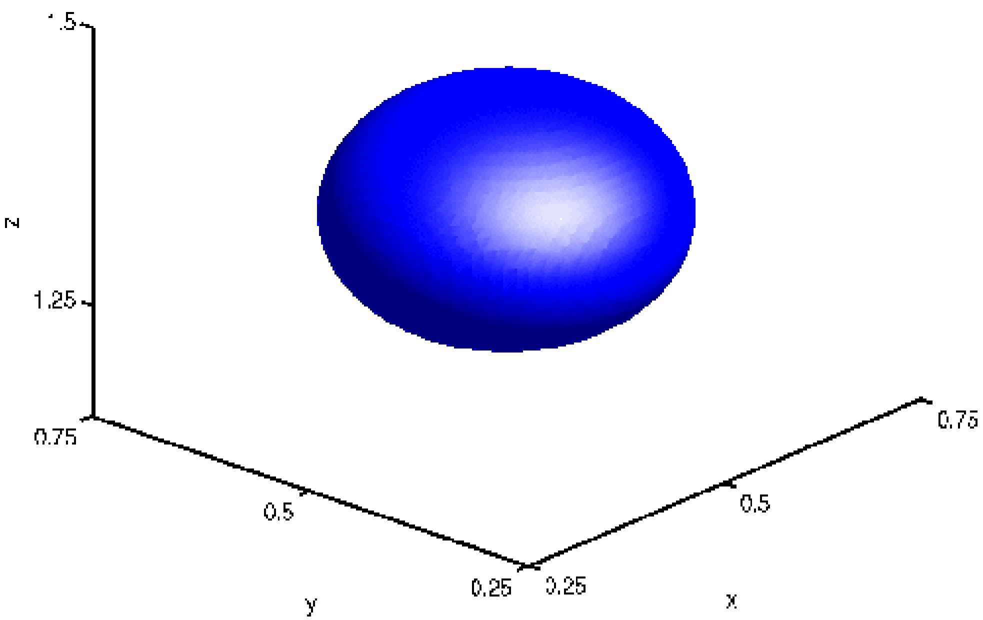


2D, B = 3

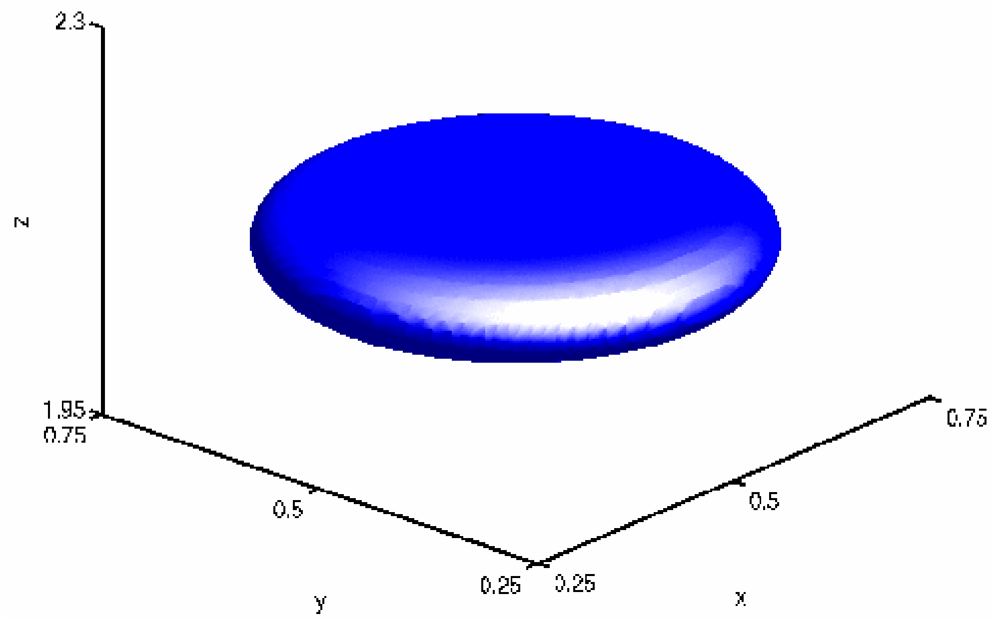


3D, Cross Sections
B = 1, 3, 5

Vertical Channel - Steady



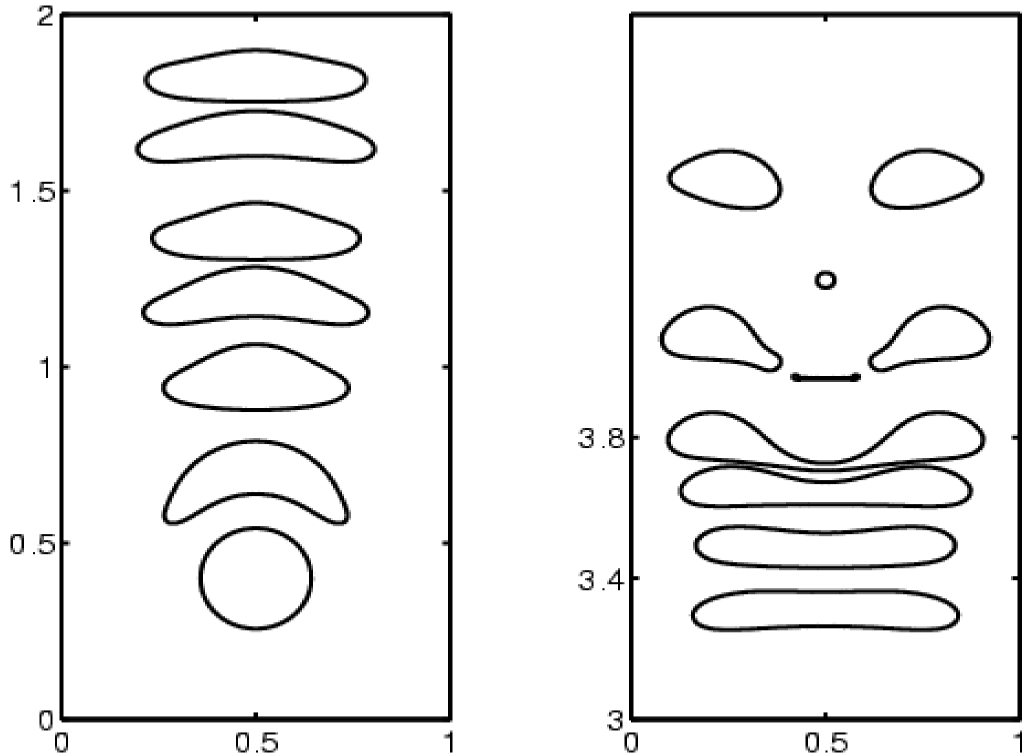
B = 1



B = 5

Vertical Channel - Rupture (1)

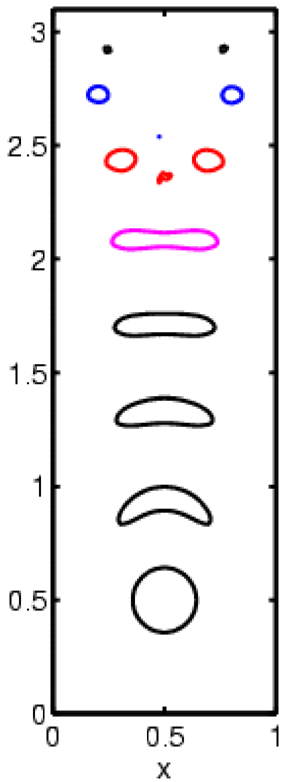
Re = 250

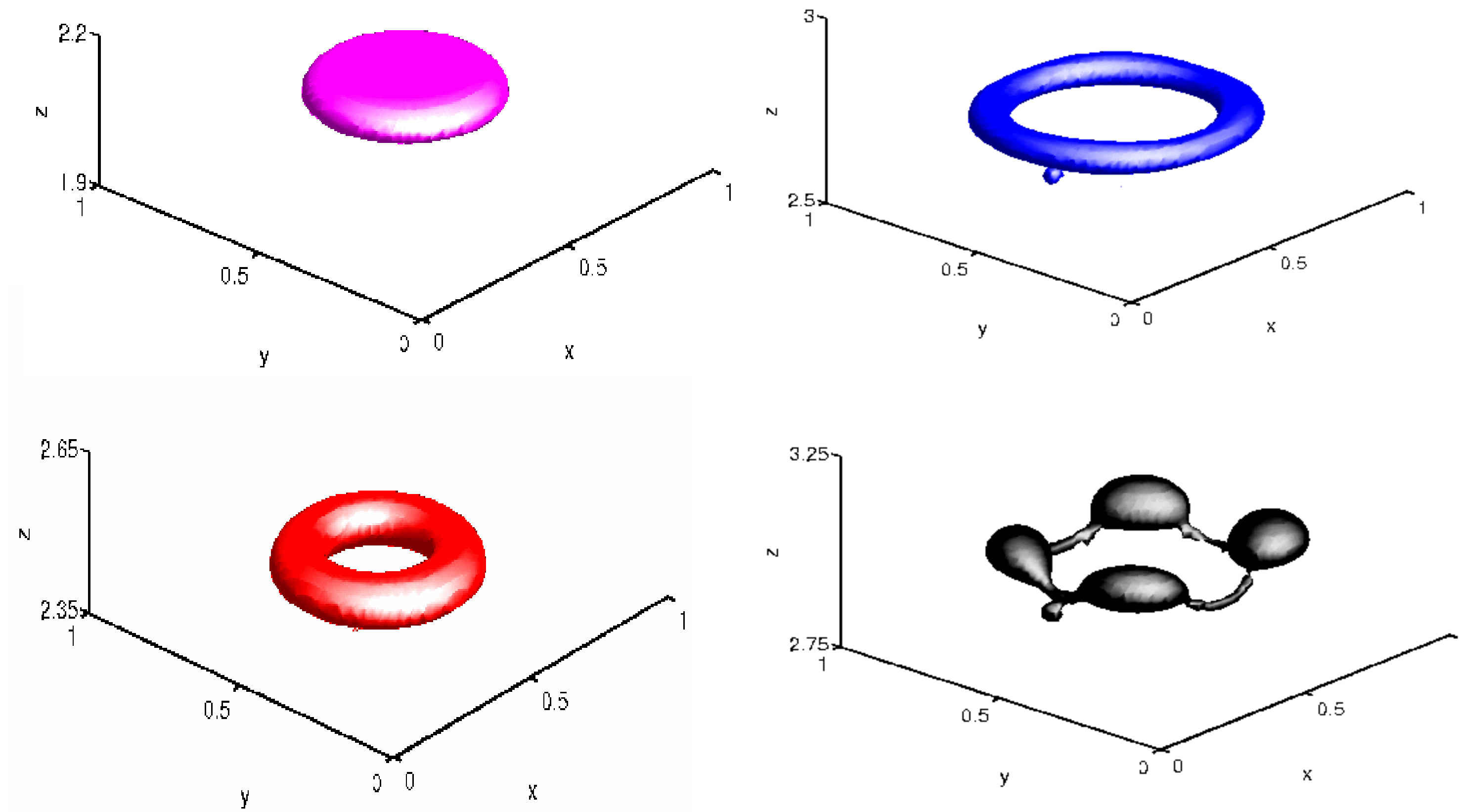


2D, B = 4

Vertical Channel - Rupture (1)

Re = 250

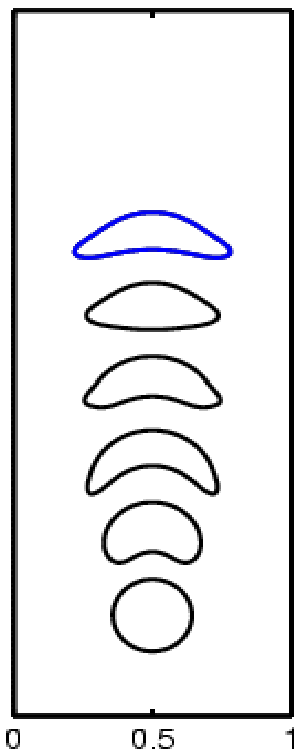




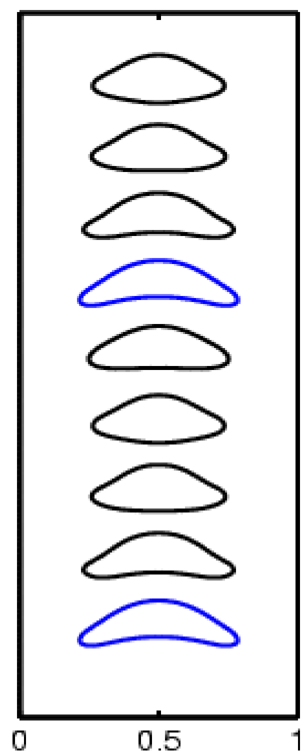
3D, B = 7

Vertical - Periodic

2D, Re = 250

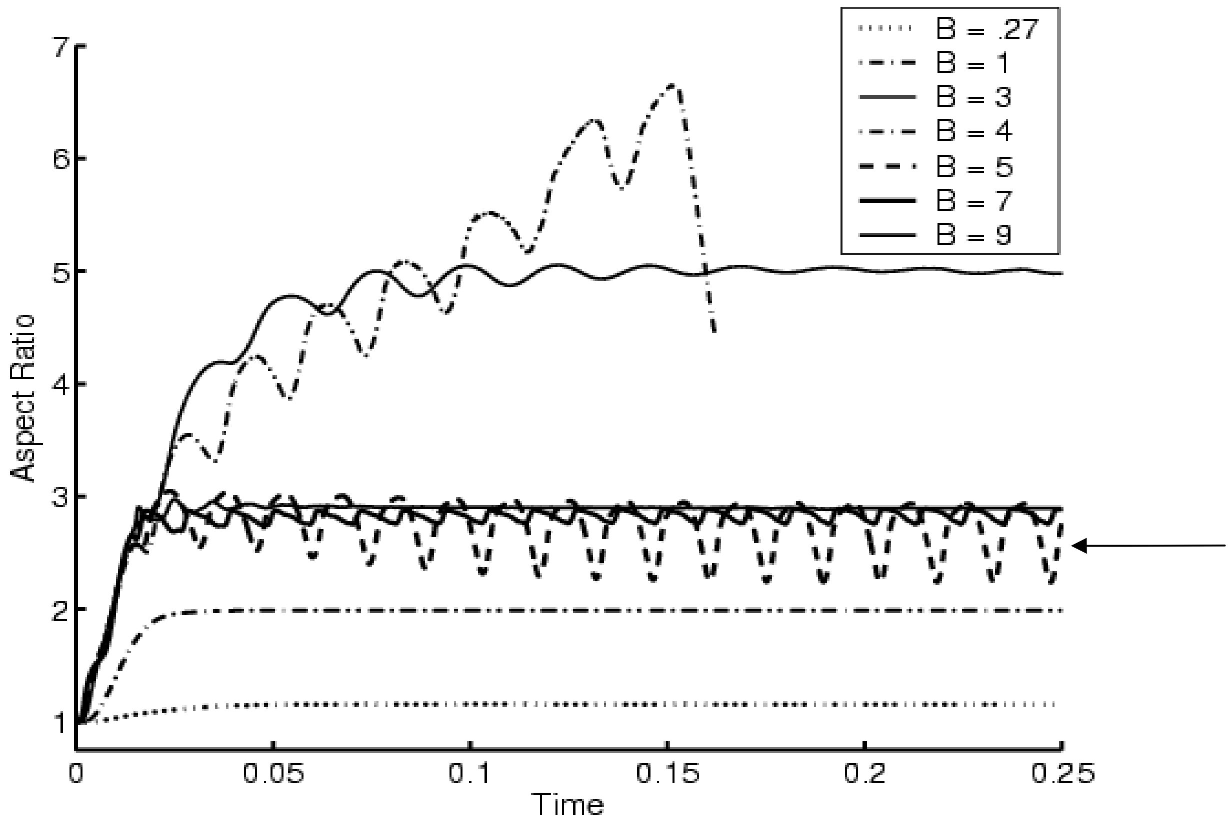


B = 5,
Initial



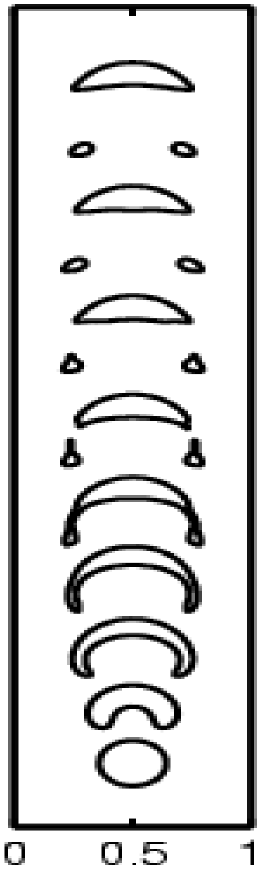
B = 5,
Periodic Shape Oscillations

Vertical Channels Summary

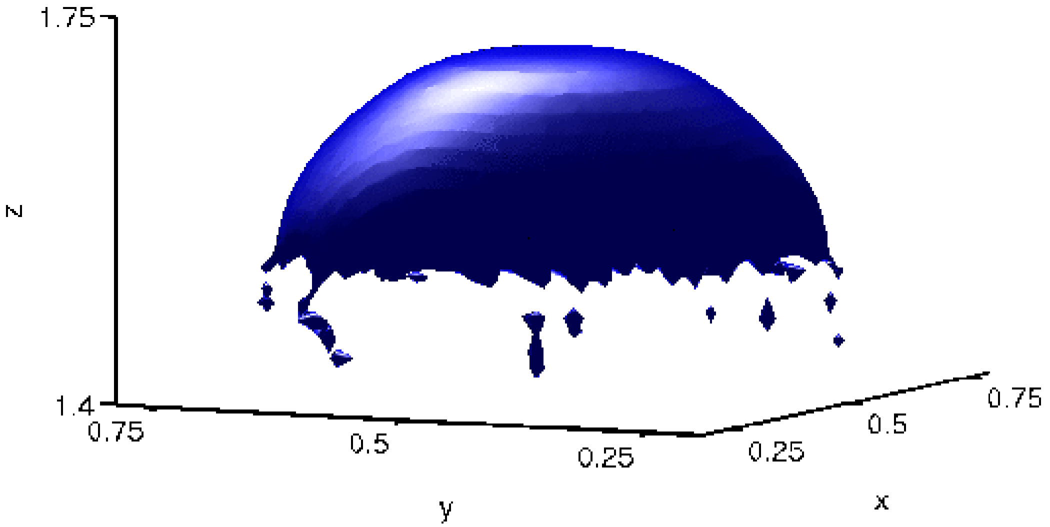


B = 5, Periodic Shape Oscillations

Vertical Channel - Rupture (2)



2D, B=15



3D, B=15

Zigzag Motion

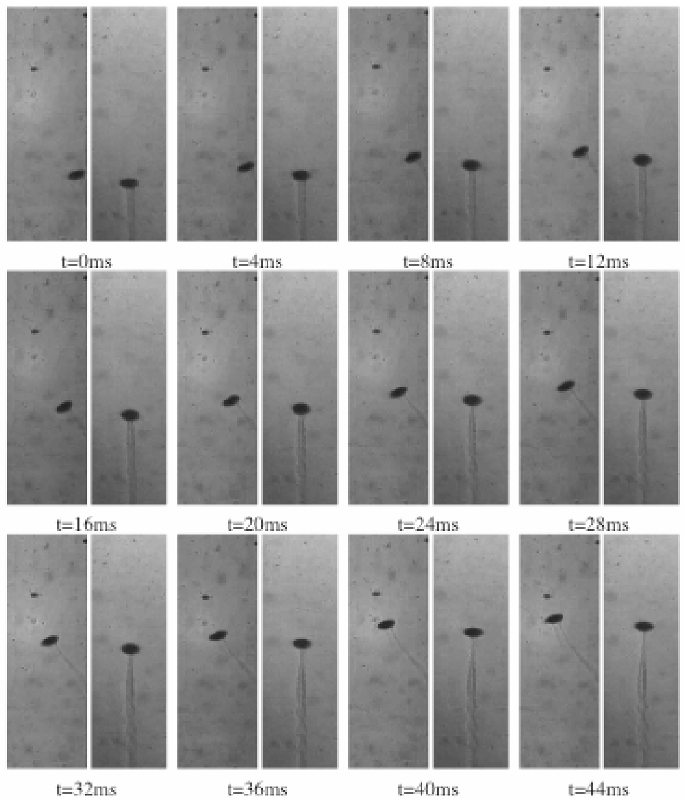


FIGURE 4.4: *Successive schlieren images of a bubble (r_{eq} = 1.00 mm) in zigzagging motion. Each pair of images contains the XZ and YZ view, respectively. Note that in the YZ-view the path is straight while it is sinusoidal in the XZ-plane. Furthermore, the wake of the bubble is a double-threaded wake, unless the curvature of the path is zero (starts at $t \approx 24$ ms) in the mean position of the zigzag. The wake reconnects and in the following the occurrence of an instability is observed in the wake. It is clear that the zigzag is* NOT *maintained by vortex shedding at the maximum of the sinus in the XZ-plane ($t \approx 64$ ms).*

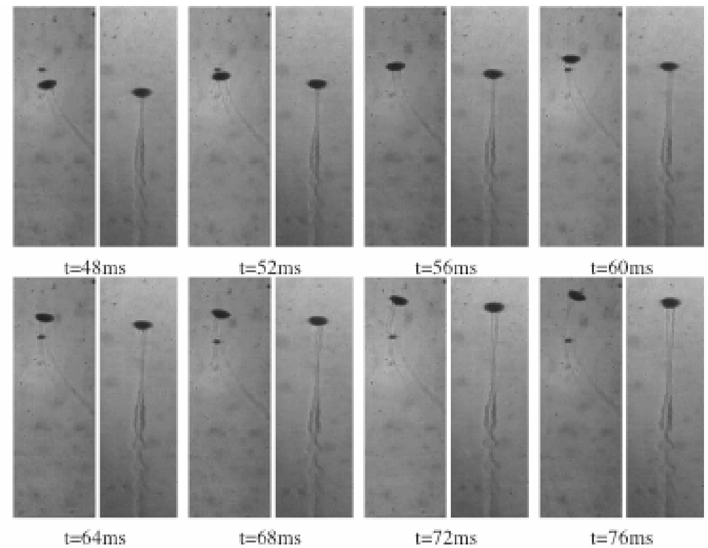


FIGURE 4.5: *Continuation of schlieren images of a zigzagging bubble*

- Larger Re number
- Zigzag in XZ
- Steady rise in YZ
- No vortex shedding

A.W.G. De Vries, Ph.D. Thesis 2001, Univ of Twente

Spiraling Motion

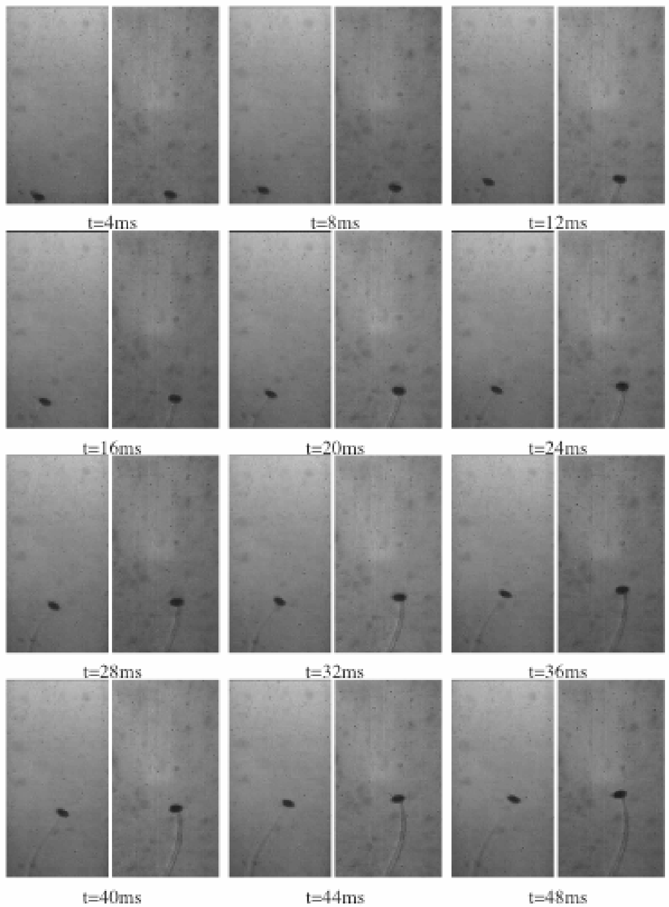


FIGURE 4.9: *Successive images, XZ and YZ views, of a bubble (r = 1.01 mm) in spiralling motion. The wake consist of a double-threaded wake, which becomes unstable far behind the bubble (not visible).*

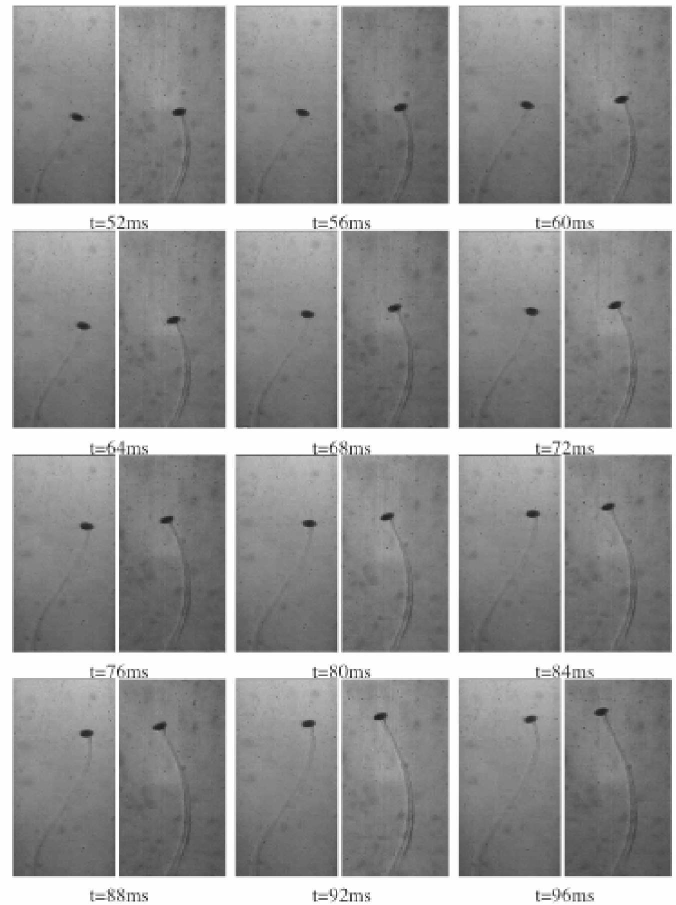
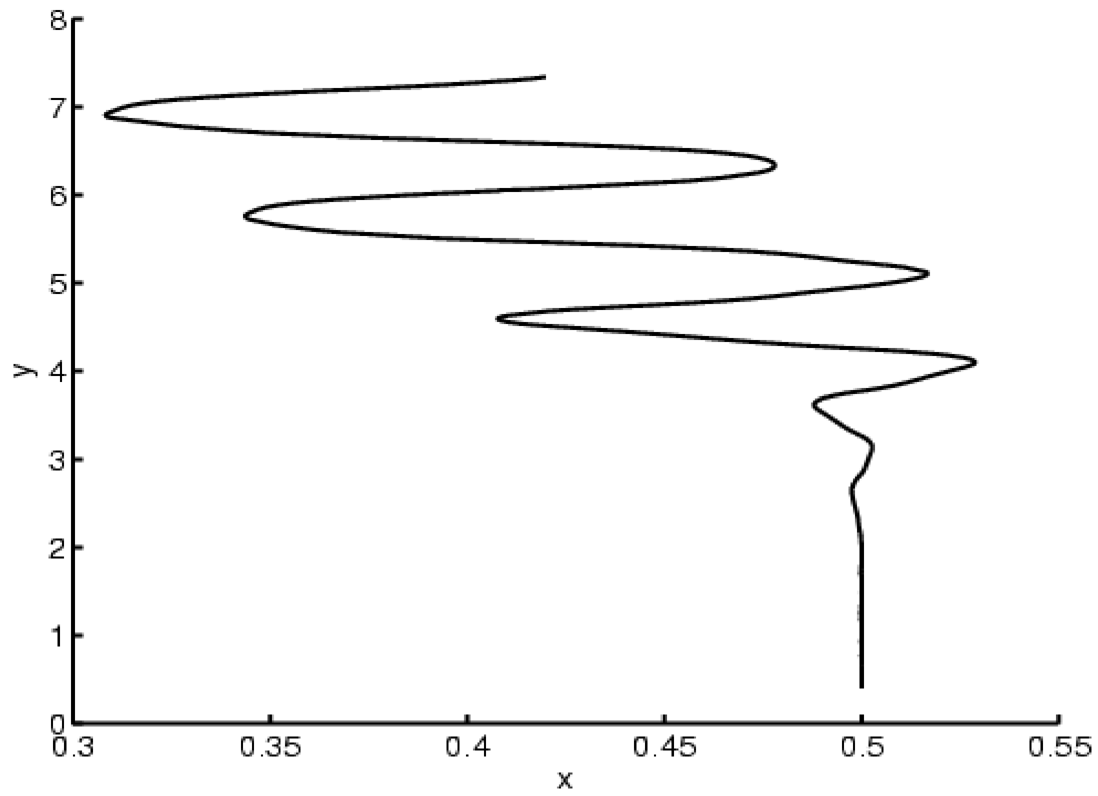


FIGURE 4.10: *Successive images of spiralling motion r_{eq} = 1.01 mm (continued).*

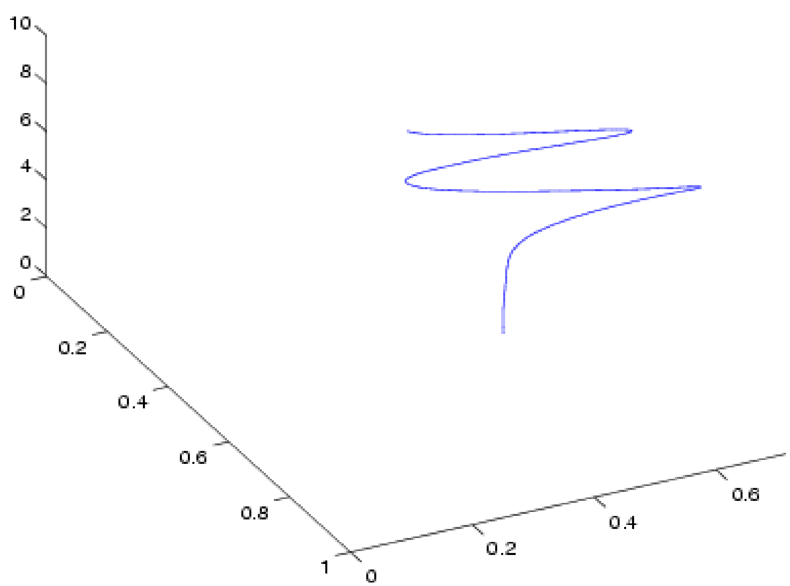
Path Instability - Zig Zag

Center of Mass oscillates due to small noise at large Re.

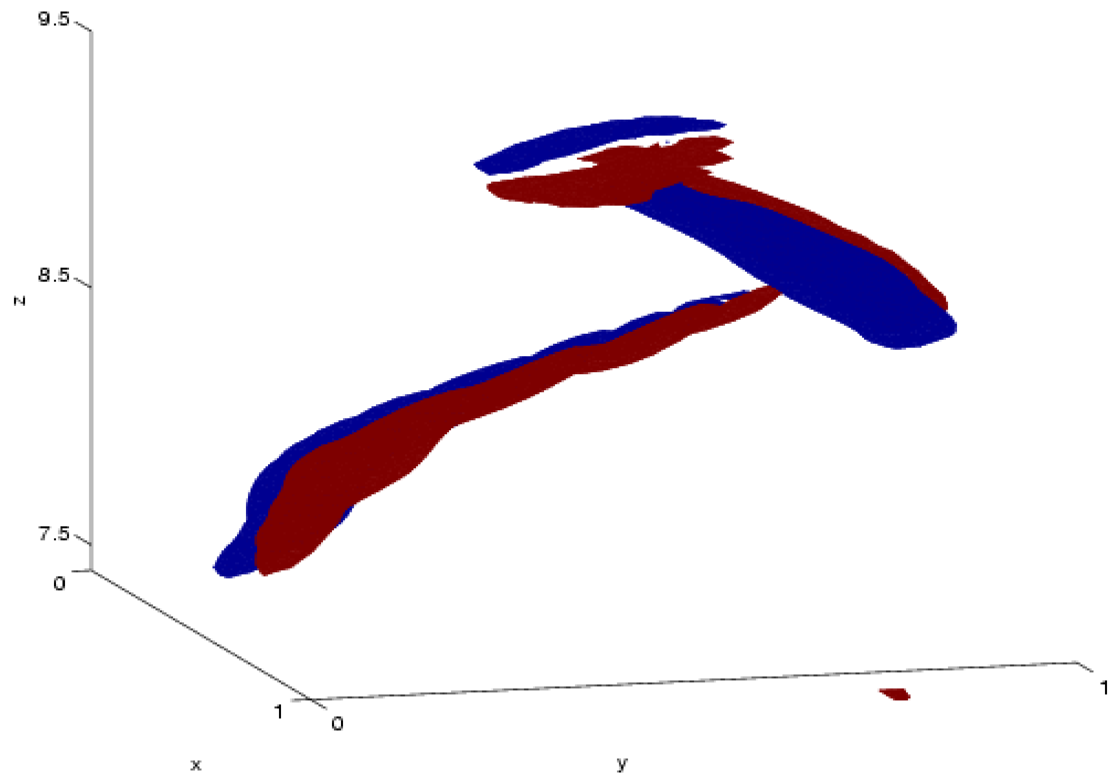


Re = 8000; B=1, 2D

Path Instability - Spiral



Path of the center of mass of the rising bubble

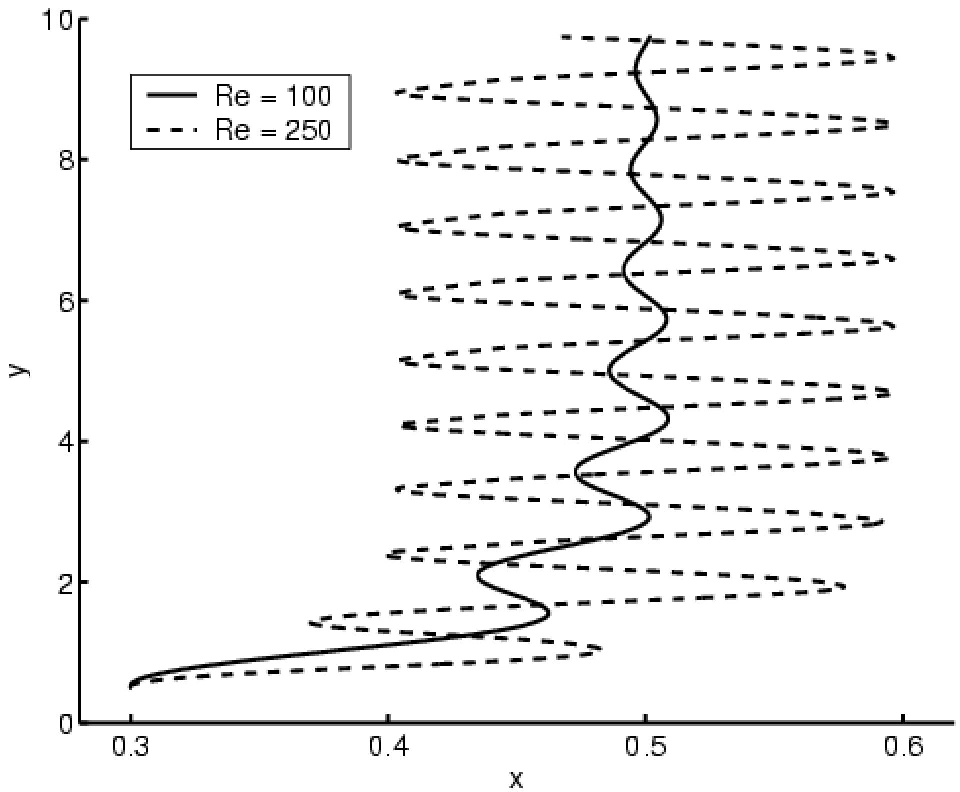


Isosurfaces of the streamwise vorticity, showing the spiral wake path

Re = 8000, B = 1

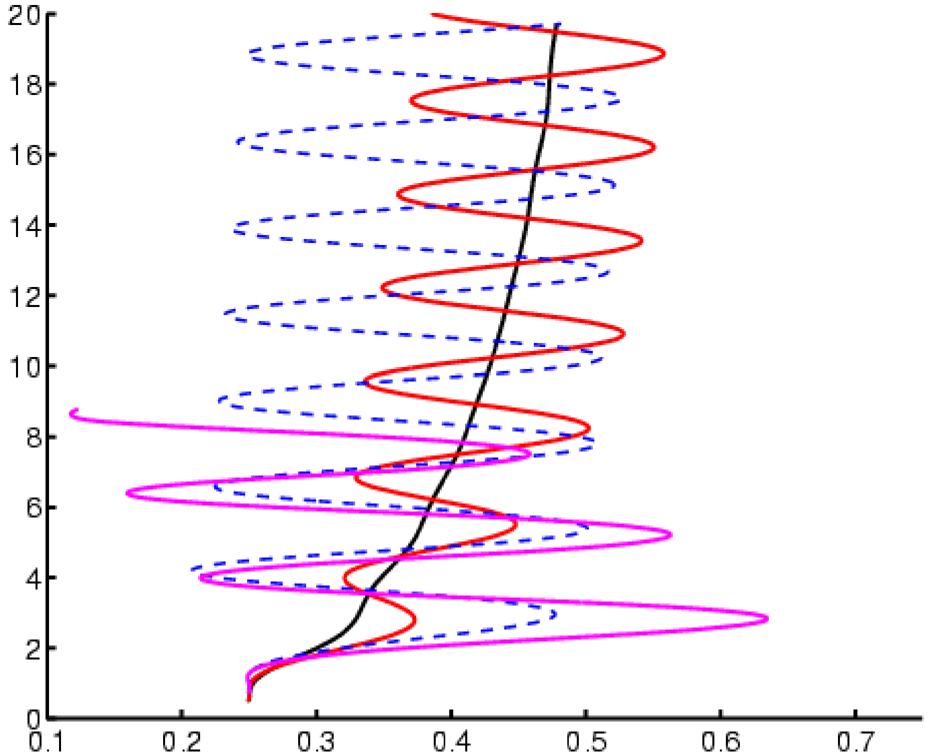
Start off Center - Zig Zag

Large displacement of initial data leads to zigzag motion in 2D, at higher Re.

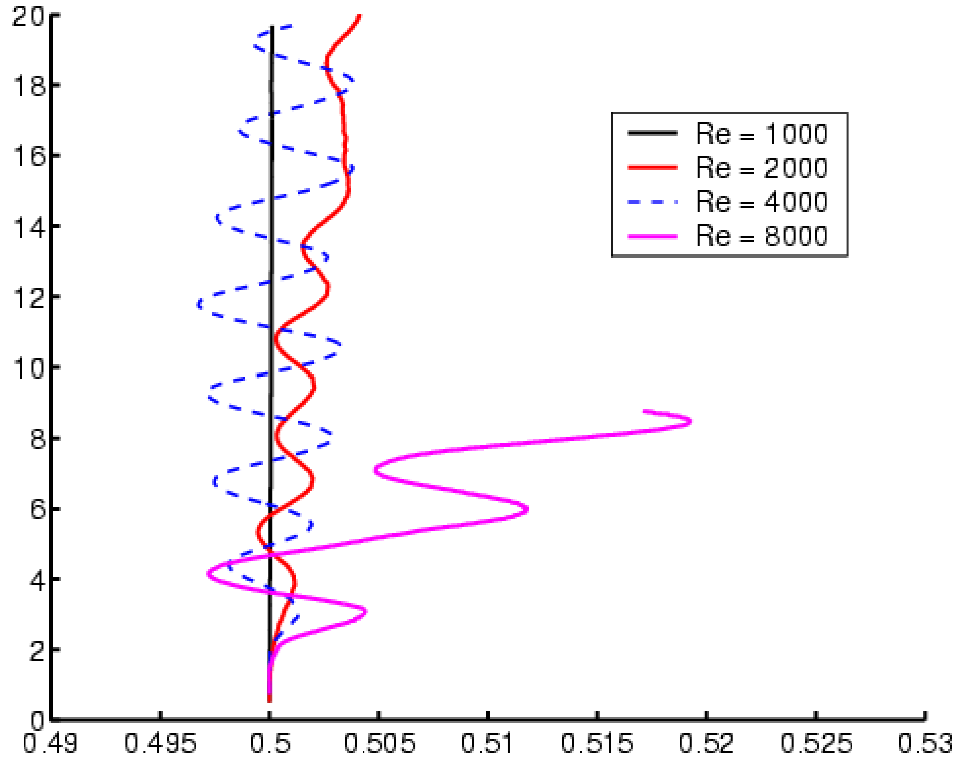


Start off Center 3D

Low Re, steady motion.
Higher Re, Zig Zag, Spiral?

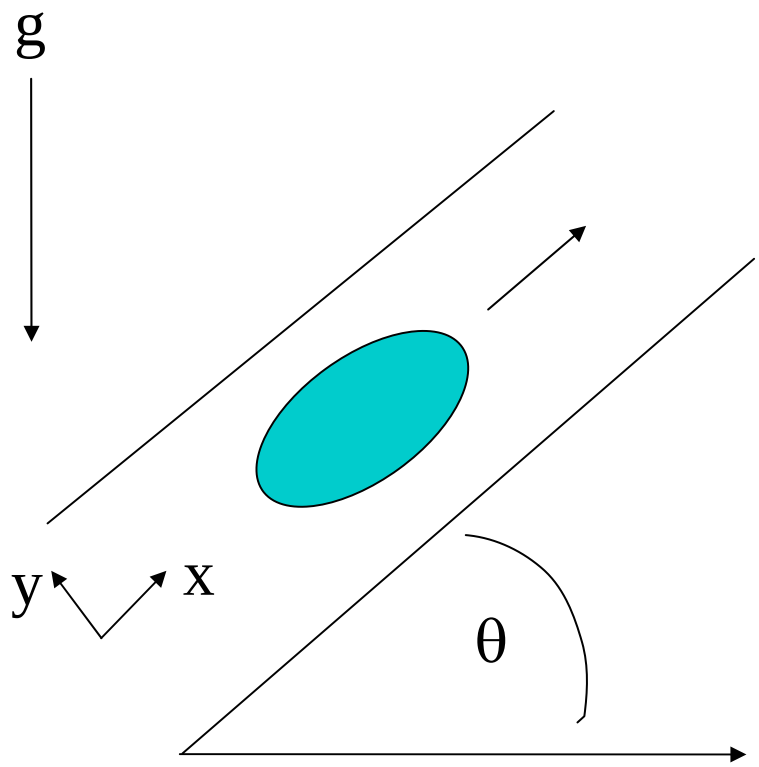


X-Z axis



Y-Z axis

Bubble rising in an inclined channel



Observed Steady Bubbles

- Deformation increases with angle

- Rise velocity &
- Distance from wall decrease with angle

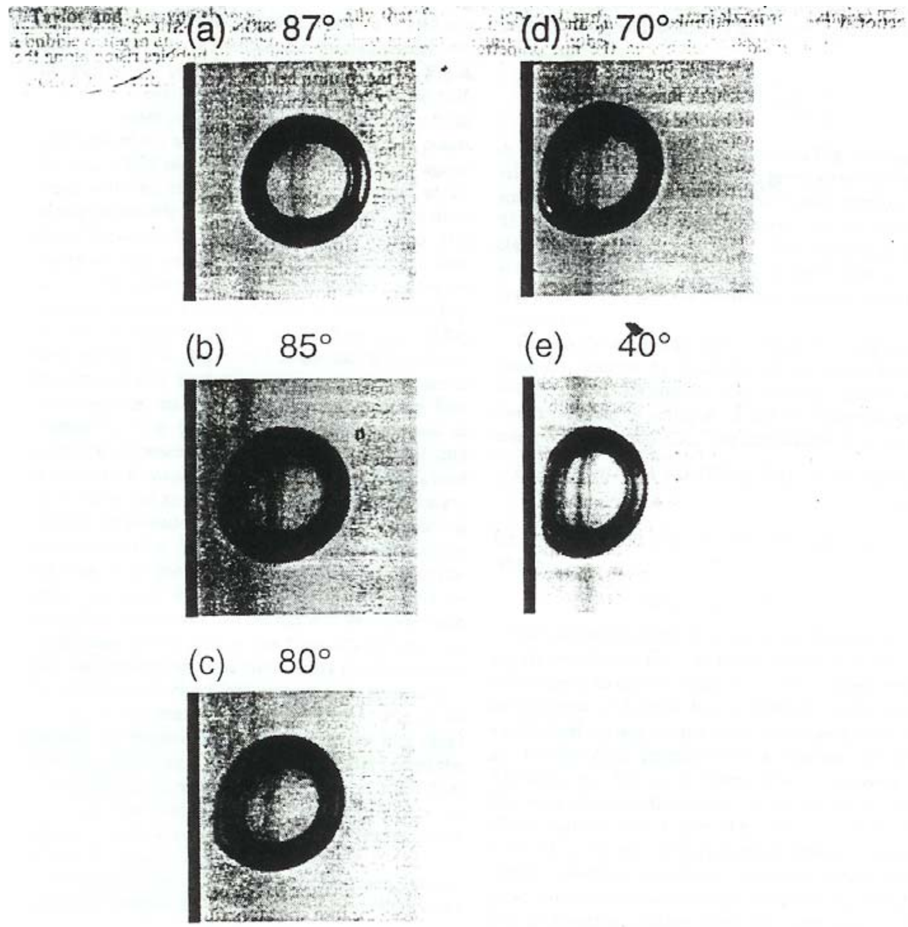
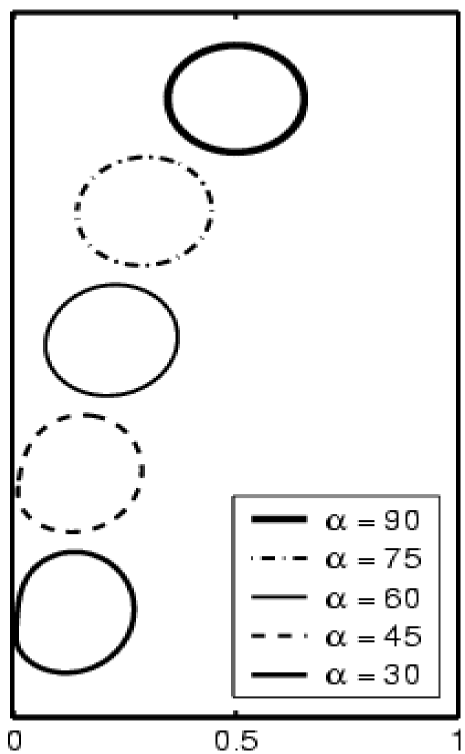


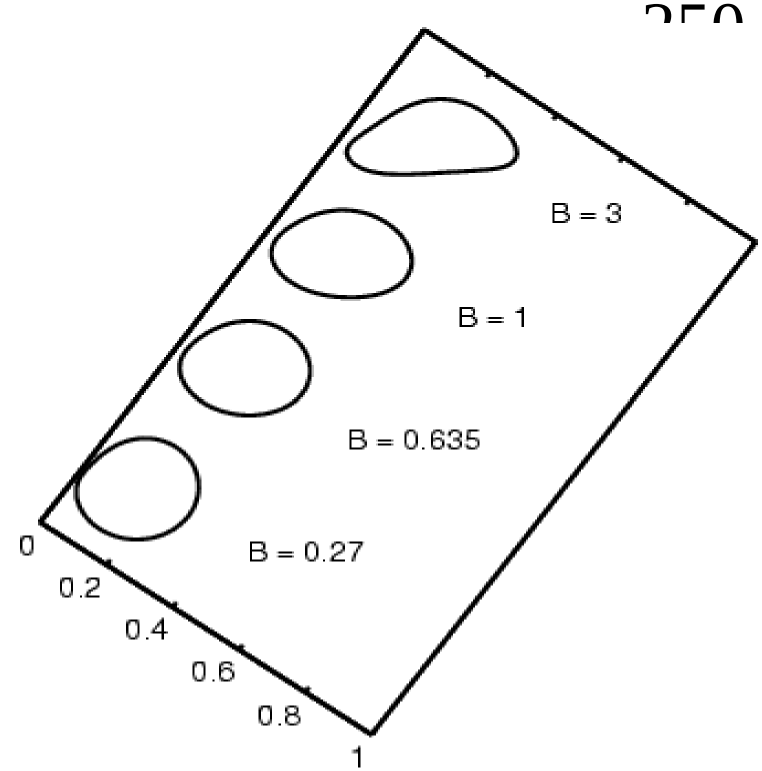
Fig. 3. Shape of air bubbles and separation gap width at various surface inclination angles from the horizontal, de ≅ 0.0028 m.

Masliyah, Jauhari & Gray '94

Effect of Angle --- Steady Bubbles

Re =
250





Distance to wall decreases with angle
(B = 0.27)

Distance to wall decreases with B
($\theta = 45^o$)

Observed Bouncing Bubbles

- Bubble bounced if angle from the horizontal > 55^o

- At large angles, bubbles bounced repeatedly without loss of amplitude

- At small angles, bubbles slid steadily along the wall

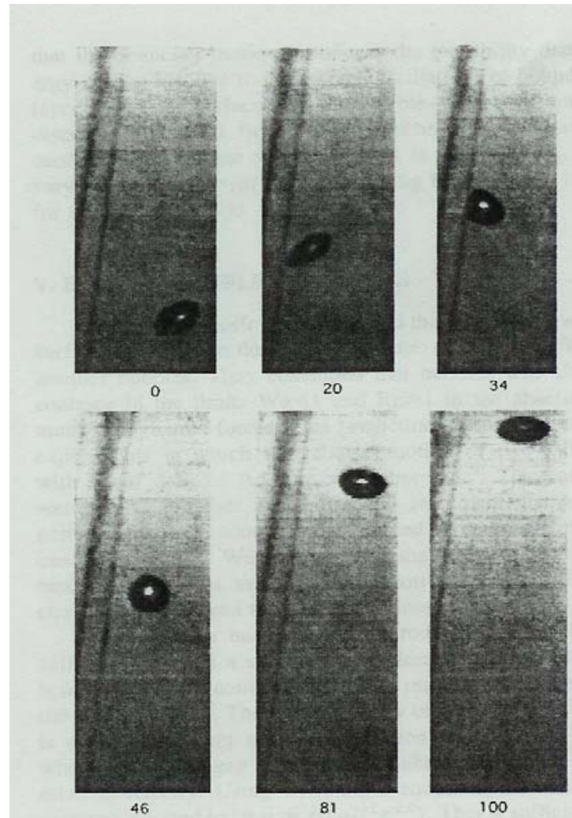
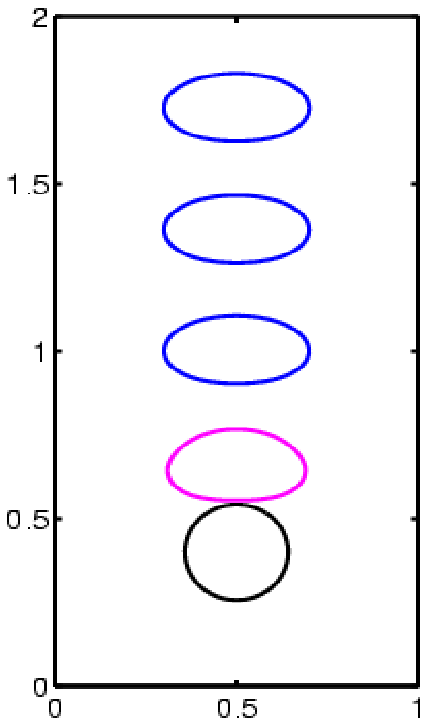


FIG. 12. Sequence of photographs illustrating one cycle of the bu bouncing motion. The bubble radius is 0.7 mm and the inclination ang 83°.

Inclination angle = 83^o

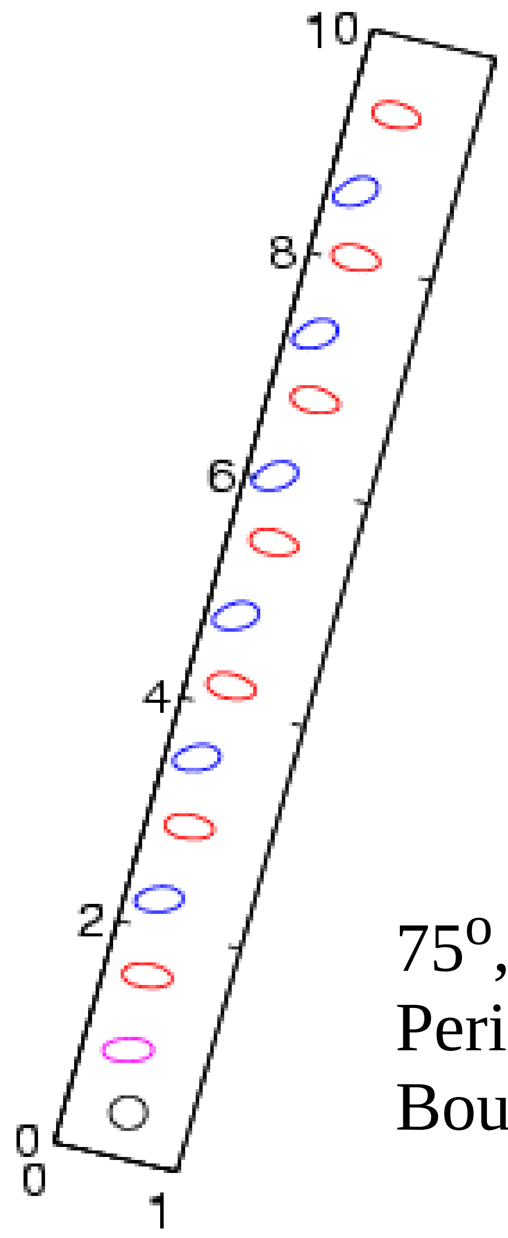
Tsao & Koch '97

Effect of Angle of Inclination

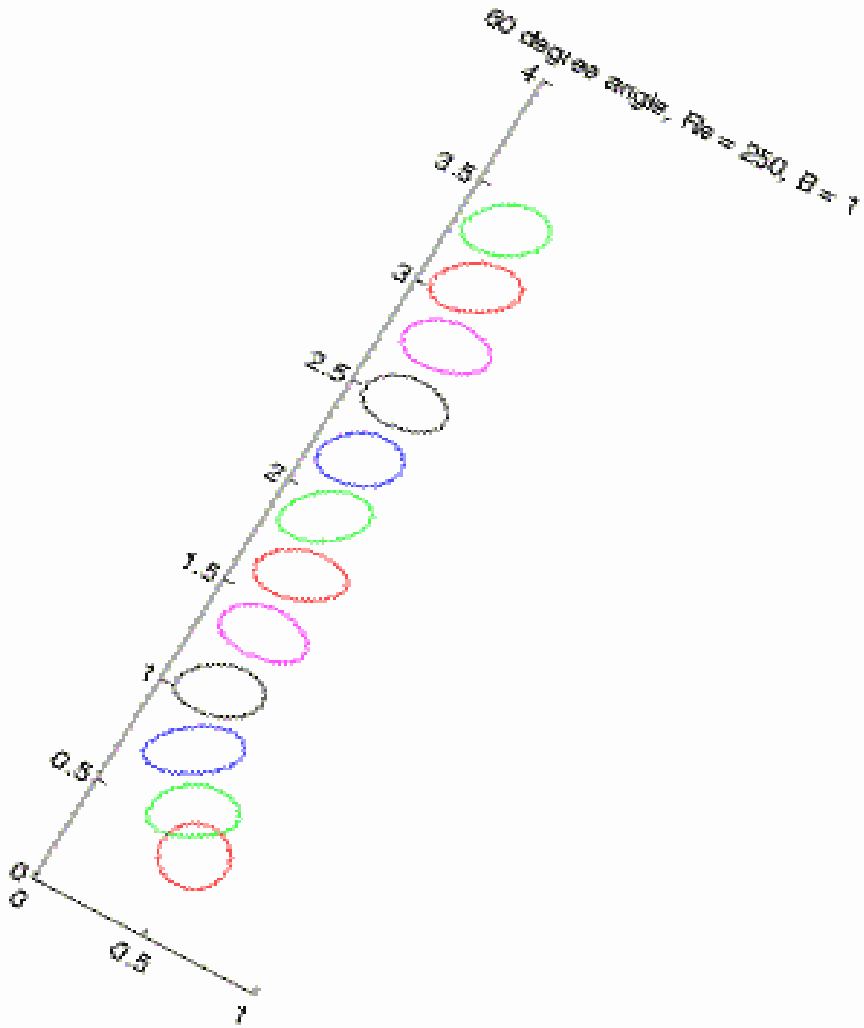


90^o, Steady

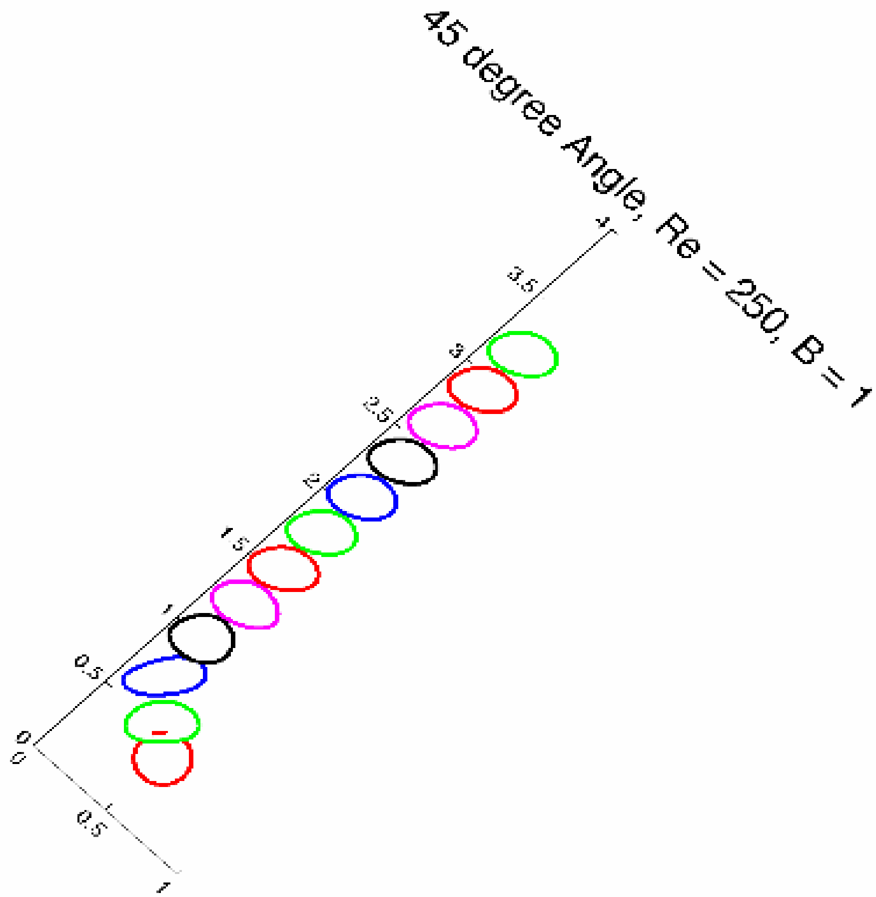
Re = 250,
B = 1



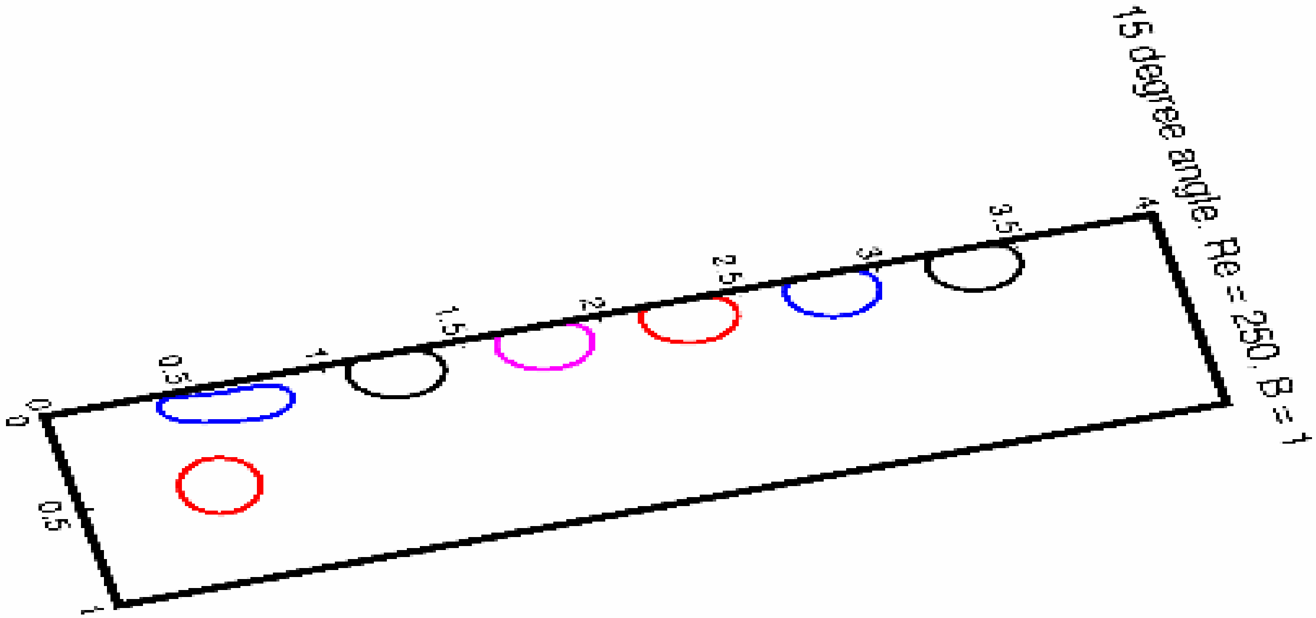
75^o, Periodic Bouncing



60^o, Small Amplitude
Damped Bouncing



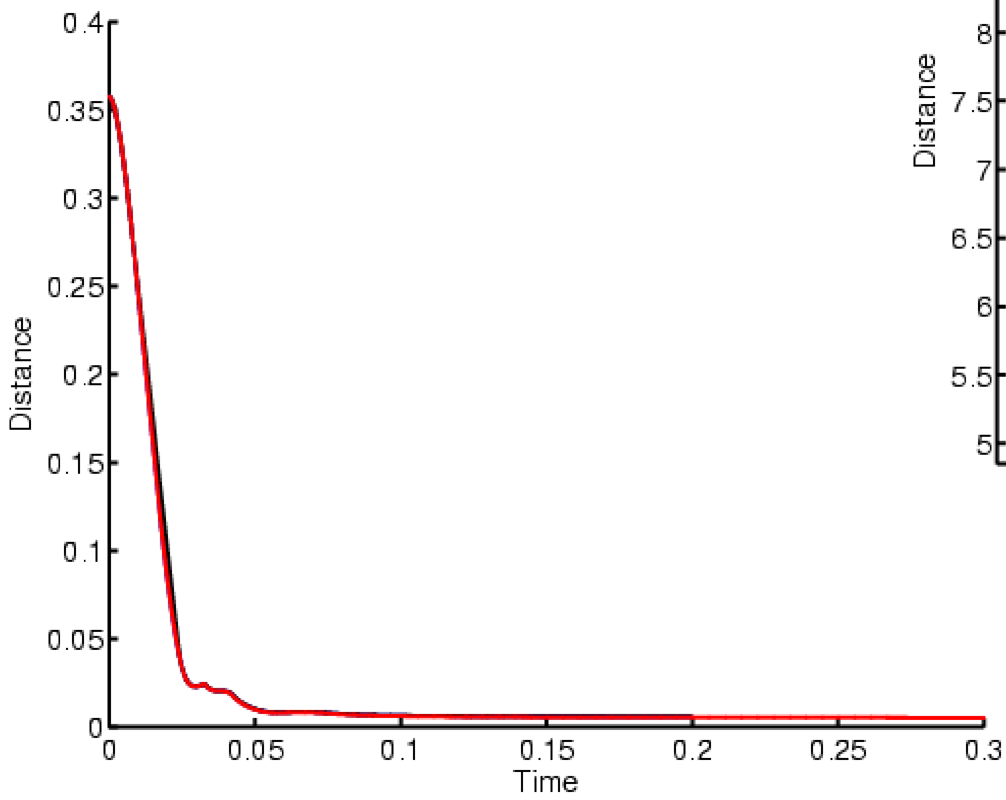
45^o, Steady

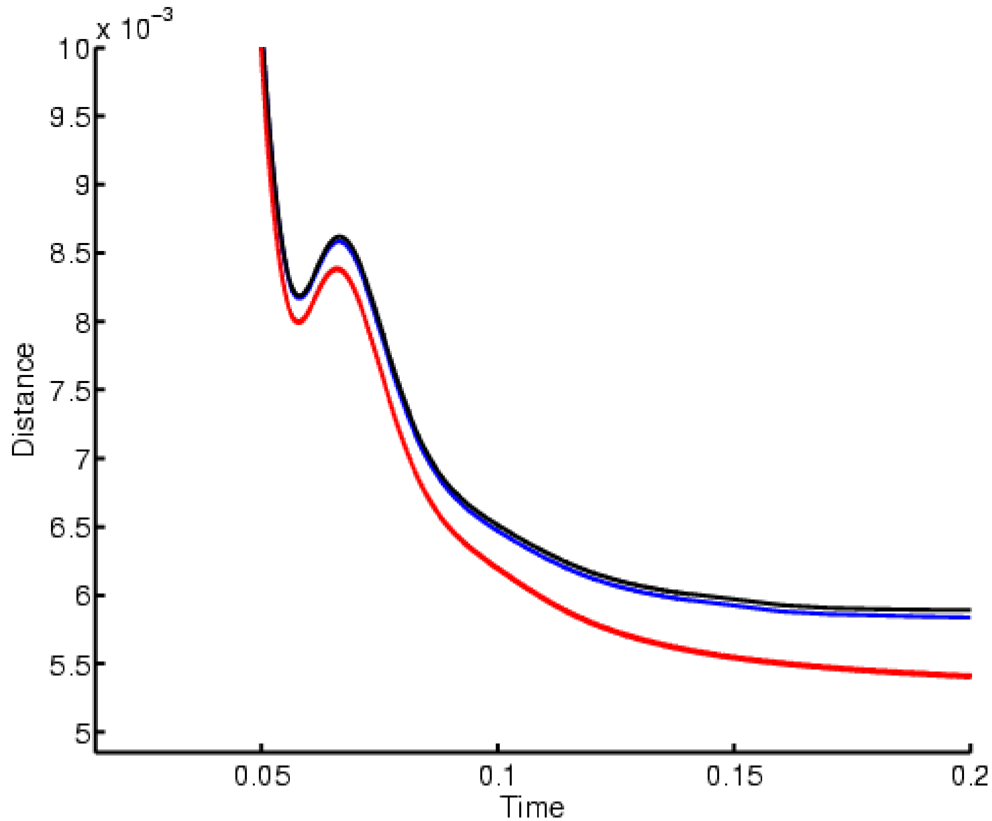


15°, Steady

Wets top of Channel? -- No.

Distance to Wall, $\theta = 15^o$



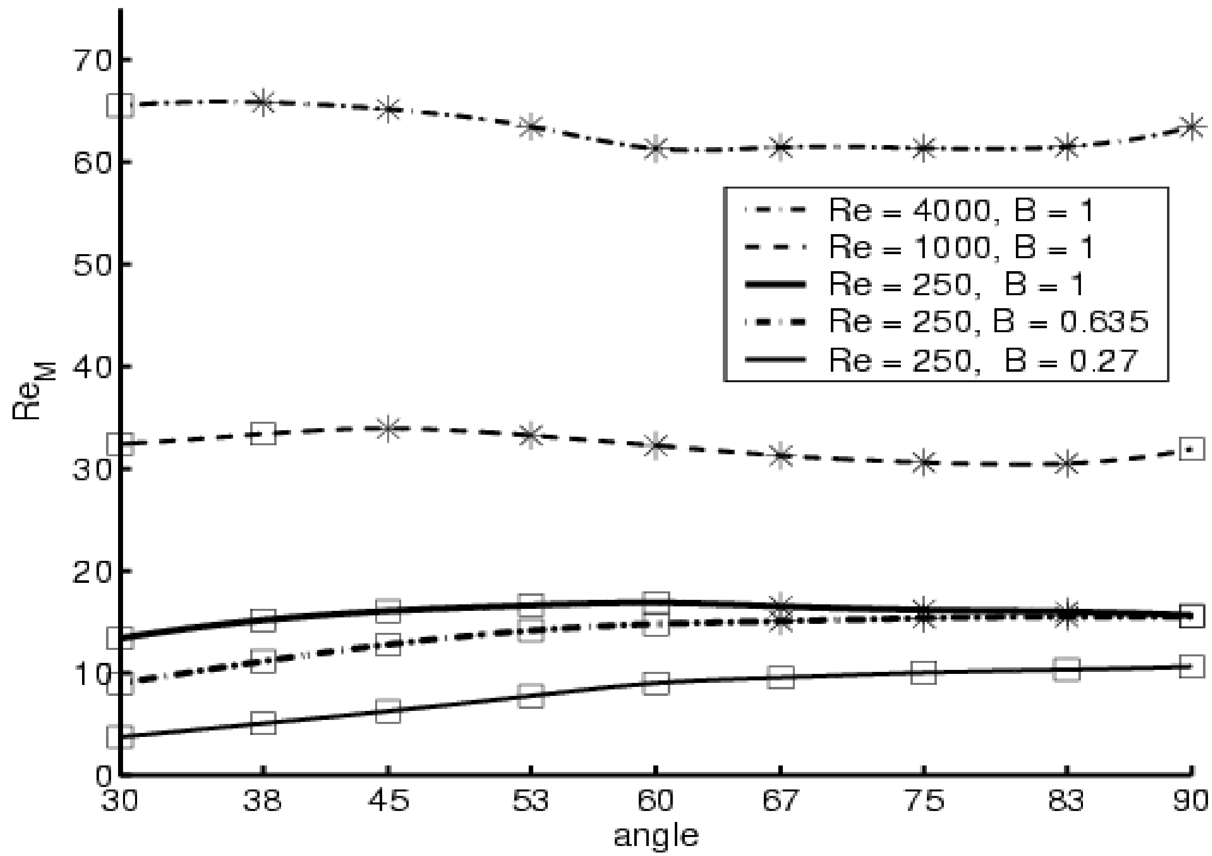


Red = lowest resolution
Black = highest

Reynolds Number vs. θ

$Re_M = \rho U r / \mu$
= Reynolds number
based on rise velocity

- U =average velocity
- Square = steady
- Star = bouncing, oscillating



Maximum in velocity (Re_M) only for Re=250, B=1
➔ Window of parameter space where Re_M vs. angle has a maximum.

Re_M increases with θ for B =0.27

Bond number vs. angle

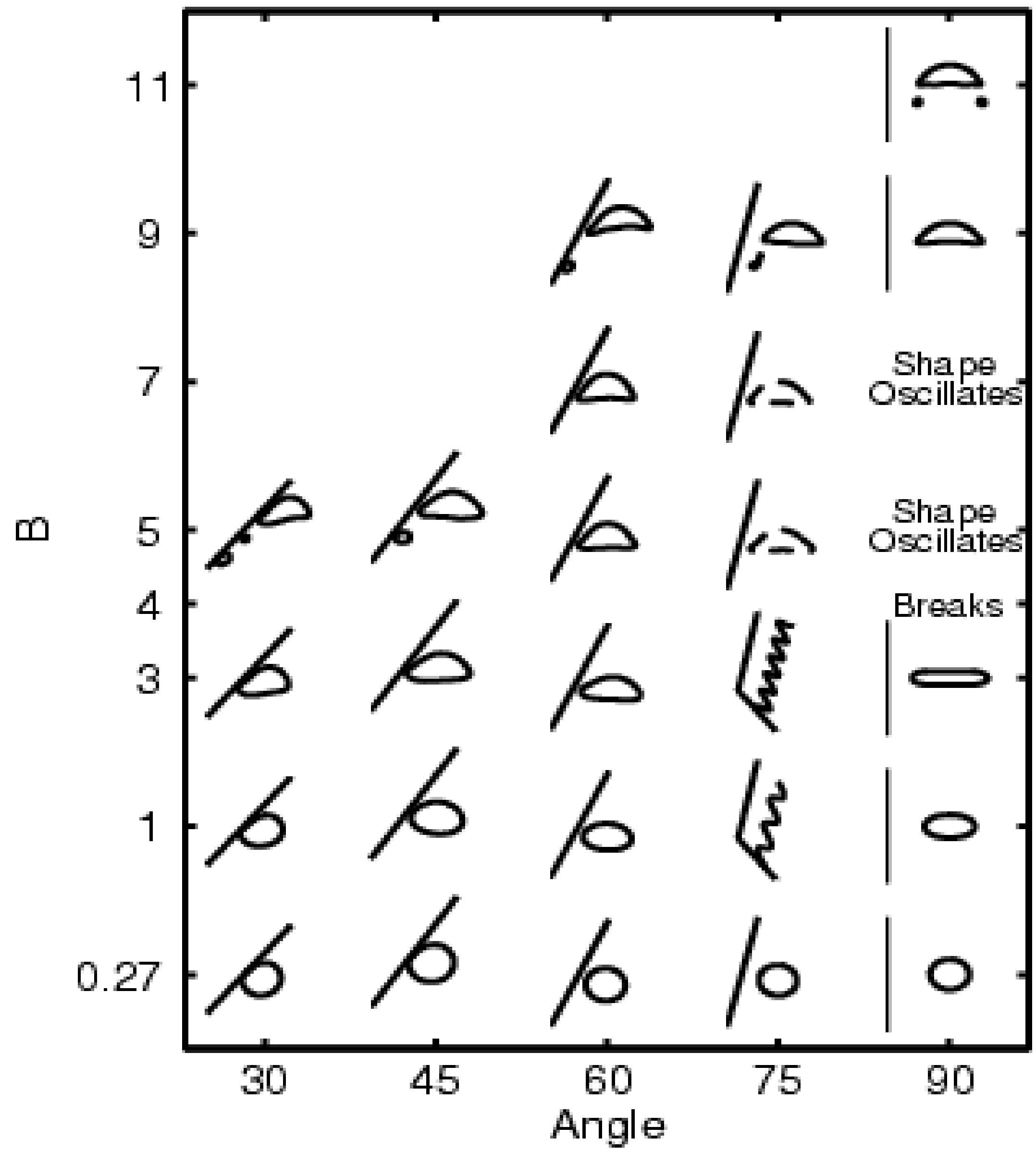
Re = 250

B = ρ g r^2 / σ

Large B = rupture

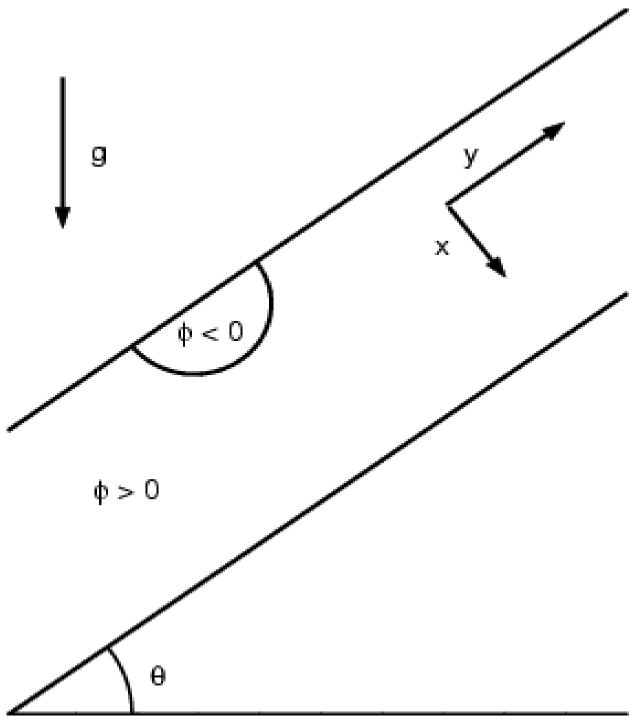
Window of angles for bouncing

Small angle = close to upper channel wall



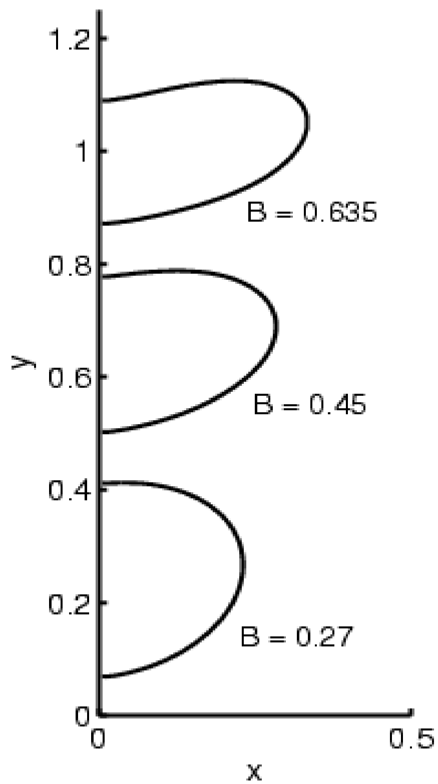
Contact Line Motion

- Navier Slip law on wall:
  $$u + \lambda\, \partial u / \partial n = 0$$
- λ = slip coefficient=0.01
- Fixed contact angle 90°

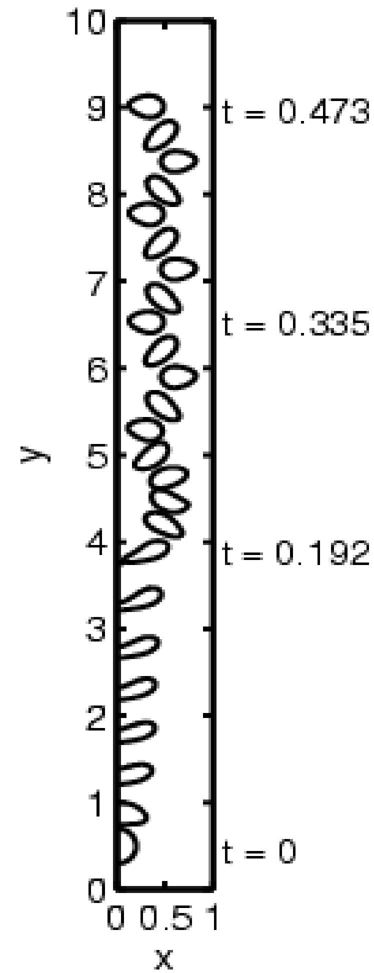


Initial Data

90 degrees, Re=1000



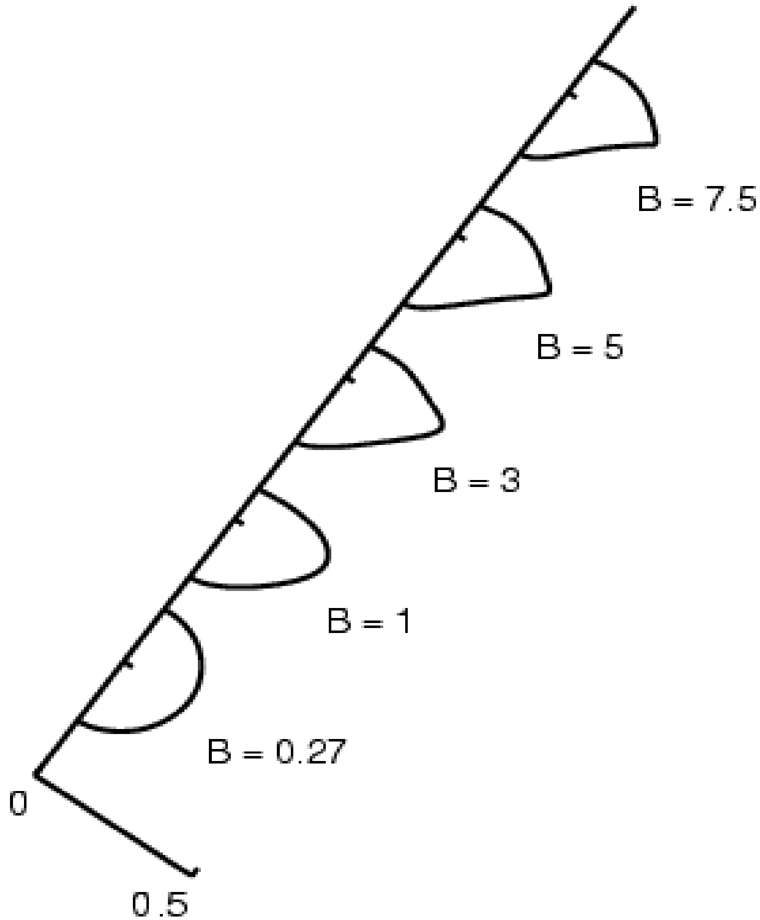
Steady shapes
for different
Bond numbers



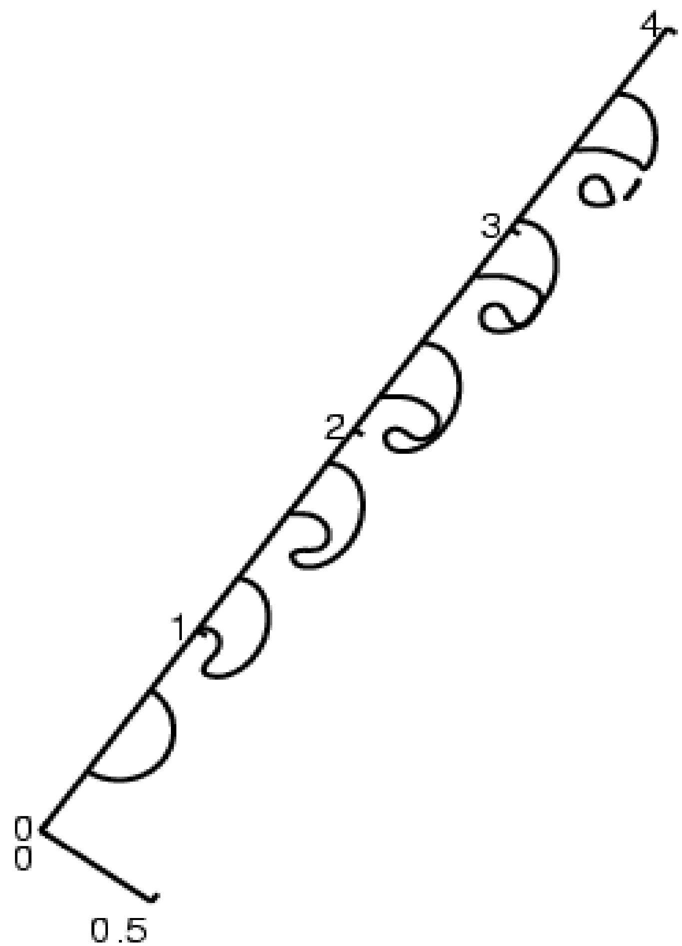
B=0.81

Free Bubble oscillates
periodically in channel

45 degrees, Re = 1000



Steady Shapes for
small Bond number



Rupture at B = 20

Summary

- Bubble dynamics consistent with experimental work
  - Shape changes with angle
  - Bouncing for large angles
  - Path instability with Re increasing in vertical channel
  - Maximum in rise velocity for $0 < \theta < 90^o$ only for a range of B and Re
- Remaining questions:
  - 3D
  - Rupture? Code allows for bubbles to break or rupture onto a wall, but this rupture is numerical, not physical.